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Experimental Verification of the Theoretical Model of Sound Radiation from an Unflanged Duct with Low Mean Flow

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The paper deals with experimental verification of results provided by theoretical approach to the problem of sound radiation from an unflanged duct with mean flow of the medium taking into account existence of all allowable wave modes and, in particular, occurrence of the so-called unstable wave, which results in decay of radiation on and in vicinity of the duct axis. The flow is assumed to be uniform with the source of flow located inside the duct, which is the case frequently occurring in industrial systems. Mathematical considerations, accounting for multimodal and multifrequency excitation and diffraction at the duct outlet, are based on the model of the semi-infinite unflanged hard duct with flow. In the experimental set-up a fan, mounted inside the duct served as the source of flow and noise at the same time modelled as an array of uncorrelated sources of broadband noise, what led to the axisymmetrical shape of the sound pressure directivity characteristics. The theoretical analysis was carried out for the root mean square acoustic pressure in the far-field conditions. Experimental results are presented in the form of the measured pressure directivity characteristics obtained for uniform flow directed inwards and outwards the duct compared to this observed for the zero-flow case. The directivity was measured in one-third octave bands throughout five octaves (500 Hz – 16 kHz) which, for a duct with radius of 0.08 m, corresponds to the range 0.74–23.65 in the reduced frequency ka (Helmholtz number) domain. The results obtained are consistent with theoretical solutions presented by Munt and Savkar, according to whom the weakening of the on-axis and close-to-axis radiation should take place in the presence of medium flow. Experimental results of the present paper indicate that this effect is observed even for the Mach number as low as 0.036.

Keywords: convected wave equation, unflanged cylindrical duct, multimodal, broadband excitation.
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1. Introduction

In many cases of practical importance, sound sources are located inside a hard cylindrical duct and radiate sound outwards through the open end. Examples include exhaust stacks, heating, ventilation and air-conditioning (HVAC) systems as well as turbofan jet engines. Growing interest in duct acoustics, results from the fact that duct openings are often sources of unwanted, harmful noise of extremely high levels, to mention only the noise generated by engines of taking off or landing jet planes. Strong pressure exerted on manufacturers to produce quieter systems resulted in extensive theoretical, numerical and experimental research work on the problem leading to elaboration of more exact and thorough description of the acoustic field phenomena reflecting the real conditions in which these appliances operate with better accuracy. This was achieved by the use of more complicated mathematical models, accounting for diffraction at the duct outlet and presence of higher wave modes (Levine, Schwinger, 1948; Weinstein, 1969; Snakowska, Wyrzykowski, 1986; Homicz, Lordi, 1975; Joseph, Morfey, 1999), flow of the medium inside the duct (Savkar, 1975; Munt, 1977; 1990) or in the whole space (Sinayoko *et al.*, 2010). In each case was accomplished by solving appropriate wave equation (either with or without flow) with adequate boundary conditions. The above-quoted theoretical approaches to the problem of open-ended acoustic duct constitute a class of "exact" models in contrast with "approximate" ones based on such simplifying assumptions as existence of plane wave only and/or a baffle at the duct outlet.

The first model to consider radiation from the duct outlet, commonly applied because of its relative simplicity, was the flanged duct, with the Neumann boundary condition imposed on the duct surface and the infinite flange situated in the outlet plane (Rayleigh, 1945). The above-described boundary condition was applied to the duct without flow (Zorumski, 1973) and with uniform flow occurring both inside the duct and in the surrounding medium (Tyler, Sofrin, 1962). The next model, assuming diffraction of the sound waves at the duct outlet, although considerably complicated and demanding application of the Wiener-Hopf method (Noble, 1958), has been frequently applied both to the single mode (Snakowska, Wyrzykowski, 1986; Weinstein, 1969) and the multimode radiation (Joseph, Morfey, 1999; Snakowska, 1993; Snakowska *et al.*, 1996; Snakowska, Idczak, 1997a, b; Snakowska, Jurkiewicz, 2010). Solving the diffraction problem is still considered a key step towards better coherence between mathematical model and the conditions observed in practice, as the unflanged duct usually better reflects the radiation conditions from the duct-like systems mentioned above. The assumption on uniform flow of the same velocity inside and outside the duct (Sinayoko *et al.*, 2010) represents a reasonable approximatation of conditions in which sound is radiated from the inlet of a jet engine during the steady flight of a plane (except for taking off or landing). Correctness of this theoretical approach consisting in application of the Prandtl-Glauert transformation to the wave equation with flow can be verified experimentally in the wind tunnel. Some early papers developing this method were presented by Carrier (1956) (for the plane wave, i.e. principal mode) and Lansing *et al.* (1970) (for higher modes). The next step towards making theoretical model closer to real-life conditions was made by Mani, Savkar and Munt, who considered acoustic transmission properties of a jet pipe with subsonic flow inside, separated from the ambient, stagnant or co-flowing fluid by a vortex layer (Mani, 1973; Savkar, 1975; Munt, 1977; 1990). Munt, in his earlier study, considered the sound radiated into the far field, and in the latter – the reflection coefficient for the plane wave and other duct modes. In his papers he assumed the ambient gas flowing slower than the jet and in the same direction, with the ambient gas/ flow speed ratio within the range $0 \leq \alpha \leq 1$. This way, by assuming $\alpha = 0$, the flow of the medium is limited to the inside of the duct and its extension along the duct axis, while the ambient gas remains still. Approximate theories considering difference in flow speed between the jet and ambient gas were also presented by Mani (1973) and Savkar (1975). In his paper, Mani considered a semi-infinite rectangular duct. The problem, studied theoretically, was formulated as this of Wiener-Hopf type and solved by an approximate method proposed by Carrier (1956) and Koiter (1954). Continuity of both transverse acoustic particle displacement and the acoustic pressure was assumed at the jet/still-air interface. Savkar, in turn, considered radiation from a circular duct, imposing the same boundary conditions as Mani's, the Wiener-Hopf method, and the Carrier-Koiter approximation. Authors of all of the above-quoted papers present the pressure directivity characteristics for a given reduced frequency ka and a single spinning/circumferential (m, l) mode.

Despite the continuous progress in mathematical modelling, analytical solutions are known for systems far simpler than those met in practice. For the sake of taking into account the specific properties of the duct (e.g. the outlet profile, finite thickness of the duct wall or variation of its radius) or the flow (heterogeneous, turbulent *etc.*), numerical methods of the so-called computational aeroacoustics (CAA) or computational fluid dynamics (CFD) have been recently applied (Dykas *et al.* 2010; Weyna, 2010). Correctness of numerical solutions is usually verified by reducing the model to such a form for which analytical solutions are known and comparing the results obtained by means of both methods. Using modern high-speed computers, numerical methods can be applied to problems involving high-frequency sound sources operating within waveguides with flow. This way the effect of mode number, source frequency, and flow Mach number on physical quantities such as the radiated power or directivity patterns can be discussed.

In this paper, we examine experimentally the radiation from the duct outlet when the source of flow (fan) is located inside the duct. Obviously, the motion of the medium exterior to the duct strongly depends on the direction of flow (in or from the duct), which in turn depends on the way the fan is operating (suction or blowing mode). In the theoretical model (Savkar, 1975; Munt, 1977; 1990) based on the solution obtained by means of the Wiener-Hopf method the diffraction of sound at the open end is accounted for but the flow outside is limited to the extension of the duct and so it is assumed to be uniform and of the same velocity as inside the duct. Nevertheless, the results obtained in the experiments carried out by the present author are consistent with theoretical solutions presented by Munt and Savkar, according to whom the weakening of the on-axis and close-to-axis radiation should take place. The data obtained indicate that this effect is observed even for the Mach number as low as 0.036.

2. Governing equations

2.1. Convected wave equation

The wave equation for the velocity potential Φ in a duct with flow of the medium can be written in the form (Lighthill, 2007)

$$\left(\triangle - \frac{1}{c^2}\frac{D^2}{Dt^2}\right)\Phi(\mathbf{r},t) = 0, \qquad (1)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla$$

is the so-called substantial derivative with respect to the flow velocity $\mathbf{u}$.

Assuming harmonic excitation of the type $\exp(i\omega t)$ with the angular frequency ω, flow of the medium in the z direction and the flow Mach number $M = u/c$, where c is the speed of sound and $k = \omega/c$ is the free space wavenumber, one obtains in the cylindrical coordinates (ϱ, φ, z) (Lighthill, 2007)

$$\frac{1}{\varrho}\frac{\partial}{\partial\varrho}\left(\varrho\frac{\partial\Phi}{\partial\varrho}\right) + \frac{1}{\varrho^2}\frac{\partial^2\Phi}{\partial\varphi^2} + \frac{\partial^2\Phi}{\partial z^2} - \left(ik + M\frac{\partial}{\partial z}\right)^2\Phi = 0. \qquad (2)$$

The axial wavenumbers γ_{mn} derived while solving (2) differ in respect to the direction of the sound wave propagation

$$\gamma_{mn}^{\pm} = \left[\mp kM + \left(k^2 - (1-M^2)\mu_{mn}^2/a^2\right)^{1/2}\right]\frac{1}{1-M^2}, \qquad (3)$$

where μ_{mn} is the n-th root of the derivative of Bessel function of order m: $J'_m(\mu_{mn}) = 0$. The upper sign "+" describes waves propagating in the direction of flow, the lower "−" the waves travelling against the flow. The plane wave, with axial wavenumber $\gamma_{00} = k$ splits into two forms with $\gamma_{00}^{+} = k/(1+M)$, for waves in the direction of flow and $\gamma_{00}^{-} = k/(1-M)$, against the flow. If $ka > \mu_{mn}\sqrt{1-M^2}$, then the axial wavenumbers $\gamma_{mn}^{\pm}$ are real and represent the so-called propagating modes (cut-on modes), otherwise the appearing imaginary part in $\gamma_{mn}^{\pm}$ represents wave attenuation and thus such modes (cut-off modes) are commonly neglected in further investigations. In the duct with flow, the cut-off frequency of mode (m,n), below which it is subject to attenuation, $\omega_{mn}^{\mathrm{fl}} = c\mu_{mn}/a\sqrt{1-M^2}$ is lower than in the absence of flow, thus at a given ka, called reduced frequency or Helmholtz number, a higher number of modes can propagate. An example showing how the number of cut-on modes varies with the reduced frequency ka is presented in Figs. 1–3.

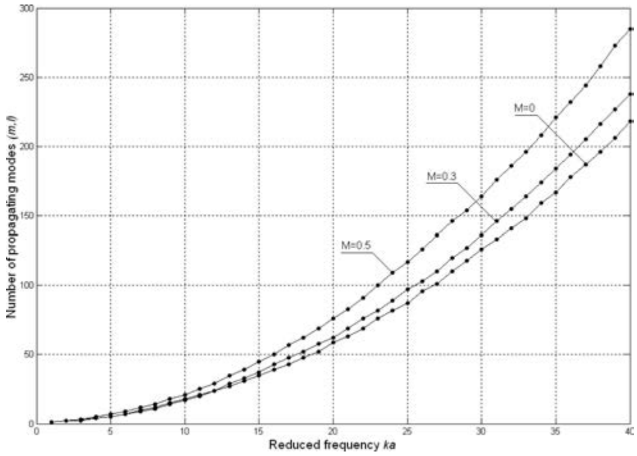

Fig. 1. Total number of cut-on modes *versus* ka for Mach numbers $M = 0, 0.3$ and 0.5.

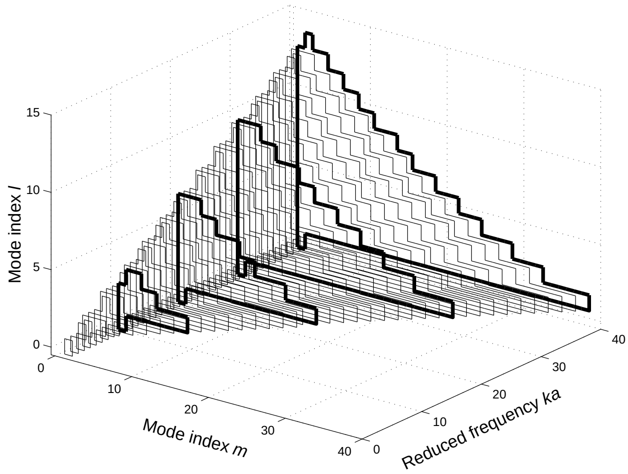

Fig. 2. Number of cut-on modes of given circumferential m and radial l order *versus* reduced frequency ka for Mach number $M = 0.1$.

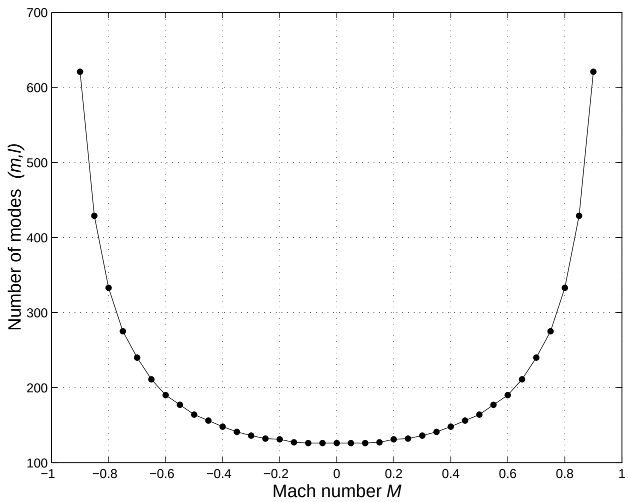

Fig. 3. Total number of cut-on modes *versus* Mach number M for reduced frequency $ka = 30$.

Solution of the diffraction problem in cylindrical coordinates $(\varrho,\ \varphi,\ z)$, is obtained for a semi-infinite duct of radius a, with the symmetry axis along the z coordinate axis and the exhaust at $z = 0$. The hard duct means that Neumann boundary condition (normal component of the acoustic velocity $v_n = \partial_\varrho \Phi = 0$) is fulfilled at the duct surface Σ. Geometry of the problem is presented in Fig. 4.

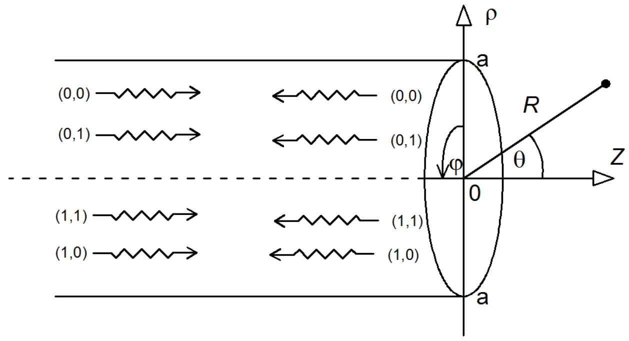

Fig. 4. Schematic representation of circumferential and radial modes heading the duct outlet.

Further considerations require adoption of additional assumptions regarding those parts of space where the flow takes place. In each case the solution is obtained by Wiener-Hopf method, in which the potential consists of two components representing the incident and the diffracted wave (Noble, 1958)

$$\Phi = \begin{cases} \Phi_< = \Phi^{\text{inc}} + \Phi^{\text{dif}}_<, & \varrho \le a, \\ \Phi_> = \Phi^{\text{dif}}_>, & \varrho \ge a. \end{cases} \tag{4}$$

and the common boundary conditions are as follows:

- the flow is in the z direction,
- the incident wave is the (m, l) mode of amplitude A_{ml} and the velocity potential $\Phi^{\text{inc}}_{ml}(\varrho, \varphi, z) = A_{ml} J_m(\mu_{ml}\varrho/a) \exp[-i(\gamma^{(+)}_{ml} z + m\varphi)]$,
- the duct is hard, so the normal component of the acoustic velocity equals zero at the duct surface Σ: ($\varrho = a$, $z < 0$), while the velocity potential undergoes a jump

$$\left.\frac{\partial \Phi}{\partial \varrho}\right|_{\varrho=a} = 0, \qquad z \le 0, \tag{5}$$

- the potential is continuous at the duct extension (called sometimes the phantom duct):

$$\Phi|_{\varrho \to a_-} = \Phi|_{\varrho \to a_+}, \qquad z > 0,$$

- the potential fulfils the Sommerfeld conditions of radiation (Rayleigh, 1945).

2.2. The inlet and outlet problem

As was mentioned before, further considerations require determination of where the flow takes place. So far we have only assumed that it is uniform, without specifying whether it takes place throughout the space or whether it is limited to some of its parts. The appropriate additional boundary conditions imposed on the solution will be discussed in the following.

When specifying the parts of space in which the medium is in motion in relation to solution of the convected wave equation, the following cases are considered most frequently:

- the flow is uniform and the Mach number is the same in the entire space – inside and outside the waveguide. This model is commonly used to describe radiation of sound from the inlet of a jet engine (the duct outlet on the turbofan side), during the steady flight of a plane and is called "the inlet problem" in the literature. In this case the solution can be obtained by applying the Prandtl-Glauert (Lidoine *et al.*, 2001) transformation and the so-called similarity variables (Chapman, 2000), which transform the convected wave equation into its ordinary form. Recently, radiation out of the duct inlet was considered in the paper by Sinayoko *et al.* (2010) for the uniform distribution of monopole, dipole and also the so-called "equal energy per mode" sound sources, under the assumption that the sources were incoherent, so the phase relations and interference effects could be neglected and the analysis of the far field was carried out for the root mean square pressure,
- the flow is assumed to be uniform in some selected parts of space, but its velocity differs. Most frequently the Mach number M_1 assumed inside the duct and its extension ($-\infty < z < \infty$, $\varrho < a$) exceeds the Mach number M_2 assumed in the remaining part of space ($\varrho > a$) (e.g. at the engine exhaust of a flying plane, when jets of gases escape into the outer space). This case, called "the exhaust problem" in the literature, was considered by Savkar (1975) and Munt (1977), who have obtained the solution by means of the Wiener-Hopf method. As one of the aims of this paper is to check whether their main finding concerning existence of a zone of relative silence (weakening of the propagation of the plane wave on the axis and in its vicinity) is observed also for the medium velocity of the range of tens of meters per second, we recall assumptions of their model below.

The convected wave equations referring to parts of space $\varrho > a$, $\varrho < a$, described by Mach numbers M_1 and M_2, respectively, take in cylindrical coordinates system (ϱ, φ, z) the form

$$\frac{\partial^2 \Phi}{\partial z^2} + \frac{1}{\varrho}\frac{\partial}{\partial \varrho}\left(\varrho \frac{\partial \Phi}{\partial \varrho}\right) + \frac{1}{\varrho^2}\frac{\partial^2 \Phi}{\partial \varphi^2} - \left(ik + M_j \frac{\partial}{\partial z}\right)^2 \Phi = 0, \tag{6}$$

where $j = 1$ for $\varrho < a$ and $j = 2$ for $\varrho > a$.

The additional boundary conditions, imposed in the "exhaust problem" on the duct wall extension and called the Kutta conditions (Munt, 1977; 1990) can be formulated as follows:

- the radial acoustic particle displacement $\eta(\varphi, z)$ must be matched along the stream lines separating the two regions $(z > 0, \varrho = a)$, so $\dfrac{D\eta}{Dt} = \dfrac{\partial \Phi}{\partial \varrho}$,

$$c\left(ik + M_1\frac{\partial}{\partial z}\right)\eta(\varphi, z) = \frac{\partial}{\partial \varrho}\Phi(a_-, \varphi, z), \quad z > 0, \quad (7)$$

$$c\left(ik + M_2\frac{\partial}{\partial z}\right)\eta(\varphi, z) = \frac{\partial}{\partial \varrho}\Phi(a_+, \varphi, z), \quad z > 0, \quad (8)$$

where $\dfrac{\partial}{\partial \varrho}\Phi(a_\pm, \varphi, z)$ means the right/left handed derivative with respect to variable ϱ calculated for $\varrho = a$,

- condition of the continuity of the acoustic pressure on the extension of the duct surface $(z > 0)$ $p|_{\varrho \to a_-} = p|_{\varrho \to a_+}$ takes the form

$$\left(ik + M_1\frac{\partial}{\partial z}\right)\Phi(a_-, \varphi, z) = \left(ik + M_2\frac{\partial}{\partial z}\right)\Phi(a_+, \varphi, z), \quad z > 0, \quad (9)$$

- in the far field outside the duct the potential of (m, l) mode must take the form of a spherical wave modified by the directivity function $d_{ml}(\theta, \phi)$

$$\Phi_{ml}(R, \theta, \varphi) \sim d_{ml}(\theta, \varphi)\frac{e^{-ikR}}{R}. \quad (10)$$

In the process of obtaining his solution, Savkar applied the Carrier-Koiter approximation method (Koiter, 1954; Carrier, 1959). This latter was skipped in Munt's two successive papers (Munt, 1977; 1990). One of Savkar's important results, apart from deriving directivity characteristics, was determination of the zone of relative silence, where the sound wave, especially the plane wave is subject to additional attenuation, which results in a cusp (cf. Fig. 5) in the directivity patterns at $\theta < \theta_s$ (Savkar, 1975). If the medium outside the duct is motionless $M_2 = 0$, the zone of relative silence reduces to

$$\theta_s = \cos^{-1}\left\{\frac{1}{1 + M_1}\right\}. \quad (11)$$

The zone of relative silence was also observed in our experiments, as will be discussed further in this paper.

The next step to make the initial conditions better reflect what is observed in practice is the assumption on propagation of a multimode incident wave, i.e. consisting of all spinning modes with the cut-on frequencies below the reduced frequency ka. The consecutive modes amplitudes and phases (complex amplitudes) depend on the sound source nature. In practice one can determine complex amplitudes of the spinning modes only for some simplest sources or under some

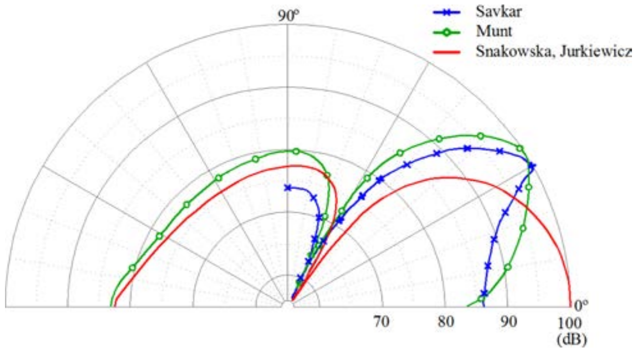

Fig. 5. Comparison of the plane wave $(m = 0,\ l = 0)$ theoretical radiation pattern at $ka = 4.48$ according to Savkar (1975), and Munt (1977) – both curves for the flow velocity $M = 0.134$, with the no-flow case (Snakowska, Jurkiewicz, 2010).

additional conditions. These conditions may specify the nature of the source, e.g.: coherent/incoherent monopole or dipole sources or the way in which the energy is distributed between the modes. In the literature, the equal energy per mode (EEpM) assumption is frequently applied to coherent (Snakowska, 1993; Snakowska, Jurkiewicz, 2010) or incoherent sources (Joseph, Morfey, 1999; Sinayoko *et al.*, 2010). If the sound sources are assumed to be coherent, the phases can be chosen in some particular way, e.g.: all modes in phase; each two consecutive modes in antiphase (Snakowska, Jurkiewicz, 2010) or all modes in random phases. In the latter case, the analysis must be carried out with the use of methods of the probability theory (Snakowska *et al.*, 1996; Snakowska, Idczak, 1995; 1997a) and the obtained results should be expressed in terms of the expected value, the variance and the standard deviation of the acoustic pressure, intensity, power (Snakowska, Idczak, 1997b; Enghardt *et al.*, 2004) *etc.*

The theoretical results we have referred to so far concerned multimodal, yet single frequency excitation. The experiments were conducted for broadband noise and the results presented in one-third octave bands. Thus it is necessary to consider how the multimodal but single-frequency directivity patterns would be summed-up into broadband, one-third octave directivity patterns.

The assumption on incoherent broadband sound sources, applied in this paper and also in papers by Joseph, Morfey (1999) and Sinayoko *et al.* (2010) allows to neglect the phase relations between modes and base the considerations on the root mean square pressure. As a result of this assumption the pressure directivity function does not depend on the azimuthal angle φ, even if all admissible circumferential modes $(m > 0)$ are taken into account and therefore

$$P_{ml}^2(R, \theta, \varphi) = |A_{ml}|^2 |d_{ml}(\theta)|^2 \frac{1}{R^2}, \quad (12)$$

where P_{ml} denotes root mean square pressure of p_{ml} of amplitude A_{ml} (see Appendix A for details).

Analysis of radiation of the higher waveguide modes in the presence of flow (Savkar, 1975; Munt, 1977) and without flow (Snakowska *et al.*, 1996; Snakowska, Jurkiewicz, 2010) leads to the conclusion that only the plane wave ($m = 0,\ l = 0$) radiates along the axis (cf. Fig. 5 and Figs. 12–13) and thus in case of flow, any change in the directivity characteristics in this direction can only be explained by excitation of a new type of wave, which was not observed in the medium at rest. Such a wave appearing in the solution of the convected wave equation (Savkar, 1975), is called the unstable wave and is responsible for weakening the on- and close-to-axis radiation (Fig. 5), a phenomenon observed also in experiments carried out by present authors.

3. Description of research work and analysis of the results

The aim of our experiments was to determine the directivity patterns of radiation from a cylindrical waveguide outlet, especially the effect of flow on their shape. The conditions under which the experiments were carried out correspond to the case described as "the exhaust problem" under the assumption that the movement of air outside the waveguide is only the result of gas discharged from its interior, which in theory corresponds to consider zero Mach number ($M_2 = 0$ for $\varrho > a$) outside the duct and its extension (cf. Eq. (6)).

In the experimental set-up (Fig. 6) a waveguide with radius $a = 0.08$ m and length $L = 2$ m was applied. The experiment was conducted in the free field conditions in an anechoic chamber at the Department of Mechanics and Vibroacoustics AGH in Cracow (Fig. 7). The flow of medium was generated by a centrifugal fan mounted at the end of the waveguide serving at the same time as the sound source. Measurements were carried out using 1/2 inch microphone (G.R.A.S.) and the sound level meter (SVAN912E) in the first class accuracy. The maximum error of the measuring set-up was estimated as equaling 1.6 dB.

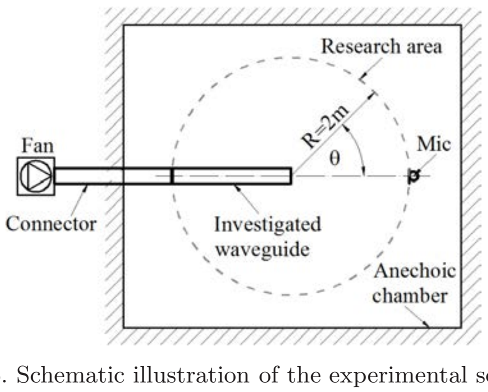

Fig. 6. Schematic illustration of the experimental set-up.

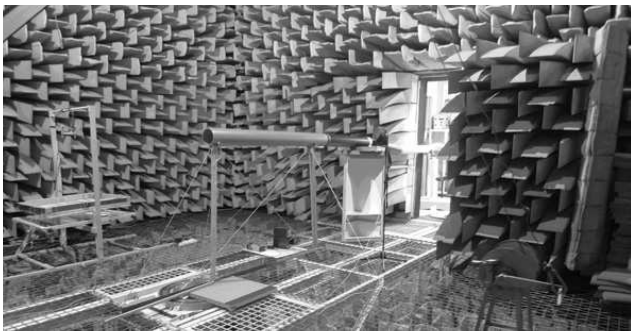

Fig. 7. View of the anechoic chamber and the experimental set-up.

The medium flow velocity, determined by means of the Prandtl tube, equalled approximately 12.3 m/s, which corresponds to the Mach number $M = 0.036$ (the value that can be considered typical for ventilation/AC ducts). To reduce the effect of excessive flow velocity, which could have affected the measurement data (mainly in the axial direction), the microphone was protected by a special kind of fabric commonly used in loudspeakers that significantly limited the flow noise and was almost transparent for sound.

To measure radiation in the absence of flow, the suction side of the fan has been obscured with an aperture which allowed to reduce the flow to the values not exceeding 5 percent of the maximum Mach number generated by the fan, without significant change of the noise produced thereby. The fan generated broadband noise. Measurements were taken in one-third octave bands, along a circle of radius 2 m, with the vertical angle step θ of 15 degrees.

The experimental results for the selected centre band frequencies and the flow directed inwards and outwards the duct, as well as without the flow, are presented in directivity patterns (Figs. 8–10). As it could be expected based on theoretical considerations, the resultant far field pressure of broadband excitation was not dependent on azimuthal angle φ. This feature was confirmed experimentally by means of a dedicated series of measurements taken at a selected value of the polar angle with azimuthal angle step of 15°.

For a given angle θ differences between max/min and the average value in the measured sound pressure level for various values of angle φ did not exceed the estimated experimental error of 1.6 dB, defined as the sum of the errors of measuring set-up, i.e. the error of class I meter (0.7 dB) and the microphone error (0.9 dB). Example results of these sampling measurements are presented in Tables 1–2. Results of this "axisymmetry test" allow to present the directivity characteristics in 2D without loss of generality. The only 3D graph presented herein, Fig. 11, depicts the pressure directivity for blowing mode and frequency 3150 Hz, $ka = 4.65$, that is for parameters close to those for which theoretical results of Savkar and Munt.

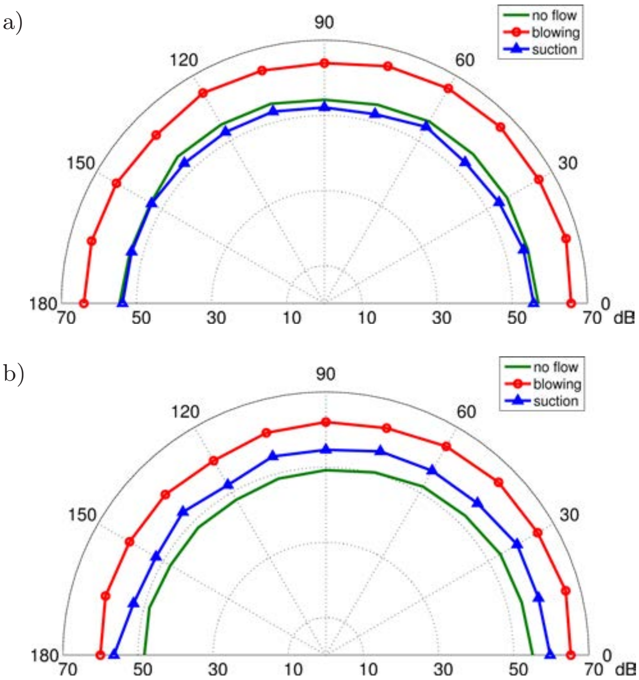

Fig. 8. Pressure directivity $d(\theta)$ of sound radiated from a rigid cylindrical duct with flow excited by means of 1/3 octave broadband noise with mid-frequencies 500 Hz (a) and 1000 Hz (b), for which the reduced frequencies are 0.74 and 1.48, respectively.

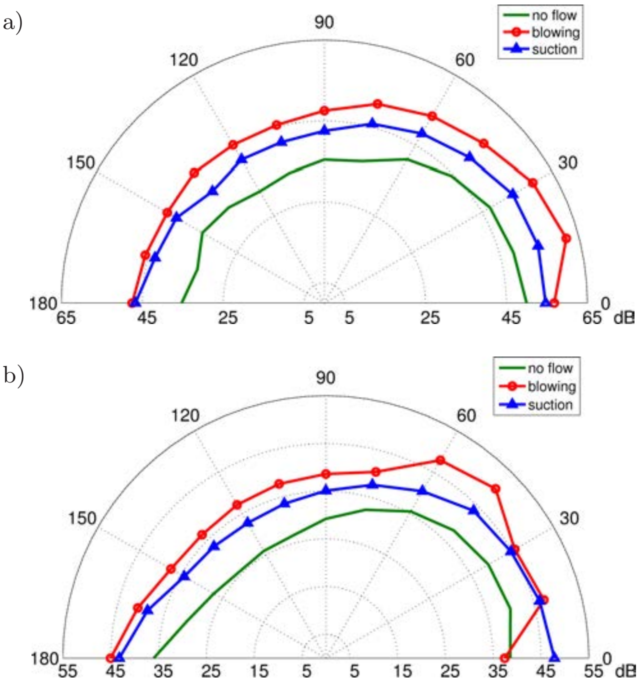

Fig. 9. The same as in Fig. 8, but for mid-frequencies 2000 Hz (a) and 4000 Hz (b) corresponding to reduced frequencies ka equal to 2.96 and 5.91, respectively.

Analyzing the data presented above it can be seen that for frequencies below 2000 Hz, the obtained directivity characteristics remind those of the omnidirectional source, while for higher frequencies the ra-

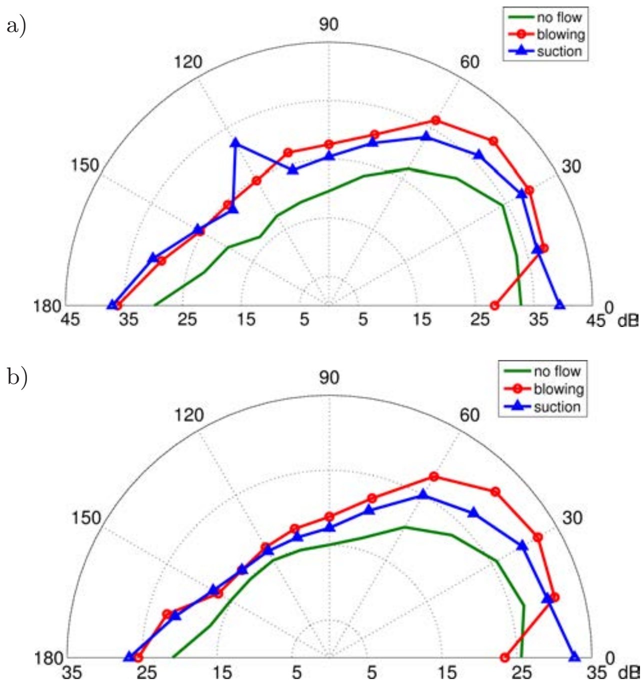

Fig. 10. The same as in Fig. 8, but for mid-frequencies 8000 Hz (a) and 16000 Hz (b) corresponding to reduced frequencies ka equal to 11.82 and 23.65, respectively.

diation patterns become significantly different. In the presence of flow of the medium, regardless of its direction, higher pressure levels and additional maxima are observed for directions deviated from the axis. For angles close to the axis, the obtained values are smaller for the fan operating in the blowing mode and larger for the sucking mode. Although the result may be unexpected and contrary to intuition, it is still consis-

Table 1. Example results of sound pressure level measured for polar angles $\theta = 26.5°$ and various values of the azimuthal angle φ. The results confirm that for broadband excitation of a system of incoherent sound sources, despite the fact that circumferential modes are excited in the duct the resultant pressure directivities are symmetrical with respect to the azimuthal angle.

Blowing					
f [Hz]	average [dB]	max [dB]	min [dB]	max-min [dB]	st. dev. [dB]
500	70.9	72.1	70.3	1.8	0.5
1000	69.7	70.4	69.2	1.2	0.3
2000	66.2	67.0	65.1	1.9	0.6
4000	54.2	55.0	52.7	2.3	0.7
Suction					
f [Hz]	average [dB]	max [dB]	min [dB]	max-min [dB]	st. dev. [dB]
500	57.2	58.8	56.5	2.3	0.6
1000	62.8	63.3	62.0	1.3	0.3
2000	58.6	59.6	58.3	1.3	0.3
4000	48.4	49.7	47.1	2.6	0.8

Table 2. The same as in Table 1, but for polar angle $\theta = 45°$.

Blowing					
f [Hz]	average [dB]	max [dB]	min [dB]	max-min [dB]	st. dev. [dB]
500	73.3	74.3	72.7	1.6	0.4
1000	71.9	72.4	71.5	0.9	0.3
2000	67.7	68.1	67.3	0.8	0.2
4000	55.4	56.5	54.6	1.9	0.5
Suction					
f [Hz]	average [dB]	max [dB]	min [dB]	max-min [dB]	st. dev. [dB]
500	60.3	60.9	59.5	1.4	0.4
1000	65.7	66.3	65.0	1.3	0.3
2000	59.9	60.8	59.6	1.2	0.3
4000	50.5	51.3	49.3	2.0	0.6

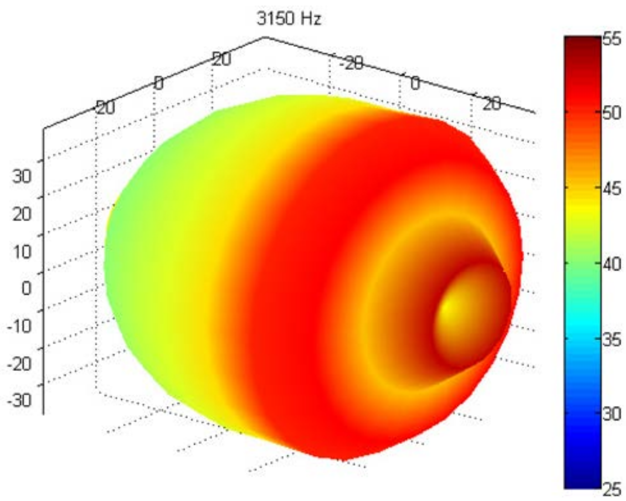

Fig. 11. Three-dimensional pressure directivity characteristic of waveguide outlet radiation *versus* polar angle θ for 3150 Hz, with the flow directed outwards, $M = 0.036$. The radiation pattern confirms theoretical results predicting weakening of radiation on the duct axis and in its vicinity as an effect of excitation of the so-called unstable wave (Savkar, 1975; Munt, 1977).

tent with the results of theoretical analysis reported by Savkar (1975) and Munt (1977) on the zone of relative silence (cf. Eq. (11) and Fig. 5), as a result of appearance of the so-called unstable wave responsible for the effect observed in the experiments we carried out. For example, for the frequencies of 4000 Hz and higher, the value of pressure sound level on the axis for the fan operating in the blowing mode was about 10 dB lower than this measured in the suction mode.

The shapes of the presented directivity characteristics can be explained in the frame of the theoretical results presented in the literature for single-frequency but multimodal excitation (Joseph, Morfey, 1999; Snakowska, Jurkiewicz, 2010). They are also consistent with experimental data reported by Simoneau (2008) who has presented directivity patterns measured in octave bands. The author used a loudspeaker which generated pink noise and a 1 m long waveguide with 0.05 m diameter. In spite of these differences in the experimental set-up and presentation of data, his results are consistent with ours.

The measured directivity patterns indicate excitation and propagation of the numerous circumferential modes, for which – as it follows from the theory (Joseph, Morfey, 1999; Snakowska, 1992; Snakowska, Jurkiewicz, 2010) – the radiation pressure on the axis equals zero and the maxima deviate from the axis with increasing order of radial and circumferential modes (Snakowska, Jurkiewicz, 2010). Thus, for the frequency 2000 Hz the maximum value occurs, approximately, at the angle of 15°, while for the frequency 4000 Hz one can observe maxima in the vicinity of 15°, 45° and 60°. The observed smoothing of directivity characteristics for mid-frequencies of 2000 Hz and above, when comparing them to those obtained for single frequency (Figs. 12–13), can be attributed to the broadband noise produced by the fan. It should be borne in mind that for each value of the reduced frequency ka many circumferential modes (m, l) are present (cf. Figs. 1–3) and their numerous maxima and minima are displayed in different directions. For example, for the mid-frequencies 8000 Hz and 16000 Hz and the duct with radius 0.08 m, i.e. those used in the experiment ($ka = 11.83$ and 23.65, respectively) the

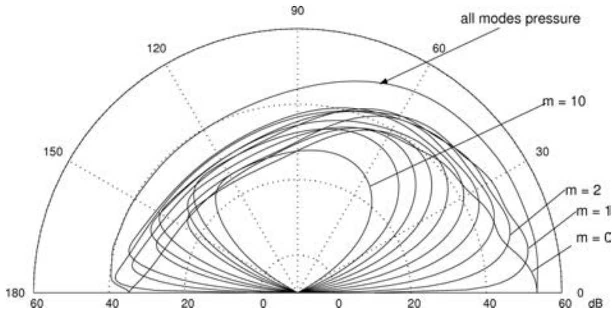

Fig. 12. The acoustic pressure directivity $d(\theta)$ for all cut-on modes for $ka = 11.83$ *versus* the circumferential index $m = 0, 1, 2, ...10$ and the resulting all-modes pressure.

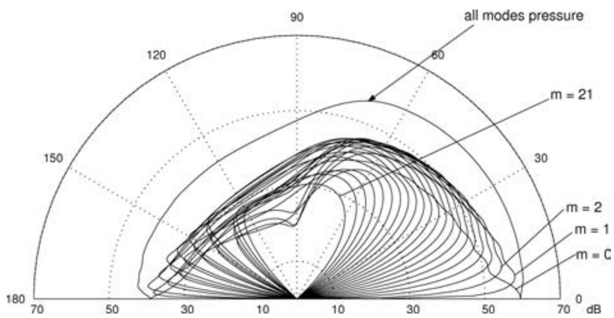

Fig. 13. The acoustic pressure directivity $d(\theta)$ for all cut-on modes for $ka = 23.65$ *versus* the circumferential index $m = 0, 1, 2...21$ and the resulting all-modes pressure.

total number of cut-on modes equals 23 and 84 (cf. Figs. 12–13).

Although for frequencies above 8000 Hz the typically desired signal to noise ratio (SNR) of 10 dB is not maintained, the effect of the presence of flow is clearly noticeable and shapes of the directivity patterns are in line with those for lower frequencies. The obtained acoustic pressure data allowed to calculate, by numerical integration, the distribution of the acoustic power radiated from the waveguide outlet as a function of frequency. The results for Mach number $M = 0$ are depicted in Fig. 14.

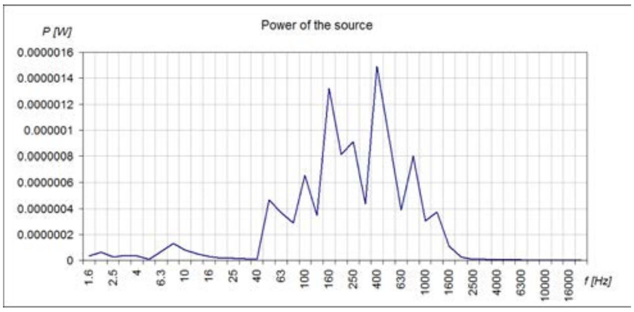

Fig. 14. The acoustic power emitted by the fan as a function of frequency.

4. Conclusions

In the paper, influence of the medium flow on the pressure directivity characteristics was discussed on the grounds of comparing theoretical, numerical and experimental results obtained for the sound field radiated from an unflanged cylindrical duct with low mean flow produced by a fan mounted inside. The fan producing a broadband noise was modeled as a system of uncorelated sound sources. Measurements were carried out in one-third octave bands starting from frequency 500 Hz up to 16 kHz ($0.74 < ka < 23.65$). The obtained experimental data of the acoustic pressure for uniform flow occurring in both directions (to and from the duct) were presented on graphs and compared to the zero-flow data. For frequencies 500 Hz and 1000 Hz (ka equal to 0.74 and 1.48, respectively, and thus the only cut-on mode is the plane wave) the directivity patterns are almost omnidirectional, as was expected for radiation of a wave much longer than the duct radius. The predicted theoretically symmetry of the pressure field *versus* azimuthal angle was confirmed. The smoothing of the curves presenting the directivity characteristics *versus* the polar angle for mid-frequencies of 2000 Hz and above when comparing them to those obtained for single-frequency was also observed, as the fan produced broadband noise and for each reduced frequency many circumferential modes were present with their numerous maxima and minima occurring in different directions. One of the main tasks of the research was to check experimentally, whether weakening of the on-axis and close-to-axis radiation reported in theoretical papers by Munt and Savkar for flow directed outwards occurs for small Mach numbers, typical for fans applied in HVAC systems. In the course of experiments the effect, called "the zone of relative silence" was clearly observed for the mid-frequencies 2000 Hz and above when the fan was operating in the blowing mode. To sum up, the presented experimental results remain consistent with "exact" theoretical solutions predicting appearance of the so-called unstable wave. As the duct with the fluid medium moving at uniform velocity can be used to model actual duct-shaped components of many appliances, the results may prove useful in constructing efficient active or passive noise attenuating systems.

Appendix A. Pressure directivity characteristics for circumferential modes and excitation by incoherent sources

Let us assume that some incoherent sound sources are distributed over the duct cross section. The sound pressure of the duct mode of circumferential order m and radial order l can be expressed inside the infinite duct as:

$$p_{ml}(\varrho, \varphi, z, t) = (A_{ml}\cos(m\varphi) f_A(t) + B_{ml}\sin(m\varphi) f_B(t)) J_m(\mu_{ml}\varrho/a). \quad \text{(A1)}$$

Thus the pressure radiated from the outlet, in the far field takes the form

$$p_{ml}(R, \theta, \varphi, t) = (A_{ml}\cos(m\varphi) f_A(t) + B_{ml}\sin(m\varphi) f_B(t)) d_{ml}(\theta)\exp(-ikR)/R, \quad \text{(A2)}$$

where

$$p^A_{ml}(t) = A_{ml} f_A(t), \qquad p^B_{ml}(t) = B_{ml} f_B(t), \quad \text{(A3)}$$

can be considered the pressure component exciting the (m, l) mode.

In order to be able to consider A_{ml} the amplitude the expected value of the function representing the pressure dependence on time should equal unity, $E\{f^2(t)\} = 1$, so

$$E\{(p^A_{ml}(t))^2\} = |A_{ml}|^2, \quad \text{(A4)}$$

$$E\{(p^B_{ml}(t))^2\} = |B_{ml}|^2 \quad \text{(A5)}$$

and

$$E\{p^A_{ml}(t) p^B_{ml}(t)\} = 0. \quad \text{(A6)}$$

As for incoherent sources $|A_{ml}| = |B_{ml}|$, the mean square pressure $P_{ml}(R, \theta\varphi)$ of the (m, l) mode pressure $p_{ml}(R, \theta\varphi, t)$

$$P^2_{ml}(R, \theta, \varphi) = E\{p^2_{ml}(R, \theta, \varphi, t))\} = |A_{ml}|^2 |d_{ml}(\theta)|^2 \frac{1}{R^2}. \quad \text{(A7)}$$

To conclude, the assumption on uncorelated system of sound sources leads to pressure directivity characteristics depending only on the polar angle θ, as the root mean square pressure depends only on R and θ variables $P_{ml}(R, \theta)$.

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