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Analysis of Current Harmonics in 3kV DC Catenary Caused by Specific Current Harmonics of an Asynchronous Motor

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Abstract

The mathematical model of vehicle supply system as well as the mathematical model of main circuit of locomotive with asynchronous motors have been described in this article. The necessity of analysis of disturbances caused by distorted current run of drive motors emerged together with the introduction of high power vehicles with power electronic converters. Analysis of the compatibility of traction high current circuits with circuits of the signal and traffic control systems requires the knowledge of current spectrum in a catenary, which has been taken by a vehicle. The author has described the algebraic method of calculating of the spectrum amplitudes in a catenary. It does not require laborious and time-consuming simulations of a system, which considerably decreases preliminary costs of designation and dimensioning of a vehicle drive system.

Keywords: electric traction system, asynchronous motor, current harmonics

1. Introduction

Electric traction system contains many sources that influence the spectrum of current harmonics in a catenary. One of the sources introducing the harmonics to the system is a traction vehicle. Traction substation is a source that introduces harmonics voltage to the power system of a vehicle. Interaction of harmonics introduced by these sources may cause serious disturbances. If one limits the subject of the article to the issues related to the impact of harmonics generated by a three-phase converter, those harmonics operate in a network system, traction-intermediate circuit converter-

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asynchronous motor. The main cause of current disturbances in a catenary, which are caused by a vehicle, is the impulse character of voltage supplying an asynchronous motor. The asynchronous motor operates in a set-point mode, which is defined by the amplitude of first harmonic of motor supplying voltage, its frequency as well as mechanical angular velocity of a motor.

Familiarity with a spectrum of current supplying an inverter is of high importance due to the necessity of adjustment of power electronic converter drive in vehicles to the existing railway infrastructure on account of a range of generated harmonics, which might interfere with trackside systems. Parameters selection of the run, which controls the inverter in order to reduce a range of generated harmonics in a catenary, requires multiple simulations of the analysed system. Time of attainment of steady values, in comparison with simulation, is considerable, so the simulation method is time-consuming. According to the author, an interesting method, because of the manner of calculation of the harmonic spectrum of a current inverter and a catenary is the description of the input current inverter with usage of the Bessel function of first kind. The parameters of the spectrum of input current inverter and current in a catenary result directly from this description.

2. Model of the System

2.1. Model of the vehicle supply system

For the purposes of system analysis of electromechanical energy conversion, this article assumes a sample system comprising: DC traction substations, catenary and an electric traction vehicle. The main circuit of the locomotive includes: four input filters, four voltage inverters, four asynchronous motors (individual supply). Functional diagram of the scheme is illustrated in the Figure 1.

The elaborated model of an assumed system was used to study the impact of current collected by the inverter on current distortion in the catenary. Voltage inverters, which supply asynchronous motors, are connected to the catenary through the input low-pass filter (LC four-terminal network of „gamma" type). The task of the input filter is to reduce pulsation current collected from the network so as the compatibility of traction current with circuits and control and protection devices of rail traffic would be provided as well as to ensure proper operation of the main circuit power electronic devices under transient conditions. The dynamics of the phenomena occurring in the electrical circuit of the subsystem consisting of four LC filters and four voltage inverters along with power supply replacement model: DC traction substation and the network is described by the equation of state (1)

$$\mathbf{E}_{\mathrm{OG}}\dot{\mathbf{X}}_{\mathrm{OG}} = \mathbf{A}_{\mathrm{OG}}\mathbf{X}_{\mathrm{OG}} + \mathbf{B}_{\mathrm{OG}}\mathbf{U}_{\mathrm{OG}} \tag{1}$$

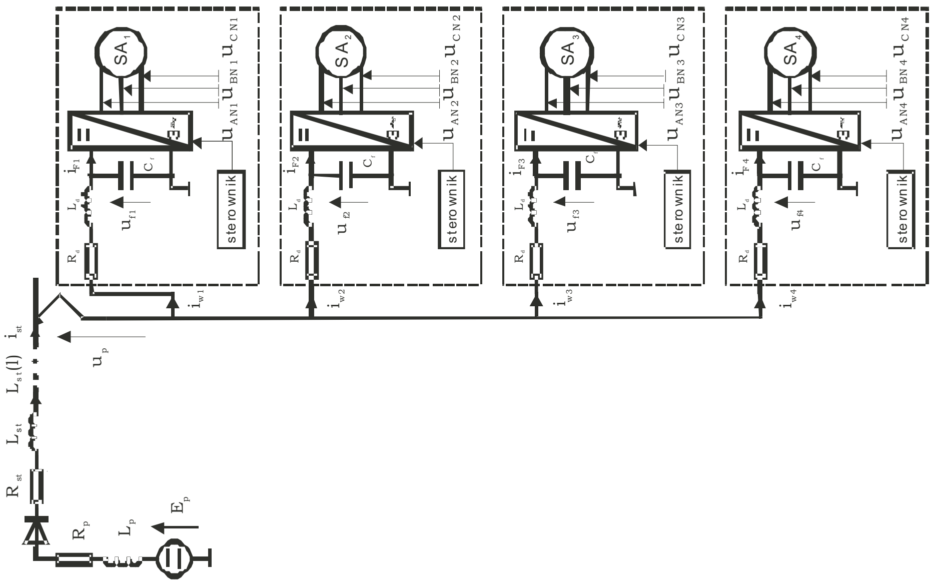

Fig. 1. Functional scheme of a model of electromechanical energy conversion system of a rail traction vehicle with an asynchronous drive

where:

$$\mathbf{E}_{OG} = \begin{bmatrix} L_{pst} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_f & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_d & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_f & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C_f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_f & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

$$\mathbf{X}_{OG} = \begin{bmatrix} i_{st} & i_{w1} & u_{f1} & i_{w2} & u_{f2} & i_{w3} & u_{f3} & i_{w4} & u_{f4} & u_P \end{bmatrix}^T \tag{3}$$

$$\mathbf{A}_{OG} = \begin{bmatrix} -R_{pst} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & -R_d & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -R_d & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -R_d & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_d & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{R_{pst}L_x}{L_{pst}} & \frac{R_d L_x}{L_d} & \frac{L_x}{L_d} & \frac{R_d L_x}{L_d} & \frac{L_x}{L_d} & \frac{R_d L_z}{L_d} & \frac{L_x}{L_d} & \frac{R_d L_x}{L_d} & \frac{L_x}{L_d} & -1 \end{bmatrix} \tag{4}$$

$$\mathbf{B}_{OG} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ \frac{L_x}{L_{pst}} & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$\mathbf{U}_{OG} = \begin{bmatrix} E_P & i_{F1} & i_{F2} & i_{F3} & i_{F4} \end{bmatrix}^T \tag{6}$$

where:

i_{st} – current in a catenary, i_{wi} – filters' input current, u_{fi} – voltage on the filters capacitance, u_p – voltage on the vehicle's pantograph, E_p – source voltage of a substation, i_{Fi} – inverters' input current, L_d, R_d, C_f – respectively: inductance, resistance, capacity of the LC filter's choke , i=1,. . . ,4 number of the subsequent circuit of input filter – motor of the electromechanical system of locomotive.

Other parameters of a system are being defined as follows:

$$R_{pst}(l) = R_p + R_{st}(l) \tag{7}$$

$R_{pst}(l)$ – resistance of traction substation and a catenary, dependent on the vehicle's location on a route, R_p – replacement resistance of a traction substation, $R_{st}(l)$ – catenary resistance, dependent on the vehicle's location on the route no 1.

$$L_{pst}(l) = L_p + L_{st}(l) \tag{8}$$

$L_{pst}(l)$ – inductance of traction substation and a catenary, dependent on the vehicle's location on a route, L_p – replacement resistance of a traction substation, $L_{st}(l)$ – inductance of catenary dependent on the vehicle's location on the route no 1. L_x inductance values are described by the following formula:

$$L_x = \frac{L_{pst} L_d}{L_{pst} + 4L_d} \tag{9}$$

2.2. Mathematical model of an inverter and a motor

Voltage inverter is a power electronics device, which static and dynamic properties depend mainly on the type of controllers as well on the type of power electronics elements (GTO thyristor. IGBT transistor), which create electric connectors (valves). Mathematical description of an electric connector as a lossless switch between two states: state of lossless on-state and off- state.

Possible states of a connector in three branches of an inverter are represented by the variables K_A, K_B, K_C Each of these three variables can take value 1 or 0. By selection of an appropriate strategy of attachment of connectors, one has an influence on the proper shaping of runs of three-phase motor supply fasteners. Change in the connector state does not cause energy loss, in the off- state value of current in a connector is equal to zero, while in the on- state voltage drop on the connector is equal to zero.

Voltage inverters used in traction vehicles must be capable of controlling the frequency and amplitude of the voltage supplying the asynchronous motor. Moreover, it is necessary to minimize the distortion of phase current in order to reduce parasitic torques adversely affecting the work of the vehicle (vibrations of drive torque) and the current harmonics in the catenary. In order to do so, different modulation methods of pulse with (PWM) are being used. Operation states of inverters' valves under impulse mode result from the formula [2].

$$y_x(t) = \left[m_0 + \frac{1}{2} m_1 \cos(\theta_{1x}(t)) + \sum_{n=1}^{n_{max}} \frac{2}{n\pi} \left\{ \sin\left(\frac{n\pi}{2} m_1 \cos(\theta_{1x}(t)) + \frac{n\pi}{2} \right) \right\} \cos(n m_f \theta_{2x}(t)) \right] \tag{10}$$

Angular position of impulses and basic harmonic of impulses' range is performed with usage of the following parameters:

ω_f – angular frequency of a basic harmonic of an inverter's output signal,

m_f – multiple impulses in relation to the frequency of a basic harmonic,

φ – phase of a basic harmonic of inverter's output signal (for phase A of a motor)

ψ – angle of a phase shift between multiphase signals

ε – angle of a phase shift of harmonics spectrum (for phase A of motor)

n_{max} – maximum value of sum index of a range, define the accuracy of measurement,

m_1 – depth of a basic harmonic modulation of the output signal,
m_0 – depth of a zero component modulation of the output signal

$$\begin{aligned} \theta_{1X}(t) &= \omega_f t + \varphi_X + \psi_X \\ \theta_{2X}(t) &= \omega_f t + \varphi_X + \psi_X + \varepsilon_X \end{aligned} \tag{11}$$

ω_{f1} = const.
Index x defines letter supplement comprising of designation for inverter's phases.

Signal y(t), which defines operation states of inverters' valves for A, B, C phases, has the following form:

$$\begin{aligned} &y_A\,(t, m_f, m_0, m_1, \varphi_A, \psi_A, \varepsilon_A) - \text{dla fazy A} \\ &y_B\,(t, m_f, m_0, m_1, \varphi_B, \psi_B, \varepsilon_B) - \text{dla fazy B} \\ &y_C\,(t, m_f, m_0, m_1, \varphi_C, \psi_C, \varepsilon_C) - \text{dla fazy C} \end{aligned} \tag{12}$$

Output voltage of each converter phases is described by the following formula:

$$\begin{aligned} u_{AN}\,(t) &= u_f\,(t)\,y_A\,(t) \\ u_{BN}\,(t) &= u_f\,(t)\,y_B\,(t) \\ u_{CN}\,(t) &= u_f\,(t)\,y_C\,(t) \end{aligned} \tag{13}$$

Description of the dynamic properties of a squirrel-cage induction motor, which provides a presentation of a three-phase motor in complex spatial vectors in a rectangular coordinate system. Using a vector description , where vectors are represented in a spatial coordinate system K rotating with an angular velocity of motor equations can be presented as follows:

– voltage equation of stator and rotator circuits.

$$\begin{aligned} \mathbf{U}_{sK} &= R_s \mathbf{i}_{sK} + \frac{d\,\boldsymbol{\Psi}_{sK}}{d\,t} + j\,\omega_K\,\boldsymbol{\Psi}_{sK} \\ \mathbf{0} &= R_r \mathbf{i}_{rK} + \frac{d\,\boldsymbol{\Psi}_{rK}}{d\,t} + j\,(\omega_K - p\,Ł_m)\,\boldsymbol{\Psi}_{rK} \end{aligned} \tag{14}$$

– flux-current equations

$$\begin{aligned} \boldsymbol{\Psi}_{sK} &= L_s\,\mathbf{i}_{sK} + L_m\,\mathbf{i}_{rK} \\ \boldsymbol{\Psi}_{rK} &= L_r\,\mathbf{i}_{rK} + L_m\,\mathbf{i}_{sK} \end{aligned} \tag{15}$$

where:

$\mathbf{U}_{sK}, \mathbf{i}_{sK}, \boldsymbol{\Psi}_{sK}$ – define respectively: vectors of voltage, current, flux linkage with stator,

$\mathbf{U}_{rK}, \mathbf{i}_{rK}, \boldsymbol{\Psi}_{rK}$ – define respectively: vectors of voltage, current and flux linkage with rotor,

L_s, L_r, and L_m – define inductance of a stator, rotor (to the stator's windings) and mutual inductance of windings,

R_S, R_r – define respectively resistances of stator and rotor circuits (to the stator's windings),

p – is a number of motor poles pairs,

ω_K – angular velocity of reference system, Ω_m – mechanical angular velocity of rotor's rotation,

m_s – number of motor's phases

In the issues related to research simulation of induction machines supplied by distorted voltage a description of an asynchronous motor in a fixed coordinate system ($\omega_K = 0$), connected to the stator, in the system (α, β) is being used. Turning to the model of two-axle motor spatial vectors describing a motor are written as components α, β.

$$\begin{aligned}
\mathbf{U}_{sK} &= u_{s\alpha} + ju_{s\beta} \\
\mathbf{i}_{sK} &= i_{s\alpha} + ji_{s\beta} \\
\mathbf{i}_{rK} &= i_{r\alpha} + ji_{r\beta} \\
\mathbf{\Psi}_{sK} &= \psi_{s\alpha} + j\psi_{s\beta} \\
\mathbf{\Psi}_{rK} &= \psi_{r\alpha} + j\psi_{r\beta}
\end{aligned} \tag{16}$$

Taking into consideration the fact that dynamics of electromagnetic changes of state variables (fluxes, current) of a modeled motor are faster than dynamics of changes of rotor's angular velocity, mathematical model of a motor described by the set of equations (14, 15) after decomposition of vectors of voltage, current, stator and rotor flux into square components α – β, can be described as a non-stationary equation of state.

Elements of state matrix are dependent on the present value of mechanical angular velocity of Ω_m rotor rotation.

Those equations would accommodate the following shape:

$$\dot{\mathbf{X}}_e = \mathbf{A}_e(\Omega_m)\mathbf{X}_e + \mathbf{B}_e\mathbf{U}_e \tag{17}$$

where:

– state matrix $\mathbf{A}_e(\Omega_m)$

$$\mathbf{A}_e(\Omega_m) = \begin{bmatrix} 0 & 0 & -R_s & 0 \\ 0 & 0 & 0 & -R_s \\ \dfrac{R_r}{\sigma_1 L_r L_s} & \dfrac{1}{\sigma_1 L_s} p_b \Omega_m & -\dfrac{1}{\sigma_1}\left(\dfrac{R_s}{L_s} + \dfrac{R_r}{L_r}\right) & -p_b\Omega_m \\ -\dfrac{1}{\sigma_1 L_s} p_b \Omega_m & \dfrac{R_r}{\sigma_1 L_r L_s} & -p_b\Omega_m & -\dfrac{1}{\sigma_1}\left(\dfrac{R_s}{L_s} + \dfrac{R_r}{L_r}\right) \end{bmatrix} \tag{18}$$

where:

$$\sigma_1 = 1 - \frac{L_m^2}{L_r L_s} \tag{19}$$

– state vector

$$\mathbf{X}_e = \begin{bmatrix} \Psi_{s\alpha} & \Psi_{s\beta} & i_{s\alpha} & i_{s\beta} \end{bmatrix}^T \tag{20}$$

– control matrix

$$\mathbf{B}_e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{1}{\sigma_1 L_s} & 0 \\ 0 & \frac{1}{\sigma_1 L_s} \end{bmatrix} \tag{21}$$

– control vector

$$\mathbf{U}_e = \begin{bmatrix} u_{s\alpha} & u_{s\beta} \end{bmatrix}^T \tag{22}$$

where:

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} = T_u u_f \begin{bmatrix} y_A \\ y_B \\ y_C \end{bmatrix} \tag{23}$$

where: matrix T_u Has the shape:

$$T_u = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix} \tag{24}$$

Phase current of motor i_A, i_B, i_C

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = T_i \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \tag{25}$$

where: matrix T_i has the shape:

$$T_i = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \tag{26}$$

Input current o fan inverter is described by the following formula:

$$i_F(t) = i_A K_A + i_B K_B + i_C K_C \tag{27}$$

$$K_x = \begin{cases} 1 & \text{gdy } y_x(t) \geq 0,5 \\ 0 & \text{gdy } y_x(t) < 0,5 \end{cases} \tag{28}$$

$x \in [A, B, C]$.

On the basis of equations (1, 17, 27) time run of current in a catenary was determined by the simulation and then using Fourier transform, spectrum of this current was set. Analysis of the impact of various system parameters as well as parameters of an inverter voltage control method for amplitudes of current harmonics in a catenary requires each time simulation calculations, which are time consuming. Figure 2 (a, b, c) shows the time run components of inverter current from individual inverter phases, and the time run of current inverter Fig. 2 (d).

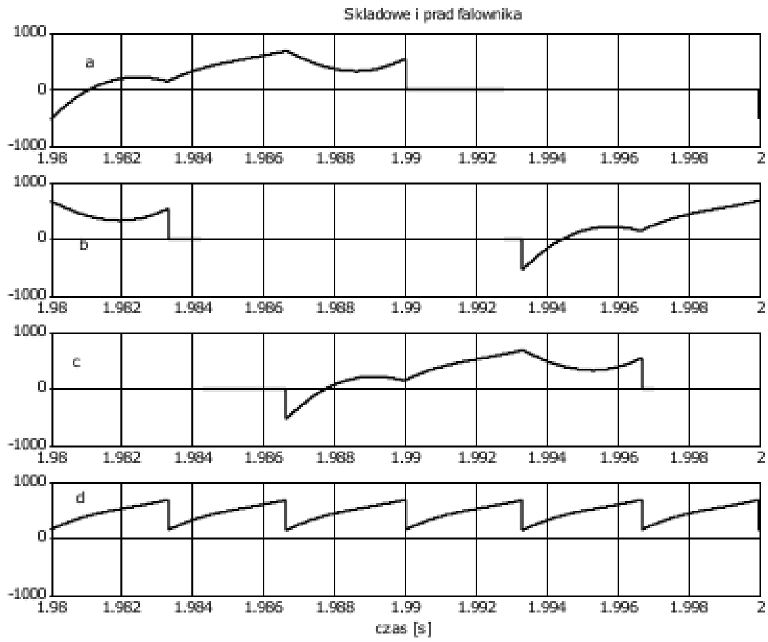

Fig. 2. Inverter current run and its components for each phase

3. Calculation of Current Inverter Spectrum from Individual Harmonics of Motor Current

Current spectrum in a catenary can be defined without necessity of simulation calculations. By $U_{AN}(f)$, $U_{BN}(f)$, $U_{CN}(f)$ complex amplitude of inverter's output voltage for the f frequency have been defined respectively for phases A, B and C, with application of Bessel's function, as author described in the article [1].

By $Z_S(f)$ impedance of a given motor's model has been described.

$$Z_S = \frac{\left(\left(\frac{R_sR_r}{2\pi f_s\omega_s}\sigma L_sL_r2\pi f_r\right)^2 + \left(L_sR_r + \frac{L_rR_sf_r}{f_s}\right)^2\right)}{(L_m^2 2\pi f_rR_r) + R_s((L_r^2f_r^2 + R_r^2)/2\pi f) - (L_s(R_r^2 + \sigma_1L_r^2 2\pi f_r^2)i),} \tag{29}$$

Complex amplitudes of motor's current for the frequency f and A, B, C phases are described by the following formulas:

$$I_A(f) = \frac{U_A(f)}{Z_S(f)},\ \ I_B(f) = \frac{U_B(f)}{Z_S(f)},\ \ I_C(f) = \frac{U_C(f)}{Z_S(f)} \tag{30}$$

Time run of h range harmonic of i_A (t) phase current of motor is described by the formula:

$$i_{Ah}(t) = I_{Ah}\cos(\theta_{3A}(t)) \tag{31}$$

where:

I_{Ah} – harmonic amplitude of motor's phase current of h range and its phase $\theta_{3A}(t) = h\omega_f t - \gamma_{Ah}$

Time run of inverter's input current i_{FAh} (t), which is generated from the h range harmonic of phase A current is described by the formula:

$$i_{FAh}(t) = y_A(t) I_{Ah}\cos(\theta_{3A}(t)) \tag{32}$$

Taking into account (10, 32), iFAh(t) time run is described by the formula:

$$i_{FAh}(t) = \left[m_0 + \frac{1}{2}m_1\cos(\theta_{1A}(t)) + \sum_{n=1}^{n_{max}}\frac{2}{n\pi}\left\{\sin\left(\frac{n\pi}{2}m_1\cos(\theta_{1A}(t)) + \frac{n\pi}{2}\right)\right\}\cos(nm_f\theta_{2A}(t))\right] \cdot I_{Ah}\cos(\theta_{3A}(t)) \tag{33}$$

After transformation:

$$\begin{aligned} i_{FAh}(t) = &\left[m_0 I_{Ah}\cos(\theta_{3A}) + \frac{1}{2}m_1 I_{Ak}\cos(\theta_{1A} - \theta_{3A}) + \frac{1}{2}s_1 I_{Ah}\cos(\theta_{1A} + \theta_{3A})\right] + \\ &+ \sum_{n}^{n_{max}}\frac{1}{n\pi}I_{Ah}J_{(0)}(m_2)Z_S[\cos(nm_f\theta_{2A} \pm \theta_{3A})] + \\ &+ \sum_{n}^{n_{max}}\frac{1}{n\pi}I_{Ah}J_{(2q-1)}(m_2)Z_N Z_C[\cos((2q-1)\theta_{1A} \pm nm_f\theta_{2A} \pm \theta_{3A})] + \\ &+ \sum_{n}^{n_{max}}\frac{1}{n\pi}I_{Ah}J_{(2q)}(m_2)Z_P Z_S[\cos((2q)\theta_{1A} \pm nm_f\theta_{2A} \pm \theta_{3A})] \end{aligned} \tag{34}$$

where:

J_l – Bessel's function of first kind and l-range

$$\begin{aligned} &Z_N = (-1)^{q-1}, Z_P = (-1)^{q+1}, Z_C = \cos\left(\frac{n\pi}{2}\right), Z_S = \sin\left(\frac{n\pi}{2}\right) \\ &q = 1, 2, 3..... \end{aligned} \tag{35}$$

On the basis of expression (33) and for t = 0, with usage of the mathematical package Matlak, one calculated the complex amplitude of inverter's current coming from the current harmonic of h range and A phase of a motor. Complex amplitudes of inverter current of the remaining phases can be calculated by analogy, on the basis of expression (34), assuming appropriate values for the phases θ1B, θ2B, θ3B for Phase B, and θ1C, θ2C, θ3C for the phase C. Complex amplitude of inverter current for f frequency is described by the sum of complex amplitudes from the individual phases and is described by the expression:

$$I_F(f) = I_{FA}(f) + I_{FB}(f) + I_{FC}(f) \tag{36}$$

Figure 3 presents time run of inverter current obtained from the simulation – (a-curve) as well as time run generated on the basis of complex amplitudes for frequency range from 1 to 60 - (b-curve). Comparison of those two runs shows that values of runs calculated according to the described methods do not differ.

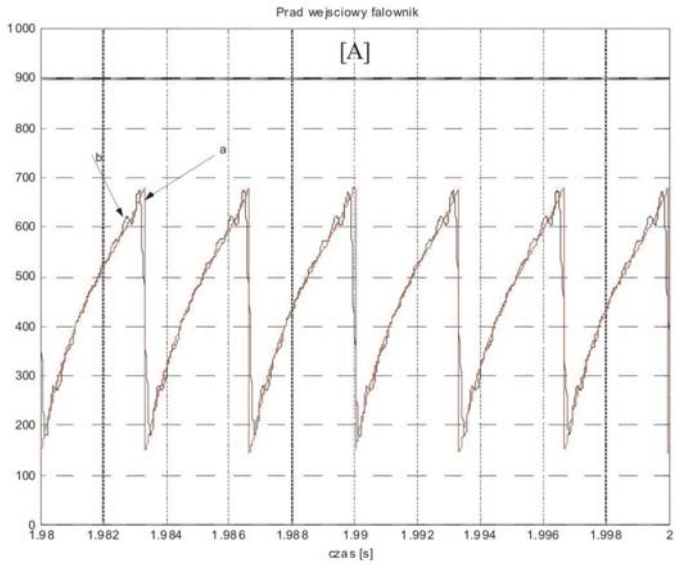

Fig. 3. Time run of inverter current, a) time run obtained as a result of a simulation of circuit, b) run determined on the basis of complex amplitudes for frequency range between 1 and 60

Table 1

Harmonic range	Amplitudes of considerable harmonics				
1	2	3	4	5	6
6	148,8	150,3	88,8	121,1	51,0
12	82,5	83,2	42,2	11,3	14,3
18	55,6	56,3	27,5	13,6	7,4
24	41,8	42,7	20,9	9,7	5,8
30	33,1	34,0	16,4	7,8	4,0

Table 1 shows amplitudes of considerable harmonics of range 6,12,18,24,30 of inverter's current time run. Column 2 presents amplitudes obtained using the FFT function. Column 3 contains amplitudes calculated on the basis of the expression (36) for motor current harmonics of range from 1 to 60. Column 4 presents amplitudes of inverter's current induced by the amplitude of basic range of current motor. Column 5 shows amplitudes of inverter's current induced by the amplitude of 5 range of motor current. Column 6 presents amplitudes of inverter's current induced by the amplitude of 7 range of motor current. Analysis of values of inverter current amplitudes from a given range of harmonics of motor current allows the appropriate selection of parameters of spectrum shaping motor supply voltage in other words establishment of m_f dependence according to the frequency of inverter supply. Current complex amplitudes of f frequency in a catenary have been calculated with assumption that four engines in a locomotive operate at the same point and that input current of inverters are equal and input voltage of inverters are of constant values. In the considered circuit, which is presented in the fig.1, the current complex amplitude in a catenary for f frequency will be calculated from the following dependence:

$$I_S(f) = 4I_F(f)\frac{Z_{C(f)}}{Z_{ST}(f) + Z_C(f)} \tag{37}$$

where:

$$\begin{aligned} Z_C(f) &= \frac{1}{j2\pi f C_f}; \\ Z_{ST}(f) &= \frac{j2\pi f}{4}\left(L_d + L_{st}(l) + \frac{L_{dp}}{4}\right) + \frac{1}{4}\left(R_d + R_{st}(l) + \frac{R_{dp}}{4}\right) \end{aligned} \tag{38}$$

Table 2 shows the amplitude of the considerable harmonics of range 6, 12, 18 of current in the catenary. In column 2 of the amplitudes obtained using the FFT function from the time run obtained as a result of simulation. Column 3 shows the amplitudes calculated from the expression (34, 36, 37) for harmonics motor current range of 1 to 60. In column 4 are the amplitudes of catenary - induced current caused by the amplitude of the basic range of the motor current. In column 5, the amplitude of the catenary current caused by the amplitude of the range of 5 motor current. In column 6 amplitudes of catenary current induced by the amplitude of 7 range of a motor's current.

Table 2

Harmonic range	Amplitudes of considerable harmonics				
1	2	3	4	5	6
6	0.362	0.355	0.215	0.290	0.124
12	0.051	0.002	0.025	0.014	0.009
18	0.015	56.3	0.007	0.004	0.002

4. Conclusions

In his article two methods of harmonics measurement in the catenary have been presented. The first method is based on the simulation of the model described by equations (1) and (13) and on calculation of the time run of current in a catenary, and the appointment of spectrum by usage of FFT. The second method consists in calculation of the complex amplitudes of motor current for f frequency on the basis of the complex amplitudes of voltage and impedance of the motor and appointment of the complex amplitudes of the current in the network. In this method, we can calculate the complex amplitudes of the current in the overhead line from the individual complex amplitudes of motor current for a particular frequency selected is relevant for determining the level of interference. This method of calculation of complex amplitudes of the current network does not require long-term simulations of the system described a rigid system of differential equations. The presented simplified methods for calculating the power spectrum in the network can be used in the initial analysis of damping efficiency of the vehicle by an input filter harmonics generated by the vehicle traction. Knowing the noise made by the inverter input current can determine the required characteristics of the input filter and the parameters forming the input voltage.

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