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ATTA OVEISI *

FREE VIBRATION OF PIEZO-NANOWIRES USING TIMOSHENKO BEAM THEORY WITH CONSIDERATION OF SURFACE AND SMALL SCALE EFFECTS

This paper investigates the influence of surface effects on free transverse vibration of piezoelectric nanowires (NWs). The dynamic model of the NW is tackled using nonlocal Timoshenko beam theory. By implementing this theory with consideration of both non-local effect and surface effect under simply support boundary condition, the natural frequencies of the NW are calculated. Also, a closed form solution is obtained in order to calculate fundamental buckling voltage. Finally, the effect of small scale effect on residual surface tension and critical electric potential is explored. The results can help to design piezo-NW based instruments.

1. Introduction

Nanowires hold great promise for a wide range of significant applications such as sensors, actuators, transistors and resonators in high-precision nano-electromechanical instruments [1, 2]. Because of the rising ratio of surface area to volume at the nanoscale, the mechanical properties of NWs show discrete size dependences [3, 4]. For the applications of NWs, both the axial buckling and the transverse vibration are considered by large number of researchers. Wang *et al.* showed that surface tension will considerably affect the effective Young's modulus of Al nanowires, which decrease with either the reduction of nanowires thickness or the increase of the aspect ratio [5]. Khajeansari *et al.* investigated the bending response of nanowires considering both elastic substrate and surface effects. They employed Euler-Bernoulli beam theory with a Winkler-Pasternak elastic type substrate medium [6]. Song *et al.* considered the effect of initial stresses in Young-Laplace Euler-

* *School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran; E-mail: atta.oveisi@gmail.com*

Bernoulli beam model and they showed that it can significantly influence the mechanical properties of nanowires [7]. Song and Huang studied the incremental deformation theory for consideration of the surface stress effects in the bending behavior of the nanowires and then the resonant shift predictions, by using both the modified Euler-Bernoulli beam and the modified Timoshenko beam theories [8]. Park quantified nanoscale surface stresses impact effect on the critical buckling strains of silicon nanowires using the nonlinear finite element method [9]. Olsson and Park presented responses from atomistic simulations of gold nanowires under axial compression. They focused their research on examining the effects of both axial and surface orientation effects on the buckling behavior [10]. Gheshlaghi and Hasheminejad investigated the nonlinear flexural vibrations of nanobeams in presence of surface effects within the framework of Euler–Bernoulli beam theory including the von Kármán geometric nonlinearity and obtained an exact solution for free vibration of nanobeams [11]. Yao and Yun investigated the effect of surface elasticity on the Euler buckling of ZnO nanowires under axial compressive loads and resulted that the surface elasticity has a significant effects on the critical stresses for given buckling ZnO nanowires under different boundary conditions [12]. Gheshlaghi and Hasheminejad adopted a sandwich-beam model with two surface layers of finite thickness to investigate the resonance frequency shift of moderately thick microbeams due to atom/molecule adsorption in presence of surface effects using Timoshenko beam theory. They obtained a framework for the optimal design of micro- and nano-based beam sensors [13, 14]. Wang and Feng examined the axial buckling and the transverse vibration of nanowires by using the refined Timoshenko beam theory [15]. Yan and Jiang studied the electromechanical coupling (EMC) behavior of piezoelectric nanowires (NWs) with the Euler-Bernoulli beam theory. They used an analytical method by taking into account the surface effects, including surface elasticity, residual surface stress and surface piezoelectricity to find the static response of the NWs [16]. Zhan and Gu utilized large-scale molecular dynamics simulations to study the dual-mode vibration of 110 Ag NWs with triangular, rhombic and truncated rhombic cross-sections. They integrated the generalized Young-Laplace equation into the Euler Bernoulli beam theory and studied the surface effects on the dual-mode vibration [17]. Wang and Feng showed the influence of surface stresses on the vibration and buckling behavior of piezoelectric nanowires by using the Euler-Bernoulli beam model by applying a curvature-dependent distributed transverse loading along the beam [18]. Hong *et al.* presented an experimental method of determining resonant frequencies and Young's modulus of nanobeams by combining finite element analysis and frequency response tests based on an electrostatic excitation and visual detection by using a laser Doppler vibrometer [19]. He and Lilley studied the influence of surface stress on the resonance frequencies of bending nanowires by using the generalized Young Laplace equation, along with Euler-Bernoulli beam theory and different boundary conditions [20]. Wang and Feng analyzed analytical relation for the critical force of axial buckling of nanowires by accounting for both the effects of surface elasticity and residual surface tension [21]. Hasheminejad and Gheshlaghi presented a dissipative surface stress model to study the effect of size-dependent surface dissipation on natural frequencies of vibrating elastic nanowires NWs using Euler-Bernoulli beam theory along with the classic Zener model for interior friction in the presence of an initial surface tension [22]. Olsson *et al.* have studied the resonant properties of unstressed and pre-stressed nanowires to explain the reason why the higher order natural frequencies in Euler-Bernoulli beam theory were not exact, and they showed the importance of shearing and rotary inertia for higher order resonant modes [23]. Zhan and Gu conducted a comprehensive theoretical and numerical study for bending properties of nanowires considering surface/intrinsic stress effects and axial extension effect based on the molecular dynamics (MD) simulation and different modified beam theories [24]. This paper investigates the influence of surface effects, nonlocal effect and electromechanical coupling due to the piezoelectric medium on free transverse vibration of piezoelectric nanowires (NWs) together with fundamental buckling voltage using Timoshenko beam model.

2. Formulation

Figure 1 shows a graphic presentation of the problem with piezoelectric NW of length L, width b and thickness $2h$. It is assumed that the NW is simply supported between a pair of substrates coated by thin, continuous layers of electrodes. In this arrangement, the electroded substrates cannot change the dynamic characteristics of the NW, effectively. For a brief consideration on the relevancy of this system in regard to realistic instruments, the reader is referred to Ref. [25].

It has been shown that the piezoelectric NWs under bending have independent electric potential of the axial coordination along the NW, except in the vicinity of two ends [26]. In other word, it was established that the deflection of the piezo-NW by application of an electric potential $\psi(x, z)$ across the width of the NW (i.e., through the converse piezoelectric effect; see [27]), creates a strain field, with the outer surface being stretched (positive strain) and the inner surface being compressed (negative strain). Consequently, the electric potential varies between $\psi(x, -h)$ and $\psi(x, h)$ across the width of the NW from the compressed to the stretched side surface, and one may

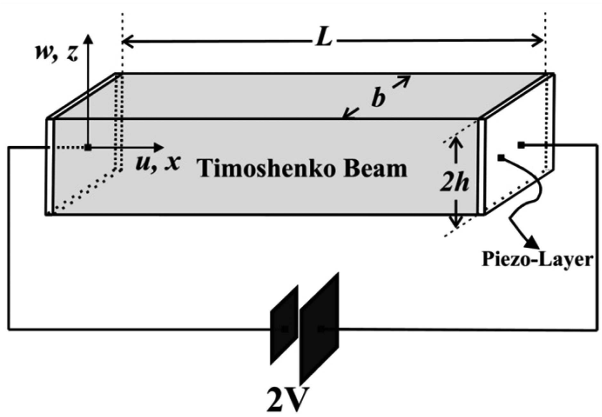

Fig. 1. Geometry of the problem

assume a uniform piezoelectric potential distribution along the NW (x-axis), implying that $\Psi_z \ll \Psi_x$. Ψ_z and Ψ_x are the electric-field components which are connected with the electric potential according to the following relations

$$\Psi_x = -\frac{\partial \psi}{\partial x}, \quad \Psi_z = -\frac{\partial \psi}{\partial z} \tag{1}$$

For a detailed interpretation of the electric-field components and electric potential, the interested reader is referred to Ref. [27]. In the absence of electric charges, the electrostatic equilibrium condition can be expressed as

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} = 0, \tag{2}$$

where D_x and D_z are the electric displacement components, given by [26]

$$D_z = e_{31}\varepsilon_{xx} + \lambda_{33}\Psi_z, \quad D_x = \lambda_{11}\Psi_x \tag{3}$$

where λ_{11} and λ_{33} are dielectric constants that state for dielectric displacement per unit electric field at constant stress in the x and z direction, respectively. In addition, e_{31} defines the ratio of the electric field strength to the effective mechanical stress induced in z direction by mechanical stress acting in x direction. As λ_{11} and λ_{33} are on the same order, with consideration of $\Psi_z \ll \Psi_x$, one can ignore the electric displacement D_x in comparison with D_z. It is also assumed that $\psi(x, -h) = \psi(x, h) = 2V$ (see Fig. 1). Here in this paper, the Timoshenko beam theory is adopted. The effect of shear deformation, in addition to the effect of rotary inertia, is considered in this theory. The Timoshenko beam theory (see Reddy [28, 29]), is based on the displacement field

$$u_1 = u(x,t) + z\phi(x,t), \quad u_2 = 0, \quad u_3 = w(x,t) \tag{4}$$

where ϕ denotes the rotation of the cross-section. The nonzero strains of the Timoshenko beam theory are then the components of strain can be found as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\frac{\partial \phi}{\partial x} \stackrel{\text{def}}{=} \varepsilon^0_{xx} + z\kappa, \ \ \varepsilon_{xz} = \phi + \frac{\partial w}{\partial x} \stackrel{\text{def}}{=} \gamma \tag{5}$$

where

$$\varepsilon^0_{xx} = \frac{\partial u}{\partial x}, \ \ \kappa = \frac{\partial \phi}{\partial x}, \ \ \gamma = \phi + \frac{\partial w}{\partial x} \tag{6}$$

and κ indicates the bending strain and γ is the transverse shear strain. The Timoshenko beam theory requires shear correction factors to balance for error due to assuming constant shear stress. This factor depends on both material and geometric factors, but also on the load and boundary conditions. The principle of virtual displacement for Timoshenko beam is given by

$$0 = \int_0^{\mathrm{T}}\int_0^{\mathrm{L}}\left[m_0\left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}\right) + m_2\frac{\partial \phi}{\partial t}\frac{\partial \delta \phi}{\partial t} - N\delta\varepsilon^0_{xx} - M\delta\kappa - \right. \\ \left. -Q\delta\gamma + f\delta u + q\delta w + \bar{N}\tfrac{\partial w}{\partial x}\tfrac{\partial \delta w}{\partial x}\right]\mathrm{d}x\mathrm{d}t \tag{7}$$

with $m_0 = \rho A$ and $m_2 = \rho I\,(1 + E/KG)$ which states the effects of rotary inertia and shear deformation, where ρE and G are density, Young's modulus and shear modulus, respectively. Moreover, κ, γ, $f(x, t)$ and $q(x, t)$ are bending strain, transverse shear strain, the axial and transverse distributed forces (measured per unit length) and $\bar{N}$ is the applied axial compressive force. The Euler-Lagrange equations are

$$\frac{\partial Q}{\partial x} + q - \frac{\partial}{\partial x}\left(\bar{N}\frac{\partial w}{\partial x}\right) = m_0, \ \ \frac{\partial M}{\partial x} - Q = m_2\frac{\partial^2 \phi}{\partial t^2} \tag{8}$$

where $\bar{N} = 2b\tau_0 + P_e(xt)$ is the total axial force with $P_e\,(x,t) = b\int_{-h}^{h}\sigma_{xx}\mathrm{d}z = 2Vbe_{32}$ (see Fig. 1) being the electric potential induced component. In this relation τ_0 is the residual surface tension that is in relation with nonlocal parameters of the piezo-NW and piezoelectric effect. According to [30-32], the stress field at a point $\mathbf{x}$ in an elastic continuum depends both on the strain field at the point (hyper elastic case) and on the strains fields of all other points of the body. Eringen qualified this fact to the atomic theory of lattice dynamics. Thus, the nonlocal stress tensor σ at point $\mathbf{x}$ is expressed as

$$\sigma = \int_{\mathrm{V}} K\left(\left|\mathbf{x}'-\mathbf{x}\right|, \tau\right) t\left(\mathbf{x}'\right)\mathrm{d}\mathbf{x}' \tag{9}$$

where $t(\mathbf{x})$ is the classical, macroscopic stress tensor at point $\mathbf{x}$ and the kernel function $K\left(|\mathbf{x}'-\mathbf{x}|, \tau\right)$ states the nonlocal modulus in terms of Euclidean norm and τ which is a material constant that depends on internal and external characteristic length (such as the lattice spacing and wave length, respectively). The macroscopic stress **t** at a point **x** in a Hookean solid is related to the strain e at the point by the generalized Hooke's law

$$\mathbf{t}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : \varepsilon(\mathbf{x}) \tag{10}$$

where **C** is the fourth-order elasticity tensor and : denotes the 'double-dot product'. The constitutive Eqs. (9) and (10) together define the nonlocal constitutive behavior of a Hookean solid. Eq. (9) represents the weighted average of the contributions of the strain field of all points in the body to the stress field at a point. However, the integral constitutive relation in Eqn. (9) makes the elasticity problems difficult to solve. Yet, it is possible (see Ref. [31]) to represent the integral constitutive relations in an equivalent deferential form as

$$\left(1 - \tau^2 l^2 \nabla^2\right)\sigma = \mathbf{t},\ \tau = \frac{e_0 a}{l} \tag{11}$$

where e_0 is a material constant, and a and l are the internal and external characteristic lengths, respectively. Using Eqs. (10) and (11), we can express stress resultants in terms of the strains in Timoshenko beam theory. As opposed to the linear algebraic equations between the stress resultants and strains in a local theory, the nonlocal theory results in differential relations involving the stress resultants and the strains. In the following, we present these relations for homogeneous isotropic beams under the assumption that the nonlocal behavior is negligible in the thickness direction. Then, the nonlocal constitutive relation in Eq. (11), with Eq. (10) for the macroscopic stress, takes the following special relations for beams

$$\sigma_{xx} - \mu\frac{\partial^2\sigma_{xx}}{\partial x^2} = E\varepsilon_{xx},\ \sigma_{xz} - \mu\frac{\partial^2\sigma_{xz}}{\partial x^2} = 2G\varepsilon_{xz},\ \left(\mu = e_0^2 a^2\right) \tag{12}$$

When the nonlocal parameter μ is zero, the constitutive relations of the local theories are obtained. In all theories, the axial force–strain relation is the same and it is given by

$$N - \mu\frac{\partial^2 N}{\partial x^2} = EA\varepsilon_{xx} \tag{13}$$

where we have used the relations

$$\mathrm{A} = \int_{\mathrm{A}} \mathrm{dA},\ \int_{\mathrm{A}} \mathrm{z\,dA} = 0,$$

Thus, the x-axis is taken along the geometric centroid of the beam. In the Timoshenko beam theory, we have M and Q, in addition to N. Then constitutive relations are given by

$$M - \mu \frac{\partial^2 M}{\partial x^2} = EI\kappa, \quad Q - \mu \frac{\partial^2 Q}{\partial x^2} = GAK_s\gamma \tag{14}$$

Here K_s denotes the shear correction factor. The equations of motion of Timoshenko beam theory now can be expressed in terms of the displacements (u, w, ϕ). This requires the use of force- and moment-deflection relationships in (13) and (14) to replace the stress resultants appearing in the equations of motion of each theory.

First, the equation of motion governing the axial displacement is derived for the nonlocal theory, as it is common to all beam theories. Substituting for the first derivative of the axial force N leads to

$$N = EA\frac{\partial u}{\partial x} + \mu\left(m_0 \frac{\partial^3 u}{\partial x \partial t^2} - \frac{\partial f}{\partial x}\right) \tag{15}$$

Substituting N from Eq. (15) into the equation of motion

$$\frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}\right) + f - \mu \frac{\partial^2 f}{\partial x^2} = m_0\left(\frac{\partial^2 u}{\partial t^2} - \mu \frac{\partial^4 u}{\partial x^2 \partial t^2}\right) \tag{16}$$

Up to this point, the equation of motion governing the axial displacement is derived for the nonlocal theory, as it is common to all beam theories. Eliminating Q in Eq. (8) leads to

$$\frac{\partial^2 M}{\partial x^2} + q - \frac{\partial}{\partial x}\left(\bar{N}\frac{\partial w}{\partial x}\right) = m_0 \frac{\partial^2 w}{\partial t^2} + m_2 \frac{\partial^3 \phi}{\partial x \partial t^2} \tag{17}$$

Substituting for the second derivative of M from Eq. (8) into the first equation in (14)

$$M = (EI)_{\text{eff}} \frac{\partial \phi}{\partial x} + \mu\left[-q + \frac{\partial}{\partial x}\left(\bar{N}\frac{\partial w}{\partial x}\right) + m_0 \frac{\partial^2 w}{\partial t^2} + m_2 \frac{\partial^3 \phi}{\partial x \partial t^2}\right] \tag{18}$$

where $(EI)_{\text{eff}} = \frac{1}{2}E_s bh^2 + \frac{E_s}{6}h^3 + \frac{Ebh^3}{12} + b\int_{-h}^{h}\left(1 + \frac{e_{31}^2}{E\lambda_{33}}\right)Ez^2 \mathrm{d}z$ is the effective bending stiffness [20, 21, 33]. Next, substituting for the second derivative of Q from Eq. (8) into the second equation in (14), we obtain

$$Q = GAK_s\left(\phi + \frac{\partial w}{\partial x}\right) + \mu \frac{\partial}{\partial x}\left[-q + \frac{\partial}{\partial x}\left(\bar{N}\frac{\partial w}{\partial x}\right) + m_0 \frac{\partial^2 w}{\partial t^2}\right] \tag{19}$$

Now substituting for M and Q from Eqs. (18) and (19), respectively, into Eq. (4)

$$\frac{\partial}{\partial x}\left[GAK_s\left(\phi+\frac{\partial w}{\partial x}\right)\right]+q-\frac{\partial}{\partial x}\left(\bar{N}\frac{\partial w}{\partial x}\right)-\mu\frac{\partial^2}{\partial x^2}\left[q-\frac{\partial}{\partial x}\left(\bar{N}\frac{\partial w}{\partial x}\right)\right]=m_0\left(\frac{\partial^2 w}{\partial t^2}-\mu\frac{\partial^4 w}{\partial x^2\partial t^2}\right) \tag{20a}$$

$$\frac{\partial}{\partial x}\left((EI)_{\text{eff}}\frac{\partial\phi}{\partial x}\right)-GAK_s\left(\phi+\frac{\partial w}{\partial x}\right)=m_2\frac{\partial^2\phi}{\partial t^2}-\mu m_2\frac{\partial^4 w}{\partial x^2\partial t^2} \tag{20b}$$

Here the exact solutions of bending, natural vibration, and buckling of simply supported beams are calculated. The boundary conditions of simply supported beams are

$$w=0,\;\; M=0,\;\; x=0,L, \tag{21}$$

The following expansions of the generalized displacements w and ϕ satisfy the simply supported boundary conditions

$$w\,(x,t)=\sum_{n=1}^{\infty}W_n\sin\left(\frac{n\pi x}{L}\right)e^{\mathrm{i}\omega_n t},\;\; \phi\,(x,t)=\sum_{n=1}^{\infty}\Phi_n\cos\left(\frac{n\pi x}{L}\right)e^{\mathrm{i}\omega_n t} \tag{22}$$

Substituting these in Eq (20). leads to

$$\begin{aligned}\frac{1}{L^4}\Big(n^2P\pi^2\mathrm{W_n}\left(L^2+n^2\pi^2\mu\right)+AL^2\left(-GK_sn\pi\left(n\pi W_n+L\Phi_n\right)+\right.\\ \left.+W_n\left(L^2+n^2\pi^2\mu\right)\rho\omega^2\right)\Big)\sin\left(\frac{n\pi x}{L}\right)e^{\mathrm{i}t\omega}=0\end{aligned} \tag{23a}$$

$$-\frac{1}{L^2}\left((EI)_{\text{eff}}\,n^2\pi^2\Phi_n+AGK_sL\left(n\pi\,W_n+L\,\Phi_n\right)\right)\cos\left(\frac{n\pi x}{L}\right)e^{\mathrm{i}t\omega}=0, \tag{23b}$$

Using orthogonality of mode shapes one can obtain

$$\Phi_n=-\frac{AGK_sLn\pi}{AGK_sL^2+(EI)_{\text{eff}}\,n^2\pi^2}W_n \tag{24}$$

Substituting Eq (24) in (23a) leads to

$$\frac{1}{L^4}W_n(n^4P\pi^4\mu+AL^4\rho\omega^2+L^2n^2\pi^2(\bar{N}-\frac{A\,(EI)_{\text{eff}}\,GK_sn^2\pi^2}{AGK_sL^2+(EI)_{\text{eff}}\,n^2\pi^2}+A\mu\rho\omega^2))\sin\left(\frac{n\pi x}{L}\right)e^{\mathrm{i}t\omega}=0 \tag{25}$$

Again using orthogonality properties of eigenfunctions leads to frequency equation as

$$\omega_n=\frac{\sqrt{L^2\left(-n^2\bar{N}\pi^2+\frac{A(EI)_{\text{eff}}GK_sn^4\pi^4}{AGK_sL^2+(EI)_{\text{eff}}n^2\pi^2}\right)-n^4\bar{N}\pi^4\mu}}{\sqrt{AL^2\left(L^2+n^2\pi^2\mu\right)\rho}} \tag{26}$$

Substituting $\bar{N} = 2b\tau_0 + 2Vbe_{32}$. By setting ω_1 to zero, one can obtain the expression for fundamental buckling voltage (i.e., $n = 1$) in the form

$$V_b = \frac{\dfrac{A(EI)_{\mathrm{eff}} GK_s L^2\pi^2}{b\left(AGK_s L^2 + (EI)_{\mathrm{eff}}\pi^2\right)\left(L^2+\pi^2\mu\right)} - 2\tau_0}{2e_{32}} \tag{27}$$

Furthermore, using the above expression, we obtain the following useful relation between the residual surface tension and the non-local parameters of a piezoelectric NW

$$\tau_0 = -e_{32}\left(V - \frac{AGh^2K_sL^2\pi^2\left(2E_sh\lambda_{33} + b\left(e_{31}^2h + 3E_s\lambda_{33} + Eh\lambda_{33}\right)\right)}{be_{32}\left(3AGK_sL^2\lambda_{33} + 2h^2\pi^2\left(2E_sh\lambda_{33} + b\left(e_{31}^2h + 3E_s\lambda_{33} + Eh\lambda_{33}\right)\right)\right)\left(L^2 + \pi^2\mu\right)}\right) \tag{28}$$

Some numerical examples are considered in order to examine both the surface and small scale effects for a simply supported piezoelectric NW of square cross section with the physical properties as $\frac{L}{2h}$= 50 (−h≤ z ≤h) E=207 GPa, ρ =7800 kg/m^3 e_{31}=-0.51 C/m^2 and $\lambda_{33} = -7.88 \times 10^{-11}$ F/m [26].

For crystalline metals, atomic simulations display that τ^0 and E^s are of the same order [34]. Furthermore, in the case of linear elastic deformation, the contribution of surface elasticity to the total surface stresses can be neglected in comparison with residual surface stress. The experimental values of surface constants τ^0 and E^s for piezoelectric materials are not available in the literature, we shall not consider the surface elasticity effects in our numerical examples, and the residual surface stress constant is assumed to be τ^0= 0.5 N/m along with two selected values for the small scale parameter. It is clear that the validation of the model is very important for reliability of the model. Fig. 2 shows the dimensionless natural frequencies compared to those available in Ref. [14] which is obtained from

$$\omega_1 = \left(\frac{\pi}{L}\right)\sqrt{\frac{L^2P + (\pi)^2\left[\left((EI)_{eff} + \mu P\right)\right]}{\rho A\left(L^2 + \mu\pi^2\right)}}, \tag{29}$$

where it is assumed that E=207 GPa, λ_{33}=−7.88 10^{-11}, τ_0=0, ρ=7800 kg/m^3, $E_s = 0$, as one can see, this comparison shows an appropriate accommodation between these two methods, so the model is reliable and accurate enough. Figure 3 showed variation of the normalized fundamental natural frequency with NW length for selected input voltages and non-local parameters with $\tau_0 = 0.5$. Here, the natural frequency is normalized with respect to the fundamental frequency calculated using the classical Euler-Bernoulli beam model i.e., neglecting the surface and small scale parameters as well as the piezoelectricity effects as

$$\omega_n = \frac{(n\pi)^2 (EI)_{\text{eff}}}{(\rho A L^4)} \tag{30}$$

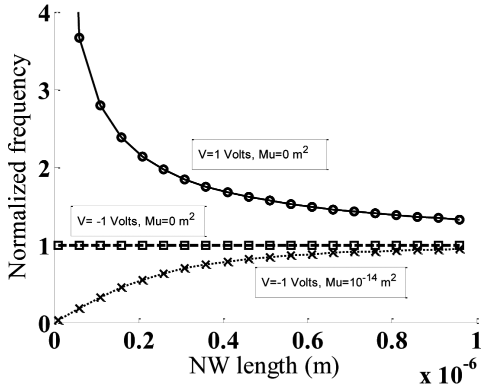

Fig. 2. Comparison of nondimensional fundamental natural frequencies between present model and results obtained from Ref. [14]: markers: Ref. [14], Lines: Present model

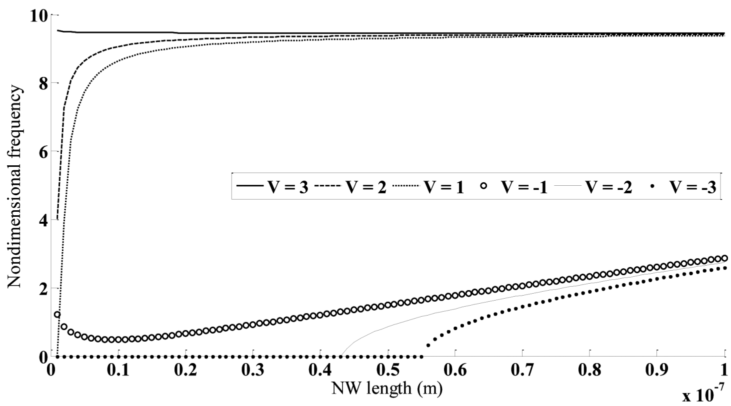

Fig. 3. Variation of the normalized fundamental natural frequency with NW length for selected input voltages and non-local parameters with $\tau_0 = 0.5$

Here, it is clear that by increasing the NW length, both surface and small scale effects gradually disappear. Also, decreasing the input voltage leads to an expected overall decrease in the calculated NW natural frequency. Furthermore, one can note that by including the non-local effects for very short NWs under a negative input voltage ($L < 50$ nm, $V = -1$ Volt, $\mu = 10^{-14}$m^2), the natural frequency is calculated to be zero, i.e., harmonic motion would not be possible for the piezoelectric NW, and buckling may occur. Moreover, one should note that the calculated natural frequency of the piezoelectric NW without the non-local effects (i.e., $\mu = 0\,\mathrm{m}^2$), perfectly match the numerical results obtained using Eq. (17) in Ref. [18]. The most interesting observation is perhaps the fact that including the non-local effects (i.e., $\mu = 10^{-14}\mathrm{m}^2$), causes a notable drop in the value of NW natural frequencies (i.e., like a damping effect), nearly regardless of input voltage.

Figure 4. shows the effect of variation of residual surface tension τ_0 and surface Young's modulus E_s on fundamental natural frequency of the NW with parameters set as $V = 1, \mu = 10^{-14}$. It is clearly seen that the fundamental natural frequency decreases with increase of both surface young's modulus and residual surface tension. Also the effect of surface Young's modulus in low values is more significant than high values specially when $E_s \ll E$.

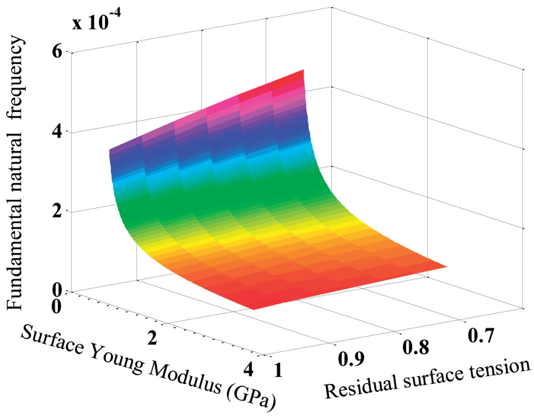

Fig. 4. The effect of variation of residual surface tension τ_0 and surface young's modulus E_s on fundamental natural frequency of the NW with parameters set as $V = 1, \mu = 10^{-14}$

Many different types of nanowires exist, including metallic (e.g., Ni, Pt, Au), semiconducting (e.g., Si, InP, GaN, etc.), and insulating (e.g., SiO_2, TiO_2). The fundamental dimensionless natural frequency for Nickel, Platinum, Gold and Silicon are compared with respect to NW length in Fig. 5. One can see that GOLD has the highest first dimensionless natural frequency which indicates that it has better performance in applied problems because of wide application of nanowires in low frequency range. Another realization which can be extracted from Fig. 5 is that fundamental dimensionless natural frequency is independent of nanowires material in long NWs.

Furthermore, by making use of Eq. (27), one can obtain a useful design chart describing the relationship between the fundamental buckling voltage and the length of the piezoelectric NW. This is done in Fig. 6 for a piezoelectric NW of the given parameters, for selected values of small scale effect. It can be seen that by increasing the NW length the variation of fundamental buckling voltage is tending to a constant value.

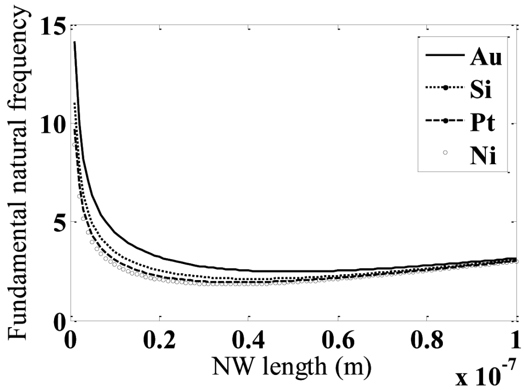

Fig. 5. The fundamental dimensionless natural frequency for Nickel, Platinum, Gold and Silicon

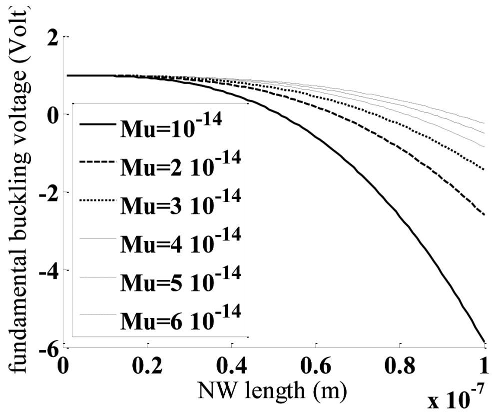

Fig. 6. Relationship between the fundamental buckling voltage and the length of the piezoelectric NW

3. Conclusion

This paper makes the first attempt to study the free vibration of piezoelectric Timoshenko NWs in presence of both the surface and the non-local elasticity effects. Furthermore, a theoretical criterion for describing the relationship between the residual surface tension and the small scale parameter of the piezoelectric NWs is presented. This information can complement the experimental measurement of the critical electric potential at which the axial buckling occurs. It is seen that the resonant frequency of piezoelectric NW can be tuned by adjusting the applied electric potential. Also, the small-scale parameter (non-local elasticity effect) can significantly affect the predicted resonant frequency of piezoelectric NW. Furthermore, an explicit expression (design chart) between the residual surface tension and the non-local parameter is presented, which can aid in experimental characterization of piezoelectric NWs.

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Badanie drgań swobodnych nanodrutów piezoelektrycznych przy zastosowaniu teorii Timoszenki z uwzględnieniem efektów małej skali i efektów powierzchniowych

S t r e s z c z e n i e

W pracy badano wpływ efektów powierzchniowych na poprzeczne drgania swobodne nanodrutów piezoelektrycznych (nanowires, NW). Model dynamiczny NW stworzono posługując się nielokalną teorią belki Timoszenki. Stosując tę teorię, przy uwzględnieniu zarówno efektów powierzchniowych i efektów nielokalnych, obliczono częstotliwości drgań własnych nanodrutu. Uzyskane rozwiązanie, o formie zamkniętej, pozwala także obliczyć podstawowe napięcie wyboczenia. Ponadto, zbadano wpływ efektów małej skali na resztkowe naprężenie powierzchniowe i potencjał elektryczny. Wyniki pracy mogą być użyteczne przy projektowaniu przyrządów wykorzystujących nanodruty piezoelektryczne.