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Multi-Modal Acoustic Flow Decomposition Examined in a Hard Walled Cylindrical Duct

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Flow fields could be of great interest in the study of sound propagation in aeroengines. For ducts with rigid boundaries, the fluid-resonant category may contribute significantly to unwanted noise. An understanding of the multi-modal propagation of acoustic waves in ducts is of practical interest for use in the control of noise in, for example, aero-engines, automotive exhaust and heating or ventilation systems. The purpose of our experiments was to test the acoustic energy transmission of duct modes based on studies carried out by the sound intensity technique. Sound intensity patterns in circular duct are discussed of modal energy analysis with particular reference to proper orthogonal decomposition and dynamic mode decomposition. The authors try to justify some advantages of the sound intensity experimental research in this area. In the paper, the wide-band sound signal propagated from source approximated with loudspeaker in hard-walled duct is imaged using a sound intensity – based approach. For a simple duct geometry, the sound intensity field is examined visually and by performing a modal decomposition greater insight into the acoustic structures is obtained. The image of sound intensity fields below and above "cut-off" frequency region are found to compare acoustic modes which might resonate in duct.

Keywords: acoustics flow, sound intensity, acoustic waveguide.

1. Introduction

Much of theoretical research concerned with acoustics provides useful information about pressure fields, but none currently offers a full mapping of the acoustic energy flow (vectorial effects) in acoustic waveguides or in the front and back of any scattering system working in three-dimensional real environmental conditions. Interference, diffractions and scattering of waves modes in the real field are very complex and difficult compared with the theoretical modelling. This is one of the reasons why the experimental investigations of acoustic field using sound intensity (SI) techniques are such effective and serviceable methods. Besides that, sound intensity investigation techniques are very useful in locating noise sources and provide the advantage that the measurements can be made in almost any environment, without the requirement of special facilities, such as an anechoic room. Researches can be made even in the presence of parasitic noise, which is a very important attribute in industry vibroacoustic investigation.

The visualization of acoustic energy flow in real-life acoustic three-dimensional space fields can explain many particular energetic effects (perturbations and vortex flow, effects of scattering in the direct and near field, *etc.*) concerning the areas, in which it is difficult to make numerical modelling and analysis with the numerical simulation methods. The sound intensity image represents a more accurate and efficient information compared to the spatial pressure acoustic field modelling.

The lack of precise understanding of flow separation in an acoustic waveguide makes some further research necessary. In the present study, the flow resulting from an acoustic wave at the transition inside duct is investigated. The acoustics flow fields inside circular duct are mainly measured using Sound Intensity (SI) technique and partly Particle Image Velocimetry (PIV). PIV is widely used in the field of fluid mechanics, in particular to study the dynamics of coherent structures. Vortex detection, Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) are among many tools used for this purpose.

Based on the research results on the cylindrical open-end acoustic waveguide, authors would like to justify the importance of experimental SI technique in the studies of vortex sound theory.

2. Proper orthogonal and dynamic mode decompositions (POD, DMD)

The POD is generally used to separate different spatial modes in a flow. POD, also known as Karhunen-Lòeve expansion (Lòeve, 1955), is a method to decompose vector fields (2D or 3D) into a set of empirical eigenfields, which describes the dominant behaviour or dynamics of a given problem. The POD can examine the series of input vector fields of a certain flow condition, each at a different instant in time. These input vector fields are used to form an eigenvalue problem that is solved to determine a set of optimal basis functions for representing the flow field. Each eigenfield has an associated energy value. This value reflects the fraction of the overall energy in the input vector fields, that is represented by this eigenfield. The eigenfields are numbered by decreasing energy values, so that the first eigenfield is the most important one.

In summary, we can conclude that POD is the most efficient way of extracting the main energetic component of the infinite-dimensional process with only a few modes. POD has been applied in various turbulent flows to extract dominant flow structures, i.e. a *coherent structure* (Moreau *et al.*, 2000; Holmes *et al.*, 1996).

The DMD is different from the proper orthogonal decomposition where the former attempts to represent a data sequence by orthogonalizing it in time (i.e. isolating distinct frequencies in the data), while the latter attempt a decomposition based on the orthogonality in space. Furthermore, the DMD applies directly to the data, while a POD analysis processes second-order statistics of the data.

DMD is based on snapshots of the flow which is equally applicable to experimental and numerical flow field data. This decomposition technique is at the basis of a Koopman analysis of nonlinear dynamical systems (Schmidt, 2010). The method extracts dynamic information from a sequence of uniformly sampled flow measurements. The resulting modes represent the relevant flow structures that contribute most to the overall ‘evolution’ captured in the measurement sequence. The technique is flexible enough to equally deal with simple flow visualizations and with time-resolved PIV measurements (Lourenco *et al.*, 1997).

Sometimes the velocity measurements are taken at some phases within one acoustic period, example on 20 phase steps (Sung, Yoo, 2001). In terms of one period of the sine wave direction for the positive half-sine (10 phases) the velocity is forward directed end is called *ejection*. The negative half-wave shows a reverse flow direction end is called *suction*. This phase- or time-resolved of phase analysis are less used in physical acoustics. Acoustic analysis are adapted to human hearing system perception of sound. The sound is always percept acoustic pressure averaged over time and space (*rms* value). It is also assumed that the signal is averaged in the frequency bands using a globally adopted octave band (usually the typical 1/1, 1/3, or 1/12 of an octave band).

3. Acoustic orthogonal decomposition (AOD) in hard walled circular ducts

Very often a modal decomposition technique is used to provide detailed information about the modal content of the sound field. In our case, we want to do experimental research on the acoustic waveguide model.

Description of mode propagation as a sound intensity stream flow in hard walled cylindrical ducts, called acoustic orthogonal decomposition (AOD), give best results when all acoustic modes are excited. For ducts with rigid boundaries, the fluid-resonant category may contribute significantly to unwanted noise. Height level of acoustic energy propagated along the duct occurs when the mode is excited above a cut-off frequency which depends on the mode eigenvalue and the duct radius. Modes excited below their cut-off frequency are evanescent and decay exponentially with distance along the duct. The dimensionless number which expresses the cut-off frequency independently of the radius is the term kr, or as a Helmholtz number He (Bennett *et al.*, 2010; Joseph *et al.*, 2003).

We can find a minimum wave-number k, or frequency, at which a given mode in an annular duct is represented in terms of plane waves. This minimum wave number, called the cut-off value, below which the simple wave breaks down is investigated in duct and marks the boundary between high-frequency propagation and low-frequency decay of duct modes.

The main object of the paper is to give an image of sound-energy transmission by higher-order duct modes. The discussion is restricted to uniform rigid duct with no flow inside. Higher order modes (not found in plane waves) transmitted a significant level of noise in the ducts. This is the “cut-off” property; each mode in a uniform duct fails to propagate if the frequency fails below critical value, the “cut-off frequency”. An approximate wave theory for annular ducts gives a physical picture of sound waves in any duct mode. The simplest mode of sound transmission in a duct involves motion of the gas only in the axial direction, along the duct. This is plane-wave transmission, in which the disturbance is uniform over a cross-section normal to the axis. Other modes of transmission have a characteristic form as a "tumble motion" shapes.

The acoustic modal decomposition proposed in this paper is based on the SI data with good qualitative agreement found for the axial flow (horizontal and azimuthal one) and incident radial modes.

4. Experimental setup

In Fig. 1 we show a model of circular acoustic waveguide where investigations with sound intensity measurement were made. The 6 m long open-end duct with internal radius 0.474 m was used as a model for an acoustic waveguide. At one end it was connected to a loudspeaker, a source of broadband acoustic signals. The method employed in this paper is based on SI technique and experimental set-ups where monopole sources are approximated with loudspeakers. The duct is excited with acoustic pink noise, so, the sound power along the duct is sent without mean flow. Measurements were made in frequency band 50–6800 Hz and analyzed in 1/3 and 1/12 octave band frequency. SI measurement were made on a duct without any obstacles present inside. Measurement region was placed at a distance about of 2.2 m from the end of the duct.

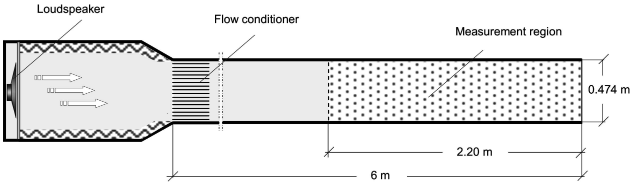

Fig. 1. Sound incident of circular cross section acoustic waveguide.

The space inside the duct was scanned with sound intensity measuring the x, y and z components of sound intensity vectors (Mickiewicz *et al.*, 2011). The image of the dipolar and quadrupolar sound generated by a flow inside a duct was obtained using a SI three-dimensional *USP Microflown* probe and our graphical *SIWin* post-processing software (Weyna, 2010).

5. Particle image velocimetry (PIV)

If a intensity probe with a nose cone is brought into the flow-field, extra sound may be generated by the probe itself. This sound can lead to a contamination of the correlation function. Hence, this effect was called "probe contamination". The "probe contamination" can be avoided completely by using a optical non-intrusive technique to measure the flow field quantity.

Due to rapid advances in computers, optics and digital image processing techniques, instantaneous acoustic particle velocity fields can be extracted using a specially implemented PIV technique. While PIV is widely used in the field of fluid mechanics, in particular to study the dynamics of turbulent flows, we adapt this technique to acoustic applications.

In our research velocity data are acquired with a 2D PIV system (Weyna *et al.*, 2013; Weyna, Mickiewicz, 2014). System consists of a high-resolution CCD camera, a dual-head Nd:Yag laser (Litron Lasers-Nano piv), a frame grabber, a synchronizing device and a computer. The high-resolution CCD camera (Imager pro X 4M) with a spatial resolution of 2048 × 2048 pixels was used to capture particle images. The 14-bit air-cooled CCD camera can capture 14 images per second at a 100% fill factor. The maximum energy of the two-head Nd:Yag laser is about 325 mJ per pulse. The CCD camera and Nd:Yag laser were synchronized using a delay generator. For the single-frame PIV measurements, two successive particle images were recorded on a single frame.

During the first exposure of the CCD camera, the particle image scattered by the first laser pulse is recorded on the CCD sensor array. The CCD sensor array is then translated by prescribed pixel lines within the time interval Δt and starts the second exposure to capture the second particle image. The two particle images are superimposed on a single-frame and the double-exposed single-frame image is then cross-correlated to extract the instantaneous velocity field. Small olive droplets were used as seeding tracers. The flow is seeded with di-ethyl-hexyl-sebacate oil (DEHS) tracer particles of approximately 1 μm in diameter. They are generated by a seeding generator with Laskin atomizer nozzles.

A thin laser light sheet was formed by passing the laser beam through a mirror and through spherical and cylindrical lenses. The CCD camera was installed perpendicularly to the laser light sheet to capture the scattered particle images of the investigated flow. In this study, the interrogation window size was 32 × 32 pixels and overlapped 50%. These instantaneous velocity fields were ensemble averaged to obtain spatial distributions of the mean velocity and turbulence statistics. A multipass algorithm with an additional image deformation correction is used. The PIV data are processed using the LaVison software (DaVis v. 8.11). Details of the PIV method including measurement accuracy were described in literature (Raffel *et al.*, 2007; Verkaik *et al.*, 2009; Epps, Techet, 2010, Mickiewicz, 2014).

6. Flow diagnostics results

The most common situation to be found in the literature is for a plane travelling acoustic wave in ducts which reflects from the end to form a standing wave. In cylindrical ducts, plane waves, only, can propagate below a characteristic frequency which is a function of the duct diameter.

In our investigation several types of axisymmetric and spiral type of vortex breakdowns have been observed experimentally. Vortex breakdown phenomenon occurs when the ratio of the azimuthal to axial momentum exceeds a certain threshold, while both quantities have to be of the same order of magnitude (Oberleithner, 2011). It can play a crucial role in a variety of technical applications. Understanding the cause of the vortex breakdown is therefore of great importance in order to develop appropriate control strategies.

Sound intensity field in a cylindrical duct was excited by a wide-band sound signal propagated from source approximated with a loudspeaker. In Fig. 2 we show some results of investigations for 1/12 octave band frequencies where the sectional streamlines show the topological flow multi-cell structure for high-order modes. With graphical form we can see the evolution process of flows in the cross-section plane. In this paper, higher order acoustic modes which are excited above cut-off frequency are considered. These modes with frequency above 817 Hz have a much more complicated pressure pattern compared to the plane wave mode below. When more than one mode has "cut-on", these modes are superimposed upon the lower frequency wave mode and can co-exist with each other (Bennett *et al.*, 2009).

The frequency of 817 Hz is critical for the test waveguide that caused vortex breakdown phenomenon in the acoustic flow. Vortex breakdown phenomenon occurs when the ratio of the azimuthal to axial momentum exceeds a certain threshold, while both quantities have to be of the same order of magnitude (Verkaik *et al.*, 2009). This frequency limiting case of vortex breakdown phenomenon for researched duct is well illustrated in Fig. 2. The vortex breakdown can play a crucial role in a variety of technical applications. Understanding the cause of the vortex breakdown is therefore of great importance in order to develop appropriate control strategies.

In our research we also found helical disturbances that characterize strongly swirled flow for mode 2440.62 Hz (Fig. 3), but their role in the dynamics of vortex breakdown is still a controversial issue. Recent quantitative investigations could significantly con-

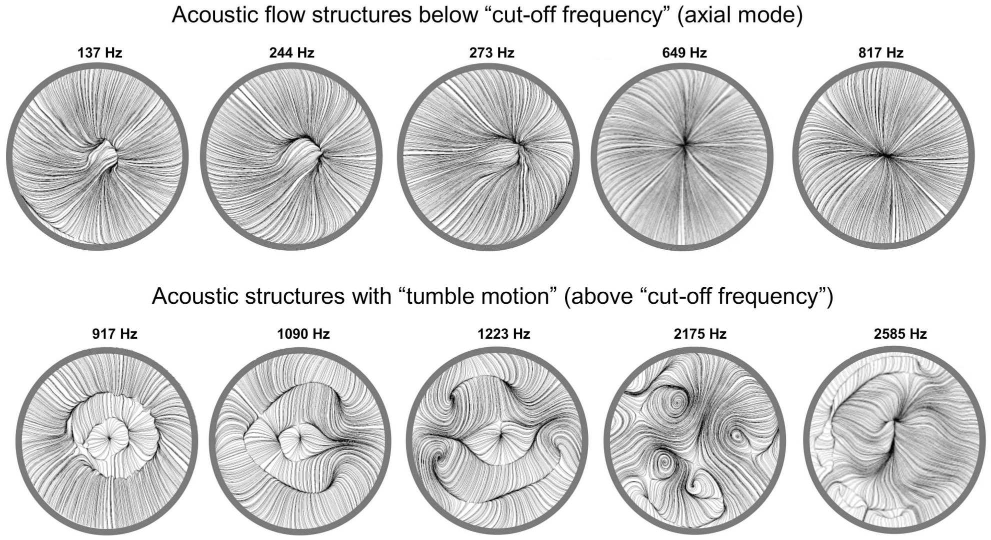

Fig. 2. Cross-sectional sound intensity streamlines fields below and above cut-off frequencies.

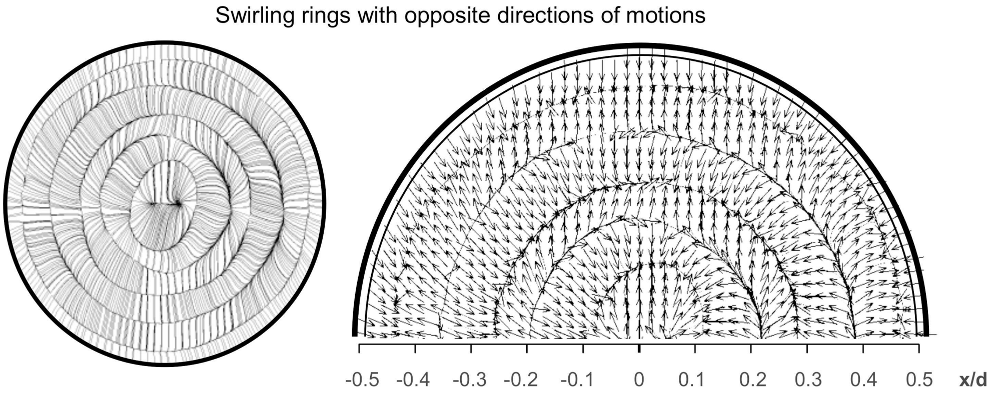

Fig. 3. Annula-helical coherent structure flow in-duct for 2440.62 Hz.

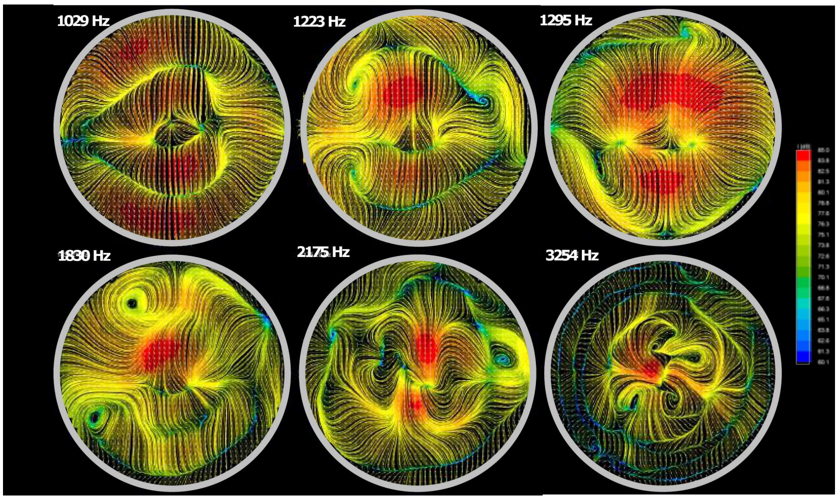

Fig. 4. The sound field in the cross section at 15 cm from the end of the waveguide – tumble motion as the effect of back-scattering reactions is shown.

tribute to the understanding of the dynamics accompanying the onset of vortex breakdown.

Sound intensity measurement and graphically imaging flow in a cylindrical duct can also well demonstrate the cut-off phenomenon and the effect of acoustic wave reflection from open-and. In open-ended ducts, waves reflected from the open end play an important part in sound transmission. The reflection properties of the opening are specified in terms of the impedance presented to each duct mode (Macdonald *et al.*, 2010), and the sound intensity along the duct is related to the mode impedance and the forward-wave amplitude. In Fig. 4 the sound field in the cross section at 15 cm from the end of the waveguide shows a tumble motion as the beck-scattering wave reactions.

Also the termination of the duct assumes an important role on the shape of the field at the end of the waveguide (see also Fig. 7), and consequently the level of the noise at the outlet of the waveguide (Weyna, 2012; Weyna *et al.*, 2013).

7. The comparison of PIV/POD with SI method

Proper orthogonal decomposition is used, most commonly in the study of turbulent flows, to identify distinct flow structure. In our experiment, for the PIV measurement, the system was excited at 2227 Hz to 4454 Hz frequency band that generated purely acoustic motion in-duct. In this frequency range, the duct will be induced to high order modes (above cut-off frequency region), which will be analyzed by POD using *DaVis* v.8.11 software. The result of modal energy decomposition shown in Fig. 5.

In Fig. 6 comparison results estimated with POD and with sound intensity field measured inside circular duct are shown. Comparison of some POD analysis results taken from the literature (Graftieaux *et al.*, 2001) confirms the usefulness of SI techniques to the decomposition of any modal distributions. Such SI studies can be carried out both in the flat and three-dimensional acoustic flow fields (Fig. 7).

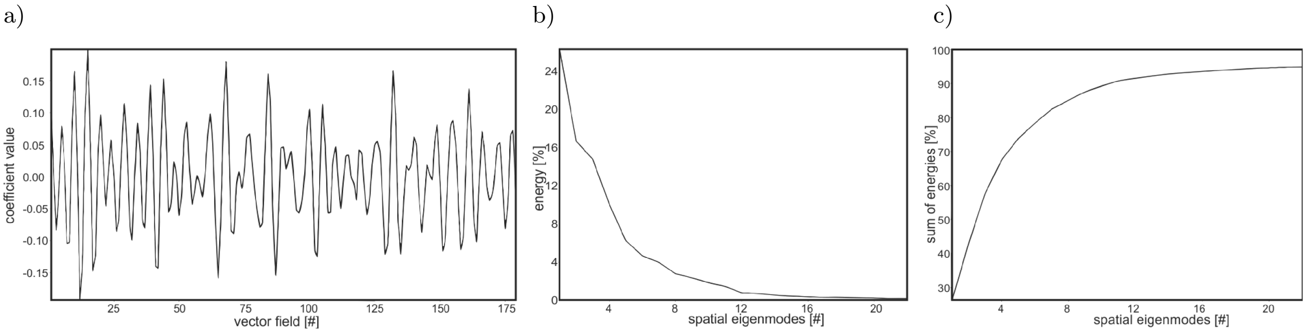

Fig. 5. Proper orthogonal decomposition of in-duct field investigate using PIV and POD technique for frequency band 2227–4454 Hz.

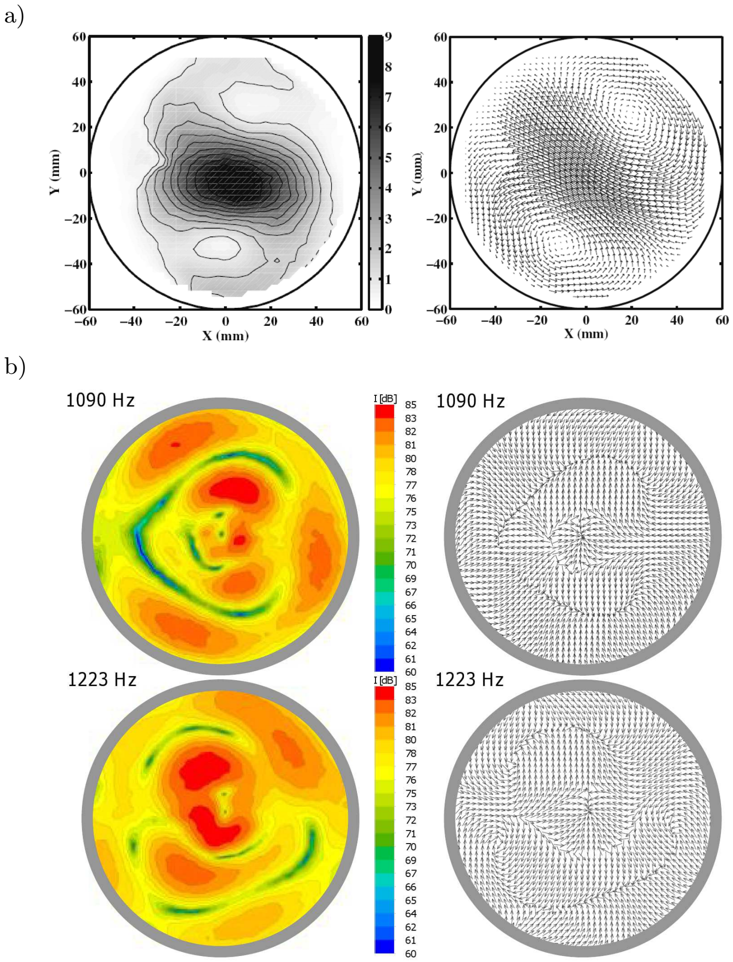

Fig. 6. Comparison results estimated with POD (a) from (Graftieaux *et al.*, 2001) and sound intensity field measured inside circular duct (b).

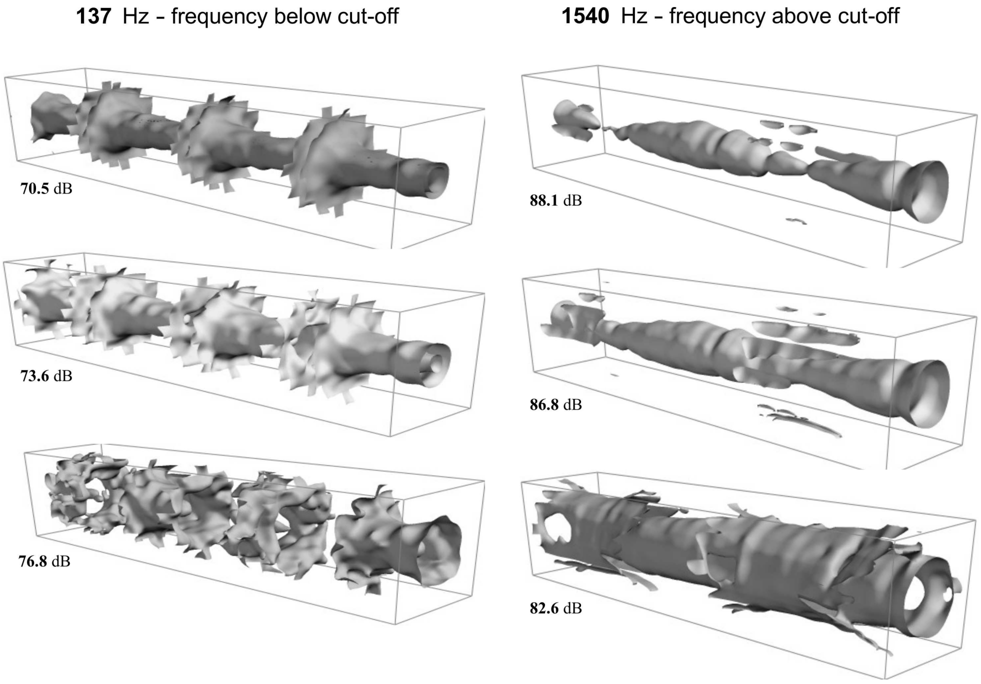

Fig. 7. Three-dimensional distribution of equal sound energy in duct (sound intensity iso-surface) in the 2.20 m from the open end.

According to the analysis above, it can be seen that the experimental decomposition of acoustic flow field using sound intensity (SI) and PIV/POD measurement techniques can be successfully applied in the acoustic waveguide duct.

8. Conclusions

The article presents the application of a SI technique to graphically show a spatial distribution of the acoustic energy flow in a hard walled cylindrical duct. As research results, the graphic analysis of the sound intensity flux in two- and three-dimensional space is shown. Visualization of the results is shown in the form of the acoustic intensity stream in space and as the shape of a flow wave or an iso-surface in space. Numerous examples illustrate the application of the SI measurement for practical problems for multi-modal flow diagnostics inside an angular duct, using flow acoustic imaginations.

Our experiment on acoustic waveguide model confirms that flow acoustic imaginations in real-life conditions are very complex, even for extremely simple modelling facility and for the sound field in a circular duct excited by loudspeaker used in the study (without flow field). These investigations provide a physical understanding of acoustic wave flow phenomena in real cylindrical duct where the measurements show both qualitative and quantitative flow diagnostics. The presentation of the vector distributions of real-life acoustic fields inside the duct areas – for which it is difficult to make a theoretical analysis of sound flow above the "cut-off frequencies" – shows that it is transmitted along the annular duct by higher-order "spinning modes", not by plane waves. Properly modelled and analyzed flow-induced sound led directly into the modern aeroacoustic approach, in which theory and experiment are inseparable.

In this paper, we investigate how SI technique may be combined with POD and DMD techniques as a tool for searching the energy dominant modes in the acoustics flow field in the interior of ducts and pipes. We attempt to show that the tested coherent structures by SI give the same opportunities to analyze the distribution of energy in the sound field that we can get from the PIV/POD methods. In summary, the acoustic signal of higher-order acoustic modes can be separated and visualized graphically. From the analysis of modes amplitude, it can be concluded which one represents the highest energy level. Experimental analysis of the distribution of the volume of sound intensity field may be equivalent to statistical methods – proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD).

Further research on the importance of energy interaction phenomena between axial and radial modes will focus for more realistic engineering applications.

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