www.czasopisma.pan.pl PAN www.journals.pan.pl

DE GRUYTER OPEN

Arch. Min. Sci., Vol. 60 (2015), No 1, p. 63–71

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.1515/amsc-2015-0005

PIOTR STRZAŁKOWSKI*

MATHEMATICAL MODEL OF FORECASTING THE FORMATION OF SINKHOLE USING SALUSTOWICZ'S THEORY

MODEL MATEMATYCZNY PROGNOZOWANIA ZAPADLISK PRZY WYKORZYSTANIU TEORII SKLEPIENIA CIŚNIEŃ SAŁUSTOWICZA

In area affected by old, shallow extraction in some cases sinkholes are formed, causing security issues in urbanized areas. Problem of working out deterministic forecast of this threat seems to be important and up-to-date. Mathematical model presented in this work let us predict the possibility of sinkhole formation. That prediction is essential for analyzing possibility of investments in such areas. Basing on presented work, it is also possible to determine dimensions of sinkhole. Considerations are based on known from literature Sałustowicz's theory, which is utilises Huber's solution of equation describing the stress state around elliptic void made in flat plate

Keywords: rock mechanics, shallow caverns, predicting sinkholes

Początki eksploatacji górniczej na Górnym Śląsku sięgają XVIII stulecia. Dawna eksploatacja prowadzona na głębokościach nie przekraczających 80 m, do dziś generuje zagrożenia bezpieczeństwa powszechnego z uwagi na możliwość wystąpienia zapadlisk. Jak to wynika z pracy (Chudek i in., 2013), obszary pod którymi prowadzono w minionych latach płytką eksploatację zajmują znaczną powierzchnię śląskich miast, które w dalszym ciągu się rozbudowują. Dlatego problem występowania zapadlisk należy w dalszym ciągu uznać za ważny i aktualny. Duża liczba zapadlisk, ze zrozumiałych względów, jest wynikiem utraty stateczności płytkich wyrobisk korytarzowych. Istniejące metody prognozowania zapadlisk pozwalają głównie określać prawdopodobieństwo wystąpienia zapadliska. Jeśli wartość prawdopodobieństwa wystąpienia zapadliska jest większa od 0, wówczas należy się liczyć z zagrożeniem bezpieczeństwa terenu i co istotniejsze ludności. Taki sposób prognozowania wystąpienia zapadlisk nie daje jednoznacznej odpowiedzi na pytanie, czy teren objęty analizą jest rzeczywiście zagrożony. Dlatego istotnym jest stworzenie możliwości deterministycznego prognozowania tego typu deformacji. W tym kierunku zmierza propozycja autora pracy, w której wykorzystano teorię sklepienia ciśnień (Sałustowicz, 1956). Teoria ta znakomicie nadaje się do tego celu, gdyż jako jedyna z wielu w tym zakresie pozwala określić, czy pustka związana z wyrobiskiem znajduje się w stanie stateczności. Znane są bowiem przypadki, gdy płytkie wyrobiska górnicze, bez obudowy przez wiele lat pozostają w stanie nienaruszonym.

* FACULTY OF MINING AND GEOLOGY, SILESIAN UNIVERSITY OF TECHNOLOGY, UL. AKADEMICKA 2, 44-100 GLIWICE, POLAND

W ramach pracy dokonano szczegółowych obliczeń pola strefy odprężonej nad wyrobiskiem, bez stosowania uproszczeń przyjętych przez autora metody. Stosując podobne założenia jak w innych, znanych z literatury rozwiązaniach, podano warunki, mówiące o tym kiedy gruzowisko skalne zapełni szczelnie pustkę, bez powstania pustki wtórnej, a kiedy pustka wtórna powstanie. Zależy to od wymiarów i głębokości lokalizacji pustki oraz własności górotworu nad pustką. Warunkiem wystąpienia zapadliska jest aby strefa odprężona, związana z pustką pierwotną lub wtórną osiągnęła wysokość, przy której obejmować będzie nadkład, zbudowany ze skał luźnych. W dalszej kolejności zaproponowano wzory umożliwiające określenie wymiarów zapadlisk. Wyróżniono przy tym dwa przypadki:

- gdy strop pustki osiąga spąg nadkładu – wzór (15),
- gdy strefa odprężona obejmuje swym zasięgiem luźne skały nadkładu – wzór (19).

Dalszym etapem badań prowadzonych przez autora jest sformułowanie warunków, pozwalających stwierdzić, kiedy eksploatacja górnicza prowadzona pod pustką może wywołać jej samopodsadzenie, a w konsekwencji spowodować powstanie zapadliska na powierzchni. Prowadzone są również prace związane z utworzeniem oprogramowania komputerowego, wykorzystującego podane wzory i z weryfikacją rozwiązania w oparciu o przypadki znane z praktyki górniczej.

Słowa kluczowe: mechanika górotworu, płytkie pustki, prognozowanie zapadlisk

1. Introduction

According to data published in articles (Chudek et al., 2013; Kowalski et al., 2012) under relatively wide area of Upper Silesia there were fossil fuels extracted, between XVIII and the beginning of XXth century, at depths lower than 80 m-100 m. Similar conditions characterize Lower Silesia Basin, Ostrava-Karviná Basin and German basins. Old, shallow excavations are also reliefs of salt (Fajklewicz et al., 2004) and ore mines in many European countries. That shallow depth of mining, according to our previous experiences, poses a threat of discontinuous deformations. It is common knowledge, that those deformations are often caused by self-filling of voids connected with mining drifts. Liquidation of shortwalls voids was carried out with roof caving or sometimes with stowing after exploitation. However, mining drifts were often left on their own, without proper liquidation, and were self-filled after being damaged, or after destruction of wooden support. Because of that, it is important to analyze the behavior of the rock mass in the vicinity of mining drifts, after their loss of stability. At first, it's worth to mention that underground galleries not always lose the stability by themselves, what is proved by the fact, that discontinuous deformations forms only in some cases over shallow mining workings. It is obvious, that in case of drift led in rock mass characterized by high strength, it appears to be in self-supporting state. That kind of underground workings can be stable for hundreds of years. As an example, heading drifted in Xth century in Złoty Stok, in gneisses, characterized by high rock mass strength, is presented in the figure. 1.

Methods of prognosis of discontinuous deformation known from literature, don't take into account case of self-supporting of void (Chudek et al., 1988; Fajklewicz et al., 2004). They a'priori assume, over cavity, cover-caving area forms, and rocks included inside this area moves into cavity, causing its lofting. Mentioned methods allow us only to predict probability of sinkhole arising. Small value of probability, even this slightly higher than "0", means possibility of sinkhole formation. In this case it is difficult to take fully responsible decision on investments in such areas. On the other side, building adequate theoretical deterministic model seems to be quite difficult.

Taking into account mentioned above remarks, including those linked with: formation of high amount of sinkholes proceeded by self-collapsing of not filled mining excavations, and possibilities of self-supporting rock mass occuring, the Sałustowicz theory of pressure arch (Sałustowicz, 1956) looks useful in preparing predictions of sinkholes arising. Despite the long history of using this solution, and new methods development (eg Jakubowski, 2011), the Sałustowicz model still has a high practical utility.

2. The proposal of sałustowicz theory use to determine the possibility of sinkhole formation

Sałustowicz theory of pressure arch assume, that stress relaxation zone forms around mining drift, and it's shape depends on rock mass natural tendency to adjust the void contour to state, in which maximal stress is equal to rock strength. The zone boundary is ellipsis-shaped, each axis of which depends on the size of mining working, initial pressure and rock strength. Author used the Huber solution of equation describing the stress state around elliptic void made in flat plate – figure 1.

Fot. 1. Mining drift from X century, without support, developed in gneisses. Author's photo

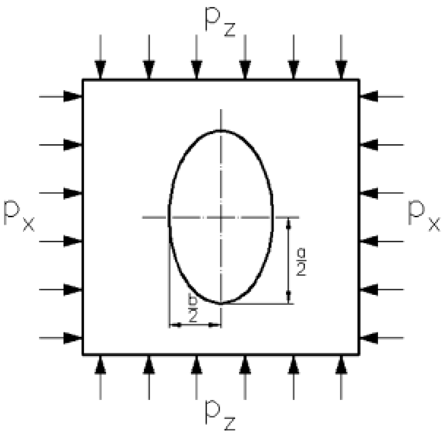

Fig. 1. The scheme illustrating load of flat plate with elliptic void

According to this solution, extreme stress values are:
– in sidewall:

$$\sigma_x = 0$$
$$\sigma_z = \sigma_{z\max} = p_z(1 + 2\frac{b}{a}) - p_x \quad (1)$$

– in roof and floor:

$$\sigma_z = 0$$
$$\sigma_x = \sigma_{x\max} = p_x(1 + 2\frac{a}{b}) - p_z \tag{2}$$

Formation of fractured zone above the roof happens when following condition is fulfilled.

$$\sigma_{x\,\max} \geq R_r \tag{3}$$

The relation between major and minor axis – n results directly from equation (1), on the assumption that $\sigma_{x\,\max} = R_r$, and is equal to :

$$\frac{a}{b} = \frac{R_r + p_z - p_x}{2p_x} = n \tag{4}$$

In case when $n < 0$ fractured zone does not form, because $\sigma_{x\,\max} < R_r$

Basing on the ellipsis equation one can calculate the length of semi-axis:

$$a = \sqrt{n^2 l + w^2}$$
$$b = \sqrt{l^2 + \left(\frac{w}{n}\right)^2} \tag{5}$$

It's worth to mention that there is good recognition of rock strength parameters, necessary for calculations (Chudek, 2002).

In order to calculate support's load, A. Sałustowicz simplified calculations and approximated ellipse's sector with a parabola's sector. He also did not take into account the area of stress relaxation zone confined by drift sidewalls. Both mentioned areas are easy to calculate using method presented below, worked out by author of this paper.

The area of upper part of ellipse over x axis can be calculated in the easiest way by using parametrical equations:

$$x = \frac{b}{2}\cos t$$
$$z = \frac{a}{2}\sin t \tag{6}$$

General equation depicting mentioned area is:

$$P = 0.5\int_{t_1}^{t_2}(x \cdot z' - z \cdot x')dt \tag{7}$$

Therefore area of upper sector of ellipsis is equal to (Fig. 2):

$$S_1 = 2 \cdot \frac{1}{2}\int_0^{\pi/2}[\frac{b}{2}\cos t \cdot \frac{a}{2}\cos t - \frac{a}{2}\sin t \cdot \frac{b}{2}(-\sin t)]dt = \frac{\pi ab}{8} \tag{8}$$

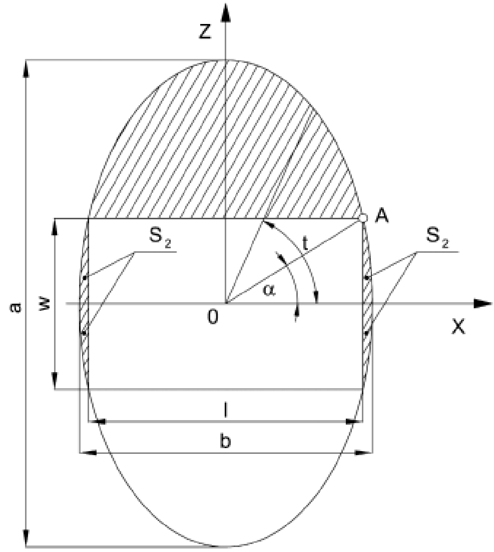

Fig. 2. The relaxed zone around void (drift) inside rock mass

Area confined by ellipsis and sidewall of drift in one quadrant of coordinate system can be described by equation below:

$$S_2 = \frac{1}{2}\int_0^k [\frac{b}{2}\cos t \cdot \frac{a}{2}\cos t - \frac{a}{2}\sin t \cdot \frac{b}{2}(-\sin t)]dt - \frac{wl}{8} = \frac{abk - wl}{8} \tag{9}$$

where:

k	—	arc measure of a angle, therefore $k = \pi\alpha/180°$ $k = \frac{\pi}{180^0} \cdot \arctan\frac{w}{l}$,
w	—	height of drift,
l	—	width of drift.

Stress relaxation region area (Fig. 2) (without the part under the drift floor), is equal to area of upper half of ellipsis – S_1, reduced by area of upper half of drift, above x axis, equal to $wl/2$, and enlarged by double area of figure confined by ellipsis and drift's sidewall below x axis, in one quadrant of coordinate system – S_2. Therefore relaxed zone area P_e is equal:

$$P_e = S_1 - \frac{wl}{2} + 2 \cdot S_2 = \frac{\pi ab}{8} - \frac{wl}{2} + \frac{2abk - 2wl}{8} = \frac{ab(\pi + 2k) - 6wl}{8} \tag{10}$$

It can be considered that in case of roof caving occurring, rocks inside relaxed zone will move toward void, filling the drift.

Let's take:

$$P_1 = P_e \cdot k_r, \qquad P_2 = P_e + w \cdot l$$

where k_r — rock loosening coefficient.

Taking into account mass conservation law, we can provide similar considerations like in case of other methods (Chudek et al., 1988). In case of $P_1 = P_2$, self-filling of void will take place, and rocks inside relaxed zone will fill tight the drift's void (assumption of Chudek-Olaszowski's method). Afterwards, void will no longer exist in rock mass. Otherwise, when $P_1 < P_2$, in the vicinity of top part of relaxed zone, secondary void will form. Its size will be equal to the difference of areas $P_2 - P_1$ (similarly like in Janusz – Jarosz method) – Fig. 3.

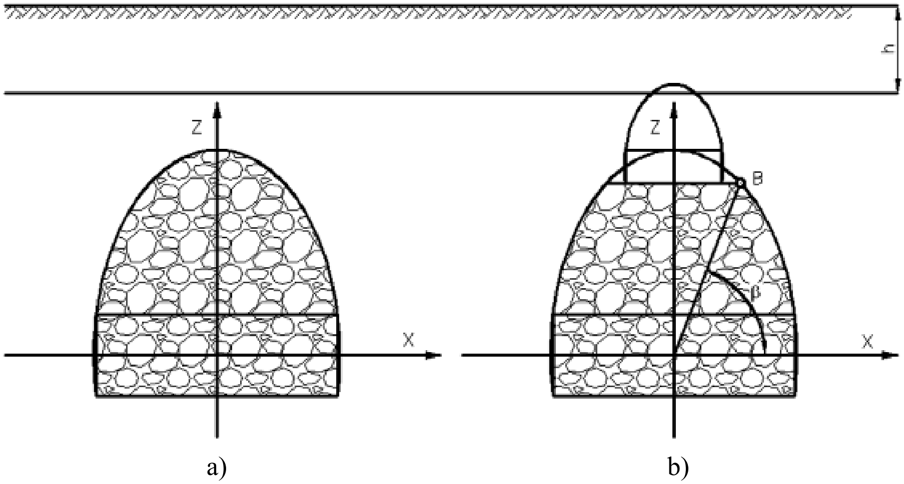

Fig. 3. Possibility of forming of caving zone in the vicinity of void. a – rocks included originally in relaxed zone, after fragmentation, tightly fill drift (void) and relaxed zone, b – after filling of void with fragmented rock from relaxed zone, secondary void forms in the vicinity of top part of ellipse

Secondary void section area is confined by ellipse contour and line parallel to x axis. Joint point of that line and ellipsis, marked as B on fig. 3, has coordinates described by parametrical equations:

$$x_B = \frac{b}{2}\cos\beta$$
$$z_B = \frac{a}{2}\sin\beta = p \tag{12}$$

Angle β describing location of point B can be calculated from equation: $\beta = \arcsin\frac{2p}{a}$.
s – arc measure of β angle, therefore $s = \pi\beta/180°$.

Thus, area of secondary void can be calculated from the equation:

$$Sw = 2\frac{1}{2}\int_{s}^{\pi/2}[\frac{b}{2}\cos t\cdot\frac{a}{2}\cos t - \frac{a}{2}\sin t\cdot\frac{b}{2}(-\sin t)]dt - 2\frac{pb\cos\beta}{4} = \frac{ab}{4}(\frac{\pi}{2} - s) - \frac{pb\cos\beta}{2} \tag{13}$$

Taking into consideration, that $S_w = P_2 - P_1$, values p and s can be simply calculated. In practice it is easier to consider rectangle with height equal to $a/2 - p$, which's area is equal to S_w. This rectangle can be treated as secondary void, which let us repeat algorithm presented above, to consider if pressure arch of secondary void will reach the level of carbon layers roof.

In case when relaxed zone connected with primary or secondary void reaches overburden of loose rock, on the surface sinkhole arises – Fig 3. In other case no sinkhole will appear on surface.

3. Determining the size of sinkhole

When determining the size of sinkhole, two cases were distinguished

1. Roof of primary or derivative void overlaps floor of overburden of loose rock – Fig. 4.
2. Relaxed zone, connected with void, exceeds the floor of overburden of loose rock – Fig. 5.

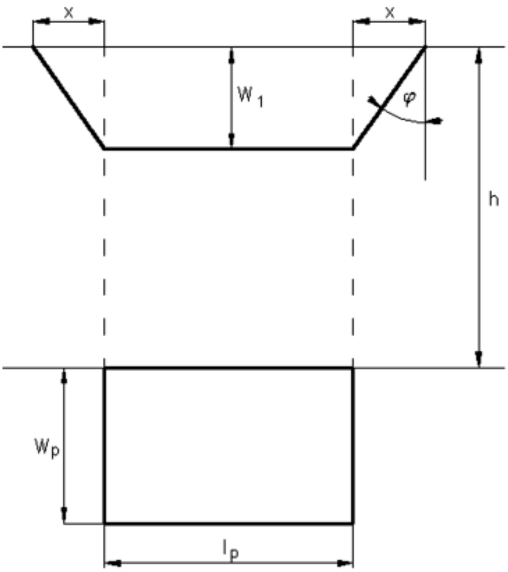

Fig. 4. Dimensions of sinkhole over void located in the roof of carboniferous layers

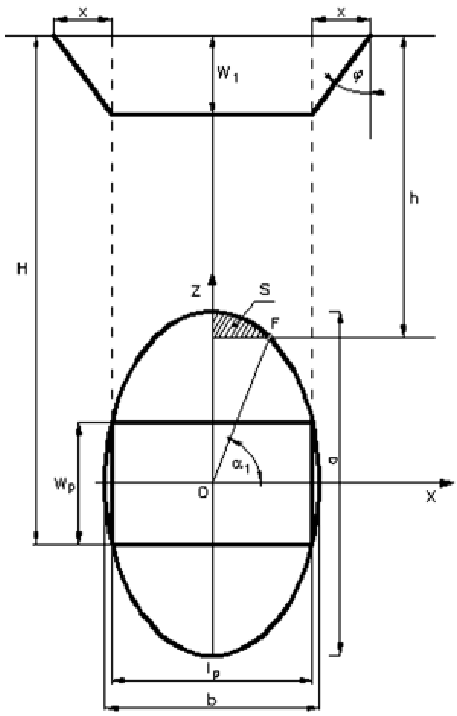

Fig. 5. Dimensions of sinkhole over void when relaxed zone exceeds roof of carboniferous layers

In both cases it was assumed, patterning on work (Bell, 1988), that diameter of sinkhole is greater than size of void which caused its formation. It was also assumed that sink hole is cut-cone-shaped.

In the first case (Fig. 4) dimensions of sinkhole were calculated in the following way:

Cross-section area of sinkhole is equal to:

$$P_z = l_p \cdot w_1 + x \cdot w_1 = l_p \cdot w_1 + w_1^2 \cdot \text{tg}\varphi \tag{14}$$

where:

- φ — internal angle of friction for overburden soil rock,
- l_p, w_p — dimensions of secondary void,
- x, w_1 — dimensions of sinkhole, $x = w_1 \cdot \text{tg}\varphi$,
- H — depth of void location,
- h — thickness of loose rock overburden.

Cross-section area of sink hole is equal to the area of section of void, provided that it is tightly filled by loose rock of overburden, so:

$$w_1^2 \cdot \text{tg}\varphi + l_p \cdot w_1 - w_p \cdot l_p = 0 \tag{15}$$

Trinomial equation, which solution is easy to calculate by well-known formulas, was obtained. Only positive root is important in our case.

Concerning second case (Fig. 5) one can point that:

Joint point F of ellipse and line marked the thickness of overburden has its coordinate z equal to $z_F = H - h - w_p/2$. Using parametrical equation of ellipse we can evaluate angle α_1 from formula: $\sin\alpha_1 = 2\dfrac{H - h - w_p/2}{a}$. Coordinate x_F is therefore equal to $\text{x}_\text{F} = \dfrac{b}{2}\cos\alpha_1$.

Area of sector of ellipse confined by angle α_1^1, arc measure of α angle and $\pi/2$ is according to formulas (7) and (8) equal:

$$P_\alpha = \frac{a \cdot b}{8}\left(\frac{\pi}{2} - \alpha^1{}_1\right) \tag{16}$$

where: α_1^1 — arc measure of α_1 angle

Part of this sector located in overburden is described by formula:

$$S = \frac{a \cdot b}{8}\left(\frac{\pi}{2} - \alpha^1{}_1\right) - 0.5x_F \cdot (H - h - w_p/2) \tag{17}$$

Part of all ellipse area located in overburden is equal to $2S = P_n$.

Taking into account the fact, that extensive strength of loose overburden is $R_r = 0$, it can be considered that void loses stability, therefore we can write down:

$$P_3 = (P_e - P_n) \cdot k_r \qquad P_4 = P_e - P_n + w_p \cdot l_p \tag{18}$$

where: P_e — the area of relaxed zone.

Difference of areas $P_4 - P_3$ should be equal to area of void cross-section:

$$P_4 - P_3 = w_1^2 \cdot \text{tg}\varphi + l_p \cdot w_1 \tag{19}$$

Just as in previous case, trinomial equation which's solution is trivial, was obtained.

4. Summary

Under Upper Silesia terrain, like in case of many basins in Europe, shallow extraction was led. Voids left in rock mass are still reason of sinkholes formation on the land surface. That occurrences are real threat for urbanized areas. From the other side, not at all rock mass voids lead to sinkholes arising. So it is of special importance to work out theoretical model, which should enable to clearly determine the conditions of sinkhole forming. In the framework of this paper such model has been proposed. Further author's research works concern:

- Verifying model on the basis of practical examples, including both cases: where sinkhole was not formed, despite of shallow extraction led, as well as those were sinkholes arised
- Extending presented model, taking into consideration case of deep extraction influence on the rock mass in vicinity void.

The results of mentioned above research results are being prepared for publication.

References

Bell F.G., 1988. *Land development. State of the art. in the location of old mine shafts*. Bull. of Int. Ass. of Eng. Geology, 37, p. 91-98.

Chudek M., 2002. *Geomechanika z podstawami ochrony środowiska górniczego i powierzchni terenu*. Wydawnictwo Pol. Śl. Gliwice.

Chudek M., Janusz W., Zych J., 1988. *Studium dotyczące rozpoznania tworzenia się i prognozowania deformacji nieciągłych pod wpływem podziemnej eksploatacji złóż*. Zeszyty Naukowe Politechniki Śląskiej, seria Górnictwo, zeszyt nr 141, Gliwice.

Chudek M., Strzałkowski P., Ścigała R., 2013. *Charakterystyka wybranych obszarów zagrożonych występowaniem deformacji nieciągłych na Górnym Śląsku*. Budownictwo Górnicze i Tunelowe, nr 1/2013, p. 27-30.

Fajklewicz Z. i in., 2004. *Badania zmian deformacyjnych w górotworze w celu odtworzenia wartości budowlanych terenów pogórniczych*. Monografia. Wydawnictwo AGH.

Jakubowski J., 2011. *Probabilistyczna analiza stateczności tuneli w strefie spękań*. Arch. Min. Sci., Vol. 56, No 3, p. 405-413.

Kowalski A., Gruchlik P., Polanin P., 2012. *Początki górnictwa węgla kamiennego w Zagłębiu Dąbrowskim i problem płytkich wyrobisk*. Przegląd Górniczy, 8, p. 130-139.

Sałustowicz A., 1956. *Zarys mechaniki górotworu*. Wydawnictwo Śląsk.

Received: 28 June 2013