A R C H I V E O F M E C H A N I C A L E N G I N E E R I N G

VOL. LXI 2014 Number 2

10.2478/meceng-2014-0013

Key words: multibody systems, model reduction, parametric reduction

MICHAEL FISCHER *, *PETER EBERHARD* *

SIMULATION OF MOVING LOADS IN ELASTIC MULTIBODY SYSTEMS WITH PARAMETRIC MODEL REDUCTION TECHNIQUES

In elastic multibody systems, one considers large nonlinear rigid body motion and small elastic deformations. In a rising number of applications, e.g. automotive engineering, turning and milling processes, the position of acting forces on the elastic body varies. The necessary model order reduction to enable efficient simulations requires the determination of ansatz functions, which depend on the moving force position. For a large number of possible interaction points, the size of the reduced system would increase drastically in the classical Component Mode Synthesis framework. If many nodes are potentially loaded, or the contact area is not known a-priori and only a small number of nodes is loaded simultaneously, the system is described in this contribution with the parameter-dependent force position. This enables the application of parametric model order reduction methods. Here, two techniques based on matrix interpolation are described which transform individually reduced systems and allow the interpolation of the reduced system matrices to determine reduced systems for any force position. The online-offline decomposition and description of the force distribution onto the reduced elastic body are presented in this contribution. The proposed framework enables the simulation of elastic multibody systems with moving loads efficiently because it solely depends on the size of the reduced system. Results in frequency and time domain for the simulation of a thin-walled cylinder with a moving load illustrate the applicability of the proposed method.

1. Introduction

Elastic multibody systems (EMBS) enable the simulation of mechanical systems which undergo nonlinear rigid body motions as well as elastic deformations. The flexibility is often small compared to the rigid body motion, which leads to the description of linear-elastic deformations in the floating

* *Institute of Engineering and Computational Mechanics, University of Stuttgart, Pfaffenwaldring 9, 70569 Stuttgart, Germany; E-mail: [michael.fischer, peter.eberhard]@itm.uni-stuttgart.de*

frame of reference approach [1]. The elastic body is spatially discretized with the Finite Element Method (FEM), which leads to many elastic degrees of freedom [2]. For complex elastic structures, e.g. engine and automotive components, car bodies and robotics, efficient simulations are only feasible if the elastic degrees of freedom are described by elastic ansatz functions which can be determined by model order reduction [3, 4].

In a rising number of applications, like gear wheel simulations, turning and milling processes, movement of cranes and steering systems, the acting forces vary their position on the elastic body. The moving load problem in structural dynamics has been state-of-the-art for many years, [5]. To achieve satisfying results, the position of the acting forces has to be considered in the determination of the ansatz functions, which is shown for many applications in [2, 4, 6]. If many nodes are potentially actuated due to the varying force position, the classically used Component Mode Synthesis (CMS) would lead to large reduced models. Although the number of possible inputs is high, in many applications, e.g. turning of thin-walled cylinders [7], only a small number of nodes are actuated at the same time. In [8], an approach to interpolate between input-dependent shape functions is proposed and applied to sliding components. This method still depends on the size of the original model because ansatz functions have to be interpolated. One needs to take into account the change of the projection space, which influences the equation of motion.

In this contribution, we present a method based on parametric model order reduction (PMOR) with matrix interpolation, which only depends on the size of the reduced systems in the online step. The input matrix is described as parameter dependent and individually reduced support systems are generated. After a necessary transformation, the interpolation between reduced system matrices allows the generation of reduced systems for any force position. In this contribution, two PMOR-methods based on matrix interpolation [9, 10] are applied to enable efficient simulations of reduced elastic bodies in an EMBS environment with moving loads. The simulation of a thin-walled cylinder with a varying force position in the PMOR-framework illustrates the applicability and quality of the interpolated reduced systems.

The paper is structured as follows. First, elastic multibody systems and the necessary MOR and the background about PMOR with matrix interpolation are explained in Section 2. Afterward, the investigated model, a thin-walled cylinder, is introduced in Section 3. In Section 4 the PMOR-framework and the process chain for simulations of EMBS with varying force positions are presented. The different PMOR-methods are compared for the thin-walled cylinder in Section 5 in frequency and time domain. This contribution is concluded by a summary and an outlook.

2. Background

2.1. Elastic Multibody Systems

For many applications, the flexible deformation can be described as linear-elastic and the floating frame of reference approach is applied, [1]. This description leads to the nonlinear differential equation of motion

$$\begin{bmatrix} m\boldsymbol{E} & m\widetilde{\boldsymbol{c}}^T(\boldsymbol{q}) & \boldsymbol{C}_t^T \\ m\widetilde{\boldsymbol{c}}(\boldsymbol{q}) & \boldsymbol{J}(\boldsymbol{q}) & \boldsymbol{C}_r^T(\boldsymbol{q}) \\ \boldsymbol{C}_t & \boldsymbol{C}_r(\boldsymbol{q}) & \boldsymbol{M}_e \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{a}(t) \\ \alpha(t) \\ \ddot{\boldsymbol{q}}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{K}_e \cdot \boldsymbol{q}(t) + \boldsymbol{D}_e \cdot \dot{\boldsymbol{q}}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_t(t) \\ \boldsymbol{h}_r(t) \\ \boldsymbol{h}_e(t) \end{bmatrix} \tag{1}$$

with the rigid-body translational and rotational acceleration $\boldsymbol{a}$, $\boldsymbol{\alpha}$, the mass m, inertia $\boldsymbol{J}$, center of mass $\widetilde{\boldsymbol{c}}$ and the acting forces $\boldsymbol{h}_t$, $\boldsymbol{h}_r$, $\boldsymbol{h}_e$. The elastic body is spatially discretized which results in the nodal deformations $\boldsymbol{q}(t)$, mass matrix $\boldsymbol{M}_e$ and stiffness matrix $\boldsymbol{K}_e$. A velocity-proportional damping matrix $\boldsymbol{D}_e$ is often applied. The matrices $\boldsymbol{C}_t$ and $\boldsymbol{C}_r$ couple the elastic deformation and nonlinear rigid body motion. See [1] for a complete description of elastic multibody systems described with the floating frame of reference. The elastic deformation can be described without regarding the rigid body movement by assuming the acting forces and reaction forces on the elastic body are described as inputs $\boldsymbol{u}(t)$ distributed by the input-matrix $\boldsymbol{B}_e$. The nodal deformations of interest are computed by the output-matrix $\boldsymbol{C}_e$ and $\boldsymbol{q}(t)$ which leads to the Linear Time Invariant (LTI) system

$$\begin{aligned} \boldsymbol{M}_e \cdot \ddot{\boldsymbol{q}} + \boldsymbol{D}_e \cdot \dot{\boldsymbol{q}} + \boldsymbol{K}_e \cdot \boldsymbol{q} &= \boldsymbol{B}_e \cdot \boldsymbol{u}, \\ \boldsymbol{y} &= \boldsymbol{C}_e \cdot \boldsymbol{q}. \end{aligned} \tag{2}$$

2.2. Model Order Reduction in EMBS

Due to the fine spatial discretization, the number of elastic degrees of freedom $\boldsymbol{q}$ in (2) can easily increase to millions, [2]. To enable an efficient simulation of the equation of motion, the elastic degrees of freedom $\boldsymbol{q}$ are projected by a Galerkin-Projection $\boldsymbol{q} \approx \boldsymbol{V}\bar{\boldsymbol{q}}$ onto a subspace $\mathcal{V}$, spanned by the columns of the projection matrix $\boldsymbol{V} \in \mathbb{R}^{N \times n}$. Inserted into (2)

$$\begin{aligned} \boldsymbol{M}_e \cdot \boldsymbol{V} \cdot \ddot{\bar{\boldsymbol{q}}}(t) + \boldsymbol{D}_e \cdot \boldsymbol{V} \cdot \dot{\bar{\boldsymbol{q}}}(t) + \boldsymbol{K}_e \cdot \boldsymbol{V} \cdot \bar{\boldsymbol{q}}(t) &= \boldsymbol{B}_e \cdot \boldsymbol{u}(t) + \boldsymbol{\epsilon}(t), \\ \bar{\boldsymbol{y}}(t) &= \boldsymbol{C}_e \cdot \boldsymbol{V} \cdot \bar{\boldsymbol{q}}(t) \end{aligned} \tag{3}$$

the contribution from the residual error $\boldsymbol{\epsilon}$, which arises because the correct solution $\boldsymbol{q}$ is not included in the subspace $\mathcal{V}$, can be eliminated by a left-projection onto a second subspace $\mathcal{W}$ which is orthogonal to $\boldsymbol{\epsilon}$ and spanned by the projection matrix $\boldsymbol{W} \in \mathbb{R}^{N\times n}$

$$\underbrace{\boldsymbol{W}^T \cdot \boldsymbol{M}_e \cdot \boldsymbol{V}}_{\bar{\boldsymbol{M}}_e} \cdot \ddot{\bar{\boldsymbol{q}}}(t) + \underbrace{\boldsymbol{W}^T \cdot \boldsymbol{D}_e \cdot \boldsymbol{V}}_{\bar{\boldsymbol{D}}_e} \cdot \dot{\bar{\boldsymbol{q}}}(t) + \underbrace{\boldsymbol{W}^T \cdot \boldsymbol{K}_e \cdot \boldsymbol{V}}_{\bar{\boldsymbol{K}}_e} \cdot \bar{\boldsymbol{q}}(t) = \underbrace{\boldsymbol{W}^T \cdot \boldsymbol{B}_e}_{\bar{\boldsymbol{B}}_e} \cdot \boldsymbol{u}(t),$$

$$\bar{\boldsymbol{y}}(t) = \underbrace{\boldsymbol{C}_e \cdot \boldsymbol{V}}_{\bar{\boldsymbol{C}}_e} \cdot \bar{\boldsymbol{q}}(t). \quad (4)$$

The reduced elastic degrees of freedom $\bar{\boldsymbol{q}}$ should approximate the original degrees of freedom $\boldsymbol{q}$ in a best way which is determined by the projection matrices $\boldsymbol{V}$ and $\boldsymbol{W}$. In this contribution, only orthogonal projection with $\boldsymbol{W} = \boldsymbol{V}$ is investigated. See [10] for a detailed description of oblique projection with $\boldsymbol{W} \neq \boldsymbol{V}$.

The dynamical behavior of the LTI-system (2) is described in the frequency domain by the transfer functions

$$\begin{aligned} \boldsymbol{H}(s) &= \boldsymbol{C}_e \cdot (s^2\boldsymbol{M}_e + s\boldsymbol{D}_e + \boldsymbol{K}_e)^{-1} \cdot \boldsymbol{B}_e, \\ \bar{\boldsymbol{H}}(s) &= \bar{\boldsymbol{C}}_e \cdot (s^2\bar{\boldsymbol{M}}_e + s\bar{\boldsymbol{D}}_e + \bar{\boldsymbol{K}}_e)^{-1} \cdot \bar{\boldsymbol{B}}_e \end{aligned} \quad (5)$$

of the original and reduced system. To determine the approximation quality of the reduced system, the absolute and relative error

$$\varepsilon_{\text{abs}}(i\omega) = ||\boldsymbol{H}(i\omega) - \bar{\boldsymbol{H}}(i\omega)||_F, \qquad \varepsilon_{\text{rel}}(i\omega) = \frac{||\boldsymbol{H}(i\omega) - \bar{\boldsymbol{H}}(i\omega)||_F}{||\boldsymbol{H}(i\omega)||_F} \quad (6)$$

are calculated.

Different reduction methods [3, 4, 11] are applicable for the reduction of the elastic degrees of freedom of EMBS. The focus of this contribution is on the parametric model reduction based on matrix interpolation and, therefore, any reduction technique can be used. Here, CMS-based [12] reduction techniques are applied for a thin-walled cylinder example. The projection matrix in this reduction framework consists of eigenmodes $\boldsymbol{\Phi}$

$$(\lambda_i^2\boldsymbol{M}_e + \boldsymbol{K}_e) \cdot \boldsymbol{\Phi}_i = \boldsymbol{0}, \quad i = 1, \ldots, N \quad (7)$$

and static modes $\boldsymbol{\Phi}_{\text{AM}}$

$$\boldsymbol{K}_e \cdot \boldsymbol{\Phi}_{\text{AM}} = \boldsymbol{B}_e. \quad (8)$$

In [13] other reduction methods are investigated in the PMOR-method with matrix interpolation.

2.3. Parametric Model Order Reduction

In a rising number of applications, the system matrices in (2) cannot be considered as constant, since they are parameter-dependent, which results in the parametric differential equation

$$\begin{aligned} \boldsymbol{M}_e(p) \cdot \ddot{\boldsymbol{q}} + \boldsymbol{D}_e(p) \cdot \dot{\boldsymbol{q}} + \boldsymbol{K}_e(p) \cdot \boldsymbol{q} &= \boldsymbol{B}_e(p) \cdot \boldsymbol{u}, \\ \boldsymbol{y} &= \boldsymbol{C}_e(p) \cdot \boldsymbol{q}. \end{aligned} \tag{9}$$

Often, the parameter dependency is supposed to be affine

$$\begin{aligned} &\boldsymbol{M}_e(p) = \sum_{i=1}^{k} \omega_i(p)\boldsymbol{M}_{e,i}, \quad \boldsymbol{D}_e(p) = \sum_{i=1}^{k} \omega_i(p)\boldsymbol{D}_{e,i}, \quad \boldsymbol{K}_e(p) = \sum_{i=1}^{k} \omega_i(p)\boldsymbol{K}_{e,i}, \\ &\boldsymbol{B}_e(p) = \sum_{i=1}^{k} \omega_i(p)\boldsymbol{B}_{e,i}, \quad \boldsymbol{C}_e(p) = \sum_{i=1}^{k} \omega_i(p)\boldsymbol{C}_{e,i} \end{aligned} \tag{10}$$

with $\sum_{i=1}^{k} \omega_i(p) = 1$ and $\omega_i(p_j) = \delta_{ij}$ for $i, j = 1, \ldots, k$. Different parametric reduction techniques, which vary in computational effort, size of the reduced system and complexity to generate a reduced system for arbitrary parameter values, are developed. These techniques are summarized in [14]. Generally, they are distinguished as global and local approaches. Global approaches often result in large reduced systems if many possibly actuated nodes are used and, therefore, suffer under the curse of dimensionality. In this contribution, local PMOR-techniques with matrix interpolation are applied to simulate EMBS with parametric force excitation positions. Two major ideas of matrix interpolation in PMOR were developed independently, [9, 15]. In [16] these methods are compared and fitted into a common framework. Here, the basic idea of both techniques and the differences between these methods are described briefly.

The goal of the methods in [9, 15] is to provide the opportunity to calculate reduced systems for any parameter value p without going back to the original system. Therefore, the interpolation between individually reduced system matrices is aspired. For certain parameter values p_i, the original system is reduced and the resulting reduced system matrices $\bar{\boldsymbol{M}}_{e,i}$, $\bar{\boldsymbol{D}}_{e,i}$, $\bar{\boldsymbol{K}}_{e,i}$, $\bar{\boldsymbol{B}}_{e,i}$, $\bar{\boldsymbol{C}}_{e,i}$ are used as support systems for the interpolation. The direct interpolation is prohibited if parameter dependent projection matrices are applied because then the locally reduced coordinates $\bar{\boldsymbol{q}}_i$ differ. To enable the interpolation between the system matrices, the reduced coordinates $\bar{\boldsymbol{q}}_i$ are transformed

$$\bar{\boldsymbol{q}}_i = \boldsymbol{T}_i \cdot \bar{\boldsymbol{q}}_i^* \tag{11}$$

to guarantee that the k reduced systems are described in the same set of coordinates

$$\bar{\boldsymbol{q}}^* = \bar{\boldsymbol{q}}_1^* = \cdots = \bar{\boldsymbol{q}}_k^*. \tag{12}$$

For orthogonal projection, the original support systems are reduced with the projection matrix $\widetilde{\boldsymbol{V}}_i = \boldsymbol{V}_i \cdot \boldsymbol{T}_i$

$$\begin{aligned} \widetilde{\boldsymbol{M}}_i \cdot \ddot{\bar{\boldsymbol{q}}}^*(t) + \widetilde{\boldsymbol{D}}_i \cdot \dot{\bar{\boldsymbol{q}}}^*(t) + \widetilde{\boldsymbol{K}}_i \cdot \bar{\boldsymbol{q}}^*(t) &= \widetilde{\boldsymbol{B}}_i \cdot \boldsymbol{u}(t), \\ \bar{\boldsymbol{y}}(t) &= \widetilde{\boldsymbol{C}}_i \cdot \bar{\boldsymbol{q}}^*(t), \end{aligned} \tag{13}$$

with

$$\begin{aligned} \{\widetilde{\boldsymbol{M}}_i, \widetilde{\boldsymbol{D}}_i, \widetilde{\boldsymbol{K}}_i\} &= \widetilde{\boldsymbol{V}}_i^T \cdot \{\boldsymbol{M}_{e,i}, \boldsymbol{D}_{e,i}, \boldsymbol{K}_{e,i}\} \cdot \widetilde{\boldsymbol{V}}_i, \\ \widetilde{\boldsymbol{B}}_i = \widetilde{\boldsymbol{V}}_i^T \cdot \boldsymbol{B}_{e,i}, &\quad \widetilde{\boldsymbol{C}}_i = \boldsymbol{C}_{e,i} \cdot \widetilde{\boldsymbol{V}}_i. \end{aligned} \tag{14}$$

In the following two subsections the calculation of the transformation matrix $\boldsymbol{T}_i$ in the two PMOR-methods with matrix interpolation [9, 15] is explained. The calculation of the interpolated reduced system matrices is described, too.

2.3.1. Method by Panzer, Mohring, Eid, Lohmann [9]

The transformation matrix

$$\bar{\boldsymbol{q}}_i = \boldsymbol{T}_i \cdot \bar{\boldsymbol{q}}_i^* \quad \text{with} \quad \boldsymbol{T}_i = (\boldsymbol{R}^T \cdot \boldsymbol{V}_i)^{-1} \quad \text{and} \quad \boldsymbol{R}^T \cdot \boldsymbol{R} = \boldsymbol{I} \tag{15}$$

enables the compatibility of the reduced coordinates $\bar{\boldsymbol{q}}_i^*$ with respect to the subspace spanned by the columns of $\boldsymbol{R}$. In [9] two different methods to determine $\boldsymbol{R}$ are proposed. In the first approach, all projection matrices $\boldsymbol{V}_i$ are combined

$$\boldsymbol{V}_{\text{all}} = [\boldsymbol{V}_1 \ldots \boldsymbol{V}_k] \tag{16}$$

and the most important directions are determined by s left vectors $\boldsymbol{R} = \boldsymbol{U}(:, 1 : s)$ of the corresponding s largest singular values of the Singular Value Decomposition (SVD) of $\boldsymbol{V}_{\text{all}} = \boldsymbol{U} \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{N}^T$. The only difference in the second approach is a weighted calculation

$$\boldsymbol{V}_{\text{all}} = [\omega_1(p)\boldsymbol{V}_1 \ldots \omega_k(p)\boldsymbol{V}_k] \tag{17}$$

which can be advantageous if large variations in the projection matrices occur and the most important dynamics around the interpolation parameter should be investigated. The largest disadvantage is the necessary update of $\boldsymbol{V}_{\text{all}}$ and calculation of $\boldsymbol{R}$ for each new interpolation parameter. Therefore, only the first unweighted approach may enable the application in a real-time framework.

The additional transformation ensures the usage of the same set of reduced coordinates which enables the interpolation between the reduced system matrices $\widetilde{\boldsymbol{M}}_i, \widetilde{\boldsymbol{D}}_i, \widetilde{\boldsymbol{K}}_i, \widetilde{\boldsymbol{B}}_i, \widetilde{\boldsymbol{C}}_i$ to generate reduced systems for any parameter value p

$$\begin{aligned} &\widetilde{\boldsymbol{M}}(p) = \sum_{i=1}^{k} \omega_i(p)\widetilde{\boldsymbol{M}}_i, \quad \widetilde{\boldsymbol{D}}(p) = \sum_{i=1}^{k} \omega_i(p)\widetilde{\boldsymbol{D}}_i, \quad \widetilde{\boldsymbol{K}}(p) = \sum_{i=1}^{k} \omega_i(p)\widetilde{\boldsymbol{K}}_i, \\ &\widetilde{\boldsymbol{B}}(p) = \sum_{i=1}^{k} \omega_i(p)\widetilde{\boldsymbol{B}}_i, \quad \widetilde{\boldsymbol{C}}(p) = \sum_{i=1}^{k} \omega_i(p)\widetilde{\boldsymbol{C}}_i. \end{aligned} \tag{18}$$

The parameter dependency in (10) is retained with this method in the reduced system, which is important in the application for the simulation of moving loads.

2.3.2. Method by Amsallem, Farhat [15]

The transformation matrix $\boldsymbol{T}_i$ in [15] should minimize the difference between the transformed individual projection matrices $\widetilde{\boldsymbol{V}}(p_i) = \widetilde{\boldsymbol{V}}_i$ and a projection matrix $\boldsymbol{V}(p_j)$ for a reference configuration at the parameter value p_j

$$\min_{\boldsymbol{T}_i \text{ orthogonal}} \|\widetilde{\boldsymbol{V}}(p_i) - \boldsymbol{V}(p_j)\|_F^2 = \min_{\boldsymbol{T}_i \text{ orthogonal}} \|\boldsymbol{V}(p_i) \cdot \boldsymbol{T}_i - \boldsymbol{V}(p_j)\|_F^2. \tag{19}$$

This minimization problem is equivalent to

$$\max_{\boldsymbol{T}_i \text{ orthogonal}} \operatorname{trace}(\boldsymbol{T}_i^T \cdot \underbrace{\boldsymbol{V}(p_i)^T \cdot \boldsymbol{V}(p_j)}_{\boldsymbol{P}_{i,j}}) \tag{20}$$

with the matrix

$$\boldsymbol{P}_{i,j} = \boldsymbol{V}(p_i)^T \cdot \boldsymbol{V}(p_j) \in \mathbb{R}^{n\times n}. \tag{21}$$

An analytical solution to this problem is given by the SVD

$$\boldsymbol{P}_{i,j} = \boldsymbol{U}_i \cdot \boldsymbol{\Sigma}_i \cdot \boldsymbol{N}_i^T \tag{22}$$

which leads to the transformation matrix for (14)

$$\boldsymbol{T}_i = \boldsymbol{U}_i \cdot \boldsymbol{N}_i^T. \tag{23}$$

The method in [15] is highly related to the Modal Assurance Criterion (MAC) [17]. The user has to define a reference configuration p_j which is problem dependent. An optimal selection is still an open topic. In contrast, in [9] the SVD of $\boldsymbol{V}_{\mathrm{all}}$ determines the most dominant dynamics for the transformation matrix $\boldsymbol{T}_i$. One advantage of the method in [15] is the fact, that the matrix $\boldsymbol{P}_{i,j}$ and its SVD can be easily precomputed and only depend on the size n of the reduced system.

In contrast to [9], the transformed system matrices $\widetilde{\boldsymbol{M}}_i, \widetilde{\boldsymbol{D}}_i, \widetilde{\boldsymbol{K}}_i, \widetilde{\boldsymbol{B}}_i, \widetilde{\boldsymbol{C}}_i$ in (13) are not interpolated directly. Instead, the interpolation on matrix manifolds is proposed in [15]. With a logarithmic mapping $\boldsymbol{\Gamma} = \mathrm{Log}_{\boldsymbol{X}}(\boldsymbol{Y})$, the matrix $\boldsymbol{Y}$ is mapped to the tangent space $T_{\boldsymbol{X}}\mathcal{M}$ which is calculated at the reference configuration $\boldsymbol{X}(p_j)$. On this tangential manifold the independent interpolation of the elements of the matrix $\boldsymbol{\Gamma}_i$ for a certain parameter value p_i is executed. The interpolated matrix $\boldsymbol{\Gamma}_i$ is subsequently mapped from the tangential manifold back to the original manifold by the exponential mapping $\boldsymbol{Y}_i = \mathrm{Exp}_{\boldsymbol{X}}(\boldsymbol{\Gamma}_i)$ which guarantees to determine interpolated matrices on the same manifold as the matrices used as support systems. In Table 1 the exponential and logarithmic mapping for symmetric positive definite and real non-quadratic matrices are summarized, see [15] for a detailed description.

Table 1.
Logarithmic and exponential mapping for matrix manifolds

manifold	$\boldsymbol{Y}_i \in \mathbb{R}^{N\times p}$	symmetric positive definite matrix
$\boldsymbol{\Gamma}_i = \mathrm{Log}_{\boldsymbol{X}}(\boldsymbol{Y}_i)$	$\boldsymbol{Y}_i - \boldsymbol{X}$	$\log(\boldsymbol{X}^{-1/2} \cdot \boldsymbol{Y}_i \cdot \boldsymbol{X}^{-1/2})$
$\boldsymbol{Y}_i = \mathrm{Exp}_{\boldsymbol{X}}(\boldsymbol{\Gamma}_i)$	$\boldsymbol{X} + \boldsymbol{\Gamma}_i$	$\boldsymbol{X}^{1/2} \cdot \exp(\boldsymbol{\Gamma}_i) \cdot \boldsymbol{X}^{1/2}$

3. Mechanical Model – Thin-Walled Cylinder

The applicability of the PMOR-techniques for large-scale systems in time simulations is investigated by a model of a thin-walled cylinder with nearly 40000 degrees of freedom. In the turning process of thin-walled cylinders, vibrations of the cutting tool and the workpiece lead to poor surfaces and the regenerative chatter mechanism might damage the workpiece and the tool, see [7] for a detailed description of the physical effects. To simulate the turning of thin-walled cylinders in EMBS environment, highly accurate descriptions of the elastic behavior are indispensable. The force between the cylinder and tool changes its position and for machining the complete surface, each node on the surface might be actuated in the simulation model. Figure 1 shows the thin-walled cylinder with the contact force F_{cyl} which is modeled as a point force. The rotational speed of the cylinder is constant and the parameter p describes the position of the force around the circumference at the tip of the cylinder. Because of the focus on moving loads in EMBS, in this contribution, no material removal is simulated, see [7] how this can be handled. The Finite Element model consists of 180 nodes around the circumference times 82 nodes in longitudinal direction. Each node only contains three translational degrees of freedom which results in $N = 43983$ degrees of freedom. The transfer function of the cylinder model with one input and output node is depicted in Figure 2 for the interesting frequency range of $I_f = [0 \;\; 3000]$ Hz. In this contribution, the force is moving around the circumference at the tip of the cylinder. Therefore, 180 models could be used as support systems, although it will be presented that the PMOR-methods with matrix interpolation are feasible to save support systems independently of the Finite Element mesh.

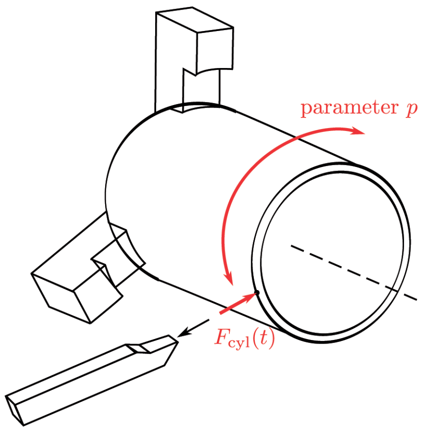

Fig. 1. Cylinder model

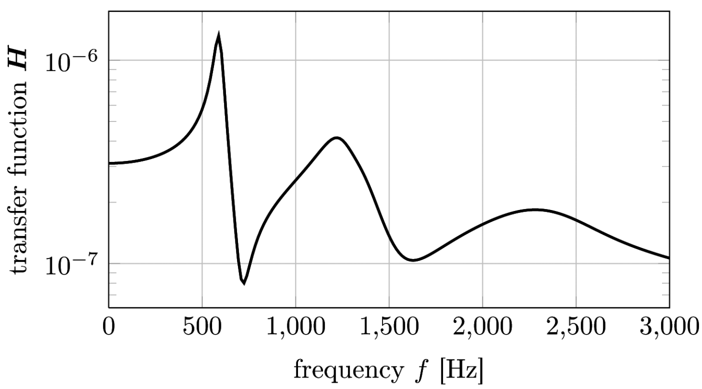

Fig. 2. Frequency response of original cylinder model

4. Moving Loads in EMBS

4.1. Input Matrix

In a rising number of applications, contact forces or applied forces vary their position on the elastic body. To generate qualitatively satisfying reduced systems, a high number of eigenmodes would be necessary which could slow down the simulation extremely. Therefore, reduction techniques which consider the acting forces enable efficient simulations. The classical approach for varying force positions would be the calculation of a force-position-dependent shape function for each possibly actuated node. In other words, for each degree of freedom of each node an input vector $\boldsymbol{b}_i$, which only contains one nonzero element for point forces, is combined to an input matrix

$$\boldsymbol{B}_e = [\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_k]. \tag{24}$$

In many applications, e.g. gear wheel simulations, simulation of turning and milling processes, the number of actuated nodes is very large or not known a-priori which might lead to large input matrices and large reduced systems although only a small number of nodes might be actuated simultaneously. To avoid this problem, the input matrix

$$\boldsymbol{B}_e(p) = \sum_{i=1}^{k} \omega_i(p)\boldsymbol{b}_i \tag{25}$$

is described as a parameter-dependent matrix, which is calculated by k support systems with $\boldsymbol{b}_i$ and the weighting functions $\omega_i(p)$. The parameter p determines the position of the acting force and is supposed to be scalar in this contribution, although an extension to multi-parameter problems is possible [18]. The dynamical behavior depending on the position of the force, its consideration in the reduction step and the arising problems with certain reduction methods are explained in [13].

To consider the parameter dependency in the reduced system, projection matrices $\boldsymbol{V}_i$ are calculated individually for given support systems with $\boldsymbol{B}_e(p)$ in (25). Applying the PMOR-methods, described in Section 2.3, these systems have to be transformed and, afterward, the interpolation of the system matrices is feasible.

4.2. Time Simulation

The application of the PMOR-methods in the EMBS environment requires some additional adaption. In the typical floating frame of reference environment, the positions of the acting forces on the elastic body are known a-priori. To determine the force on the reduced elastic body, the projection matrix is applied

$$\bar{\boldsymbol{B}}_e = \boldsymbol{V}^T \cdot \boldsymbol{B}_e. \tag{26}$$

If the parameter-dependent input matrix $\boldsymbol{B}_e(p)$ is used and parameter-dependent shape functions $\boldsymbol{V}_i$ for each support system are calculated, the idea of interpolating the projection matrix arises. In [10], a method for the interpolation of system matrices is proposed which transforms the individually generated projection matrices $\boldsymbol{V}_i$ similar to the method explained in Section 2.3. Here, this would require the interpolation of matrices with the number of rows N and, therefore, still depends on the size of the original model. For each parameter value, the original system matrices have to be reduced which is not feasible for large-scale models. Therefore, the interpolation of system matrices is applied for all system matrices which means, (26) has to be modified to distribute the force onto the reduced elastic body. As described in (18) the parameter dependent reduced input matrix $\widetilde{\boldsymbol{B}}(p)$ is determined by the individually reduced and transformed input matrices $\widetilde{\boldsymbol{B}}_i$. This means, the reduced input-matrix is calculated by matrix interpolation and the physical meaning of a force which acts on the reduced body is not necessary anymore. This does not mean that this approach is physically incorrect. In fact, it enables the dissociation of the moving load of the original Finite Element mesh and element shape functions are not necessary anymore to distribute forces which act between nodes. It allows one to save support systems by not using each potentially loaded node as a support system.

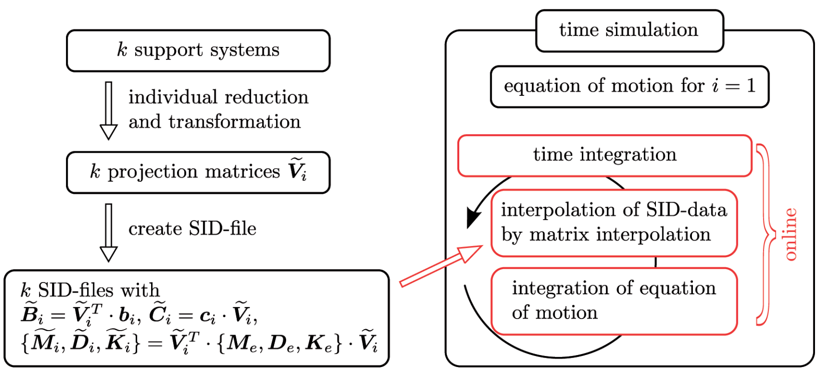

Fig. 3. Online-offline decomposition in time simulation of moving loads with PMOR-methods with matrix interpolation

The same method has to be applied for the calculation of the reduced output $\bar{\boldsymbol{y}}$. Normally, the reduced coordinate $\bar{\boldsymbol{q}}$ is backprojected onto the original elastic coordinates $\boldsymbol{q}$ by the projection matrix $\boldsymbol{V}$. As described above, the projection matrix is not calculated for each parameter value and, therefore, the interpolated reduced output matrix $\widetilde{\boldsymbol{C}}$ has to be applied to determine $\bar{\boldsymbol{y}}(p)$. Only by applying the matrix interpolation approach to all system matrices $\bar{\boldsymbol{M}}_e, \bar{\boldsymbol{D}}_e, \bar{\boldsymbol{K}}_e, \bar{\boldsymbol{B}}_e, \bar{\boldsymbol{C}}_e$ this leads to meaningful results.

One important issue in PMOR-methods and the application of moving load concepts is the possibility to simulate these systems efficiently by an online-offline decomposition. Many calculation steps are executed prior to the time simulation. The proposed application of these PMOR-methods enables an efficient simulation, as it is depicted in Figure 3, with the online-offline decomposition.

5. Numerical Results

The model of the thin-walled cylinder described in Section 3 is examined in frequency and time domain. In this contribution, the solution in time domain is focused and illustrated. A detailed description of the applicability of PMOR with matrix interpolation in frequency domain can be found in [13].

As described, 180 models are available around the circumference. In this example, the applied force moves around half the circumference which results in 91 potential support systems. To show the quality of the interpolated reduced systems, only every fifth node is used as an input for the support systems. The support systems are reduced individually by a CMS-reduction with 20 eigenmodes and one static mode for the force pointing in a constant direction onto the varying node which leads to $n = 21$. Figure 4 illustrates the relative error $\varepsilon_{\mathrm{rel}}$ for all individually reduced support systems, which are transformed by the techniques described in [9], and the relative error for the interpolated systems which are compared to the original system for each particular parameter value p_i. Here, cubic splines are applied for the calculation of the weighting function ω_i. From 0 Hz to 1500 Hz, the interpolation error dominates but for larger frequencies, the error between the original system and the interpolated reduced system is determined by the reduction error of the support systems itself. Although only 19 systems are used as support systems, the interpolated reduced systems still provide satisfying results. If a higher amount of support systems are considered, the quality can be improved drastically, see [13] for other parameter samplings.

In the following, the applicability of the PMOR-methods with matrix interpolation for the simulation of moving loads is presented. To evaluate the interpolation quality in time domain, first a force

$$F_{\mathrm{cyl}}(t) = \begin{cases} F_{\max}(1 - \cos(2\pi t)) & \text{if } 0 < t \leq 0.5 \\ F_{\max} & \text{if } t > 0.5 \end{cases} \tag{27}$$

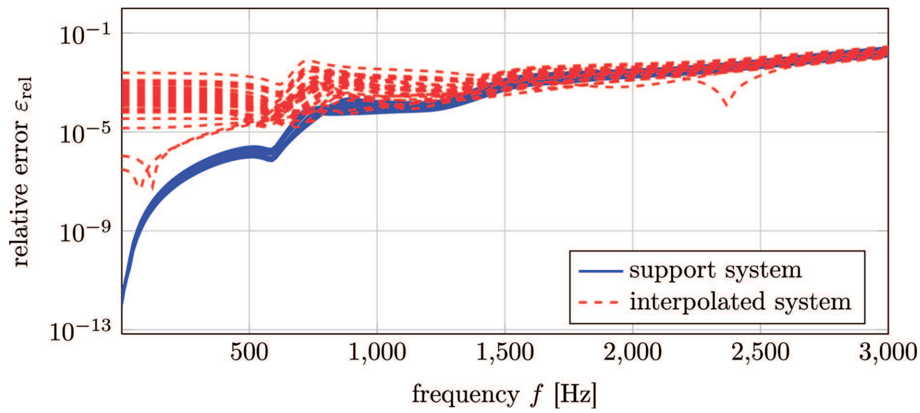

Fig. 4. Relative error for cubic spline interpolation of CMS-reduced support systems with $\boldsymbol{b}_i,\ i = 1(5)91$

which rises smoothly to its static value $F_{\max} = 200$ N, is applied. The acting force will lead to elastic deformations less than 1 mm but the high demands on the surface quality of the thin-walled cylinder requires very precise simulation results. Different reduced models are compared and summarized in Table 2 for this example with the load F_{cyl} moving around the circumference of the elastic cylinder.

Table 2.

Reduced models for cylinder model, reduction method:
*=classical CMS, o=few support systems, #=individual CMS, distribution of force:
x=neighboring nodes with FE-shape functions, &=interpolation of $\widetilde{\boldsymbol{B}}$

Name	n	red.	modes	PMOR	distr.
reference	N	-	-	original model	x
classical	200	*	20 eigenm., 180 static	global model	x
CMS supp.	38	o	20 eigenm., 1(10)180 static	global model	x
interpPa1	21	#	20 eigenm., 1 staticmode	1(1)180 [9]	&
interpAm1	21	#	20 eigenm., 1 staticmode	1(1)180 [15]	&
interpPa2	21	#	20 eigenm., 1 staticmode	1(5)180 [9]	&
interpPa3	21	#	20 eigenm., 1 staticmode	1(10)180 [9]	&
interpAm2	21	#	20 eigenm., 1 staticmode	1(10)180 [15]	&

Figure 5 shows the elastic deformation at the moved marker where the force is acting for the reference model, the CMS-classical model and the interpolated system where every node is used to generate a support system. The reference solution shows small oscillations which do not appear for the interpolated system. They occur because the nodes only contain translational degrees of freedom. Therefore, the Finite Element shape functions distribute the force which acts between two nodes onto the neighboring nodes as forces and the necessary torque cannot be applied. By using a different numerical model including nodes with rotational degrees of freedom, this behavior is eliminated. The interpolated elastic deformation does not consist of these oscillations, because the force acting between nodes is not distributed by the shape functions. Instead, in the PMOR-framework the interpolated reduced input matrix $\widetilde{\boldsymbol{B}}(p)$ is used directly as described in Section 4.2.

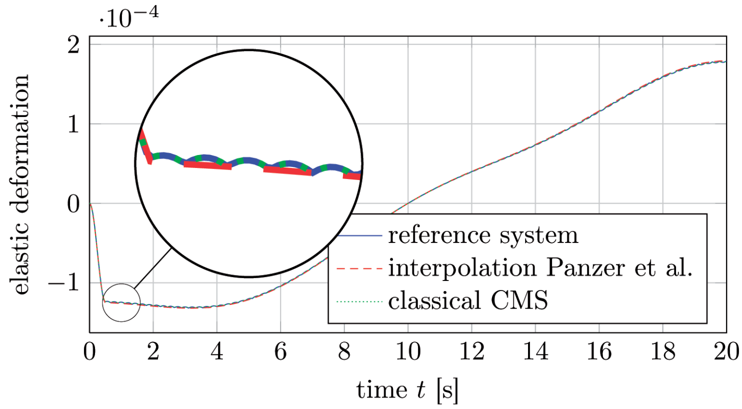

Fig. 5. Elastic deformation of reduced cylinder model actuated with moving force $F_{\mathrm{cyl}}(t)$

To illustrate the quality of the other reduced models listed in Table 2, the elastic deformation between 0.3 s and 5 s is investigated. In Figure 6, both PMOR-methods show very satisfying results if every available support system is used in the interpolation step. One major benefit of the matrix interpolation framework is the independence of the mesh of the original model. The number of support systems does not have to coincide with the

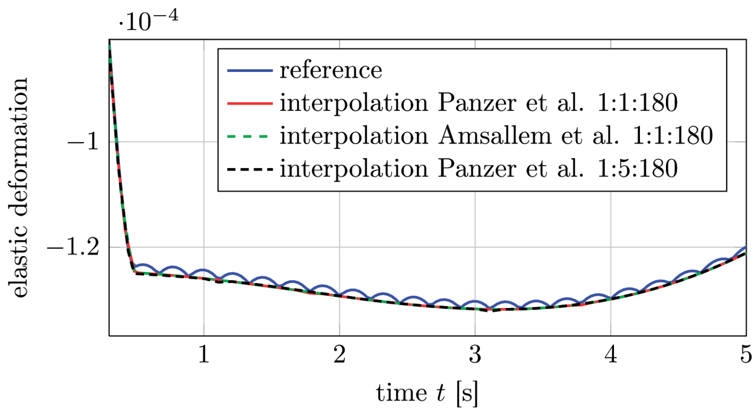

Fig. 6. Elastic deformation for reduced cylinder models with both PMOR-methods

number of nodes on the FE-mesh. Therefore, a smaller number of support systems could be applied to save computational effort in the offline step. A reduction of the necessary space to store all precomputed reduced models is achievable, too. Figure 6 illustrates the high quality of the interpolated systems, although only every fifth system is used. A difference between the interpolated system with all support systems and every fifth system cannot be observed, which fits to the results in frequency domain in Figure 4 and in [13].

The usage of a highly decreased number of support systems is also illustrated in Figure 7 where only every tenth model is used as a support system. The quality is still good and the elastic deformation of the interpolated systems is very accurate at the position of the support systems at $t = 2.2$ s and $t = 4.4$ s, because the force is applied smoothly and the position of the force changes slowly. Figure 7 also shows the benefit of using matrix interpolation in PMOR instead of combining the CMS-reduced support systems to one reduced system. This combination of the static shape functions provides good results at the support systems, but for other actuated nodes the quality is unsatisfying. This is discussed in detail with results in frequency domain in [13].

The quality of the interpolated models depends on the quality of the reduced systems and the interpolation method. Unfortunately, the interpolation and reduction error are not independent because better reduced systems lead to a more complex dependency of the matrix elements which are interpolated. A general advice, how to determine the reduced systems to get the best interpolated results, is highly problem-dependent and still an open topic. In [13], additional explanations for both PMOR-methods with matrix interpolation for reduction techniques based on Rational Interpolation and Krylov subspaces are provided.

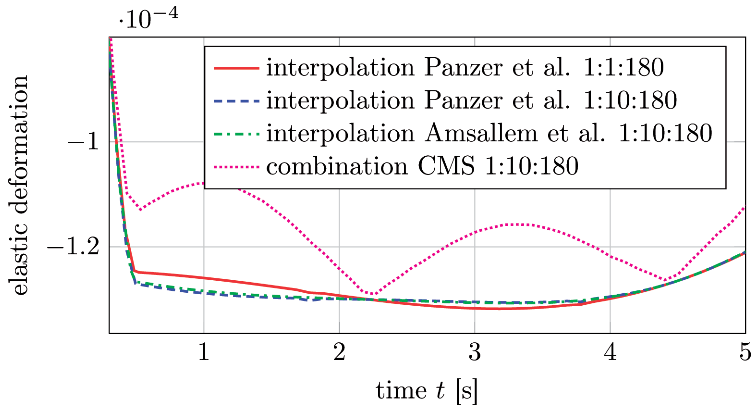

Fig. 7. Elastic deformation for reduced cylinder models with varying number of support systems

6. Conclusion

The simulation of moving loads on reduced elastic bodies in EMBS is enabled by using parametric model order reduction techniques. The input matrix in the model order reduction framework is described as a parameter-dependent matrix and the parameter dependency is retained in the reduced systems. Therefore, local support systems are reduced individually and the reduced coordinates are transformed to enable the interpolation of the system matrices in the offline step. In the online step, these system matrices are interpolated to generate a reduced model for any position of the moving force. Two PMOR-methods [9, 15] are applied and compared in frequency and time domain for a thin-walled cylinder. The application of the matrix interpolation provides the independence of the Finite Element mesh by interpolation of the reduced input matrix. This enables saving support systems which reduce the computational effort and the space to store the individually reduced systems.

If complete multibody systems are simulated, additional matrices in the floating frame of reference framework are parameter dependent and have to be interpolated. First applications of simple mechanical structures with rigid body degrees of freedom show promising results and will be investigated more extensively for large-scale systems.

Acknowledgments

The authors would like to thank the German Research Foundation (DFG) for the support of this research work within the project EB 195/11-1. We also gratefully thank Prof. Dr. Tatjana Stykel, University of Augsburg, for the helpful suggestions and discussions about parametric model order reduction and Dipl.-Ing. Achim Fischer, University of Stuttgart, who provided the model of the thin-walled cylinder.

Manuscript received by Editorial Board, December 16, 2013;
final version, April 27, 2014.

REFERENCES

[1] Schwertassek R., Wallrapp O.: Dynamik flexibler Mehrkörpersysteme (in German). Braunschweig: Vieweg, 1999.

[2] Fischer M., Eberhard P.: Model Reduction of Large Scale Industrial Models in Elastic Multibody Dynamics. Multibody System Dynamics, Vol. 31, pp. 27-46, 2014.

[3] Lehner M.: Modellreduktion in elastischen Mehrkörpersystemen (in German), Vol. 10 Dissertation, Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart. Aachen: Shaker Verlag, 2007.

[4] Fehr J.: Automated and Error-Controlled Model Reduction in Elastic Multibody Systems, Vol. 21 Dissertation, Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart. Aachen: Shaker Verlag, 2011.
[5] Fryba L.: Vibration of Solids and Structures Under Moving Loads. Groningen: Noordhoff International Publishing, 1999.
[6] Nowakowski C., Fehr J., Fischer M., Eberhard P.: Model Order Reduction in Elastic Multibody Systems using the Floating Frame of Reference Formulation. In Proceedings MATHMOD 2012 – 7th Vienna International Conference on Mathematical Modelling, Vienna, Austria, 2012.
[7] Fischer A., Eberhard P.: Simulation-Based Stability Analysis of a Thin-Walled Cylinder During Turning with Improvements Using an Adaptronic Turning Chisel. The Archive of Mechanical Engineering, Vol. 58, No. 4, pp. 367-391, 2011.
[8] Tamarozzi T., Heirman G., Desmet W.: An On-line Time Dependent Parametric Model Order Reduction Scheme with Focus on Dynamic Stress Recovery. Computer Methods in Applied Mechanics and Engineering, 2013. DOI:10.1016/j.cma.2013.09.021.
[9] Panzer H., Mohring J., Eid R., Lohmann B.: Parametric Model Order Reduction by Matrix Interpolation. At – Automatisierungstechnik, Vol. 58, No. 8, pp. 475-484, 2010.
[10] Amsallem D., Farhat C.: An Interpolation Method for Adapting Reduced-Order Models and Application to Aeroelasticity. AIAA Journal, Vol. 46, pp. 1803-1813, 2008.
[11] Antoulas A.: Approximation of Large-Scale Dynamical Systems. Philadelphia: SIAM, 2005.
[12] Craig R.: Coupling of Substructures for Dynamic Analyses: An Overview. In Proceedings of the AIAA Dynamics Specialists Conference, Paper-ID 2000-1573, Atlanta, USA, April 5, 2000.
[13] Fischer M., Eberhard P.: Application of Parametric Model Reduction with Matrix Interpolation for Simulation of Moving Loads in Elastic Multibody Systems. Advances in Computational Mathematics, 2013. (submitted for publication)
[14] Benner P., Gugercin S., Willcox K.: A Survey of Model Reduction Methods for Parametric Systems. Max Planck Institute Magdeburg Preprint MPIMD13-14, 2013.
[15] Amsallem D., Farhat C.: An Online Method for Interpolating Linear Parametric Reduced-Order Models. SIAM Journal on Scientific Computing, Vol. 33, pp. 2169-2198, 2011.
[16] Geuss M., Panzer H., Lohmann B.: On Parametric Model Order Reduction by Matrix Interpolation. In Proceedings of the European Control Conference, Zürich, 2013.
[17] Allemang R.: The Modal Assurance Criterion – 20 Years of Use and Abuse. In Proceedings of the 20th International Modal Analysis Conference, Los Angeles, USA, pp. 14-21, 2002.
[18] Neubauer M.: Implementierung multivariater Interpolation für parametererhaltende Modellreduktion (in German). Bachelor Thesis BSC-007, Institute of Engineering and Computational Mechanics, University of Stuttgart, 2012.

Zastosowanie techniki parametrycznej redukcji modelu do symulacji ruchomych obciążeń w sprężystym układzie wieloczłonowym

S t r e s z c z e n i e

W sprężystym układzie wieloczłonowym rozważane są duże, nieliniowe ruchy ciał sztywnych oraz małe odkształcenia sprężyste. W rosnącej liczbie zastosowań, np. w przemyśle motoryzacyjnym, procesach toczenia i frezowania, pozycja sił działających na ciało sprężyste jest zmienna. Redukcja rzędu modelu, niezbędna by umożliwić efektywną symulację, wymaga wyznaczenia funkcji *ansatz*, które zależą od zmiennej pozycji sił. Przy wielkiej liczbie możliwych punktów interakcji rozmiar zredukowanego systemu rósłby gwałtownie, gdyby stosować klasyczny schemat syntezy modów składowych (Component Mode Synthesis). Jeśli wiele węzłów jest potencjalnie obciążonych, lub obszar styku nie jest znany *a priori*, w ujęciu przedstawionym w artykule system opisuje się przy pomocy zależnej od parametru pozycji sił. Umożliwia to zastosowanie metod parametrycznej redukcji rzędu modelu. W artykule opisano dwie techniki oparte na interpolacji macierzowej, które transformują poszczególne systemy zredukowane, umożliwiają interpolację macierzy zredukowanych systemów i pozwalają wyznaczyć systemy zredukowane dla dowolnej pozycji sił. Zaprezentowano dekompozycję w trybie *online-offline* oraz opis rozkładu sił na zredukowanym ciele sprężystym. Zaproponowany schemat postępowania umożliwia efektywną symulację sprężystych układów wieloczłonowych z ruchomymi obciążeniami, gdyż zależy ona wyłącznie od rozmiaru zredukowanego systemu. Przedstawiono wyniki symulacji w dziedzinie czasu i częstotliwości dla cienkościennego cylindra z ruchomym obciążeniem, które ilustrują możliwości zastosowań proponowanej metody.