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GEODESY AND CARTOGRAPHY
Vol. 67, No. 1, 2018, pp. 57–69

DOI: https://doi.org/10.24425/118696

Original research paper

Study on detection of gross error in geodetic network adjustment

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Received: 13 October 2017 / Accepted: 2 February 2018

Abstract: Generally, gross errors exist in observations, and they affect the accuracy of results. We review methods to detect the gross errors by Robust estimation method based on L_1-estimation theory and their validity in adjustment of geodetic networks with different condition. In order to detect the gross errors, we transform the weight of accidental model into equivalent one using not standardized residual but residual of observation, and apply this method to adjustment computation of triangulation network, traverse network, satellite geodetic network and so on. In triangulation network, we use a method of transforming into equivalent weight by residual and detect gross error in parameter adjustment without and with condition. The result from proposed method is compared with the one from using standardized residual as equivalent weight. In traverse network, we decide the weight by Helmert variance component estimation, and then detect gross errors and compare by the same way with triangulation network In satellite geodetic network in which observations are correlated, we detect gross errors transforming into equivalent correlation matrix by residual and variance inflation factor and the result is also compared with the result from using standardized residual. The results of detection are shown that it is more convenient and effective to detect gross errors by residual in geodetic network adjustment of various forms than detection by standardized residual.

Keywords: geodetic network adjustment, gross error, Robust estimation, equivalent weight

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1. Introduction

Generally, the errors exist in observations and affect the accuracy of results. Especially, if the observations include gross errors, it could bring about bad result in the adjustment of geodetic network. Therefore, many researchers have proposed a variety of approaches for detecting gross errors in order to remove the influence as possible. In general, there are two typical approaches to process observed values affected by gross error. The first approach is to detect gross error by robust estimation methods (Cen et al., 2003; Ding and Coleman, 1996; Gui et al., 2007, Guo and Wang, 2010; Kern et al, 2005; Liu et al., 2002) and the second one is to detect gross error by quasi-accurate detection (QUAD) (Guo et al., 2007; Ou, 1999). Detection of gross error by robust estimation is easier than by QUAD in realization of algorithm. Therefore, it is used frequently in practice of adjustment computation.

The purpose of this paper is to investigate effectiveness of equivalent weight based on L_1-estimation theory for detecting the gross errors in the robust estimation method. Robust estimation methods use various types of equivalent weight. In this case, standardized residual is frequently used and threshold value is taken within range 1.0~1.5 (Yang et al., 2002). However, since standardized residual of observation uninfected by gross error may belong to range of threshold value in practice of adjustment computation, it could decrease effectiveness that detect the gross errors by standardized residual. In this paper, we review effectiveness of gross error detection by equivalent weight using residual of observation and by permissible value of residual.

2. Detection of gross error by Robust estimation method

2.1. Determination of equivalent weight of observation

2.1.1. Case of independent observation information

The main principle of Robust Estimation is to transform the observation weight into the equivalent weight reasonably, which could suppress maximally gross error contained in observation information (Yun, 2003; Yang, et al., 2002). We take the equivalent weight of independent observation information, based on estimation theory, as follows (Huber and Ronhetti, 2009; Zumberge et al., 1997):

$$w_i = \begin{cases} p_i & |v_i| \leq c_0 \\ p_i|v_i|^{-1} & |v_i| > c_0 \end{cases}, \tag{1}$$

where: p_i is weight of observation information; v_i is residual of observation; c_0 is permissible value of residual.

If the equivalent weight is taken like Equation (1), threshold value, c_0 can be decided easily corresponding to accuracy of given observation condition.

2.1.2. Case of correlative observation information

When correlation matrix of observation information is given, we introduce the variance expansion factor and transform it into equivalent correlation matrix as follows (Guo and Wang, 2010):

$$\overline{Q}_{ii} = k_{ii} \cdot Q_{ii}\,, \qquad \overline{Q}_{ij} = k_{ij} \cdot Q_{ij}\,, \tag{2}$$

where

$$k_{ii} = \begin{cases} 1 & |v_i| \le c_0 \\ |v_i|^{-1} & |v_i| > c_0 \end{cases}\,,$$

$$k_{ij} = \sqrt{k_{ii} \cdot k_{jj}}\,.$$

2.2. Detection algorithm of gross errors by Robust estimation

The residual equation of parameter adjustment can be written as follow:

$$V = AX + L\,, \tag{3}$$

where: X – correction vector of unknown parameter; L – vector of constants; A – coefficient matrix; V – residual vector.

Considering the equivalent weight in Equation (3), normal equation of parameter would be obtained according to principle of least squares as follows:

$$A^T WAX + A^T WL = 0\,, \tag{4}$$

where:

$$W = \begin{pmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & w_n \end{pmatrix} \text{ – Equivalent weight matrix.}$$

Correction vector of unknown parameter would be obtained from Equation (4) as follows:

$$X = -\left(A^T WA\right)^{-1} A^T WL\,. \tag{5}$$

Since the equivalent weight w_i is related with residual v_i in Equation (5), it could be calculated through iterative computation and thus correction vector of unknown parameter X would be also calculated through iterative computation.

Iterative computation Algorithm to determine the correction vector of unknown parameter X is as follows.

Step 1. Introducing the principle of least square into residual Equation (3), calculate the initial value of X and V as follows:

$$X^{(0)} = -(A^T PA)^{-1} A^T PL\,, \qquad V^{(0)} = AX^{(0)} + L\,. \tag{6}$$

Step 2. Determine the equivalent weight w_i using Equation (1) or Equation (2) and calculate the vector X and V as follows:

$$X^{(k)} = -(A^T WA)^{-1} A^T WL\,, \qquad V^{(k)} = AX^{(k)} + L\,, \tag{7}$$

where k – number of iteration.

Step 3. Repeat the step 2 until the following condition is satisfied.

$$\left| X^{(k)} - X^{(k-1)} \right| < \varepsilon , \tag{8}$$

where ε is a parameter related with accuracy of computation and it is small positive number.

When satisfies the condition (8), unknown parameter vector determined using vector X would be stable estimation value.

Finally, we can detect the gross errors by residual as follows:

$$|v_i| > c_0 , \tag{9}$$

where c_0 is permissible value of residual and it is taken as the same as Equation (1).

Appropriateness of detected results can be estimated as variance ratio, which is obtained by results of adjustment before and after consideration of gross errors. That is:

$$F_0 = \frac{\sigma_1^2 \cdot trace(Q_1)}{\sigma_2^2 \cdot trace(Q_2)} , \tag{10}$$

where σ_1^2 is variance of unit weight and Q_1 is correlation matrix of unknown parameter after consideration of gross errors, σ_2^2 is variance of unit weight and Q_2 is correlation matrix of unknown parameter before consideration of gross error.

When $F_0 < 1$ in Equation (10), it seems that gross errors are detected correctly.

3. Application of gross error detection method in geodetic network adjustment

3.1. Detection of gross error in triangulation network adjustment

In this section the results of gross error detection is compared and analyzed by parameter adjustment and parameter adjustment with condition in simulation geodetic network (Figure 1).

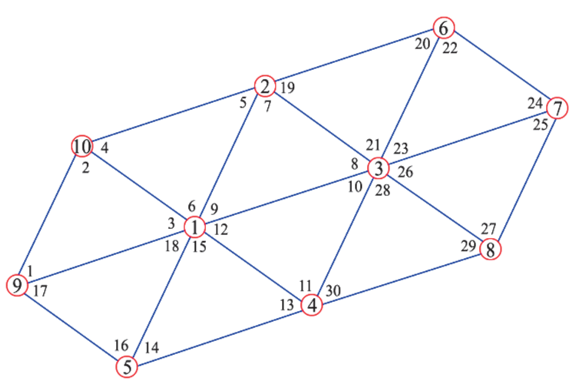

Fig. 1. Simulated triangulation network

In simulation geodetic network, coordinates of given point and simulation angle are presented in Table 1 and Table 2.

Table 1. Given data

Coordinates		
Point	X, m	Y, m
9	2000.000	0.000
10	4449.48973	1414.21356
Direction	Base line, m	Directional angle (° ′ ″)
7–8	2828.427	210 – 00 – 00

Observation data are random values obtained using function $randn(\cdot)$ in MATLAB. Then we add $10''$ to angles 2, 5, 14, 26, 30 in order to make gross errors.

Table 2. Simulated observation data

No	Observed horizontal angle (° ′ ″)	No	Observed horizontal angle (° ′ ″)
1	45 – 00 – 0.44	16	89 – 59 – 59.73
2	90 – 00 – 8.41	17	45 – 00 – 0.65
3	44 – 59 – 59.30	18	44 – 59 – 58.76
4	44 – 59 – 58.93	19	44 – 59 – 59.41
5	45 – 00 – 11.00	20	44 – 59 – 59.52
6	90 – 00 – 1.72	21	90 – 00 – 0.98
7	90 – 00 – 0.70	22	90 – 00 – 1.76
8	44 – 59 – 59.26	23	45 – 00 – 1.42
9	45 – 00 – 0.22	24	45 – 00 – 0.91
10	44 – 59 – 59.78	25	45 – 00 – 0.32
11	89 – 59 – 59.15	26	45 – 00 – 10.06
12	45 – 00 – 0.34	27	89 – 59 – 58.51
13	45 – 00 – 0.10	28	89 – 59 – 59.59
14	45 – 00 – 8.87	29	44 – 59 – 59.98
15	89 – 59 – 59.32	30	45 – 00 – 10.22

In triangulation network, we detected gross errors in two ways.

First: Transforming observation weight into equivalent one using residual in Equation (1), detect gross errors in both cases without conditions and with conditions (that is direction angle condition and baseline condition).

Second: Transforming weight into equivalent one using standardized residual, detect gross errors in both cases without conditions and with conditions. In first way, permissible value of residual is taken as $c_0 = 3''$. As a result, proposed algorithm detected number of angles 2, 5, 14, 26, 30, which contains gross errors (Figure 2 a, b).

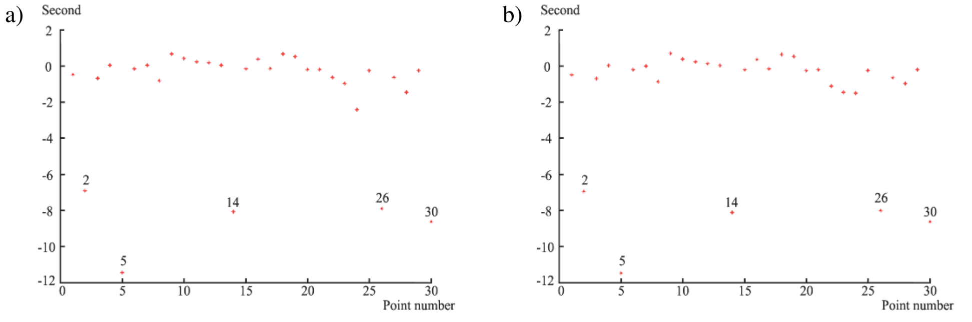

Fig. 2. Residuals of detection result in first method, a) without condition, b) with conditions

In second method, threshold of standardized residual is taken as $c_0 = 1''$ and detected gross errors. Figures 3 a, b shows the result of detection.

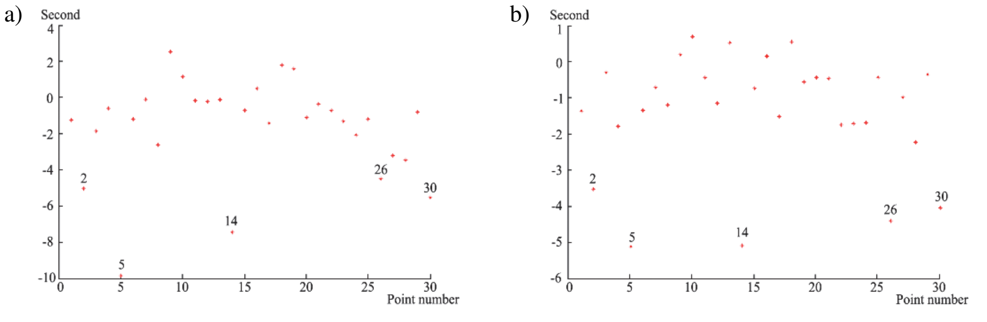

Fig. 3. Residuals of detection result in second method a) without condition, b) with conditions

As Figure 2 and Figure 3 show, it is clear that baseline condition and direction angle condition don't give effect on detection of gross errors of observed angle when given baseline and direction angle were determined with high accuracy.

It is effective to transform into equivalent weight by Equation (1) using residuals.

3.2. Detection of gross error in traverse network adjustment

Here, when weights of observed data are decided using Helmert variance component estimation in adjustment of 3-dimensional traverse network, we applied Robust estimation method in three ways and detected gross errors. Then we compared those results. For traverse network presented in Figure 4, coordinates of given points 17~22 are shown in Table 3 and observed data are presented in Table 4.

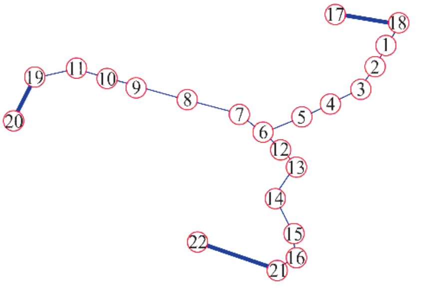

Fig. 4. Traverse network

Table 3. Coordinate of given points

Coordinate	Point					
	17	18	19	20	21	22
X, m	43661.158	43610.014	43322.896	43099.714	42303.259	42612.019
Y, m	42115.571	42321.616	41135.578	41068.370	41925.049	41096.747
Z, m	12.592	13.653	8.191	6.430	25.269	100.253

Table 4. Observed data

No	Horizontal angle (° ′ ″)	No	Vertical angle (° ′ ″)	No	Distance, m
1	277 – 00 – 12	21	0 – 34 – 23	39	123.558
2	177 – 45 – 55	22	1 – 59 – 25	40	110.583
3	181 – 52 – 15	23	1 – 01 – 20	41	129.699
4	210 – 53 – 35	24	−1 – 21 – 46	42	131.286
5	179 – 49 – 30	25	−1 – 24 – 19	43	115.081
6	187 – 28 – 0	26	−1 – 29 – 29	44	151.396
7	263 – 54 – 23	27	0 – 03 – 10	45	121.684
8	150 – 47 – 2	28	−0 – 13 – 25	46	183.603
9	181 – 06 – 25	29	0 – 18 – 28	47	181.539
10	179 – 59 – 30	30	0 – 50 – 52	48	106.440
11	182 – 52 – 28	31	0 – 20 – 44	49	109.467
12	131 – 43 – 52	32	−2 – 09 – 48	50	147.505
13	127 – 38 – 35	33	1 – 04 – 34	51	113.220
14	179 – 30 – 48	34	−0 – 01 – 33	52	97.915
15	166 – 42 – 2	35	0 – 00 – 38	53	175.742
16	134 – 46 – 31	36	1 – 06 – 53	54	210.976
17	218 – 21 – 37	37	2 – 15 – 0	55	115.956
18	166 – 3 – 47	38	2 – 04 – 11	56	92.278
19	137 – 50 – 28				
20	108 – 54 – 21.8				

In traverse network, horizontal angle are measured with standard deviation $\sigma = \pm 3''$ and vertical angle are measured with standard deviation $\sigma = \pm 3''$. Standard deviation of distance measurement is $\sigma = \pm 0.02$ m.

We detected gross errors in three ways.

First: After determination of weight of observed data using Helmert variance component estimation, transformed into equivalent weight by Equation (1) and detected gross errors.

Second: After determination of weight of observed data using Helmert variance component estimation, transformed into equivalent weight using standardized residuals and detected gross errors.

Third: Detect gross errors applying Helmert variance component estimation and Robust estimation simultaneously for each iteration (Liu and Ma, 2002).

In detection of gross errors for above three methods, initial weight are taken as follows.

$$\begin{aligned} p_\beta &= \frac{m_\beta^2}{m_\beta^2} = 1, \\ p_s &= \frac{m_\beta^2}{m_s^2}, \\ p_\delta &= \frac{m_\beta^2}{m_\delta^2}. \end{aligned}$$

In first way, permissible value c_0 for horizontal angle, distance and vertical angle are taken as $3''$, 0.02 m and $6''$. As a result, six angles of number 1, 4, 7, 14, 16, 20 were detected as the ones containing gross errors (Figure 5).

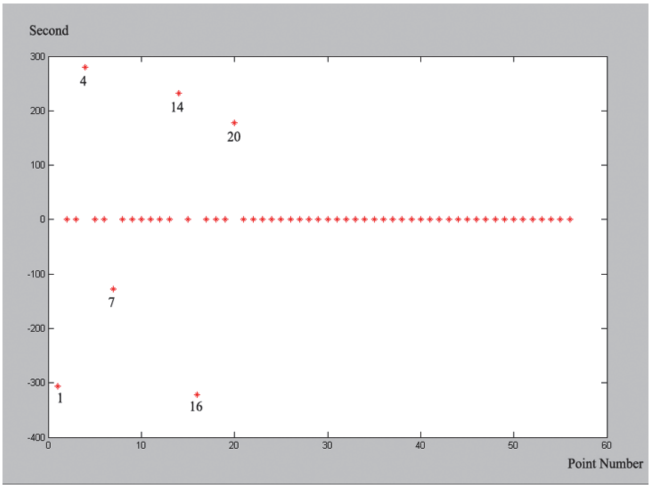

Fig. 5. Residuals of detection result in first method

As a result of second way, five angles of number 2, 3, 7, 14, 17 were detected as the ones containing gross errors under permissible value of standardized residual $c_0 = 1$ (Figure 6).

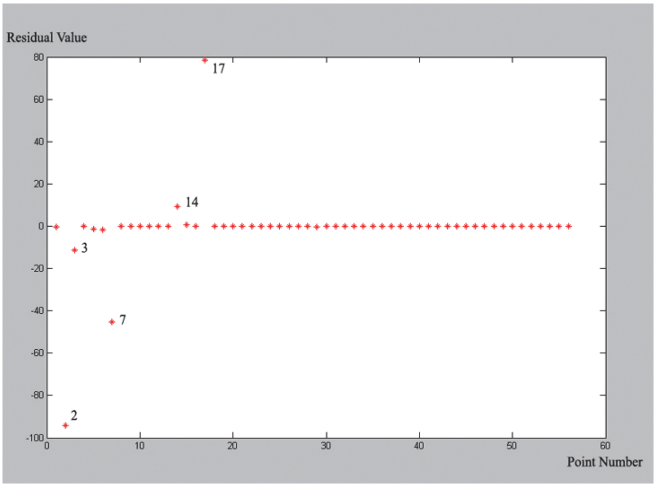

Fig. 6. Residuals of detection result in second method

There is a little difference in two method for detection of gross error, however, as a result of adjustment considering detected gross errors in traverse network, there is a significant(clear) difference in effect on standard deviation of coordinates (Figures 7–9). In Figures 7–9, curve 1 presents standard deviation of adjusted coordinate in first way and curve 2 presents standard deviation in second way. Variance ratio of adjusted result is $F_0 = 0.24$ in first way and $F_0 = 1.11$ in second way.

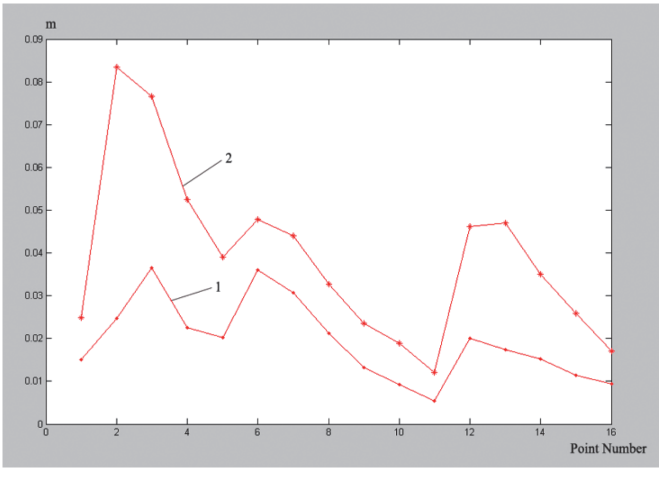

Fig. 7. Change curve of Standard deviation in X coordinates

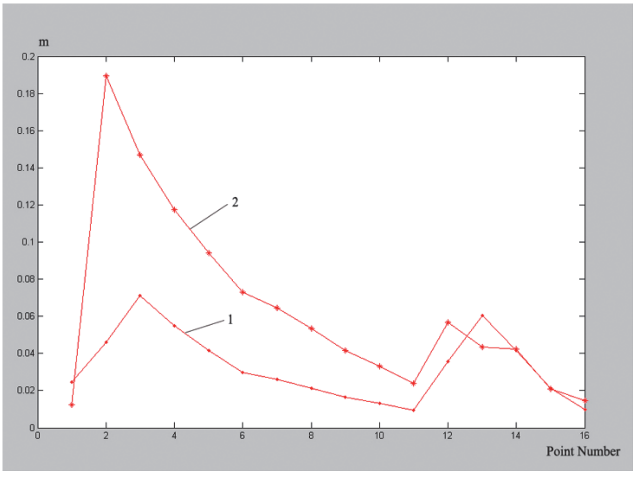

Fig. 8. Change curve of Standard deviation in Y coordinates

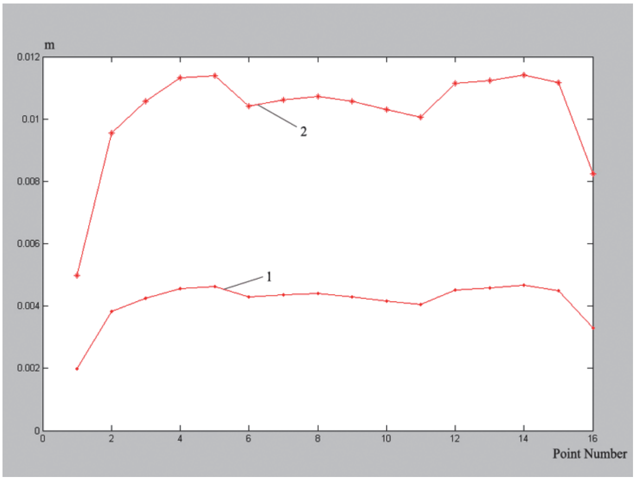

Fig. 9. Change curve of Standard deviation in Z coordinates

Result of detection of gross error in third way with same condition of first way is equal to result of first way. However, there is a difference in iteration number. In first way, iteration number is 29, but in third way it is 160.

3.3. Detection of gross error in GPS network adjustment

There is independent between baseline vectors observed in satellite geodetic network, however, since elements of baseline vectors are correlative, in GPS network adjustment, we detected gross errors by Robust estimation method using Equation (2) and compared with method by standardized residuals.

In satellite geodetic network presented in Figure 10, coordinates of given points and observed data were obtained from (Charles and Ghilant, 2006). Coordinates of given points 5, 6 are presented in Table 5. Observed baseline vector are presented in Table 6 and correlation matrix of baseline vector are presented in Table 7.

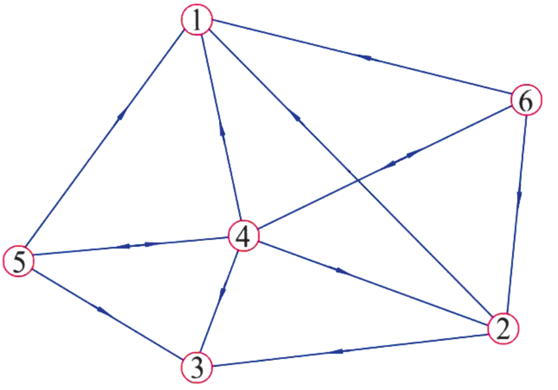

Fig. 10. GPS survey network

Table 5. Coordinate of given points

No	X, m	Y, m	Z, m
5	402.35087	−4652995.30109	4349760.77753
6	8086.03178	−4642712.84739	4360439.08326

Table 6. Baseline data observed for the network of Figure 10

No	Base line	ΔX, m	ΔY, m	ΔZ, m
1	5→1	11644.2232	3601.2165	3399.255
2	5→3	−5321.7164	3634.3754	3173.6652
3	6→1	3960.5442	−6681.2467	−7279.0418
4	6→2	−11167.6076	−394.5204	−907.9593
5	2→1	15128.6647	−6286.7054	−6371.0583
6	2→3	−1837.7459	−6253.8534	−6596.6697
7	4→5	−1116.4523	−4596.161	−4355.9062
8	4→1	10527.7852	−994.9377	−956.6246
9	4→3	−6438.1364	−962.0694	−1182.2305
10	4→2	−4600.3787	5291.7785	5414.4311
11	4→6	6567.2311	5686.2926	6321.9917
12	6→4	−6567.2310	−5686.3033	−6322.3807
13	5→4	1116.4577	4596.1553	4355.9141

Table 7. Covariance matrix elements

Base line	Q_{11}	Q_{12}	Q_{13}	Q_{22}	Q_{23}	Q_{33}
5→1	$9.88 \cdot 10^{-4}$	$-9.58 \cdot 10^{-6}$	$9.52 \cdot 10^{-6}$	$9.33 \cdot 10^{-4}$	$-9.52 \cdot 10^{-6}$	$9.82 \cdot 10^{-4}$
5→3	$2.15 \cdot 10^{-4}$	$-2.10 \cdot 10^{-6}$	$2.16 \cdot 10^{-6}$	$1.91 \cdot 10^{-4}$	$-2.10 \cdot 10^{-6}$	$2.00 \cdot 10^{-4}$
6→1	$2.30 \cdot 10^{-4}$	$-2.23 \cdot 10^{-6}$	$2.07 \cdot 10^{-6}$	$2.54 \cdot 10^{-4}$	$-2.23 \cdot 10^{-6}$	$2.25 \cdot 10^{-4}$
6→2	$2.70 \cdot 10^{-4}$	$-2.75 \cdot 10^{-6}$	$2.85 \cdot 10^{-6}$	$2.72 \cdot 10^{-4}$	$-2.72 \cdot 10^{-6}$	$2.72 \cdot 10^{-4}$
2→1	$1.46 \cdot 10^{-4}$	$-1.43 \cdot 10^{-6}$	$1.34 \cdot 10^{-6}$	$1.61 \cdot 10^{-4}$	$-1.44 \cdot 10^{-6}$	$1.30 \cdot 10^{-4}$
2→3	$1.23 \cdot 10^{-4}$	$-1.19 \cdot 10^{-6}$	$1.22 \cdot 10^{-6}$	$1.27 \cdot 10^{-4}$	$-1.21 \cdot 10^{-6}$	$1.28 \cdot 10^{-4}$
4→5	$7.47 \cdot 10^{-5}$	$-7.90 \cdot 10^{-7}$	$8.80 \cdot 10^{-7}$	$6.59 \cdot 10^{-5}$	$-8.10 \cdot 10^{-7}$	$7.61 \cdot 10^{-5}$
4→1	$2.56 \cdot 10^{-4}$	$-2.25 \cdot 10^{-6}$	$2.40 \cdot 10^{-6}$	$2.16 \cdot 10^{-4}$	$-2.27 \cdot 10^{-6}$	$2.39 \cdot 10^{-4}$
4→3	$9.44 \cdot 10^{-5}$	$-9.20 \cdot 10^{-7}$	$1.04 \cdot 10^{-6}$	$9.95 \cdot 10^{-5}$	$-8.90 \cdot 10^{-7}$	$8.82 \cdot 10^{-5}$
4→2	$9.33 \cdot 10^{-5}$	$-9.90 \cdot 10^{-7}$	$9.00 \cdot 10^{-7}$	$9.87 \cdot 10^{-5}$	$-9.90 \cdot 10^{-7}$	$1.20 \cdot 10^{-4}$
4→6	$6.64 \cdot 10^{-5}$	$-6.50 \cdot 10^{-7}$	$6.90 \cdot 10^{-7}$	$7.46 \cdot 10^{-5}$	$-6.40 \cdot 10^{-7}$	$6.04 \cdot 10^{-5}$
6→4	$5.51 \cdot 10^{-5}$	$-6.30 \cdot 10^{-7}$	$6.10 \cdot 10^{-7}$	$7.47 \cdot 10^{-5}$	$-6.30 \cdot 10^{-7}$	$6.62 \cdot 10^{-5}$
5→4	$6.61 \cdot 10^{-5}$	$-8.00 \cdot 10^{-7}$	$9.00 \cdot 10^{-7}$	$8.11 \cdot 10^{-5}$	$-8.20 \cdot 10^{-7}$	$9.37 \cdot 10^{-5}$

In order to detect gross errors using Robust estimation method in correlation observation, we add 0.1 m to x coordinate component (5^{th}) of baseline vector 2→1, 0.2 m to y coordinate component (13^{th}) of baseline vector 5→3 and -0.1 m to z coordinate component (33^{th}) of baseline vector 4→6.

Then transformed into equivalent correlation matrix by Equation (2) and detected gross errors. Here, number of observation 5, 13, 33 were detected certainly for any permissible value of $c_0 = 0.04$~0.2. While, in case of using standardized residual, same gross errors were detected for any threshold in interval of $k = 1.7$~3.2, not $k = 1.5$.

From experiment results, it is realized that detection of gross error using equivalent correlation matrix by residual is more convenient than by using standardized residual.

4. Conclusion

The detection of gross errors by Equation (1) and Equation (2) using equivalent weight based on L_1-estimation theory is effective and convenient more than detection using standardized residuals in geodetic networks adjustment.

Since main observations are angle in triangulation network, gross errors can be detected by only one algorithm in triangulation networks adjustment with different condition.

It is effective to use Robust estimation after determination of weight of observation data using Helmert variance component estimation.

Acknowledgement

The authors are indebted to anonymous editors and reviewers for their numerous constructive comments and helpful suggestions to improve the manuscript considerably. The authors wish to express his gratitude to all his colleagues and friends who helped to contribute this paper in various ways. I finally wish to acknowledge the assistance of School of Engineering Sciences of university in translation of my manuscript.

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