Central European Journal of Economic Modelling and Econometrics

Bayesian Analysis of Weak Form Polynomial Reduced Rank Structures in VEC Models

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Submitted: 3.12.2012, Accepted: 15.04.2013

Abstract

The main goal of the paper is the Bayesian analysis of weak form polynomial serial correlation common features together with cointegration. In the VEC model the serial correlation common feature leads to an additional reduced rank restriction imposed on the model parameters.
After the introduction and discussion of the model, the methods will be illustrated with an empirical investigation of the price-wage nexus in the Polish economy.
Additionally, consequences of imposing such additional short-run restrictions for permanent-transitory decomposition will be discussed.

Keywords: cointegration, Bayesian analysis, polynomial common cyclical features, permanent-transitory decompostion

JEL Classification: C11, C32, C53

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1 Introduction

While modelling time series we try to capture their most important features such as trends, serial correlation, seasonality etc., and while analysing a group of series we try to include in the model the most important properties of those series and − as they are modelled together − to find such features which are common. Following the idea of Engle and Kozicki (1993), we will focus our attention on features which are present in the analysed series, but there exists at least one linear combination of these series which does not possess these features. One of the most famous examples of this idea is cointegration. When a group of series share common stochastic trends, we say that they are cointegrated, so there exists a linear combination of them which lowers the order of integration of the analysed series. Another example of a cofeature is a serial correlation common feature. In such a case there exists a linear combination of the series which is an innovation with respect to past information. In 1993 Vahid and Engle considered cointegration for the levels of I(1) series and a serial correlation common feature for their first differences, jointly in one model. In the VEC model the serial correlation common feature leads to an additional reduced rank restriction imposed on the model parameters.
Let us consider the n-dimensional cointegrated process x_t and write it in the VEC form:

$$\Delta x_t = \alpha\beta' x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Phi D_t + \varepsilon_t = \alpha\beta' x_{t-1} + \Gamma' z_t + \Phi D_t + \varepsilon_t, \qquad (1)$$

where $z_t' = (\Delta x_{t-1}', \Delta x_{t-2}', \ldots, \Delta x_{t-k+1}')$, $\Gamma = (\Gamma_1, \ldots, \Gamma_{k-1})'$, $\varepsilon_t \sim iiN^n(0,\Sigma)$, $t = 1, 2, \ldots, T$ and the vector D_t contains deterministic terms.
In the case of the common serial correlation among the first differences of the series all Γ's and α must have less than full rank and their left null spaces must overlap (Vahid, Engle 1993), which leads us to the following model:

$$\Delta x_t = \gamma^* \delta_0^{*'} \beta' x_{t-1} + \sum_{i=1}^{k-1} \gamma^* \delta_i^{*'} \Delta x_{t-i} + \Phi D_t + \varepsilon_t = \gamma^* \delta^{*'} z_t^* + \Phi D_t + \varepsilon_t, \qquad (2)$$

where $\delta^{*'} = (\delta_0^{*'}, \delta_1^{*'}, \ldots, \delta_{k-1}^{*'})$, $z_t^* = (x_{t-1}'\beta, \Delta x_{t-1}', \ldots, \Delta x_{t-k+1}')'$. Matrices $\gamma^*_{n\times(n-s)}$ and $\delta^*_{(n(k-1)+r)\times(n-s)}$ have full rank.
There exist s linear combinations of the process $\Delta x_t - \Phi D_t$ which are innovations: $\gamma_\perp^{*'}(\Delta x_t - \Phi D_t) = \gamma_\perp^{*'}\varepsilon_t$, where $\gamma_\perp^*$ denotes a full column rank $n \times s$ matrix such that $\gamma^{*'}\gamma_\perp^* = 0$.
By our assumption, the first differences of the series x_t have the following Wold representation (see, e.g., Johansen 1996, Centoni, Cubadda 2003, Lütkepohl 2007): $\Delta x_t = \tilde{\Phi} D_t + C(L)\varepsilon_t$, where $\tilde{\Phi} = C(L)\Phi D_t$, and $C(L) = I_n + \sum_{j=1}^{\infty} C_j L^j$ is such that $\sum_{j=1}^{\infty} j||C_j|| < \infty$, where $||C_j||$ denotes the Euclidean norm of C_j. As a consequence

any cointegrated process can be written as a sum of a multivariate random walk (τ_t), a stationary process (κ_t) and deterministic values (δ_t), which is known as a multivariate Beveridge-Nelson decomposition:

$$x_t = \delta_t + \tau_t + \kappa_t = \delta_t + C(1)\sum_{i=0}^{t-1}\varepsilon_{t-i} + C^*(L)\varepsilon_t, \tag{3}$$

where $C(1) = \beta_\perp(\alpha'_\perp(I_n - \sum_{i=1}^{k-1}\Gamma_i)\beta_\perp)^{-1}\alpha'_\perp$, $C^*(L)\varepsilon_t = \sum_{j=0}^{\infty} C_j^*\varepsilon_{t-j}$ and $C_j^* = -\sum_{i>j} C_i$.
Vahid and Engle (1993) showed that $\gamma_\perp^{*'}\kappa_t = 0$, so $\gamma_\perp^*$ cancels both past information of the series Δx_t and the stationary part of x_t, which is called the cycle (κ_t), so the process x_t has s common cycles. For this reason, this type of comovement is an example of the Common Cyclical Features idea.
The above-presented type of short-run comovements is very strong and the number of common serial correlation features cannot exceed the number of common trends ($n - r$). In 2006 Hecq, Palm and Urbain introduced a model with the so called weak form reduced rank structures, which do not place limitations on the number of common features. In this case there exists a linear combination of the first differences adjusted for long-run effects, which is an innovation. This restriction implies the reduced rank structures only on the matrices of the short-term part of the model, i.e. on Γ's:

$$\begin{aligned}\Delta x_t &= \alpha\beta' x_{t-1} + \gamma\delta'_1\Delta x_{t-1} + \cdots + \gamma\delta'_{k-1}\Delta x_{t-k+1} + \Phi D_t + \varepsilon_t \\ &= \alpha\beta' x_{t-1} + \gamma\delta' z_t + \Phi D_t + \varepsilon_t,\end{aligned} \tag{4}$$

where $\delta' = (\delta'_1,\delta'_2,\ldots,\delta'_{k-1})$. Matrices $\gamma_{n\times(n-s)}$ and $\delta_{n(k-1)\times(n-s)}$ are of full column rank.
There exist s linear combinations of the process $\Delta x_t - \alpha\beta' x_{t-1} - \Phi D_t$ which are innovations. In the case of the weak common cyclical features the short- and long-run dynamics are unrelated contrary to the strong case, where they are similar.
Hecq, Palm and Urbain (2006) showed that such a definition is not invariant to alternative VEC models reparameterisations (such as the ones where $\beta' x_{t-p}$ appears instead of $\beta' x_{t-1}$).
Cubadda (2007) showed that in the case of the weak form serial correlation common feature there exists a first-order polynomial matrix $\gamma_\perp(L) \equiv \gamma_\perp - (\beta\alpha' + I_n)\gamma_\perp L$ such that $\gamma_\perp(L)' x_t = \gamma'_\perp\Phi D_t + \gamma'_\perp\varepsilon_t$ and $\gamma_\perp(L)'\kappa_t = \gamma'_\perp(I_n - C(1))\varepsilon_t$, so it cancels the dependence upon the past of both the cycles (κ_t) and the series x_t adjusted for deterministic terms (see also Centoni, Cubadda 2011). The definition based on the polynomial matrix $\gamma_\perp(L)$ is invariant to the above-mentioned VEC model reparameterisations (Cubadda 2007).
Ericsson (1993) pointed out that it would be useful to consider also non-contemporaneous relations among the analysed times series, because under the assumptions of the serial correlation common features their analysis is excluded and

they might be important. In 1997 Vahid and Engle proposed the model and the test for the non-synchronised comovement of the processes. Later, Cubadda and Hecq (2001) introduced the concept of the polynomial serial correlation common features, which allows us to describe non-contemporaneous short-run comovements among the first differences of the integrated processes.

The series Δx_t has s polynomial serial correlation common features of order 1, iff there exists an $n \times s$ full column matrix ψ_P^* such that $\psi_P^{*'}\Gamma_1 \neq 0$, and the VEC model can be rewritten in the following form:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \gamma_P^* \delta_P^{*'} (x'_{t-1}\beta, \Delta x'_{t-2}, \ldots, \Delta x'_{t-k+1})' + \Phi D_t + \varepsilon_t, \qquad (5)$$

where $\gamma_P^* = \psi_{P\perp}^*$. The matrices $\gamma^*_{P\ n\times(n-s)}$ and $\delta^*_{P\ (n(k-2)+r)\times(n-s)}$ have full column rank.

In this case there exists a polynomial matrix $\psi^*(L) = \psi_P^* - \Gamma_1' \psi_P^* L$ which cancels the dependence upon the past of the process Δx_t, i.e. $\psi^*(L)'\Delta x_t = \psi_P^{*'}\Phi D_t + \psi_P^{*'}\varepsilon_t$. The same polynomial matrix transforms the cyclical part of the series x_t into an innovation process: $\psi^*(L)'\kappa_t = -\psi_P^{*'}\Gamma_1 C(1)\varepsilon_t$ (see Cubadda, Hecq 2001 and Centoni, Cubadda 2011).

Of course, it is possible to merge the weak form serial correlation common feature and the polynomial serial correlation common feature. This way the weak form polynomial serial correlation common feature is obtained (Cubadda 2007).

The series Δx_t has s weak form polynomial serial correlation common features of order 1 (WFP(1)), iff there exists an $n \times s$ full column matrix ψ_P such that $\psi_P^{'}\alpha \neq 0$, $\psi_P^{'}\Gamma_1 \neq 0$, and the VEC model can be rewritten in the following form:

$$\Delta x_t = \alpha\beta' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \gamma_P \delta_P^{'} (\Delta x'_{t-2}, \ldots, \Delta x'_{t-k+1})' + \Phi D_t + \varepsilon_t, \qquad (6)$$

where $\gamma_P = \psi_{P\perp}$. Matrices $\gamma_{P\ n\times(n-s)}$ and $\delta_{P\ n(k-2)\times(n-s)}$ are of full column rank.

In this case there exists a polynomial matrix $\psi_P(L) = \psi_P - (\beta\alpha' + I_n + \Gamma_1')\psi_P L + \Gamma_1'\psi_P L^2$ such that $\psi_P(L)'x_t = \psi_P'\Phi D_t + \psi_P'\varepsilon_t$, so it cancels the dependence of x_t upon the past. This polynomial matrix also transforms the cycles of the series x_t into a VMA(1) process, so into a process with shorter memory:

$$\psi_P(L)'\kappa_t = \psi_P'[I_n - C(1)]\varepsilon_t - \psi_P'\Gamma_1 C(1)\varepsilon_{t-1},$$

see Cubadda 2007 and Centoni, Cubadda 2011).

Definitions of polynomial serial correlation common features may be extended for higher orders (see e.g. Cubadda, Hecq 2001).

In the present paper we are interested in the Bayesian analysis of the last form of the above-described common features. In the next section the Bayesian VEC-WFP model is introduced, Section 3 describes one type of the permanent-transitory decomposition for the analysed series, Section 4 presents the analysis of the price-wage nexus in the Polish economy based on that model. The final section concludes.

2 The Bayesian VEC model with weak form polynomial serial correlation common features

In this section we will focus our attention on the Bayesian analysis of the weak form polynomial serial correlation common features of order p, which will be conducted via the model of the following form:

$$\Delta x_t = \alpha\beta_+' x_{t-1}^+ + \Gamma' w_t + \gamma_P \delta_P^{'} z_t + \Phi D_t + \varepsilon_t,\ \varepsilon_t \sim iiN^n(0,\Sigma),\ t = 1, 2, \ldots, T \quad (7)$$

where $w_t = (\Delta x_{t-1}', \ldots, \Delta x_{t-p}')'$, $z_t = (\Delta x_{t-p+1}', \ldots, \Delta x_{t-k+1}')'$, $\beta_+ = (\beta,'\phi')'$, $x_{t-1}^+ = (x_{t-1}',\ d_t)'$. The term d_t incorporates deterministic components into cointegrating relations. To simplify the notation let us write the basic model (7) in a matrix form:

$$Z_0 = Z_1\beta_+\alpha' + W\Gamma + Z_2\delta_P\gamma_P' + Z_3\Gamma_s + E = Z_1\Pi' + W\Gamma + Z_2\Gamma_P + Z_3\Gamma_s + E, \quad (8)$$

where $Z_0 = (\Delta x_1, \Delta x_2, \ldots, \Delta x_t)'$, $Z_1 = (x_0^+, x_1^+, \ldots, x_{T-1}^+)'$, $W = (w_1, w_2, \ldots, w_T)'$, $Z_2 = (z_1, z_2, \ldots, z_T)'$, $Z_3 = (D_1, D_2, \ldots, D_T)'$, $\Gamma_s = \Phi'$ and $E = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T)'$.
In this model we have two reduced rank matrices (Π and Γ_P) which are decomposed as the products of full rank matrices, i.e. $\Pi = \alpha\beta_+'$ and $\Gamma_P = \delta_P\gamma_P'$. It is commonly known that such a decomposition is not invariant, i.e. for any full rank matrices of adequate dimensions, C_Π and C_Γ, the following equalities are fulfilled: $\alpha\beta' = \alpha C_\Pi C_\Pi^{-1}\beta'$, $\gamma_P\delta_P' = \gamma_P C_{\Gamma_P} C_{\Gamma_P}^{-1}\delta_P'$. For this reason we should estimate spaces spanned by the matrices β and δ_P (which are elements of the Grassmann manifolds) rather than these matrices. We have decided to use the scheme of estimation proposed by Koop, León-González and Strachan (2010) for the VEC models, which takes into account the curved geometry of the parameters and at the same time allows for the use of the parameter-augmented Gibbs sampling scheme to sample from the posterior distribution. In the context of the VEC models with an additional reduced rank restriction this scheme was employed by Wróblewska (2011).
For Π and Γ_P two parameterisations will be used:

$$\alpha\beta' = (\alpha M_\Pi)\left(\beta M_\Pi^{-1}\right)' \equiv AB', \quad (9)$$

where M_Π is an $r \times r$ symmetric positive definite matrix, and

$$\gamma\delta' = (\gamma M_{\Gamma_P})\left(\delta M_{\Gamma_P}^{-1}\right)' \equiv G_P D_P', \quad (10)$$

where M_{Γ_P} is a $q \times q$ symmetric positive definite matrix.
We assume that A, B, G_P and D_P are unrestricted matrices ($A \in \mathbb{R}^{nr}$, $B \in \mathbb{R}^{mr}$, $G_P \in \mathbb{R}^{nq}$, $D_P \in \mathbb{R}^{lq}$), whilst β and δ_P have orthonormal columns, i.e. they are elements of the Stiefel manifolds: $\beta \in V_{r,m}$, $\delta_P \in V_{q,l}$. Through these matrices we want to get information about the spaces. The relationship between the Stiefel and Grassmann manifolds is many-to-one, i.e. in each point of the Grassmann manifold

there is contained a set of elements of the Stiefel manifold. To weaken this drawback we additionally assume that elements of the first row of β and δ_P are positive: $\beta \in \tilde{V}_{r,m}$, $\delta_P \in \tilde{V}_{q,l}$, where $\tilde{V}_{r,m}$ denotes the 2^{-r}th part of $V_{r,m}$ and $\tilde{V}_{q,l}$ – the 2^{-q}th part of $V_{q,l}$ (Chikuse 2002). The invariant measures over $\tilde{V}_{r,m}$ and $\tilde{V}_{q,l}$ differ from the invariant measures over $V_{r,m}$ and $V_{q,l}$ by multiplicative constants 2^r and 2^q, respectively.
Imposing matrix Normal distributions on B and D_P ($mN(0,I_r,P_B)$, $mN(0,I_q,P_D)$) leads to Matrix Angular Central Gaussian distributions for the orientations of B and D_P: $B(B'B)^{-\frac{1}{2}} \sim MACG(P_B)$ and $D_P(D_P'D_P)^{-\frac{1}{2}} \sim MACG(P_D)$, which are defined on the Stiefel manifold. Through matrices P_B and P_D a researcher may incorporate prior information about the estimated spaces. Assuming $P_{\cdot} = I$ we get a uniform distribution. Prior distributions for the remaining parameters are standard: $A|\Sigma,\nu_A,r \sim mN(0,\nu_A I_r,\Sigma)$, $G_P|\Sigma,\nu_{G_P},q \sim mN(0,\nu_{G_P} I_q,\Sigma)$, $\nu_A \sim iG(s_{\nu_A},n_{\nu_A})$, $\nu_{G_P} \sim iG(s_{\nu_{G_P}},n_{\nu_{G_P}})$, $\Sigma \sim iW(S,q_\Sigma)$, $\Gamma|\Sigma,h \sim mN(0,\Sigma,hI)$, $h \sim iG(s_h,n_h)$, $\Gamma_s|\Sigma,h_s \sim mN(0,\Sigma,h_s I)$, $h_s \sim iG(s_{h_s},n_{h_s})$.
The joint prior distribution is truncated by the stability condition imposed on the process parameters:

$$p\left(A,B,G_P,D_P,\Sigma,\Gamma,\Gamma_s,\nu_A,\nu_{G_P},h,h_s\right) \propto \\ f\left(A,B,G_P,D_P,\Sigma,\Gamma,\Gamma_s,\nu_A,\nu_{G_P},h,h_s\right) I_{[0,1]}\left(|\lambda|_{max}\right),$$

where λ denotes the eigenvalue of the companion matrix.
After incorporating into the model the information contained in the data one gets the following full conditional posterior distributions (for the parameterisation with B and D_P):

$iW(S + \frac{1}{h}\Gamma'\Gamma + \frac{1}{h_s}\Gamma_s'\Gamma_s + \frac{1}{\nu_A}AA' + \frac{1}{\nu_{G_P}}G_P G_P' + E'E, q_\Sigma + pn + l_s + r + q + T)$
for Σ, where $E = Z_0 - Z_1 BA' - Z_2 D_P G_P' - W\Gamma - Z_3\Gamma_s$,

$mN(\mu_{\Gamma_s},\Sigma,\Omega_{\Gamma_s})$, for Γ_s,
where $\mu_{\Gamma_s} = (\frac{1}{h_s}I_{l_s} + Z_3'Z_3)^{-1}Z_3'(Z_0 - Z_1BA' - Z_2D_PG_P' - W\Gamma)$,
$\Omega_{\Gamma_s} = (\frac{1}{h_s}I_{l_s} + Z_3'Z_3)^{-1}$

$mN(\mu_\Gamma,\Sigma,\Omega_\Gamma)$, for Γ,
where $\mu_\Gamma = (\frac{1}{h}I_{np} + W'W)^{-1}W'(Z_0 - Z_1BA' - Z_2D_PG_P' - Z_3\Gamma_s)$,
$\Omega_\Gamma = (\frac{1}{h}I_{np} + W'W)^{-1}$

$mN(\mu_A,(\frac{1}{\nu_A}I_r + B'Z_1'Z_1B)^{-1},\Sigma)$, for A,
where $\mu_A = (Z_0 - Z_2DG' - W\Gamma - Z_3\Gamma_s)'Z_1B(\frac{1}{\nu_G}I_r + B'Z_1'Z_1B)^{-1}$,

the Normal with variance $\Omega_{vB} = ([(A'\Sigma^{-1}A) \otimes (Z_1'Z_1)] + [I_r \otimes P_B^{-1}])^{-1}$ and mean $\mu_{vB} = \Omega_{vB} vec(Z_1'(Z_0 - Z_2DG' - W\Gamma - Z_3\Gamma_s)\Sigma^{-1}A)$ for $vec(B)$,

$mN(\mu_{G_P},(\frac{1}{\nu_{G_P}}I_q + D_P'Z_2'Z_2D_P)^{-1},\Sigma)$ for G_P, where
$\mu_{G_P} = (Z_0 - Z_1BA' - W\Gamma - Z_3\Gamma_s)'Z_2D_P(\frac{1}{\nu_{G_P}}I_q + D_P'Z_2'Z_2D_P)^{-1}$,

the Normal with variance $\Omega_{vD} = ([(G_P'\Sigma^{-1}G_P) \otimes (Z_2'Z_2)] + [I_q \otimes P_D^{-1}])^{-1}$ and mean $\mu_{vD} = \Omega_{vD}vec(Z_2'(Z_0 - Z_1BA' - W\Gamma - Z_3\Gamma_s)\Sigma^{-1}G_P)$ for $vec(D_P)$,

inverted gamma distributions: $iG(s_{\nu_A} + \frac{1}{2}tr(\Sigma^{-1}AA'),n_{\nu_A} + \frac{nr}{2})$ for ν_A, $iG(s_{\nu_{G_P}} + \frac{1}{2}tr(\Sigma^{-1}GG'),n_{\nu_{G_P}} + \frac{nq}{2})$ for ν_{G_P}, $iG(s_h + \frac{1}{2}tr(\Sigma^{-1}\Gamma\Gamma'),n_h + \frac{np}{2})$ for h, and $iG(s_{h_s} + \frac{1}{2}tr(\Sigma^{-1}\Gamma_s\Gamma_s'),n_{h_s} + \frac{nl_s}{2})$ for h_s.

Draws from the posterior distributions of β and α are obtained as $\beta = B(B'B)^{-\frac{1}{2}}O_\Pi$, $\alpha = A(B'B)^{\frac{1}{2}}O_\Pi$, whereas δ_P and γ_P are obtained as $\delta_P = D_P(D_P'D_P)^{-\frac{1}{2}}$, $\gamma_P = G_P(D_P'D_P)^{\frac{1}{2}}$, with O_Π and O_P denoting diagonal matrices with 1 or -1 on their main diagonals, i.e. $O_\Pi = diag(\pm 1)$, $O_P = diag(\pm 1)$.
Having the sample from the posterior distribution the mean of β and δ_P can be computed with the method proposed by Villani (2006), i.e. by constructing the loss function which takes the curved geometry of the Grassmann manifold into account, e.g. with the projective Frobenius distance between spaces.

3 The permanent-transitory decomposition

One of the aims of this paper is to analyse the sources of variability of the time series modelled within the VEC(-WF) framework. We are particularly interested in the proportion of the transitory and permanent shocks. In order to check the importance of the above-mentioned types of shocks we have to isolate them. With the aim of decomposing time series into uncorrelated permanent and transitory components we will use the method proposed by Centoni and Cubadda (2003). They specified the decomposition characterised by the following definition:

Definition 1 *Let the permanent and transitory shocks, respectively, be* $u_t^P = \alpha_\perp'\varepsilon_t$ *and* $u_t^T = \alpha'\Sigma^{-1}\varepsilon_t$. *Then the associated polynomial coefficient matrices are, respectively, defined as*
$P(L) = C(L)\Sigma\alpha_\perp(\alpha_\perp'\Sigma\alpha_\perp)^{-1}$,
$T(L) = C(L)\alpha(\alpha'\Sigma\alpha)^{-1}$,
where $C(L)$ *denotes the polynomial matrix from the Wold decomposition of* Δx_t.

Such a decomposition belongs to the class of permanent-transitory representation with the following properties (Centoni, Cubadda 2003):

$$x_t = \delta_t + P_t + T_t,$$

$$\Delta P_t = P(L)u_t^P,$$

$$\Delta T_t = T(L)u_t^T,$$

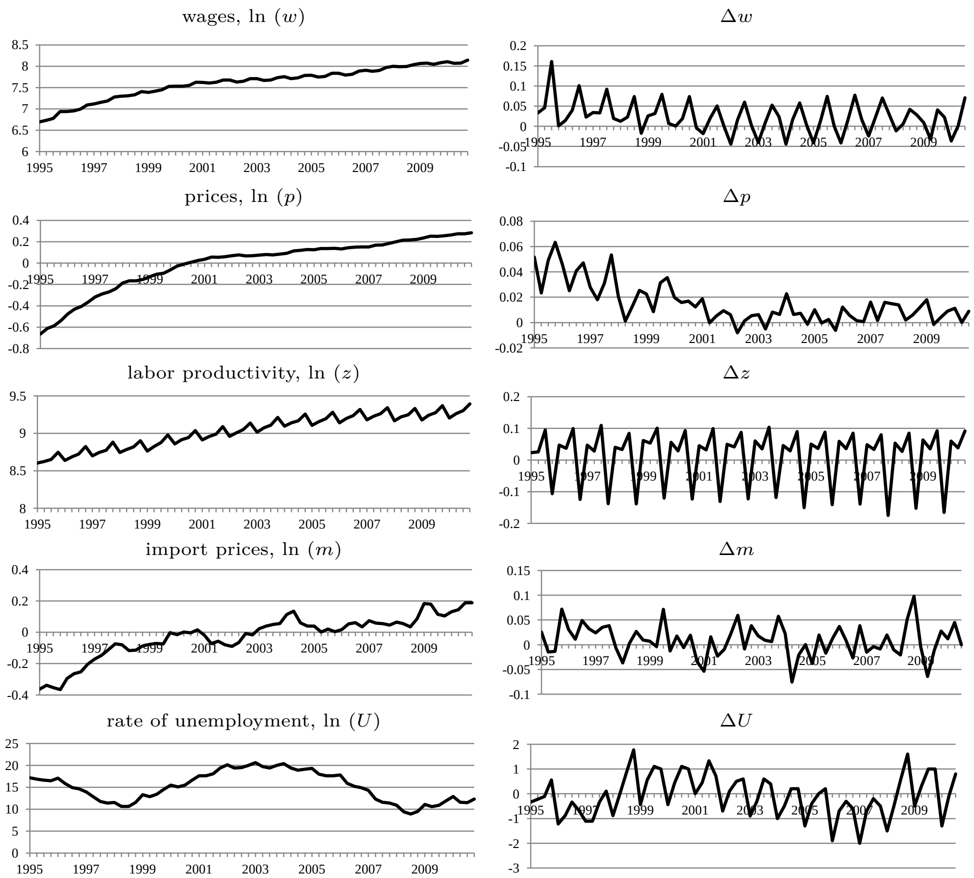
$(u_t^{P'}, u_t^{T'})' = F\varepsilon_t$, where F is a full-rank $n \times n$ matrix such that $T(1) = 0$ (i.e. transitory shocks u_t^T have no permanent effects on x_t) and $E(u_t^T u_t^{P'}) = 0$ (i.e. P_t and T_t are uncorrelated).

Obviously, in the case of the considered decomposition $F = (\alpha_\perp, \Sigma^{-1}\alpha)'$.
Additionally, we scale the permanent and transitory shocks to make them have unit variances. As $V(u_t^P) = \alpha_\perp' \Sigma \alpha_\perp$ and $V(u_t^T) = \alpha'\Sigma^{-1}\alpha$ ($V(.)$ denotes a variance-covariance matrix), the shocks $\tilde{u}_t^P = (\alpha_\perp' \Sigma \alpha_\perp)^{-\frac{1}{2}} u_t^P$ and $\tilde{u}_t^T = (\alpha'\Sigma^{-1}\alpha)^{-\frac{1}{2}} u_t^T$ have unit variances and their components are uncorrelated.
The polynomial matrices are of the form: $\tilde{P}(L) = C(L)\Sigma\alpha_\perp(\alpha_\perp'\Sigma\alpha_\perp)^{-\frac{1}{2}}$ and $\tilde{T}(L) = C(L)\alpha(\alpha'\Sigma\alpha)^{-\frac{1}{2}}$.
Knowing this we can obtain contributions of the permanent and transitory shocks by calculating them within each cycle of the Gibbs sampler with the use of commonly known methods and equations (see, e.g., Lütkepohl 2007).

4 An empirical illustration: the price-wage nexus in the Polish economy

The presented methods will be illustrated with the analysis of the price-wage spiral in the Polish economy. The seasonally unadjusted quarterly data represent five variables: average wages (current prices, W_t), price index of consumer goods (P_t), labour productivity (constant prices, Z_t), price index of imported goods (M_t) and the unemployment rate (U_t). They are collected in the vector $x_t = (m_t, U_t, p_t, z_t, w_t)'$, where lower case letters denote natural logarithms of the original variables. The analysed data cover the sixteen-year period ranging from 1995Q1 to 2010Q4. The data are plotted in Figure 1. Visual inspection of the analysed variables suggests that they may be realisations of the integrated processes, but they appear to move together in the long-run, so we can expect cointegration. The first differences of the series also seem to display a similar short-run behaviour, so it is reasonable to verify the hypothesis of the additional reduced rank restriction imposed on the short-run parameters of the VEC model. As shown by Fischer (1977) and Taylor (1980), the comovement between wages and prices may be unsynchronised (see also Vahid, Engle 1997), which is caused by wage contracts lasting more than one period. Our task is to verify this hypothesis for the Polish economy.
We will consider a set of models which differ in the number of lags $k \in \{3,4,5\}$, deterministic terms $d \in \{1,2\}$, where $d = 1$ stands for an unrestricted constant, $d = 2$ – a constant restricted to cointegrating relations (see, e.g., Juselius 2007 for further details), the number of cointegrating relations $r \in \{1,2,3,4\}$, the number of (polynomial) weak common cyclical features $s \in \{0,1,2,3,4\}$ (i.e the rank of Γ_P: $n - s = q \in \{5,4,3,2,1\}$, for $s = 0$ we have a VEC model) and the number of quarters previous to the short-run comovements $p \in \{0,1\}$ (for $p = 0$ the weak common cyclical features are synchronised).

Figure 1: The analysed data



Seasonality of the analysed series will be modelled in a deterministic manner, i.e. via the zero-mean seasonal dummies.

Altogether we will compare 216 different specifications of the VEC-(P)WF model: 24 VEC, 96 VEC-WF and 96 VEC-PWF(1) models. As we want to treat them as equally possible we impose on each of them the same prior probability: $p(M_{(k,d,q,r,p)}) = \frac{1}{216} \approx$ ≈ 0.0046.

We specify the following priors on the model parameters:

$$\Sigma \sim iW(S, 10+n+1),\ S = 10\begin{pmatrix} 0.05 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.05 \end{pmatrix},$$

$B|r, m \sim mN(0, m^{-1}I_r, I_m)$, which leads to $\beta|r \sim MACG(I_m)$,

$A|\nu_A, r, \Sigma \sim mN(0, \nu_A I_r, \Sigma)$,

$D_P|q, l \sim mN(0, l^{-1}I_q, I_l)$, which leads to $\delta_P|q \sim MACG(I_l)$,

$G_P|\nu_{G_P}, q, \Sigma \sim mN(0, \nu_{G_P} I_q, \Sigma)$,

$\Gamma|\Sigma, h \sim mN(0, \Sigma, hI)$,

$\Gamma_s|\Sigma, h_s \sim mN(0, \Sigma, h_s I)$,

$\nu_A \sim iG(2,3)\ (E(\nu_A) = 1,\ Var(\nu_A) = 1)$,

$\nu_{G_P} \sim iG(2,3)\ (E(\nu_{G_P}) = 1,\ Var(\nu_{G_P}) = 1)$,

$h \sim iG(20,3)\ (E(h) = 10,\ Var(h) = 100)$,

$h_s \sim iG(20,3)\ (E(h_s) = 10,\ Var(h_s) = 100)$.

The joint prior resulting from this specification has been truncated by the stability condition imposed on the parameters of the cointegrated process.
The 25 most probable models are presented in Table 1. The sum of posterior probabilities of the listed models equals 0.505. Table 2 presents marginal probabilities of the model features.

Table 1 reveals considerable posterior model uncertainty, but from Table 2 we can draw the conclusion that, in the analysis of the price-wage spiral in the Polish economy, we should take into consideration the possibility of both the long- and short-run comovements. Contrary to our presumption, models with immediate short-run comovements achieved higher posterior probabilities than the ones with delayed short-run common behaviour.
Consequences of the short-run restrictions on the shape of impulse responses, forecast accuracy and precision of some non-Bayesian estimation methods were investigated, e.g., by Hecq, Palm, Urbain (2006), Anderson, Vahid (2010) and, in the Bayesian framework, by Wróblewska (2011). In this paper we will focus our attention on the permanent-transitory decomposition. We will examine how these additional restrictions affect the importance of permanent and transitory shocks to analysed macroeconomic variables. This issue has been already discussed, among others, by Issler, Vahid (2001).

Table 1: The most probable models

k	d	q	r	p	$P\left(M_{(k,d,q,r,p)}\|x\right)$	k	d	q	r	p	$P\left(M_{(k,d,q,r,p)}\|x\right)$
4	4	1	3	0	0.034	4	4	4	3	0	0.017
4	4	1	2	0	0.030	3	4	1	4	0	0.017
4	4	1	4	0	0.029	3	4	1	1	1	0.017
3	4	3	1	0	0.027	3	4	1	3	1	0.016
4	4	3	3	0	0.027	3	4	4	4	0	0.016
3	4	2	4	0	0.025	3	4	1	1	0	0.015
3	4	3	2	0	0.024	3	4	1	2	1	0.015
3	4	4	1	0	0.024	3	4	1	3	0	0.015
3	4	2	2	0	0.023	3	3	1	1	0	0.015
3	4	3	3	0	0.023	3	3	1	4	0	0.014
3	4	2	3	0	0.018	4	4	1	2	1	0.014
3	4	4	2	0	0.018	4	4	2	3	0	0.014
3	3	1	2	0	0.018						

Table 2: Marginal posterior probabilities of the model features

k	3	4	5		
$p(k\|x)$	0.460	0.362	0.178		
d	1	2			
$p(d\|x)$	0.159	0.841			
r	1	2	3	4	
$p(r\|x)$	0.226	0.268	0.314	0.192	
q	1	2	3	4	5
$p(q\|x)$	0.403	0.237	0.203	0.146	0.011
p	0	1			
$p(p\|x)$	0.732	0.268			

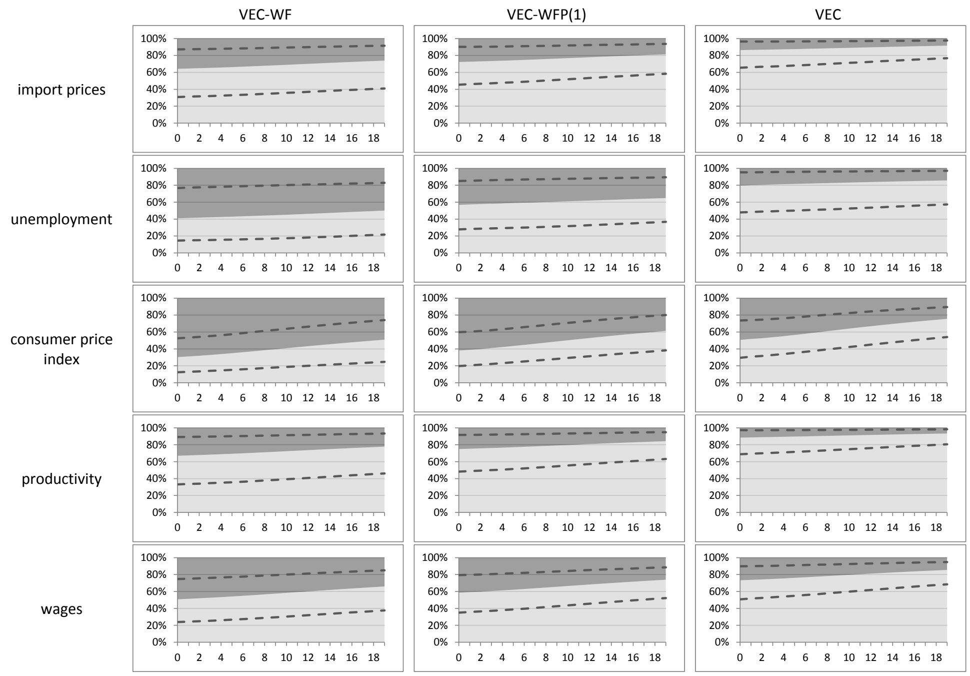
Figure 2 compares results of the permanent-transitory decompositions obtained with the Bayesian model averaging technique in the analysed model classes, i.e. VEC, VEC-WF and VEC-WFP(1).

As one could expect, the group of models which take into account the short-run common dynamics of the analysed series attribute more importance to the transitory shocks.

In Table 1 we have listed the most probable models and there is no model without short-run restrictions among them, so if we did not take into consideration the possibility of common short-run behaviour of the analysed series, the permanent-transitory decomposition results would be misleading. Let us look at Figure 3, presenting results of this decomposition obtained in the group of the 25 most probable models and in the group of the VEC models only.

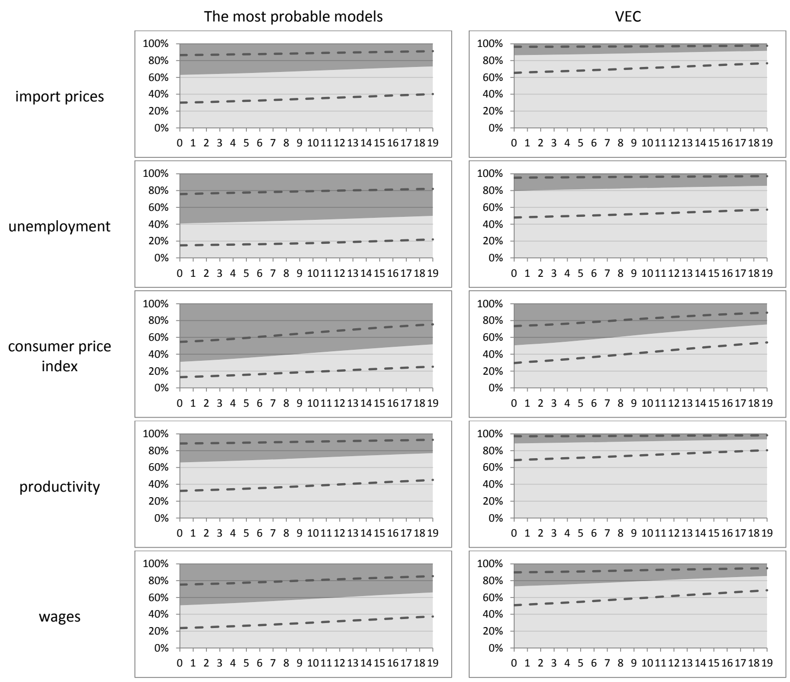
The largest difference (of about 35%) could be noted for the innovation variance decomposition of unemployment. For example, when we take common cycles into account the transitory shocks explain more than a half of the forecast error variance

Figure 2: Comparing results of variance decomposition of innovations. Light grey area represents % of the forecast-error variance attributed to the permanent component (posterior median) and the dark grey area is % of the forecast-error variance attributed to the transitory component (posterior median). The dotted lines represent 25^{th} and 75^{th} percentiles.



at all of the considered horizons and no more than 20% when we focus our attention on the models with the common trends only. In fact, visual analysis of the unemployment time path (see Figure 1) suggests that the transitory shocks could be important in explaining variation of this variable.
The under-estimation of the contribution of the transitory shocks to variability of the other variables fluctuates around 20%.
The results imply that the importance of the transitory shocks in explaining variability of the analysed series cannot be neglected. In the infinite horizon the variance-decomposition results will be the same and, in fact, the observed differences decrease quarter by quarter, but the decay is very slow, so the conclusions drawn about the nature of the macroeconomic variables should be especially interesting for decision makers who plan and judge possible results of their activities at business-cycle horizons.

Figure 3: Comparing results of variance decomposition of innovations. Light grey area represents % of the forecast-error variance attributed to the permanent component (posterior median) and the dark grey area is % of the forecast-error variance attributed to the transitory component (posterior median). The dotted lines represent 25^{th} and 75^{th} percentiles.



5 Concluding remarks

In this paper we developed Bayesian framework (i.e. estimation and model comparison) of the VEC models with an additional weak form (polynomial) reduced rank restriction imposed on the short-run parameters of such models. In the empirical example we used the proposed method to analyse the price-wage spiral in the Polish economy. The Bayesian comparison of the models confirmed the hypothesis of the presence of long-run and short-run relations among the analysed variables, but the non-synchronised short-run comovement is only weakly supported.
Additionally, we showed the consequences of such restrictions for permanent-transitory decomposition analysis in the VEC-WF(P) system. In general, it could be said that omitting the short-run comovement restrictions leads to notable distortions of the importance proportions of the transitory and permanent shocks to the variability of the analysed time series.

Acknowledgement

Research supported by a grant from Cracow University of Economics.
I would like to thank the participants of the 13^{th} Workshops on Statistics and Econometrics (Trzynaste Warsztaty Doktorskie z zakresu Statystyki i Ekonometrii) for valuable discussion on the previous draft of this paper.
Useful comments and suggestions by an anonymous Referee, Paolo Bonomolo, Łukasz Kwiatkowski and Jacek Osiewalski are gratefully acknowledged. All errors and omissions are mine.

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