A description of the sources of magnetic field using edge values of the current vector potential

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Abstract: The paper discusses the method of a description of the magnetic field sources in systems with the stranded windings. The sources are determined on the basis of an obtained distribution of edge values of the current vector potential T_0. The formulas describing the magnetic field sources in the finite element (FE) space for the formulations using the scalar potential Φ and the vector potential A have been given. The approach for determining the T_0 distribution in the stranded windings of simple and complex geometries has been proposed.

Key words: numerical modeling; edge element analysis, current vector potential T_0, stranded windings

1. Introduction

One of the important aspects ensuring high credibility of the numerical calculations of systems with a magnetic field is the proper and accurate method for the field sources description. It can be stated that among the groups of the magnetic field sources in FE space, the most difficult aspect is to describe properly the sources for windings of complex geometry. For such problems the commonly used approaches, typically implemented in the commercial FE packages, are based on the Biot–Savart's law [1–3]. Despite the fact that the algorithm of the field sources formulation is very simple, the numerical complexity is high and in systems with complex geometry of the windings the calculations are time-consuming. An alternative approach has been proposed in [4] and it is based on a decomposition algorithm of the current vector potential T_0 regarding three independent components assigned to the nodes of the applied FE mesh. Although this approach is faster and requires less memory than the typical methods using Biot–Savart's law, it is suitable for classical – nodal formulation of the finite element method (FEM). Moreover, additional

computations to satisfy the Coulomb condition are needed. The vector potential T_0 has also been used for calculation of the sources of magnetic field in the methods where the field quantities are described by edge values, i.e. the Edge Element Method. In this method, the formulation is based on a minimization of energy functional $\mathfrak{I} = \int_\Omega |\nabla \times T_0 - J_0|^2 \, d\Omega$ over domain Ω and is often used to determine the distribution of the potential T_0 [5]. Nevertheless, such an approach requires the distribution of the current density vector J_0 to be defined a priori. A more general methodology has been given in [6, 7]. In order to describe the field sources, the method based on the definition of the turn density vector has been proposed. The turn density vector distribution is determined on the basis of a given distribution of the current density vector J_0. Nevertheless, as discussed in these papers there are only examples that cover windings of simple geometry for which the current density distribution can be defined analytically. For systems with windings of complex geometry, first the distribution of J_0 must be determined numerically [8]. Other interesting ways for describing the stranded windings in the FE space for the 3D systems have been proposed in [9]. The author of this contribution considers two approaches. The main idea of the methods proposed is based on the description of windings by lines (loops) in the FE space and by surfaces in the edge element formulation. Despite the universality of the discussed methods, the proposed algorithm of defining winding distribution strongly depends on the number of turns in the considered coil.

In the paper the authors present and describe an alternative way to define the sources of magnetic field that is universal and computationally effective for both simple and complex current paths in the windings. In the discussed approach, it is assumed that the distribution of turns in the system can be treated as the distribution of the current paths inside the region described by the contour of a winding defined by means of current flow field equations [10]. For the description of the winding, the current vector potential T_0 can be successfully applied, and on the basis of the obtained T_0, the distribution of the magnetic field sources can be determined for both the scalar potential Φ and the vector potential A formulations. The expressions describing the magnetic field sources for the above formulations have been given. The background for the proposed methodology of the winding representation and description in the FE space using the current vector potential T_0 has been given in [11].

2. Description of the winding using T_0 formulation

In the proposed approach, the distribution of the current vector potential T_0 for a winding region, for example a simple coil shown in Fig. 1, can be determined using the following formulas:

$$\nabla \times \boldsymbol{\rho} \nabla \times T_0 = 0 \big|_{\Omega_c} \tag{1a}$$

and

$$\nabla \times \nabla \times T_0 = 0 \big|_{\Omega_n}, \tag{1b}$$

taking into account the boundary conditions:

$$T_0 = \boldsymbol{\tau}_0 \big|_{\Gamma_{T_0}} \tag{2a}$$

and

$$\boldsymbol{T}_0 = 0\big|_{\Gamma_0 + \Gamma_\Omega},\tag{2b}$$

where: $\boldsymbol{\rho}$ is the tensor describing the material resistivity, and $\boldsymbol{\tau}_0$ is the function that describes the distribution of $\boldsymbol{T}_0$ on the boundary surface $\Gamma_{\boldsymbol{T}_0}$. Equations (1) have been derived on the basis of the equations of a static current flow field for the considered domain Ω, i.e.:

$$\nabla \times \boldsymbol{E} = 0,\tag{3a}$$

$$\nabla \circ \boldsymbol{J}_0 = 0,\tag{3b}$$

$$\boldsymbol{E} = \boldsymbol{\rho}\boldsymbol{J}_0,\tag{3c}$$

and the relation between the current density vector $\boldsymbol{J}_0$ inside a coil and the current vector potential $\boldsymbol{T}_0$ is as follows:

$$\boldsymbol{J}_0 = \nabla \times \boldsymbol{T}_0,\tag{4}$$

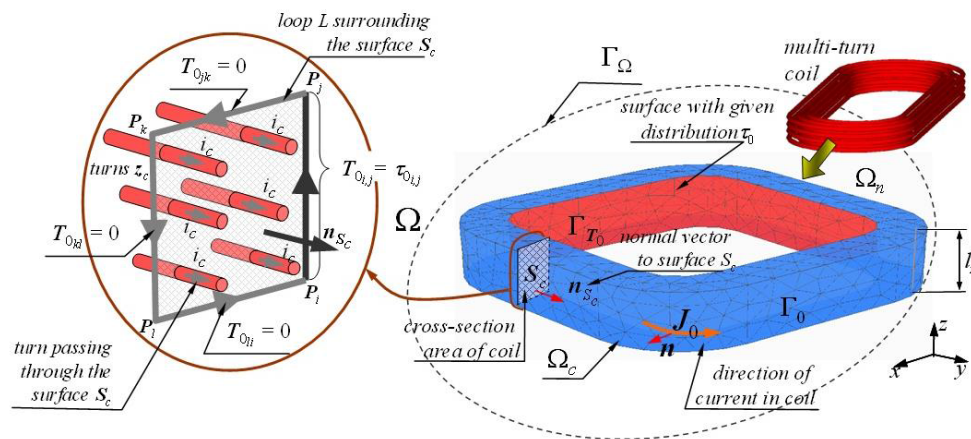where: $\boldsymbol{E}$ is the vector of electric field intensity.

Fig. 1. A considered multi-turn coil [11]

Taking into account the boundary conditions (2) in (1), the distribution of the current vector potential $\boldsymbol{T}_0$ in the domain Ω can be obtained from:

$$\nabla \times \boldsymbol{\rho}\nabla \times \boldsymbol{T}_0\big|_{\Omega_c} + \nabla \times \boldsymbol{\rho}\nabla \times \boldsymbol{\tau}_0\big|_{\Gamma_{T_0} \to \Omega_c} = 0,\tag{5a}$$

for a sub-domain Ω_c, and

$$\nabla \times \nabla \times \boldsymbol{T}_0\big|_{\Omega_n} + \nabla \times \nabla \times \boldsymbol{\tau}_0\big|_{\Gamma_{T_0} \to \Omega_n} = 0,\tag{5b}$$

for a sub-domain Ω_n.

The values of the boundary function $\boldsymbol{\tau}_0$ can be determined numerically. The simplest way is to use the relation between the magnetomotive force θ, the current density vector $\boldsymbol{J}_0$ passing through the cross-section surface of the coil $\boldsymbol{S}_c$ and the potential $\boldsymbol{T}_0$ along the line L surrounding the surface $\boldsymbol{S}_c$ (see, Fig. 1), i.e.:

$$\theta = z_c i_c = \int\limits_{S_c} \boldsymbol{J}_0 \, \mathrm{d}\boldsymbol{s} = \oint\limits_{L(S_c)} \boldsymbol{T}_0 \, \mathrm{d}\boldsymbol{l} = \boldsymbol{\tau}_{0i,j} l_z, \tag{6}$$

and the following condition:

$$(\nabla \times \boldsymbol{\tau}_0) \circ \boldsymbol{n} = 0, \tag{7}$$

where: i_c is the value of the current in the winding, z_c is the number of turns, $\tau_{0i,j}$ is the value of the boundary function assigned to the edge $P_i P_j$ of loop L (Fig. 1), l_z is the length of the edge $P_i P_j$ representing the height of the winding, and $\boldsymbol{n}$ is the normal vector to the considered boundary surface. The values of the function τ_0 are defined on the basis of at least one chosen cross-section of the coil $\tau_{0i,j}$ according to (6) and next, the obtained results are delivered to (7). In the case of representation of coils of complex geometry it is beneficial to use more $\tau_{0i,j}$ values for several different cross-sections of the coil. An increasing number of the base cross-sections enables faster determination of the τ_0 function distribution.

The approach presented above is applicable when the current value in the coil i_c is known in advance. When the current value is unknown a priori, e.g.: when the field-circuit co-simulations are performed for a system with a nonlinear magnetic circuit, it is advantageous to introduce the linear wire density vector $\boldsymbol{K}_c$ [7] which describes the edge density of turns, where:

$$\boldsymbol{K}_c i_c = \boldsymbol{T}_0. \tag{8}$$

Assuming that the test current value i_c is equal to 1 A and then solving Equation (5), the distribution of $\boldsymbol{T}_0$ is determined that corresponds to the distribution of the vector $\boldsymbol{K}_c$. On the basis of the determined distribution of the vector $\boldsymbol{K}_c$, the wire density vector $\boldsymbol{N}_c$ describing the turn density in the coil space can be calculated using the following formula:

$$\boldsymbol{N}_c = \nabla \times \boldsymbol{K}_c, \tag{9}$$

where:

$$\boldsymbol{J}_0 = \boldsymbol{N}_c i_c. \tag{10}$$

The determined vectors $\boldsymbol{K}_c$ and $\boldsymbol{N}_c$ are very often used in the analysis of systems with an electromagnetic field and external supply as well as load circuits for development of 3D field-circuit models. The discussed vectors are also used for determining the flux linkages and the inductances of the coils defined in the 3D space.

The obtained equations for $\boldsymbol{T}_0$ formulation are solved using numerical methods that are based on a space discretization. The most commonly used one is the FEM. However, other field methods can also be applied, i.e.: the Finite Integration Technique, the Cell Method or the Equivalent Resistance Network.

3. The edge element equations for T_0 formulation

In the paper, in order to solve equations describing the distribution of the current vector potential T_0 in the windings, the edge element approach of the FEM has been applied. The approach presented in [12] and [13] has been introduced. It has been respected that edge element (EE) equations for the current vector potential T_0 correspond to the loop equations for the electric facet network (EFN) with branches crossing the element faces and connecting the midpoints of the elements [12]. The following forms of EE equations have been obtained:

$$k_e R_{\rho g_c} k_e^T i_{0c} = R_{\rho g_c} i_{0\tau} \quad \text{in} \ \Omega_c \tag{11a}$$

and

$$k_e R_{g_n} k_e^T i_{0n} = R_{g_n} i_{0\tau} \quad \text{in} \ \Omega_n. \tag{11b}$$

The vectors i_{0c}, i_{0n} appeared in (11) represent the edge values of the potential T_0 in domains Ω_c and Ω_n respectively; the vector $i_{0\tau}$ describes the known source edge values of T_0 on the surface Γ_{T_0}, and k_e is the full loop matrix of the considered EFN. $R_{\rho g_c}$ and R_{g_n} are matrices of coefficients of the EE equations for domains Ω_c and Ω_n respectively; and can be determined on the basis of:

$$R_{\rho g_c} = \int\limits_{V_e} w_{fq} \rho w_{fp} \, dv \quad \text{and} \quad R_{g_n} = \int\limits_{V_e} w_{fq} w_{fp} \, dv, \tag{12}$$

where: w_{fq}, w_{fp} are the base functions for interpolation of the facet element [12].

The values of $i_{0\tau}$ on the boundary surface Γ_{T_0} have been determined at the beginning and used in (11) as source values. The values of vector $i_{0\tau}$ have been determined on the basis of the condition (7), which will take for the applied EE the following form:

$$k_e i_{0\tau} = 0 \big|_{\Gamma_{T_0}}. \tag{13}$$

As a RHS for solving (13), the edge values of the vector T_0 (defined by the function τ_0) $i_{0\tau, P_u P_v}$ for mesh edges $P_u P_v$ have been considered. It is obvious that $i_{0\tau, P_u P_v}$ can be different from zero only for edges that lay on the considered cross-section of the coil where the source function is defined, i.e. that considered $P_u P_v$ edge corresponds to the edge $P_i P_j$ on a chosen cross-section of the coil (see, Fig. 2). For example for the studied case from Fig. 1 the value of $i_{0\tau, P_u P_v}$ can be calculated using:

$$i_{0\tau, P_u P_v} = \int\limits_{P_u}^{P_v} T_0 \, dl = \int\limits_{P_u}^{P_v} \tau_0 \, dl = \frac{z_c i_c}{l_z} \cdot l_{u,v}, \tag{14}$$

where: $l_{u,v}$ is the length of the edge $P_u P_v$ (Fig. 2).

It can be noticed that a multiplication $k_e i_{0\tau}$ appearing in (13) represents the current i_{s0} that corresponds to the facet values of the current density vector J_0 [14] and it is equal to zero for surfaces on the border between the considered domains.
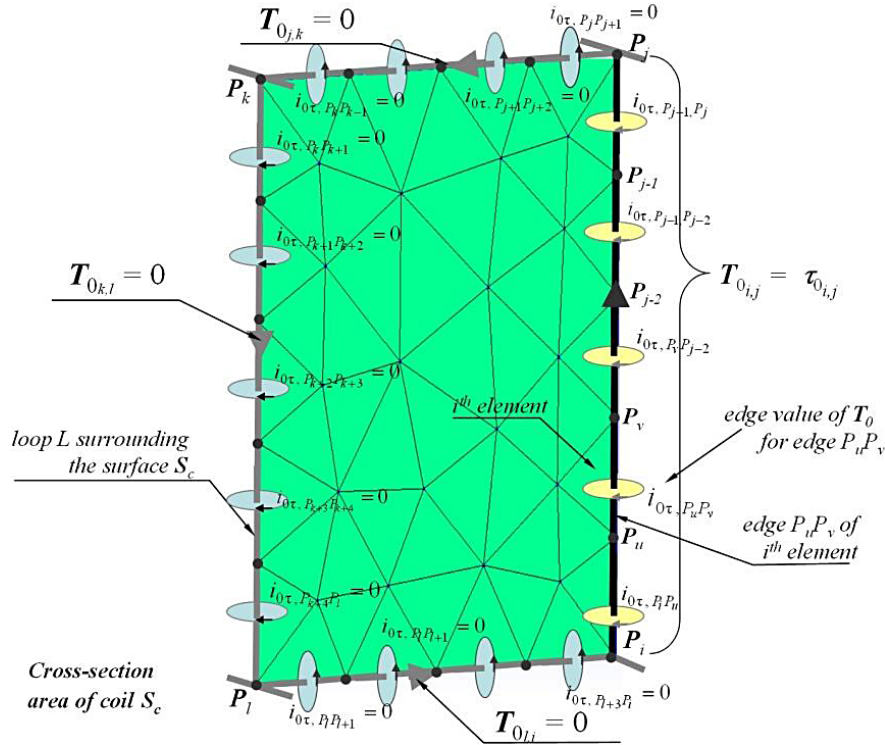
Fig. 2. The source function definition for a considered cross-section of the coil

4. Magnetic field source calculation

The starting point for the determination of the magnetic field sources is the obtained distribution of the current vector potential, i.e. the distribution of the edge values i_0 of the vector T_0, where $i_0 = [i_{0c}, i_{0\tau}, i_{0n}]$. In the case of formulation using the scalar potential Φ, the FE equations areequivalent to the nodal equations of the magnetic edge network (MEN) [12]. In the MEN the field sources, i.e. magnetomotive forces (*mmf*'s), are assigned to edges of elements. Because the edge values i_0 are also assigned to the branches of the MEN, the vector of the branch *mmf*'s Θ is directly calculated from:

$$\Theta = i_0. \tag{15}$$

The field sources description is different for a case when the magnetic field in considered system is described by the magnetic vector potential A. In such a case the FE equations are equivalent to the loop equations of the magnetic facet network (MFN) [12]. The field sources are described by the loop *mmf*'s θ_o assigned to the loops of the MFN. In order to calculate the loop *mmf*'s on the basis of the edge values of the T_0 potential, the transformation of the edge quantities to the values assigned to the facets of elements must be applied first. In the next step,

these values are transformed to the magnetomotive forces assigned to the loops of the MFN. The source vector of the loop *mmf*'s $\boldsymbol{\theta}_o$ can be determined using the following formulas:

$$\boldsymbol{\theta}_o = \boldsymbol{k}_s^T \boldsymbol{C} \boldsymbol{i}_0 = \boldsymbol{C}^T \boldsymbol{k}_s \boldsymbol{i}_0 \,, \qquad (16)$$

where: $\boldsymbol{k}_s$ is the matrix transforming the values assigned with an element edge into the values assigned to the element facets in the MFN; whereas $\boldsymbol{C}$ is the matrix converting quantities associated with edges of the elements to quantities associated with the facets of elements [12].

5. Numerical examples

In the first stage of research, the large number of calculations has been performed to test the proposed method of the winding description using the current potential vector $\boldsymbol{T}_0$ and the EE formulation of the FEM. For each test, the EE equations have been formulated and solved using the Incomplete Cholesky Conjugate Gradient (ICCG) algorithm. In the paper, determined distributions of the current density vector $\boldsymbol{J}_0$ inside of two types of coils of the high-voltage asynchronous motor have been calculated using the proposed method and presented in Fig. 3. The coils of single (Fig. 3a) and double layer windings (Fig. 3b) have been considered. In order to prove the accuracy and the quality of the determined current density distributions, there have been two test tasks worked out: in the first task (a) – the flow continuity equation $\nabla \circ \boldsymbol{J}_0 = 0$ has been checked; and in the second task (b) the values of magnetomotive force θ on the chosen cross-sections of the considered test coil have been compared. In the case of the first task, the numerical integration of $\nabla \circ \boldsymbol{J}_0$ gives a result on a level of 10^{-13} for both the coils. The second

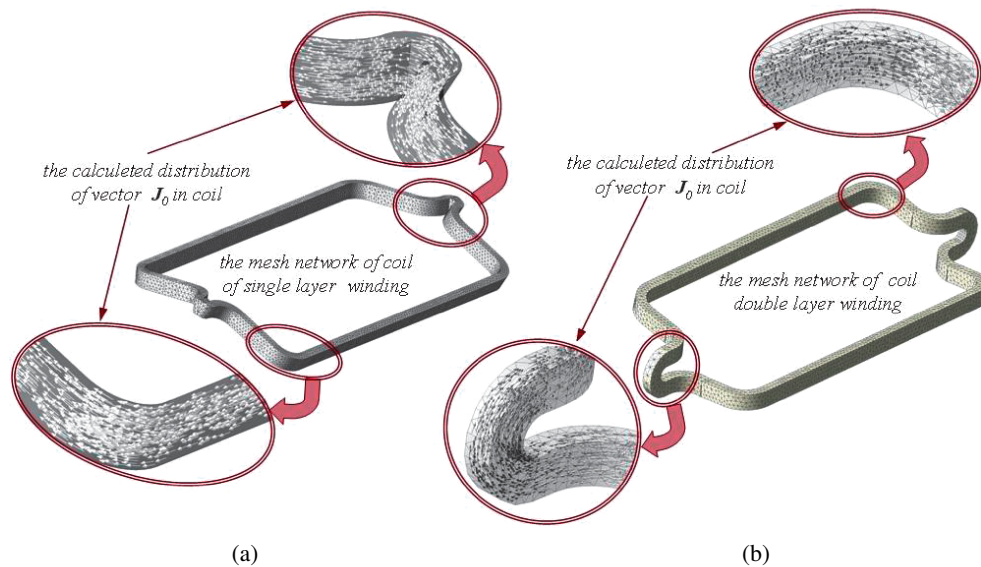(a)   (b)

Fig. 3. The distribution of the current density vector $\boldsymbol{J}_0$ in a coil of: (a) a single layer winding;
(b) a double layer winding

test showed that the magnetomotive forces at different cross-sections of the coils differ in the 7–8th decimal place.

In the next stage of conducted studies, the authors focused on a comparative analysis of the field distributions and the integral parameters of the selected systems with stranded windings. As a case study problem, the authors decided to investigate a known benchmark problem TEAM (*Testing Electromagnetic Analysis Methods*) Workshop No. 7 (see, Fig. 4) proposed by the International Compumag Society. Here, the quasi-static problem for the system with supply frequency f equal to 50 Hz has been considered. For the considered problem the results are known and the measurement data are available in [15]. To determine the field distribution in the studied system, the developed in-house software has been used. The elaborated computer code has been based on $A-V-T_0$ formulation and the edge element approach. In the applied formulation, the distribution of the magnetic field has been described using the edge quantities of the magnetic vector potential A, while the distribution of induced eddy currents has been described by nodal quantities of the electric scalar potential V. The supply current in the coil has been described by the discussed approach of using the current vector potential. The field sources in the coil domain have been determined according to the formula (16). Finally, the system of coupled equations describing the electromagnetic field can be expressed in the following form:

$$\begin{bmatrix} \boldsymbol{k}_n^T \boldsymbol{G} \boldsymbol{k}_n & -\mathrm{i}\omega \boldsymbol{k}_n^T \boldsymbol{G} \\ -\boldsymbol{G} \boldsymbol{k}_n & \boldsymbol{k}_s^T \boldsymbol{R}_\mu \boldsymbol{k}_s + \mathrm{i}\omega \boldsymbol{G} \end{bmatrix} \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{\varphi}_e \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{C}^T \boldsymbol{k}_s \boldsymbol{i}_0 \end{bmatrix}, \tag{17}$$

where: i represents the imaginary unit, ω is the electric angular velocity, V is the vector of nodal potential of V, $\boldsymbol{\varphi}_e$ represents the vector of the loop fluxes, G is the matrix of branch conductances for the electrical edge network (EEN) [12], $\boldsymbol{R}_\mu$ is the matrix of branch reluctances of the MFN's, $\boldsymbol{k}_n$ is the transposed nodal incidence matrix of the EEN.
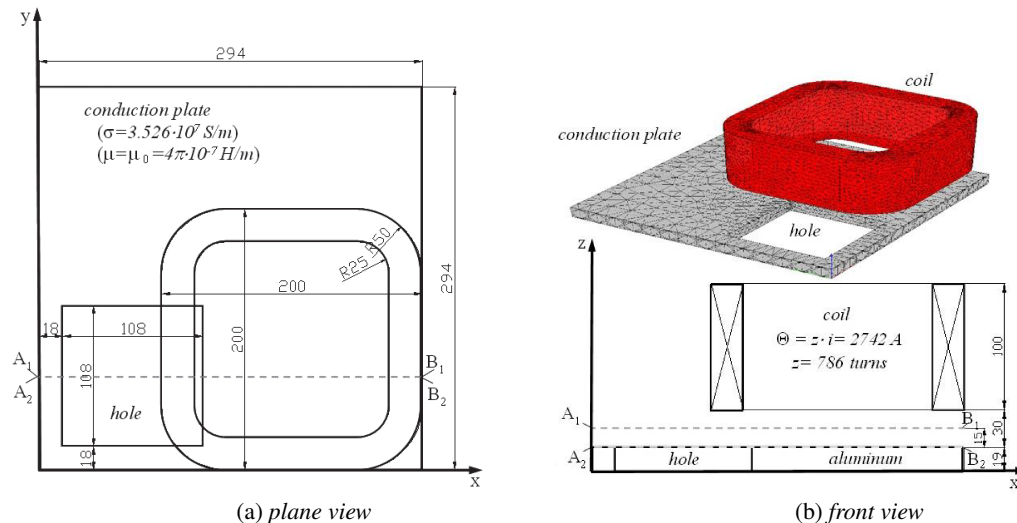

Fig. 4. The system considered containing induced currents TEAM Workshops Problem No. 7 [15]

The comparison between the measurements and the simulation of the selected distributions of (a) magnetic flux density z component B_z along the line A_1-B_1 and (b) current density y component J_y along the line A_2-B_2 for supply frequency f equal to 50 Hz has been shown in Fig. 5. An arrangement of the test lines A_1-B_1 and A_2-B_2 has been shown in Fig. 4. Comparing the results, the satisfactory concordance between measurements and simulations has been achieved.
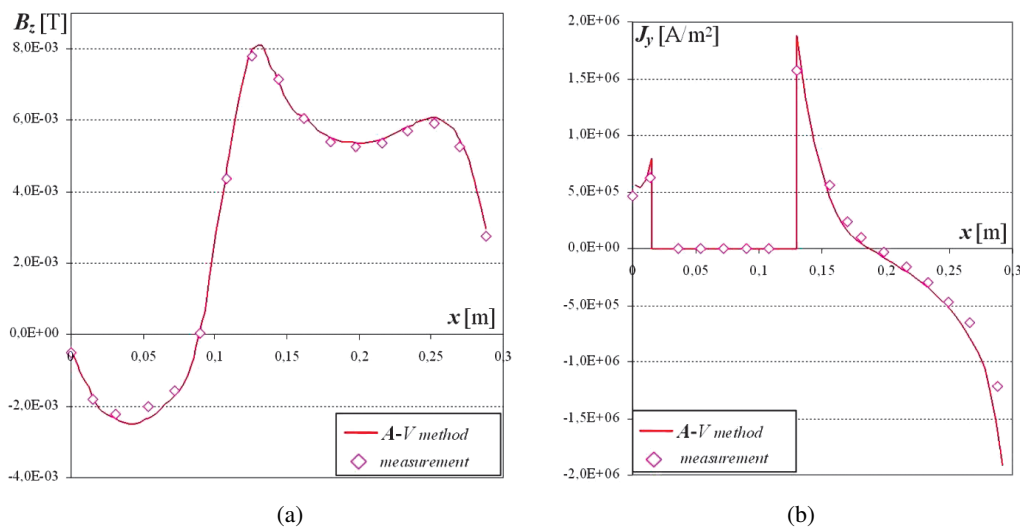

Fig. 5. The magnitude of: (a) a component B_z of a magnetic flux density along the line A_1-B_1; and (b) a component J_y of a current density vector along the line A_2-B_2

6. Conclusions

In the paper, the representation of the stranded coils in the FE space using the current vector potential T_0 has been presented and discussed. The formulas for calculations of the sources of magnetic field on the basis of the determined distribution of T_0 have been given for both the scalar potential Φ and the vector potential A formulations. The approach for determining the distribution of T_0 inside a coil domain on the basis of a given distribution on the boundary surfaces has been proposed. The advantage of the proposed method is its universality; the presented way of the source description can be applied for coils of simple geometries and of complex one as well (current flow paths). The usefulness and accuracy of the presented approach has been confirmed by several numerical examples preformed using the FEM code developed by the authors. The single and double layer coils of high voltage asynchronous motor have been tested. The method and developed software have been also benchmarked on the basis of TEAM Workshop Problem No. 7. The satisfactory concordance between the measurements and the simulations has been achieved.

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