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PROBABILISTIC ANALYSIS OF THE CYCLICAL FLUCTUATIONS ON THE BUSINESS CYCLE CLOCK: THE CASE OF VISEGRAD GROUP[1]

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STRESZCZENIE

Ł. Lenart. *Analiza probabilistyczna wahań cyklicznych na zegarze cyklu: Analiza dla państw grupy wyszehradzkiej.* Folia Oeconomica Cracoviensia 2017, 58: 105–126.

W artykule przedstawiono metodologię wyznaczania rozkładu predyktywnego dla położenia punktów na zegarze cyklu koniunkturalnego. Metodologia ta oparta jest na kilkustopniowej procedurze, polegającej na wyznaczeniu rozkładu predyktywnego analizowanej zmiennej oraz zastosowaniu metod filtracji. Podejście to umożliwia, miedzy innymi, wyznaczenie prawdopodobieństw związanych z położeniem punktów zegara w danej ćwiartce układu współrzędnych. Wyniki empiryczne przedstawiono dla gospodarek grupy wyszehradzkiej. W przypadku rozważanych gospodarek, podejście probabilistyczne pozwoliło na oszacowanie szans wejścia w daną fazę aktywności gospodarczej dla przeszłych, obecnych oraz przyszłych chwil czasowych. Ze względu na zastosowanie klasy modeli SARIMA uzyskane wyniki prognozy poza okres rozważanej próby nie wskazują na wysokie prawdopodobieństwa osiągnięcia danej fazy cyklu. W większości przypadków prawdopodobieństwa wystąpienia danej fazy cyklu stabilizują się na równym poziomie wraz ze wzrostem horyzontu prognozy (po ok. 1/4). Sugeruje to potrzebę wykorzystania modeli zawierających komponenty cykliczne o charakterze stochastycznym lub deterministycznym.

ABSTRACT

The paper investigates a methodology for determining the predictive distribution for the position of the points on a business cycle clock. The presented methodology is based on a multi-step procedure, which consists in determining the predictive distribution of the analyzed variable and applying the filtration methods. The presented approach allows, among other things, to determine the probabilities associated with the position of the clock points in a quadrant of the coordinate

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system. Empirical results were presented for the Visegrad Group economies. For the economies considered, the probabilistic approach allowed us to estimate the probabilities of entering a given phase of economic activity for past, present and future time. The predictive distributions, beyond the period under consideration, do not indicate the high probability of achieving a particular phase of the cycle. In most cases, the probability of a given cycle phase stabilizes at the same level, as the forecast horizon increases (approximately 1/4). This suggests the need to use models containing stochastic or deterministic cyclical components.

SŁOWA KLUCZOWE — KEY WORDS

zegar cyklu koniunkturalnego, filtr HP, prognoza wahań cyklicznych, model SARIMA

business cycle clock, HP filtration, prediction for cyclical fluctuations, SARIMA model

1. INTRODUCTION

Business Cycle Clock (BCC in short) is a popular tool for monitoring business activity in an intuitive way. The main purpose of the classical BCC is to clearly visualize the dynamics of economic activity (expansion, slowdown, recession and recovery). This is achieved by using the simple (two-dimensional) clock type graph. For Europe's economies the most popular BCC is the clock presented by Eurostat. Based on the gross domestic product, this tool determines the growth cycle for selected European economies and graphically illustrates the current cyclical position using the business cycle clock. They consider the six phases of the cycle clock: sector 1 — expansion, with decelerating growth; sector 2 — slowdown; sector 3 — recession; sector 4 — recession, with accelerating growth; sector 5 — recovery; sector 6 — expansion, with accelerating growth (see to Mazzi et al. (2015) for more details).

Note that, the business cycle clock can be directly determined by the deviation cycle. Therefore, only the correctly determined deviation cycle can be useful from a practical point of view. Unfortunately, in such a case, a controversial procedure called *detrending* may be the source of uncertainty for the points position on the BCC. Note that, there are several different filtration methods to extract the business cycle form a observed time series. The most popular is the Hodrick Prescott filter (HP) — see Hodrick and Prescott (1997), Baxter King filter (BK) — see Baxter and King (1999) and Christiano Fitzgerald filter (CF) — see Christiano and Fitzgerald (2003). Uncertainty related to the points position on the BCC can have different sources. One source is the arbitrary selection of the filtration method. However, in this article we are not comparing the results for different filtration methods. We only consider the HP filtration method. The problem of choosing the filtration method was discussed by de Haan et al. (2008), Canova (1998), Estrella (2007), Burnside (1998), and many others.

Another source of uncertainty on the business cycle clock is the uncertainty related to the future economic situation. Let us note that the popular methods that extract the deviation cycle use full sample. Therefore, new observations can significantly change the points position on the BCC. This may cause other conclusions regarding current or past cyclical position.

The main purpose of this article is to analyze the uncertainty associated only with the future economic situation. In this paper we would like to shed only light on this problem. We use a simple SARIMA model (without classical cyclical component) to predict (observed time series) future economic situation. Finally, we determine the predictive distribution for the growth cycle and the predictive distribution for the points position on the BCC. We are not interested in details in choosing the best predictive model.

Chapter 2 presents the data used in the empirical analysis. Chapter 3 presents the methodology for determining the predictive distribution for the deviation cycle and the predictive distribution for the points position on the BCC. Chapter 4 provides an analysis of empirical data, where the time series on GDP for the Visegrad Group economies are used.

2. DATA PRESENTATION AND PREPARATION

We consider GDP and main components (output, expenditure and income), chain linked volumes (index 2010=100), unadjusted data (i.e. neither seasonally adjusted nor calendar adjusted data) in Visegrad Group (V4) — the Czech Republic, Hungary, Poland and Slovakia. The series start in the first quarter of 2002 and ends in the last quarter of 2016 (60 observations, 15 years). Before the main analysis we take the natural logarithm of the considered time series, to stabilize the amplitude. In order to eliminate the seasonal fluctuations we use simple centered moving average (2x4MA). The Figure 1 presents the logarithm of the data with centered moving average 2x4MA.

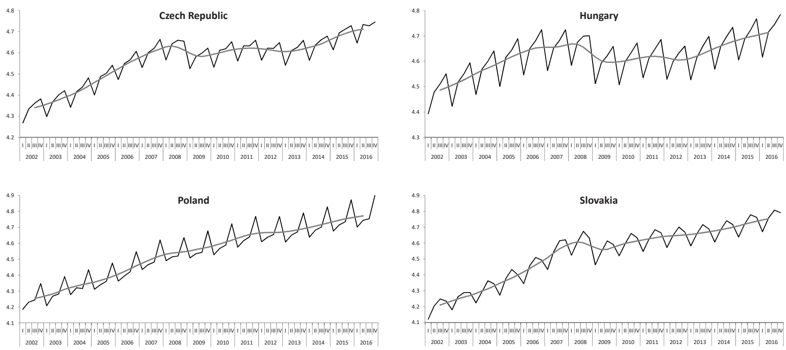


Figure 1. The logarithm of GDP and main components (index 2010=100), in Visegrad Group countries from Q1 2002 to Q4 2016 (dark line) with centered moving average 2x4MA (gray line)

3. THE BUSINESS CYCLE FLUCTUATIONS AND THE FORECASTING METHODOLOGY

Let $\{P_t: t \in \mathbb{Z}\}$ be observed time series and let $\{\hat{P}_t: t \in \mathbb{Z}\}$ be the 2x4MA process for the logarithm of $\{P_t: t \in \mathbb{Z}\}$, i.e.,

$$\hat{P}_t = [\ln(P_{t-2}) + 2\ln(P_{t-1}) + 2\ln(P_t) + 2\ln(P_{t+1}) + \ln(P_{t+2})]/8.$$

To obtain the cyclical fluctuations we use standard Hodrick and Prescott — see Hodrick and Prescott (1997) — filter for the process $\{\hat{P}_t: t \in \mathbb{Z}\}$, with smoothing parameter denoted by λ. For simplicity, we denote the Hodrick and Prescott filter, with smoothing parameter λ, by $HP(\lambda)$. The cyclical process obtained by $HP(\lambda)$ filter we denote by $\{C_{HP,\lambda,t}: t \in \mathbb{Z}\}$.

For the finite sample $\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_n$ we use standard HP filtration procedure, by using simple matrix multiplication. Under the matrix notation

$$\boldsymbol{C}_{HP,\lambda,n} = [C_{HP,\lambda,1}, C_{HP,\lambda,2}, \ldots, C_{HP,\lambda,n}]'$$

and

$$\hat{\boldsymbol{P}} = [\hat{P}_1, \hat{P}_2, \ldots \hat{P}_n]',$$

the cyclical component $\boldsymbol{C}_{HP,\lambda}$ is obtained via

$$\boldsymbol{C}_{HP,\lambda,n} = \boldsymbol{HP}_{\lambda,n} * \hat{\boldsymbol{P}},$$

where $\boldsymbol{HP}_{\lambda,n} = [r_{ij}]$ is $n \times n$ symmetric matrix with all non-zero elements given by

$$r_{ij} = \begin{cases} 6 & \text{for } i = j \text{ and } i = 3, 4, \ldots, n-2 \\ -4 & \text{for } |i-j| = 1 \text{ and } i = 2, 3, \ldots, n-1 \\ 1 & \text{for } |i-j| = 2 \text{ and } i = 1, 2, \ldots, n \\ 5 & \text{for } i = j = 2 \\ -2 & \text{for } |i-j| = 1 \text{ and } i \in \{1, n\} \\ 1 & \text{for } i = j \text{ and } i \in \{1, n\}. \end{cases}$$

Note that, the cyclical component $\boldsymbol{C}_{HP,\lambda,n}$ depends on considered sample $\hat{\boldsymbol{P}}$ and the smoothing parameter λ. Based on cyclical component $\boldsymbol{C}_{HP,\lambda,n}$ the path on the business cycle clock is defined via bivariate sequence of the points

$$\boldsymbol{BCC}_{HP,\lambda,n} = [(C_{HP,\lambda,2} - C_{HP,\lambda,1}, C_{HP,\lambda,2}), (C_{HP,\lambda,3} - C_{HP,\lambda,2}, C_{HP,\lambda,3}), \ldots, (C_{HP,\lambda,n} - C_{HP,\lambda,n-1}, C_{HP,\lambda,n})]'$$

(see to Lenart and Pipień (2016) and Lenart and Pipień (2013) for the related problems). Therefore, if the sample is expanded the actual path on the business cycle clock may change compared with the past path (for fixed time interval). To clarify this problem we consider some illustrative example. We evaluate the path on the business cycle clock based on GDP in Poland for two samples. The first ends in the last quarter of 2016 and the second ends at the first quarter of 2017. Taking into account used methodology (described above), for the first sample the last point for the path on business cycle clock is related to the second quarter of 2016, while for the second sample to the third quarter of 2016. In both cases we fix λ = 5500.

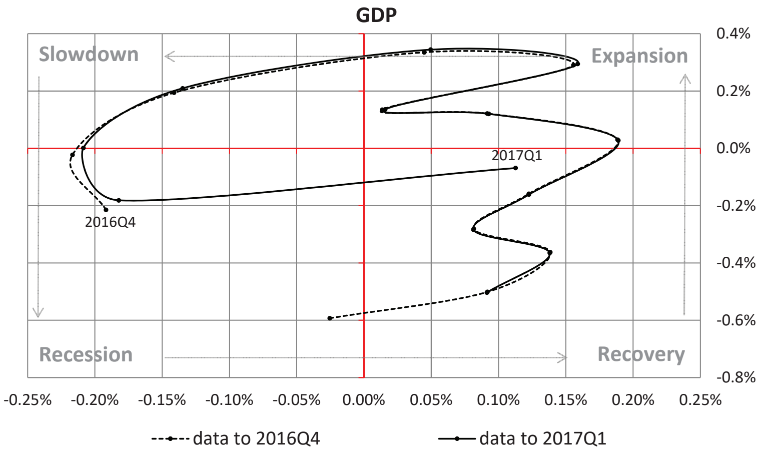


Figure 2. The paths on the business cycle clock for two samples. The case of Poland

The results shown in Figure 2 clearly indicate that the points position on a business cycle clock at a given time is determined by the range of the sample. Note that, the above paths on the business cycle clock are smooth enough to assess the economic situation. Unfortunately, the future unknown economic situation is not taken into account here. This is a source of uncertainty for the present points position and as well as for the past and future.

To investigate the uncertainty for the points position on the business cycle clock we evaluate the forecast for the future observations of observed time series. Let $\bar{\bar{\boldsymbol{P}}}_{\boldsymbol{h}} = [\bar{\bar{P}}_{n+1}, \bar{\bar{P}}_{n+2}, \ldots, \bar{\bar{P}}_{n+h}]$ be the h — variate predictive distribution (conditional on $\hat{\boldsymbol{P}}$) for the horizons 1,2, …, h. Then, the distribution of the cyclical fluctuations can be defined in the natural way via:

$$\boldsymbol{C}_{HP,\lambda,n,h} = \boldsymbol{HP}_{\lambda,n+h} * [\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_n, \bar{\bar{P}}_{n+1}, \bar{\bar{P}}_{n+2}, \ldots, \bar{\bar{P}}_{n+h}]'.$$

In the same manner we define the distribution for the path on the business cycle clock $\boldsymbol{BCC}_{HP,\lambda,n,h}$. Note that, if the distribution of the vector $\bar{\bar{P}}_h$ is normal, then both $\boldsymbol{C}_{HP,\lambda,n,h}$ and $\boldsymbol{BCC}_{HP,\lambda,n,h}$ has multivariate normal distributions.

Note that the methodology presented above differs from very well-known model of cyclical fluctuations based on unobserved components; see for example to Harvey and Jaeger (1993) and Hervey et al. (2007). The above-mentioned construct assumes that:

$$\begin{cases} P_t = \boldsymbol{\mu}_t + \boldsymbol{\psi}_{n,t} + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_{t-1} \\ \boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\zeta}_t, \end{cases}$$

where $\boldsymbol{\varepsilon}_t \sim NID(0, \boldsymbol{\Sigma}_{\varepsilon})$ and $\boldsymbol{\zeta}_t \sim NID(0, \boldsymbol{\Sigma}_{\zeta})$, and the vector cycle $\psi_{n,t}$ is a generalization of cycle $\psi_{1,t}$ presented in Harvey and Jeager (1993). The higher n the stronger mass concentration for the spectral density in an neighborhood of unknown frequency. To obtain smooth path for cyclical fluctuations the appropriate high n should be considered. For more details see to Trimbur (2006), Harvey and Trimbur (2003), Koopman and Shephard (2015), Pelagatti (2016), Azevedo et al. (2006), Harvey et al. (2007), Koopman and Azevedo (2008).

4. ILLUSTRATIVE EMPIRICAL EXAMPLE

The predictive distribution for logarithm of the observed process $\{P_t : t \in \mathbb{Z}\}$ is evaluated using simple class of seasonal ARIMA models. We consider *SARIMA* (p, d, q) $(P, D, Q)_4$ models with the following restrictions: $0 \le p, q \le 5$, $0 \le P, Q \le 3$, $0 \le d, D \le 1$. For each particular model we calculate the value of AIC. Finally, we chose model with minimal AIC. It is known that a better fit in the sample does not necessary provide more accurate forecasts for cyclical fluctuations. But the aim of this paper is to show only the illustrative example, rather than comprehensive discussion. Note that such class of models is not popular in business cycle fluctuations analysis. We use this class to show only a simple illustrative example. Table 1 shows the final models with minimal AIC, for each country respectively.

Table 1

The model with minimal AIC

Country	Model specification (with minimal AIC)
Czech Republic	SARIMA(0,1,0)(1,1,2)$_4$
Hungary	SARIMA(1,0,3)(0,1,0)$_4$
Poland	SARIMA(0,1,0)(0,1,1)$_4$
Slovakia	SARIMA(3,1,0)(0,1,1)$_4$

For each country, the predictive distribution has been evaluated, with $h = 12$. Then, the 50 thousands samples from predictive distribution was determined. The sample period covers the period from first quarter of 2002 to fourth quarter of 2016. The prediction period covers the period from the first quarter of 2017 to the fourth quarter of 2019. In the next step, the predictive distribution for centered moving average (2x12MA) process was evaluated, by using previous 50000 samples. Hence, the forecast period for centered moving average process covers the period from the third quarter of 2016 to the second quarter of 2019. Finally, 50000 samples for the predictive distribution for the vectors $\boldsymbol{C}_{HP,\lambda,n,h}$ and $\boldsymbol{BCC}_{HP,\lambda,n,h}$ has been evaluated.

Based on the above predictive distributions, for each point from the path on business cycle clock $\boldsymbol{BCC}_{HP,\lambda,n,h}$, the probability of being in the first (expansion), second (slowdown), third (recession) and fourth (recovery) quadrant was approximated. The results are shown in Figure 3 for Czech Republic, Figure 6 for Hungary, Figure 9 for Poland and Figure 12 for Slovakia. The median of the cyclical fluctuations (obtained by HP procedure) with 60% and 90% prediction intervals are presented in Figure 4 for Czech Republic, Figure 7 for Hungary, Figure 10 for Poland and Figure 13 for Slovakia. Finally, for each time series, we present the boundary of the 90% prediction area for the points position on business cycle clock concerning period from the second quarter of 2016 to the second quarter of 2017 (Figures: 5,8,11,14). For each predictive distribution we consider four smoothing parameters λ of HP filtration, which corresponds to 5, 8, 10 and 12 years, respectively. Presented results may indicate the most likely scenario for the economic situation. Below, we formulate the exemplary interpretation for the obtained results.

In the case of Czech Republic, based on the predictive distribution, the most likely in the first, second and third quarter of 2016 is the slowdown. For each considered smoothing parameters λ, the probability of slowdown is higher than 0.56. The probability of the recession and recovery is less likely for the higher smoothing parameter λ. For smoothing parameter λ, which corresponds to 12 years, the probability of the recession or recovery is less than 0.25 at each quarter during period from 2016 to 2017.

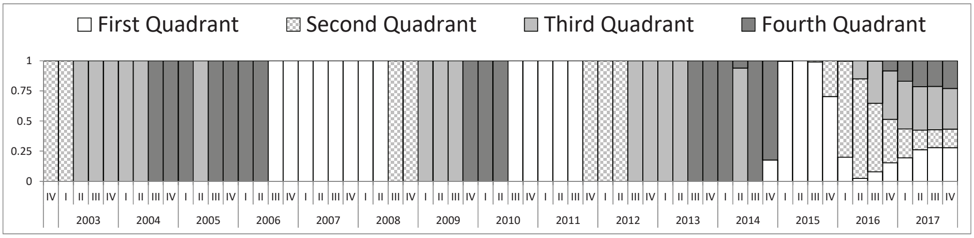
In the case of Hungary, the bigger parameter λ, the smaller probability of recession and recovery in 2016 and 2017. In addition, the bigger parameter λ, the higher correlation in (bivariate) predictive distribution for the points position on business cycle clock for second quarter of 2016 and for third quarter of 2016 (see Figure 8, first and second column). This fact is observed also for other analyzed countries. In the fourth quarter of 2016 the results for λ corresponding to 8, 10, 12 years indicate that the less likely is the recession (probability below 0,2) and the recovery (probability below 0,05).

For Poland the predictive distribution for cyclical fluctuations (see Figure 10) shows that during the period from the third quarter of 2016 to the fourth quarter of 2017 the probability of expansion, slowdown, recession and recovery stabilize on the level approx. 1/4. Although the amplitude of cyclical fluctuations for Poland is stable over time, the predictive distributions beyond the sample do not indicate a clear entry into any phase of the business cycle. Probably, this is a consequence of simplicity of model that was chosen.

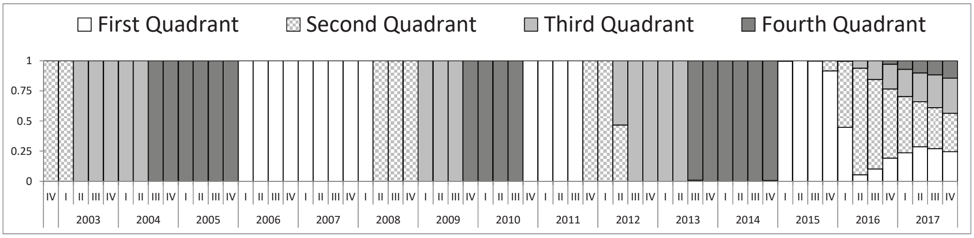
After a great crisis, the growth cycle for Slovakia was flattened. Predictive distributions indicate a much lower amplitude value than during the great crisis and directly before it. This distinguishes Slovakia from Poland, for which the amplitude of cyclical fluctuations throughout the analysis period is stable over considered time. In the case of Slovakia, the predictive distributions for the points position on the business cycle clock (in the period under consideration from the second quarter of 2016 to the second quarter of 2017) are characterized by the largest dispersion (measured by variance) compared to the distributions for other considered economies.

In the fourth quarter of 2016, only for the Czech Republic and Hungary, the estimated probabilities indicate a greater chance of expansion or slowdown than recession or recovery. In addition, these probabilities increase with the increase of the smoothing parameter λ. For Poland and Slovakia these probabilities have comparative values.

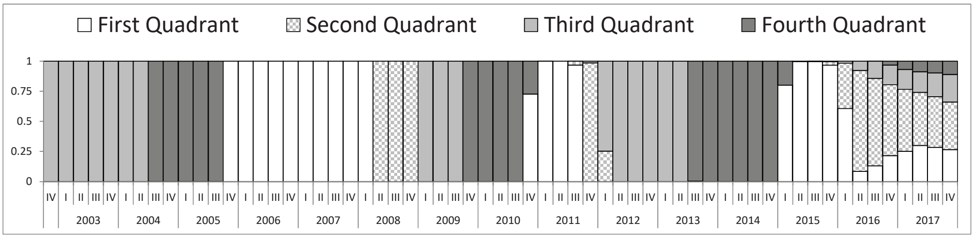
The above results suggest the need to include more sophisticated predictive models. The extended analysis should take into account the cyclical nature of the variables being analyzed. Both models with stochastic as well as deterministic nature — see Lenart and Mazur (2016), Lenart et al. (2016), Mazur (2016), Mazur (2017a), Mazur (2017b) — of cyclical fluctuations can be used.



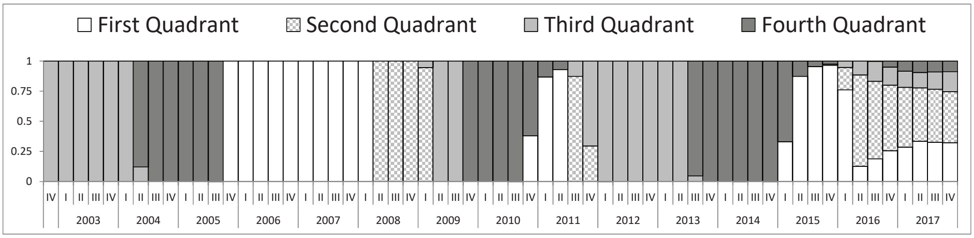
a) λ corresponds to 5 years



b) λ corresponds to 8 years

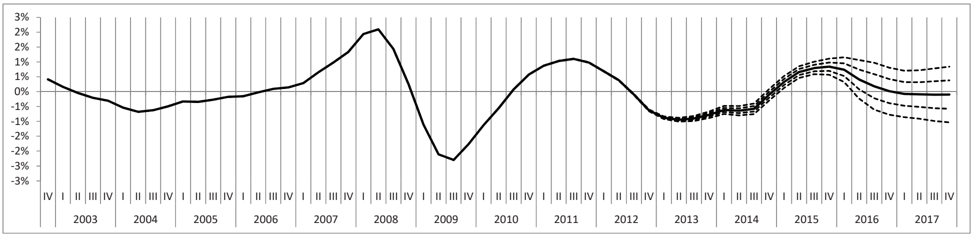


c) λ corresponds to 10 years

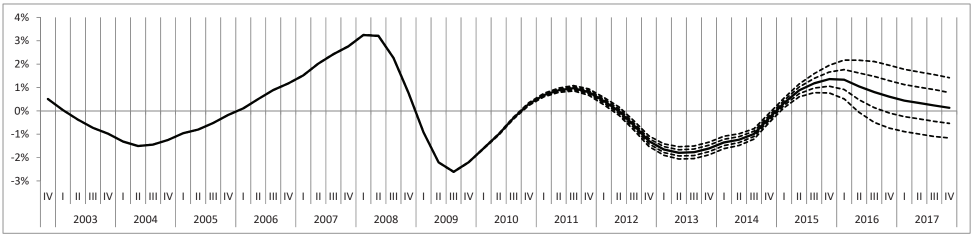


d) λ corresponds to 12 years

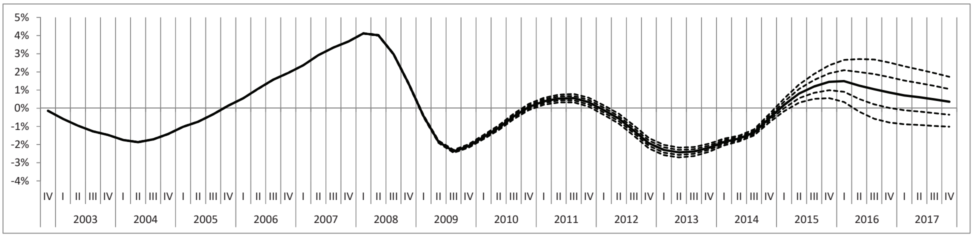
Figure 3. The probability of expansion (first quadrant), slowdown (second quadrant), recession (third quadrant) and recovery (fourth quadrant) for Czech Republic



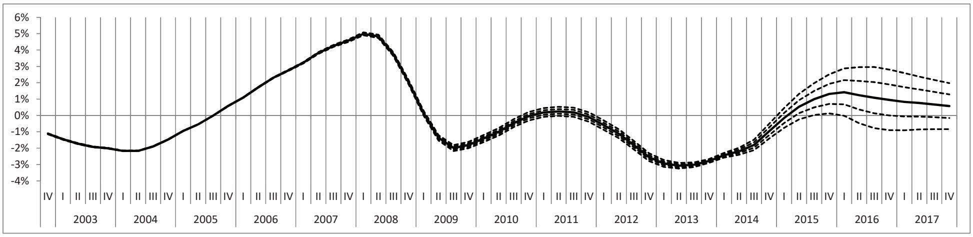
a) λ corresponds to 5 years



b) λ corresponds to 8 years



c) λ corresponds to 10 years



d) λ corresponds to 12 years

Figure 4. The median (solid line) of cyclical fluctuations obtained by HP filtration with 60% and 90% prediction bounds (dotted lines). The case of Czech Republic

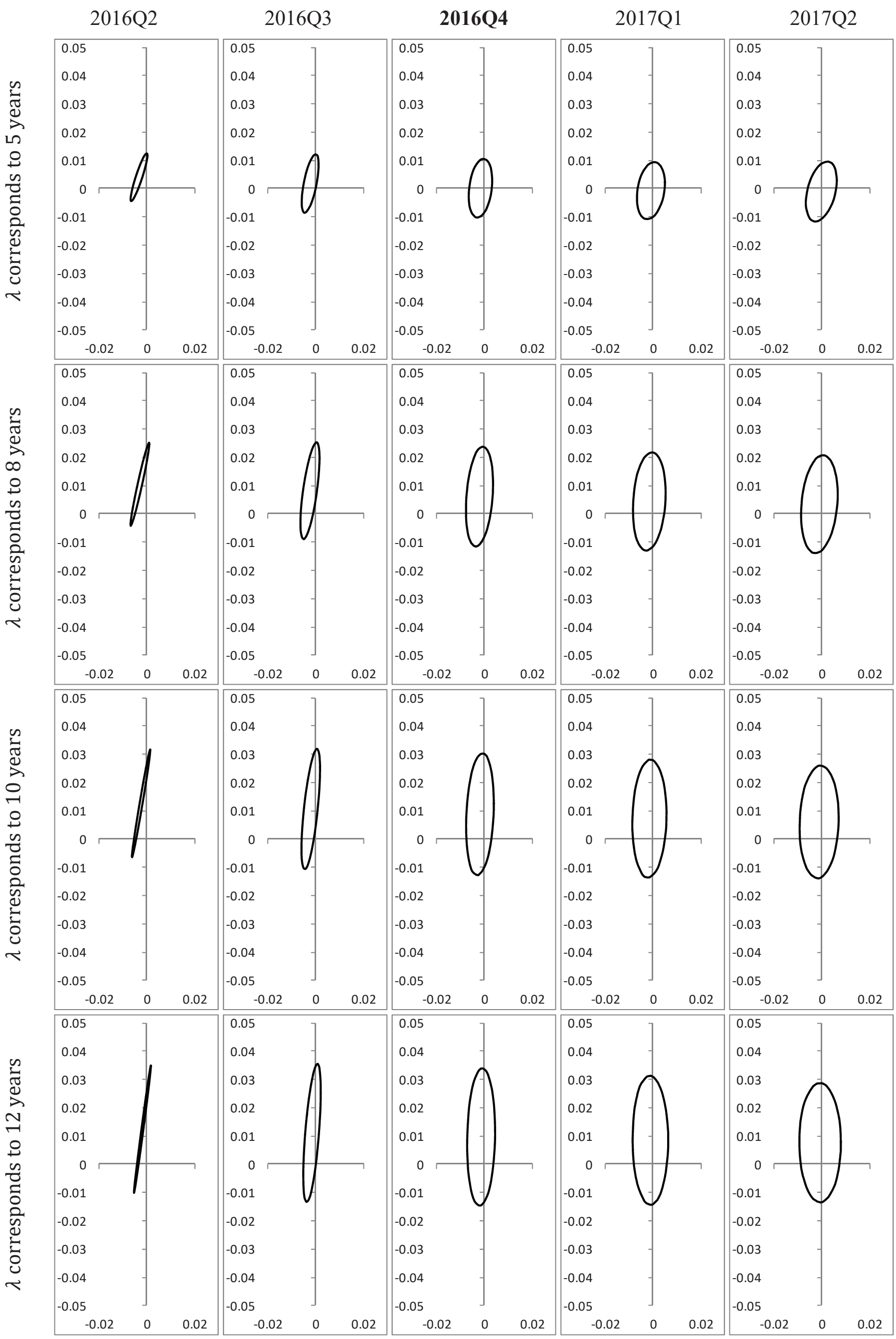
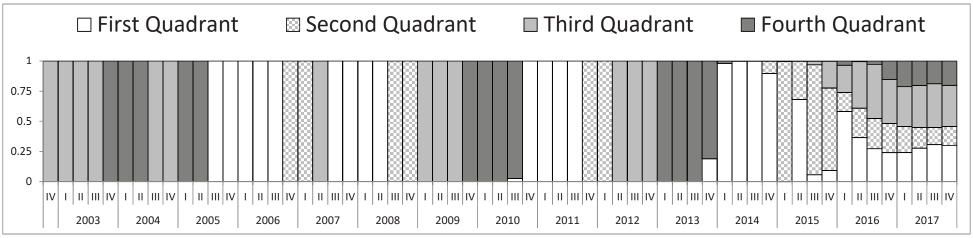
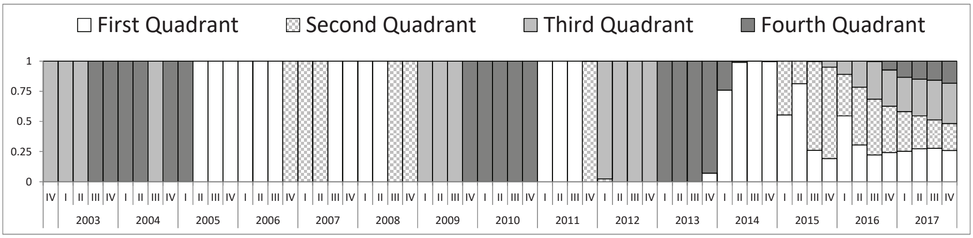


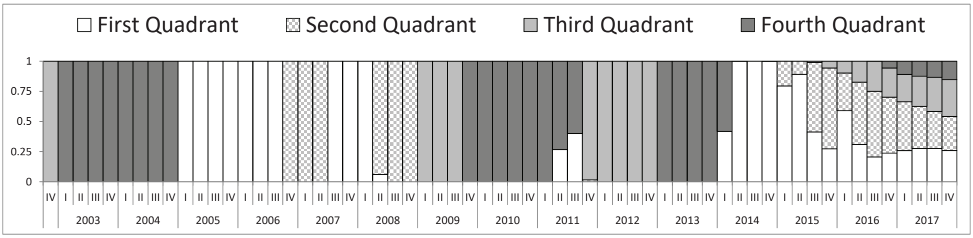
Figure 5. The boundary (solid line) of 90% prediction area for the points position on business cycle clock. The case of Czech Republic



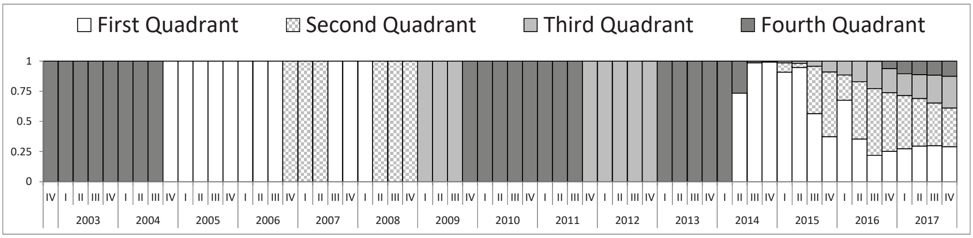
a) λ corresponds to 5 years



b) λ corresponds to 8 years

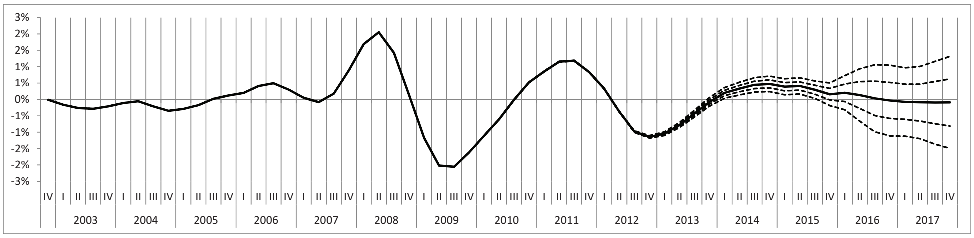


c) λ corresponds to 10 years

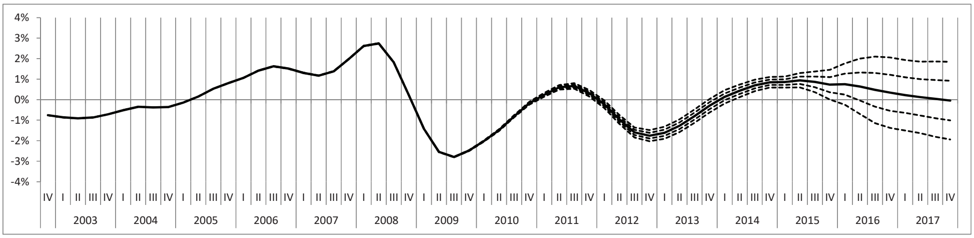


d) λ corresponds to 12 years

Figure 6. The probability of expansion (first quadrant), slowdown (second quadrant), recession (third quadrant) and recovery (fourth quadrant) for Hungary



a) λ corresponds to 5 years



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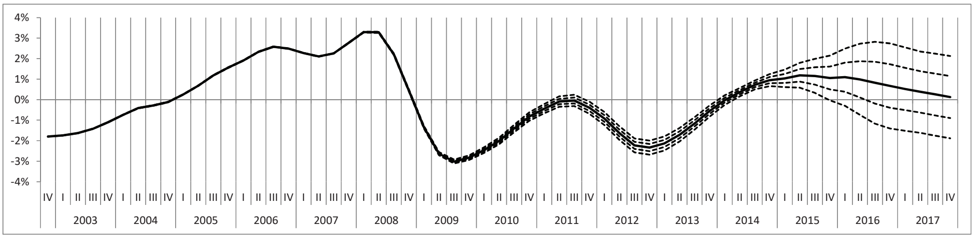

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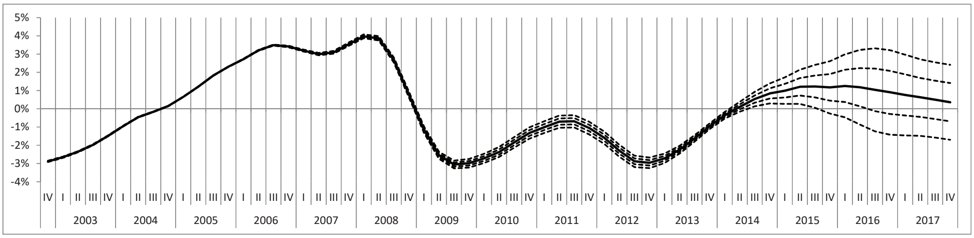


d) λ corresponds to 12 years

Figure 7. The median (solid line) of cyclical fluctuations obtained by HP filtration with 60% and 90% prediction bounds (dotted lines). The case of Hungary

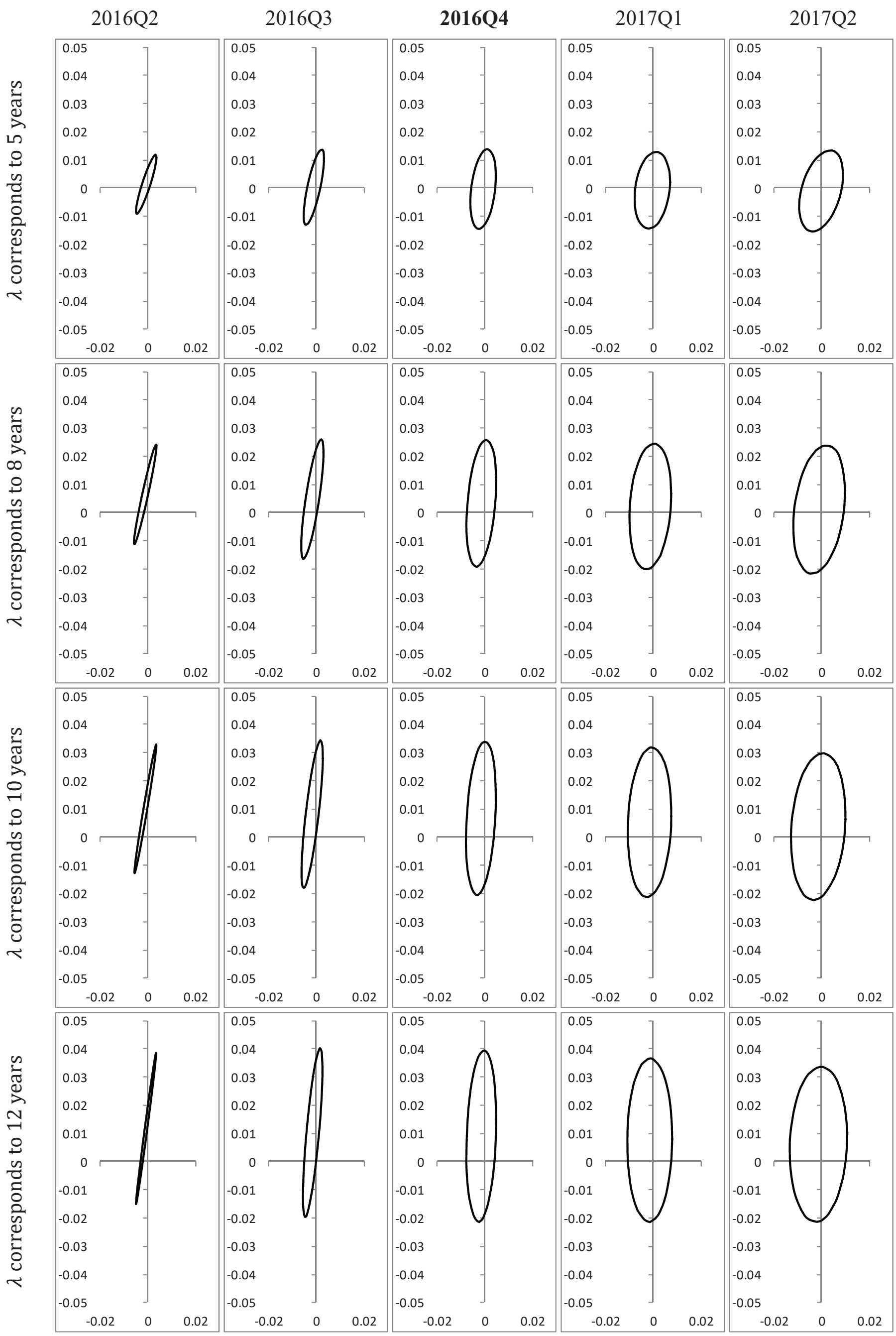
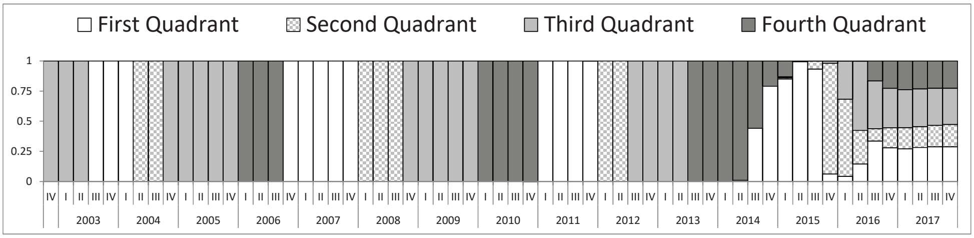
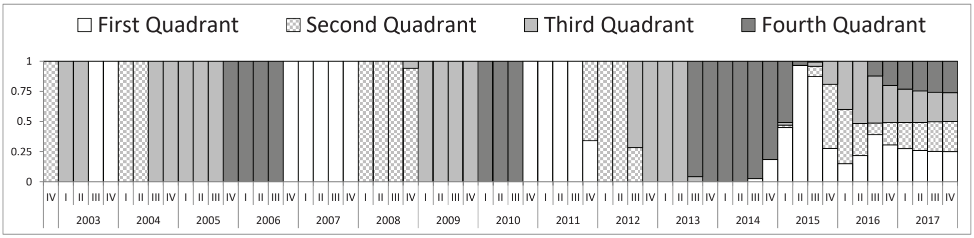


Figure 8. The boundary (solid line) of 90% prediction area for the points position on business cycle clock. The case of Hungary



a) λ corresponds to 5 years



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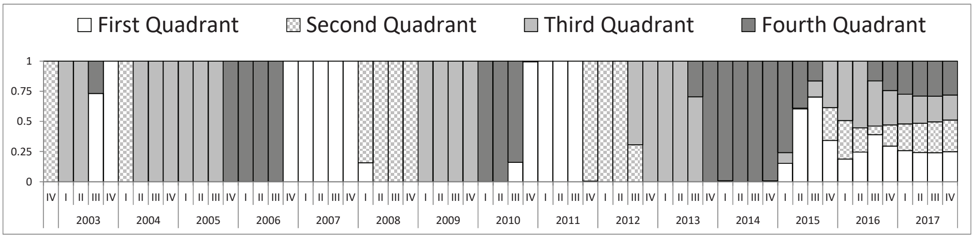

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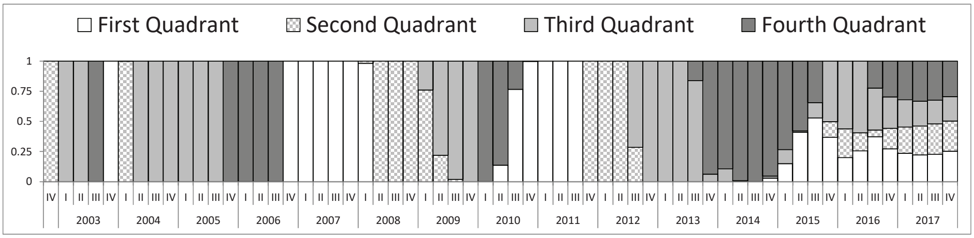
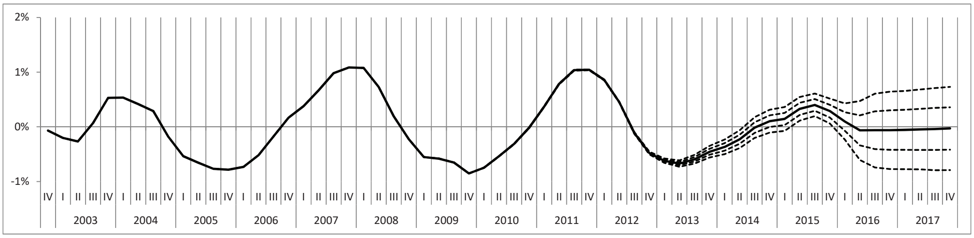
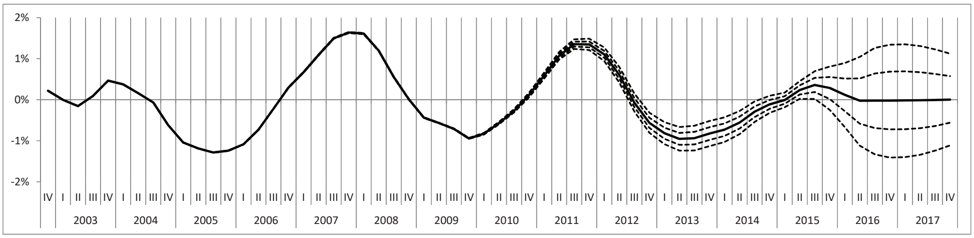


d) λ corresponds to 12 years

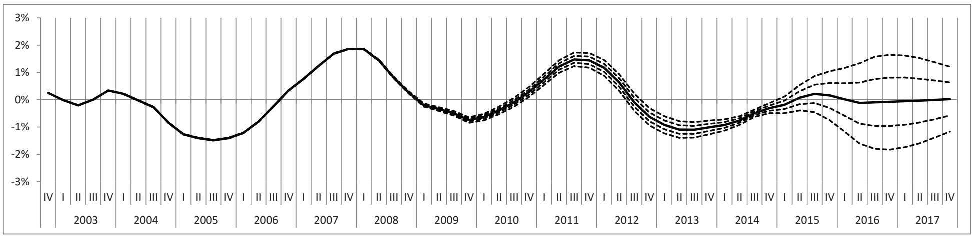
Figure 9. The probability of expansion (first quadrant), slowdown (second quadrant), recession (third quadrant) and recovery (fourth quadrant) for Poland



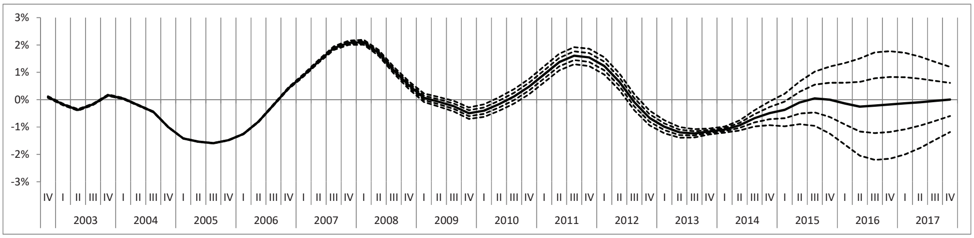
a) λ corresponds to 5 years



b) λ corresponds to 8 years



c) λ corresponds to 10 years



d) λ corresponds to 12 years

Figure 10. The median (solid line) of cyclical fluctuations obtained by HP filtration with 60% and 90% prediction bounds (dotted lines). The case of Poland

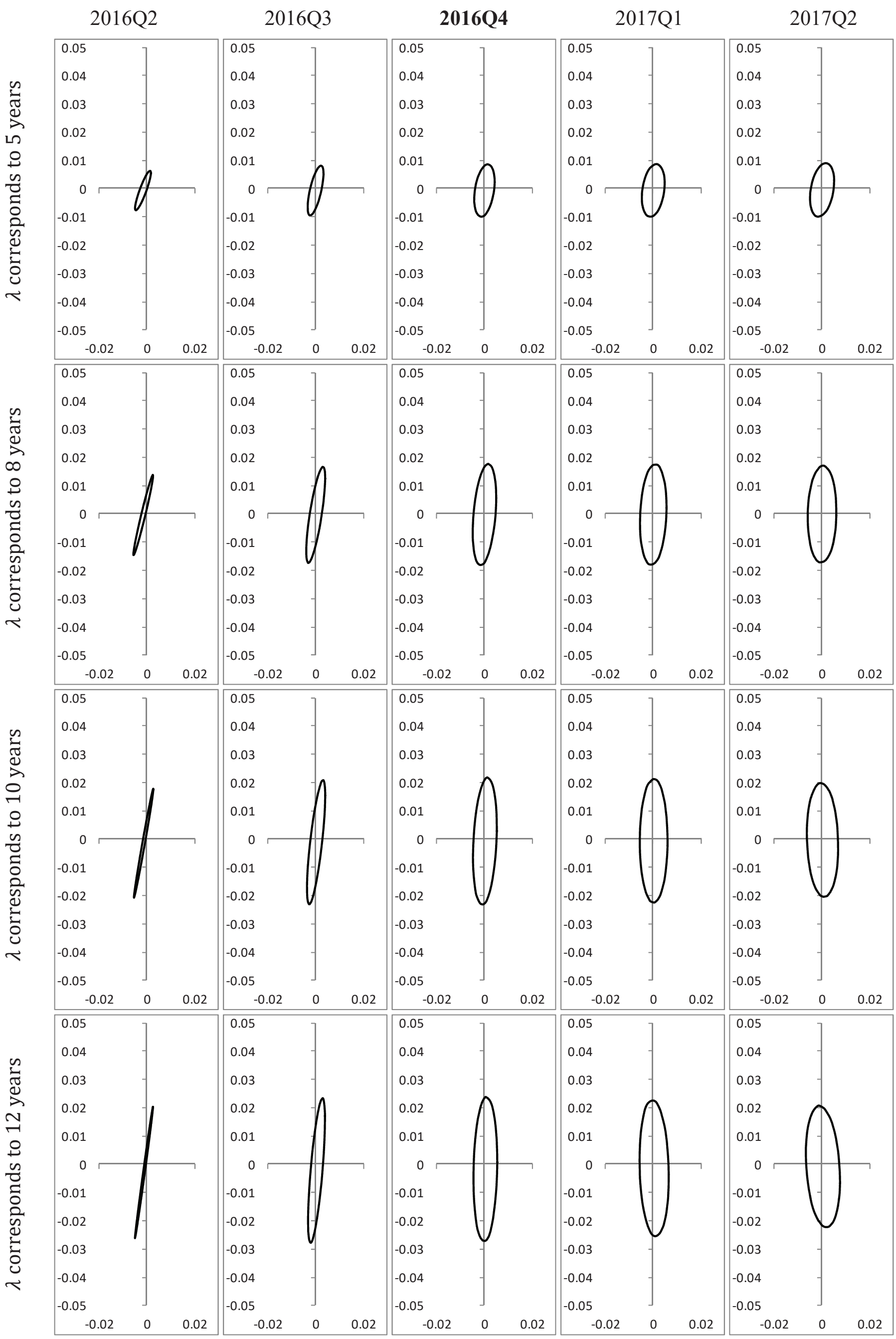
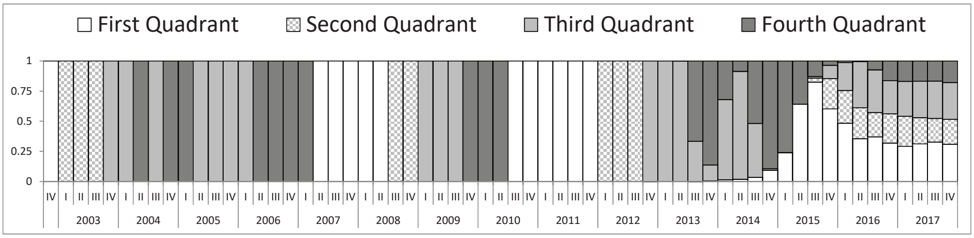
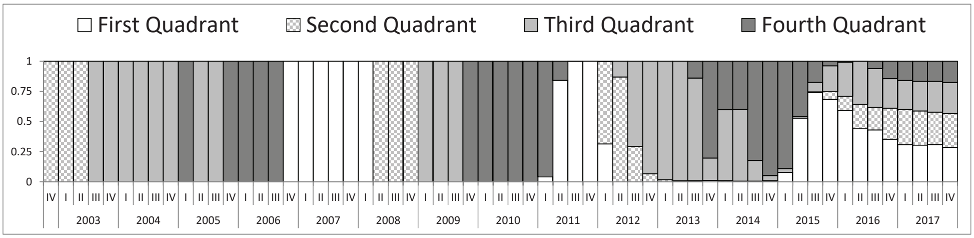


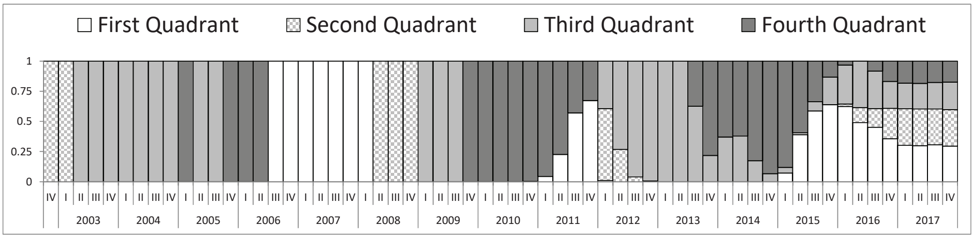
Figure 11. The boundary (solid line) of 90% prediction area for the points position on business cycle clock. The case of Poland



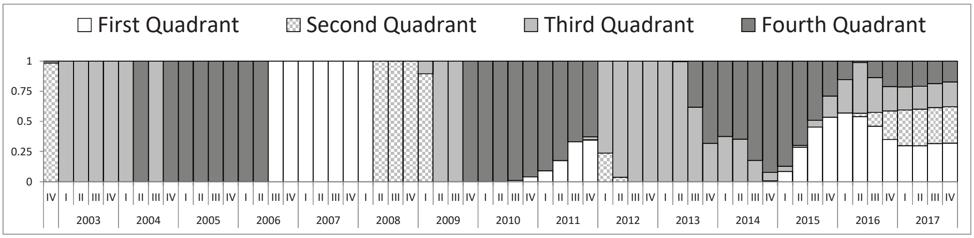
a) λ corresponds to 5 years



b) λ corresponds to 8 years

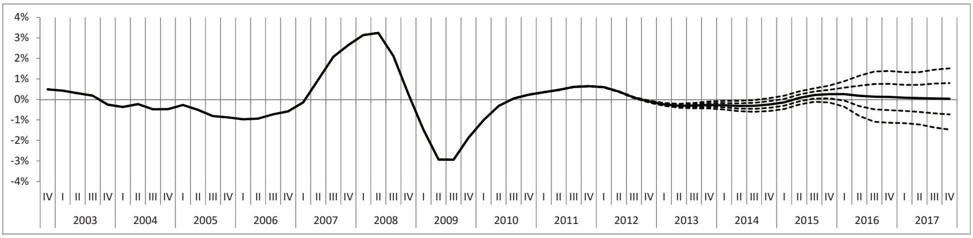


c) λ corresponds to 10 years

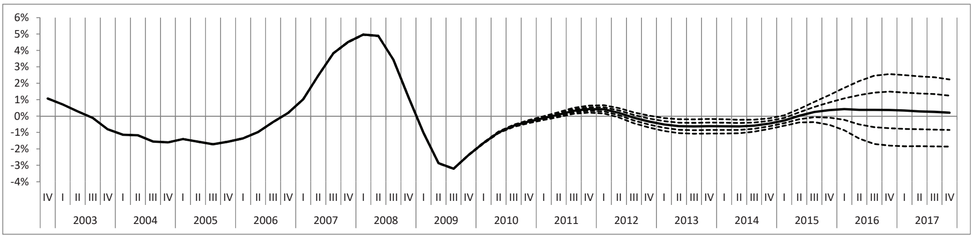


d) λ corresponds to 12 years

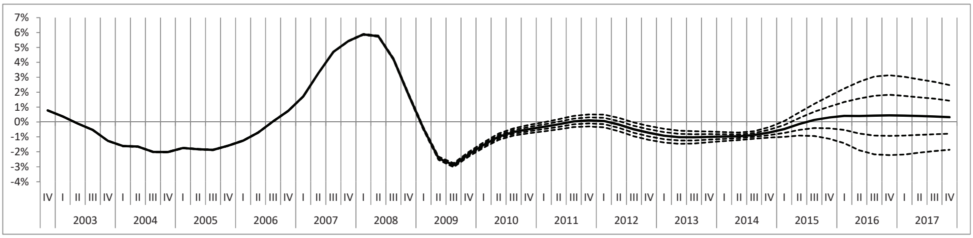
Figure 12. The probability of expansion (first quadrant), slowdown (second quadrant), recession (third quadrant) and recovery (fourth quadrant) for Slovakia



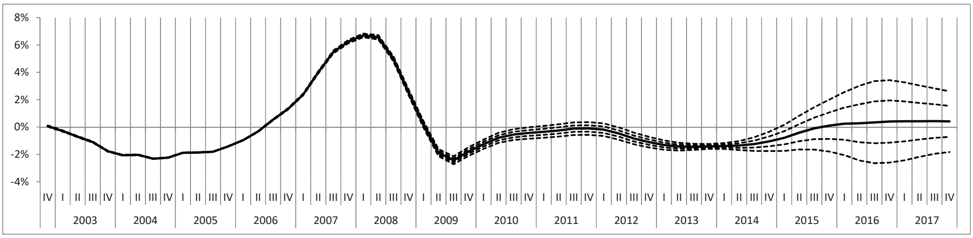
a) λ corresponds to 5 years



b) λ corresponds to 8 years



c) λ corresponds to 10 years



d) λ corresponds to 12 years

Figure 13. The median (solid line) of cyclical fluctuations obtained by HP filtration with 60% and 90% prediction bounds (dotted lines). The case of Slovakia

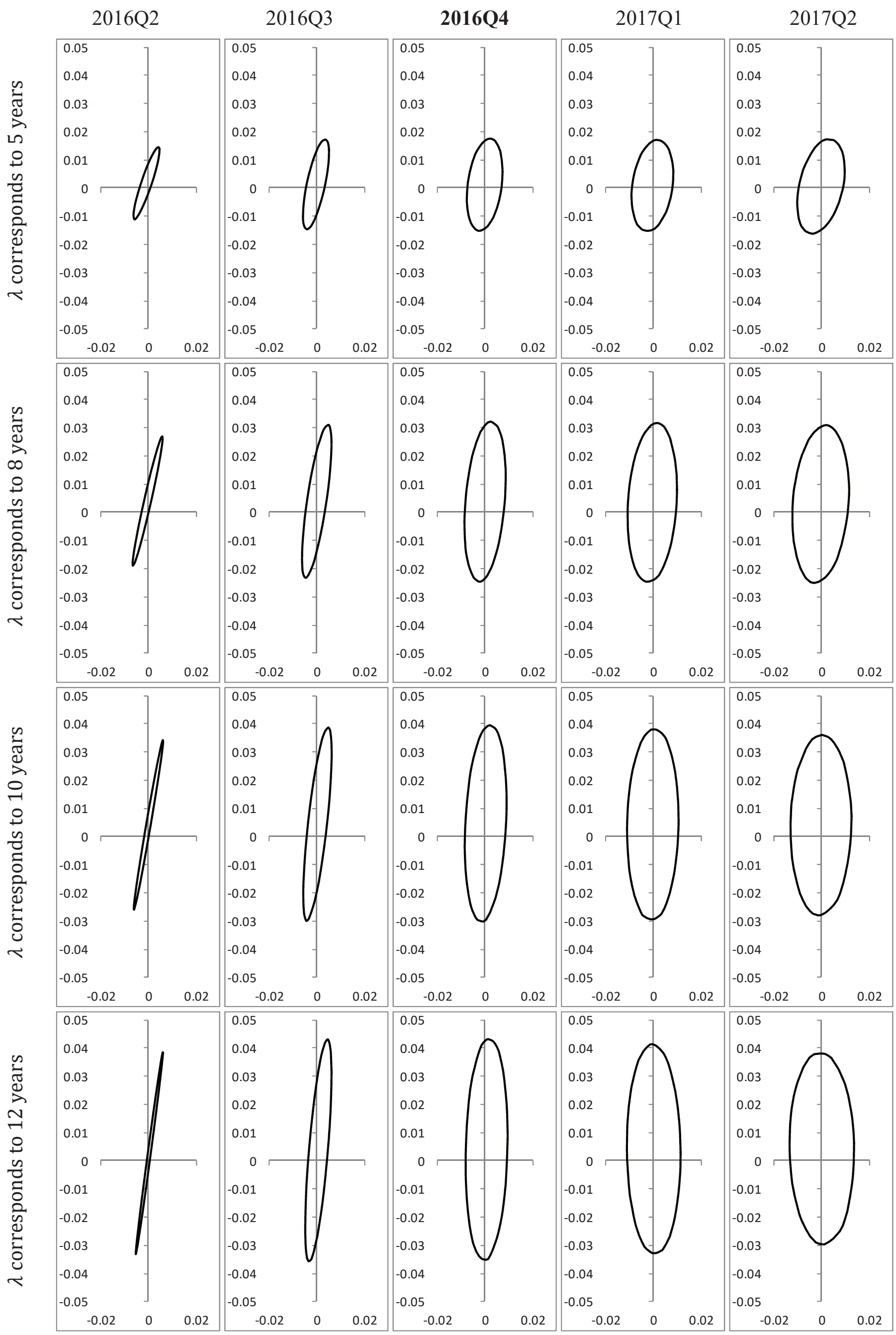


Figure 14. The boundary (solid line) of 90% prediction area for the points position on business cycle clock. The case of Slovakia

5. CONCLUSIONS

In this paper we investigate the problem of determining the prediction distributions for points position on business cycle clock. Our finding is that for present and past time the prediction distribution for points position on BCC may be marked by strong correlation between coordinates. This finding may help us to assess the actual and the past economic situation much better. In the empirical example the only SARIMA models were considered. By only applying this class of models, the ability to formulate conclusions is limited. The more advanced predictive distributions with cyclical component should be considered. Furthermore, the multivariate forecasting model for the whole Visegrad Group would allow to determine the probability of the same cycle phase for all analyzed economies. This give the opportunity to determine the degree of synchronization of cyclical fluctuations in probabilistic categories.

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