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APPLICATION OF THREE-PARAMETER DISTRIBUTION TO APPROXIMATE THE PARTICLE SIZE DISTRIBUTION FUNCTION OF COMMINUTION PRODUCTS OF DOLOMITIC TYPE OF COPPER ORE

ZASTOSOWANIE ROZKŁADU TRÓJPARAMETROWEGO DO APROKSYMACJI KRZYWYCH SKŁADU ZIARNOWEGO PRODUKTÓW ROZDRABNIANIA DOLOMITYCZNEGO TYPU RUDY MIEDZI

The paper presents the results of analyze the particle size distribution function of comminution products of dolomitic type of copper ore. The breakage tests for single irregular particles were performed with using a hydraulic press device. The authors prepared five particle size fractions of each material, within ranges: 16-18 mm, 18-20 mm, 20-25 mm, 25-31,5 mm and 31-45 mm. The particle size distribution function of single-particle breakage test was calculated separately for each size fraction. In addition, the cumulative particle size distribution function for five particle size fractions was presented. In theoretical part the study of applied functions of particle size distribution for comminution a set of particles and models of crushing single particles was performed. In that paper the curves of the particle size distribution were approximated by the three-parameter function, which parameters depend on the particle strength and material type. For conformity assessment the model distribution function to the empirical distribution function a residual deviation and non-linear correlation coefficient were calculated. The three-parameter function approximating agrees well with the particle size distribution obtained from experimental data. The dependence of the parameters of a particle size distribution function on the dolomite particle strength was presented. The results indicate the identity of single particle grinding mechanism by slow compression of irregular particles of dolomitic type of copper ore, regardless of the initial particle size

Keywords: particle size distribution, three-parameter function, dolomite

W artykule przedstawiono wyniki analizy krzywych składu ziarnowego produktów rozdrabniania próbek dolomitycznego typu rudy miedzi. Materiał został rozdrobniony w prasie hydraulicznej poprzez powolne jednoosiowe ściskanie pojedynczych nieregularnych ziaren. Do testów zostało przygotowanych pięć klas ziarnowych surowca: 16-18 mm, 18-20 mm, 20-25 mm, 25-31,5 mm, 31,5-40 mm. Wyliczono równania krzywych składu ziarnowego produktów rozdrabniania oddzielnie dla każdej klasy ziarnowej oraz podano jedno równanie wspólne dla pięciu klas. W części teoretycznej została przeprowadzona analiza stosowanych funkcji krzywych składu ziarnowego dla rozdrabniania zbioru ziaren oraz modele

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elementarnych procesów rozdrabniania. W tej pracy do aproksymacji krzywych składu ziarnowego został użyty rozkład trójparametrowy, którego parametry uzależniono od wytrzymałości ziaren na rozciąganie. Do oceny zgodności przyjętego rozkładu modelowego z rozkładem empirycznym zastosowano współczynnik korelacji krzywoliniowej oraz odchylenie resztowe. W wyniku aproksymacji uzyskano wysokie wskaźniki dopasowania modelu do rozkładu, biorąc pod uwagę że badania przeprowadzono na losowo wybranych ziarnach o nieregularnych kształtach. Uzyskane wyniki świadczą o dobrym wyborze modelu krzywej składu ziarnowego jak również o identyczności mechanizmu rozdrabniania pojedynczych ziaren dolomitu poprzez zgniatanie, niezależnie od początkowych wymiarów ziarna.

Słowa kluczowe: krzywe składu ziarnowego, rozkład trójparametrowy, dolomit

1. Introduction

The literature on the grinding process grained material presents two categories of research: study of the grinding process for large sets of particles (bounded comminution) in the specified grinding devices (called a convention crushing macro-process) and study the process of crushing the individual particle (micro-processes of crushing or free comminution) (Brożek et al., 1995). For both case the researching to aiming to identify patterns that describe curves of particle size distribution function of crushed products are conducted (Saramak & Tumidajski, 2006; Gawenda et al., 2005). These approximation formulas can be applied both in description of grained materials and in construction of suitable models of comminution processes course as well as in optimization of size reduction operations (Napier-Munn et al., 1996; Saramak, 2012; Saramak et al., 2010; Svedensten & Evertsson, 2005; Tavares, 2005; Tumidajski, 2012).

1.1. Particle size distributions of bounded comminution

Several types of distribution functions were developed in order to describe mathematically the particle size distribution of a crusher product, such as the Gates-Gaudin-Schuhmann (GGS) distribution, the Rosin-Rammler-Sperling-Bennett (RRSB) distribution, Gaudina-Meloy distribution.

The most commonly used particle size distributions function obtained from the sieve analysis is Gates-Gaudin-Schuhmann (GGS) power-law distribution:

$$\Phi(d) = Cd^k, \quad d \in (0, d_{\max}) \tag{1}$$

where:

d — relative or absolute particle diameter,
$\Phi(d)$ — yield of particles whose size is less than or equal to d,
C and k — distribution parameters ($C, k > 0$).

This type of distribution cannot be use to describe the particle size distribution in the sample with a predominance of particle with intermediate size in range $(0, d_{\max})$.

Another example of a power-law distribution as a function of particle size distribution is Gaudin-Meloy model (1962):

$$\Phi(d) = 1 - \left(1 - \frac{d}{d_{\max}}\right)^k, \quad d \in (0, d_{\max}) \tag{2}$$

where $d_{\max}$ and k are parameters of distribution.

To approximate the curve of particle size distribution with a predominance of small particles commonly used is the distribution of Rosin-Rammler-Sperling-Bennet, also called Rosin-Rammler-Bennett distribution (Sokolowski, 1995) or Weibull distribution:

$$\Phi(d) = 1 - \exp\left(-\frac{d}{d_o}\right)^k, \quad d \in (0, \infty) \tag{3}$$

where:

d — relative or absolute particle diameter,
d_o, k — parameters of distribution.

Distribution of R-R-B well approximated particle size of fine coal (Brożek, 1986).

A good approximation is achieved by using Kolmogorov distribution (Kolmogorov, 1941) to the curves of particle size of products obtained as a result of long-term grinding:

$$\Phi(d) = \frac{1}{\delta\sqrt{2\pi}} \int_0^d \frac{1}{x} \exp\left[-\frac{(\ln x - m)^2}{2\delta^2}\right] dx, \quad x \in (0, \infty) \tag{4}$$

where: $x = d - d_{\min}$, m and δ – distribution parameters.

The next distribution used to describe a particle size curve is Gauss distribution:

$$\Phi(d) = \frac{1}{\delta\sqrt{2\pi}} \int_{-\infty}^{d} \exp\left[-\frac{1}{2}\left(\frac{d-\mu}{\delta}\right)^2\right] dx \tag{5}$$

where:

d — particle diameter,
μ — average value,
δ — standard deviation of the variable d.

The quality of the compliance empirical function $\Phi^d(d_i)$ to mathematical function $\Phi(d)$ can be measured by mean square deviation compared function (Malewski, 1981):

$$D\Phi = \left[\frac{1}{g} \sum_i (\Phi^d(d_i) - \Phi(d_i))^2\right]^{1/2} \tag{6}$$

where: g – number of the compared points (particle size fraction).

Models describing the size distribution of bounded comminution (Rosin-Rammler, Gates-Gaudin-Schuhmann and others) are the result of empirical research and no principles, binding the distribution parameters with material properties or the crushing device, are known (Brożek et al., 1995). This law can be find by models of breakage individual particles.

1.2. Models of elementary processes of crushing

The first physical model of the grinding process had been given by Gilvarry. Considering the process of crushing (by compression) a single particle of a predetermined volume he used the Griffith's theory of presence of congenital defects in the structure and microscopic flaws in the bulk material around which concentrate the stress under the applied load crushing. Gilvarry expanded this theory taking into account the cracks at the edges of the particles, its surface and inside the particles. The general formula for the function of particle size distribution of the product crushing of individual particles is (Gilvarry, 1961):

$$\Phi(d) = 1 - \exp\left\{ -\left[\frac{d}{n} + \left(\frac{d}{j} \right)^2 + \left(\frac{d}{i} \right)^3 \right] \right\} \tag{7}$$

where:

d — particle size is obtained from the screening,
n, j, i — constants with linear dimension.

In specific cases where d is small:

$$\Phi(d) = 1 - e^{-d/n}$$

which takes the form of exponential distribution, whereas when $d \rightarrow 0$ can be assumed:

$$\Phi(d) = \frac{d}{n}$$

that is a special case of distribution of Gates-Gaudin-Schuhmann.

The second physical model of the grinding process is model proposed by Meloy-Gumtz. (1969). It is based on mental reconstruction of particle from fragments of product crushing and definition of density function of crushing surface along any direction. It is assumed, that particle is crushed by any method can be reconstructed, ie. to build the newly formed of particle to get the original form. By this reconstructed particle a straight line in any direction is performed and receives a cross section with a length d_0, which is cut by r number of newly established crushing surfaces. Assumed, that the function $r(y)$ is a function of the probability density distribution of the crushing surface of the section $(0, d_0)$. The function $r(y)$ characterize the comminution process and depends on many factors related to the physical and chemical properties of particle and its history. The general form of the equation of the curve of particle size depends on this function and has the form (Meloy, 1963):

$$\Phi(d) = 1 - \left[1 - \frac{d}{d_0} \right]^r \tag{8}$$

Each of the mentioned distribution has a defined range of applicability. The range is the result of the process leading to obtain a grained material or the method used crushing and mining of raw materials in the process of mining. Generally it is 2 – parameters distributions and parameters of the distribution are constants determined empirically.

Research conducted by a team led by M. Brożek (1993; 1994; 1995; 1996; 1996; 2003a,b,c; Nad et al., 2012; Nad, 2014) the comminution of individual particles by a hydraulic press machine showed that the curves of particle size of cru shing product can be accurately approximated by a function of the form:

$$\Phi(x) = ax^b e^{kx} \tag{9}$$

where: x is a variable expressing the ratio of the particle size d of the crushing product to the arithmetic value of the limits of feed size D, $x = \frac{d}{D}$ while a,b,k distribution parameters, whose values are determined based on the results of experiments.

The observations of single particle breakage show that through the process of the disintegration of the particles is formed a small amount of fine particles (spall) which is formed surrounded by the contact of the particle and crushing surface as well as a pair of compression fractures forming a large particles. According to this, the curve of particle size distribution is the cumulative distribution which should be considered as a superposition of two distributions: one describes the distribution of the obtaining the fine particles and the second describes the obtaining the coarse particles. Mentioned function (9) is the distribution function of these distributions. Differentiating the function (9) is obtained for the density distribution:

$$\Phi'(x) = f(x) = abx^{b-1}e^{kx} + kax^{b}e^{kx} \tag{10}$$

This function is the combination of two linear functions:

$$f(x) = \alpha_1 f_1(x) + \alpha_2 f_2(x) \tag{11}$$

where:

$$f_1(x) = \frac{ab}{\alpha_1} x^{b-1} e^{kx} \tag{12}$$

$$f_2(x) = \frac{ka}{\alpha_2} x^{b} e^{kx} \tag{13}$$

are the densities of distributions components. The function $f_1(x)$ is the density distribution of fine particles, and $f_2(x)$ is the density distribution of coarse particles. The characters of functions f_1 and f_2 let to the hypothesis that the distributions of components describe two different processes of crushing which were mentioned earlier.

2. Experimental

The experiments were performed on polish copper ore from minig Rudna. The sample was taken from the ROM material after comminution in a crusher hammer in 2010. The sieve analyze was made with square mesh 16 mm, 18 mm, 20 mm, 25 mm, 31,5 mm and 45 mm. After screening the five particle size fraction were obtained: 16-18 mm, 18-20 mm, 20-25 mm, 25-31,5 mm, 31,5-45 mm. Selected particle fractions were manually divided into three lithological types of copper ore, namely dolomitic ore, sandstone ore and shale ore. For the analysis of the curves of particle size distribution in this article sample of dolomitic ore has been selected. The flat and elongated particles were rejected from the population. In each fraction of particle size were randomly selected about 100 particles.

Individual particles, of all the above-mentioned size fractions, were slowly compressed at a constant rate, until the first fracture across the particle occurred. The value of the destruction force of each particle was recorded. Single-particle breakage tests were performed in the Institute of Ceramic and Building Materials in Warsaw, department of Glass and Building Materials in

Krakow, with using the compression-testing machine Toni Technik (universal testing strength machine with hydraulic drive). The load range was adjustable; 600; 300 and 120 kN. For the testing programme, the load force was 120 kN, with the accuracy of destructive force registration up to 0.1 kN. The particles obtained from each single-breakage tests were collected separately. The entire range of destructive stress value (0, $F_{\max}$) for each sample was divided into several narrow fractions (partitions). In each narrow destructive force fraction, a sieve analysis of broken irregular particles was made. To get the general form of the equation of the particle size distribution function, the cumulative yield from the each narrow destructive force fractions for particular size fraction was calculated.

3. The experimental results and analysis

The equations of particle size distribution for samples of dolomitic type of copper ore are given below. For each equation the residual deviation (S_r) and the coefficient non-linear correlation (r_k), were determined.

The non-linear correlation coefficient was calculated according to:

$$r_k = \sqrt{1-\varphi^2} \tag{14}$$

where

$$\phi = \frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{n}\left(y_i - y_{sr}\right)^2} \tag{15}$$

y_{sr} — average value $\Phi\Phi(x_i)$.

The residual deviation value was calculated by:

$$S_r = \sqrt{\frac{\sum_{i=1}^{p_i}\left(\Phi_e(d_i) - \Phi_t(d_i)^2\right)}{p_s - 2}} \tag{16}$$

where:

p_s — the number of used sieves with mesh sizes d_i,

Φ_e — value of the empirical distribution,

Φ_t — the value of the distribution calculated with using the approximating equation for particle size d_i.

All pictures was made by using STATISTICA software.

Below, the equations of particle size distribution curves, approximated by function (9) for five particle size fractions, were presented:

a) particle size 31,5-45 mm, $D = 38{,}25$ mm

$$\Phi\left(x\right) = 2{,}122x^{0{,}446}e^{4{,}999\,x} \tag{17}$$

$$S_r = 1{,}707$$

$$r_k = 0{,}996$$

b) particle size 25-31,5 mm, $D = 28{,}25$ mm

$$\Phi\left(x\right) = 3{,}790x^{0{,}414}e^{3{,}524\,x} \tag{18}$$

$$S_r = 2{,}029$$

$$r_k = 0{,}994$$

c) particle size 20-25 mm, $D = 22{,}5$ mm

$$\Phi\left(x\right) = 3{,}314x^{0{,}346}e^{3{,}825\,x} \tag{19}$$

$$S_r = 3{,}921$$

$$r_k = 0{,}991$$

d) particle size 18-20 mm, $D = 19$ mm

$$\Phi\left(x\right) = 5{,}585x^{0{,}474}e^{3{,}307\,x} \tag{20}$$

$$S_r = 4{,}979$$

$$r_k = 0{,}989$$

e) particle size 16-18 mm, $D = 17$ mm

$$\Phi\left(x\right) = 2{,}567x^{0{,}255}e^{4{,}037\,x} \tag{21}$$

$$S_r = 3{,}351$$

$$r_k = 0{,}987$$

Empirical data for five particle size fraction were fitted to theoretical distribution function, expressed by the formula (9). Calculations were performed with using the least squares method:

$$\Phi\left(x\right) = 3{,}479x^{0{,}392}e^{\,3{,}788\,x} \tag{22}$$

$$S_r = 4{,}287$$

$$r_k = 0{,}984$$

Figure 1 illustrates the combined curve of particle size distribution for five particle size fractions of dolomite type of ore. The theoretical function, calculated by the equation (22), is marked by continuous line.

Figures 2 to 4 show the dependence of the parameter a, b and k of a particle size distribution function (9) on the dolomite particle strength. To determine the dependence of the parameters a, b and k of average strength in narrow destructive force fraction the analyze of particle size distribution for each destructive force fraction was done. The particle size distribution functions were approximated by function (9) and basis on this using the method of least squares the individual values of a, b, and k parameters was determined. According to the literature (Brożek,

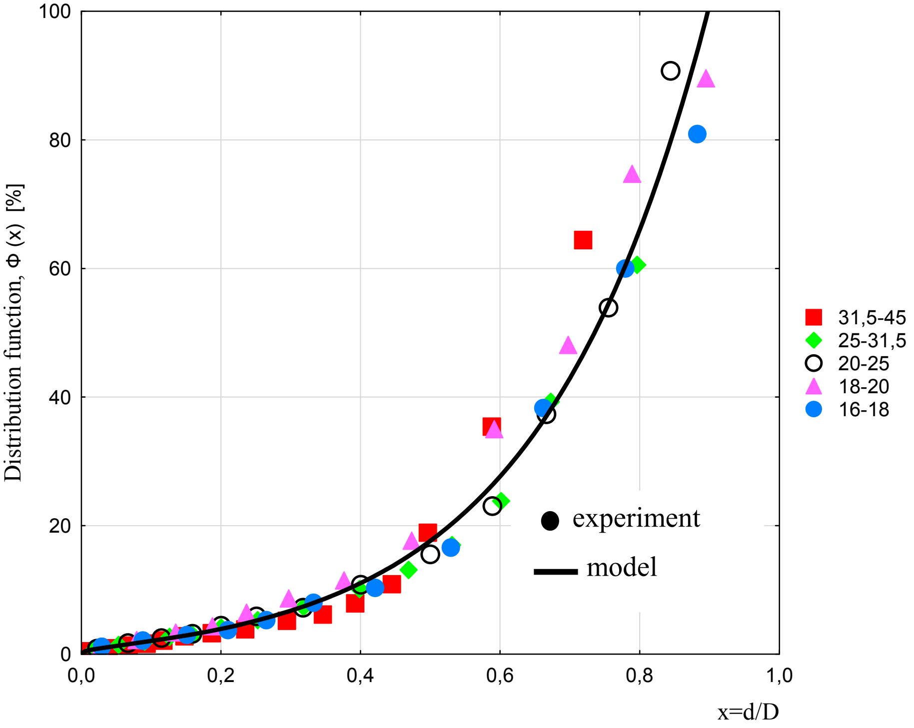

Fig. 1. Particle size distribution function of dolomite for five particle size

1996; Brożek & Tumidajski, 1996) the dependence of parameters *a* and *k* in equation (9) from the tensile strength of the material can be described by the following equation:

$$a(\sigma) = \propto_1 (\sigma - \sigma_{\min})^{\omega} \tag{23}$$

$$k(\sigma) = \frac{\propto_2}{\sigma^u} \qquad \sigma \geq \sigma_{\min} > 0 \tag{24}$$

where:

$\propto_1, \propto_2, \omega, u$ — constants dependent on the kind of material,

σ — average value of the stress in the force fraction,

$\sigma_{\min}$ — minimum value of stress at which the particle is fractured.

Specific forms of these depending for the test material are:

$$a(\sigma) = 0,87(\sigma - 3,59)^{0,72} \tag{25}$$

$$\sigma_{\min} = 3,59 \text{ [MPa]}$$

$$k(\sigma) = \frac{8,42}{\sigma^{0,32}} \qquad (\sigma) \geq 3,59\,[\text{MPa}] \tag{26}$$

In Figures 2 and 3 theoretical distribution function were drawn with using the continuous lines according to equations (25) and (26). The parameter *a* increases with increasing strength value, whereas the parameter *k* decreases with strength increasing. Figure 4 shows the relationship of the parameter *b* of a distribution function (9) on the dolomite particle strength

As can be seen from this figure, this parameter is practically independent of the strength of the particle. Large scatter of results is due to the irregular shape of the particles and the relatively small sample size.

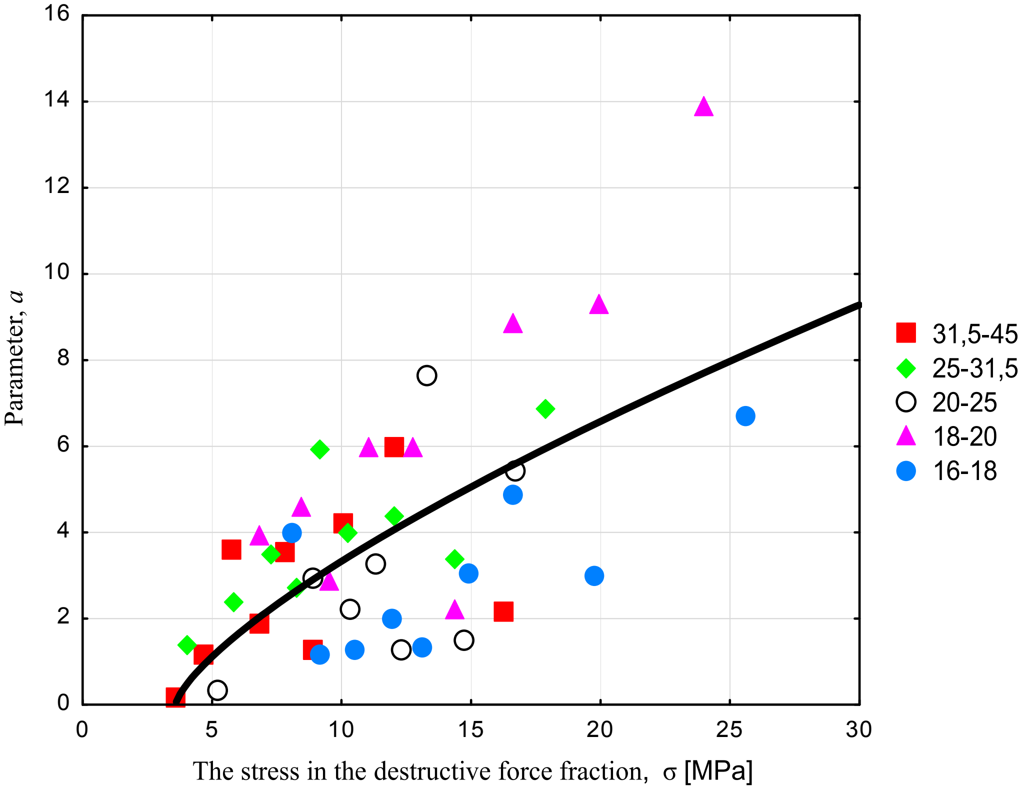

Fig. 2. The dependence of the parameter *a* of a particle size distribution function (9) on the dolomite particle strength

4. Conclusions

The conducted research indicates that the distribution model fits well with experimental data. The three-parameters function fits to empirical data obtained by sieve analysis of broken single irregular particles of dolomitic type of copper ore, regardless the size fraction. This is reflected by relatively small variation of empirical data (Fig. 1), which are arranged according to the theoretical model. The particle size distribution indices S_r and r_k indicate that the compliance

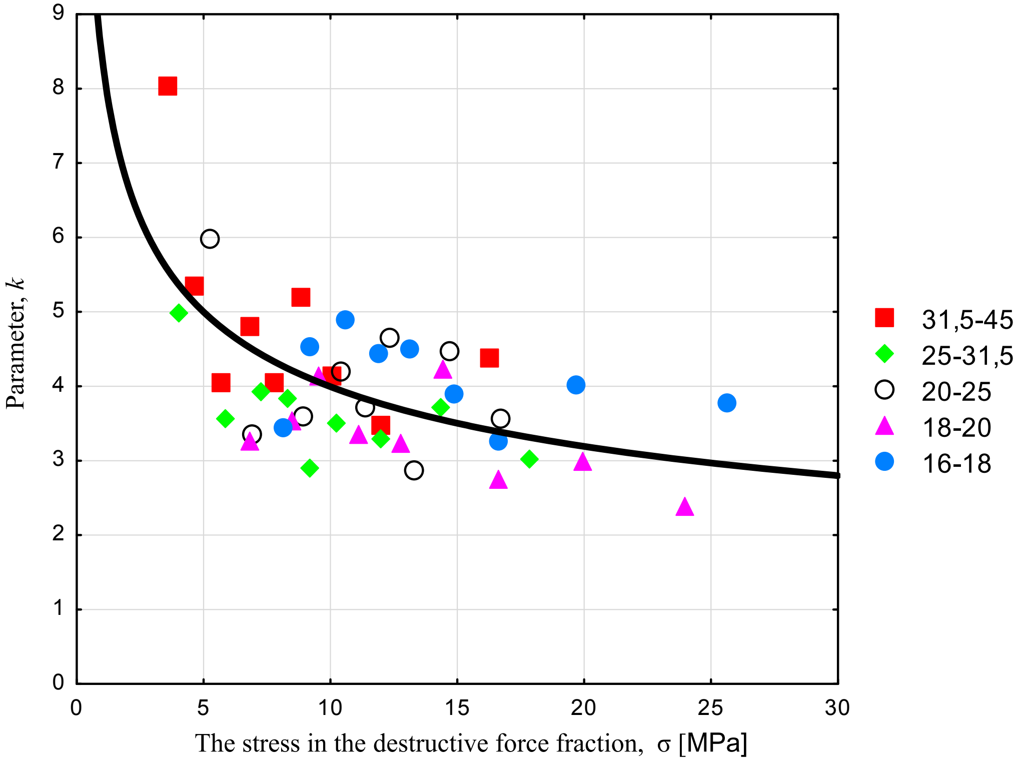

Fig. 3. The dependence of the parameter *k* of a particle size distribution function (9) on the dolomite particle strength

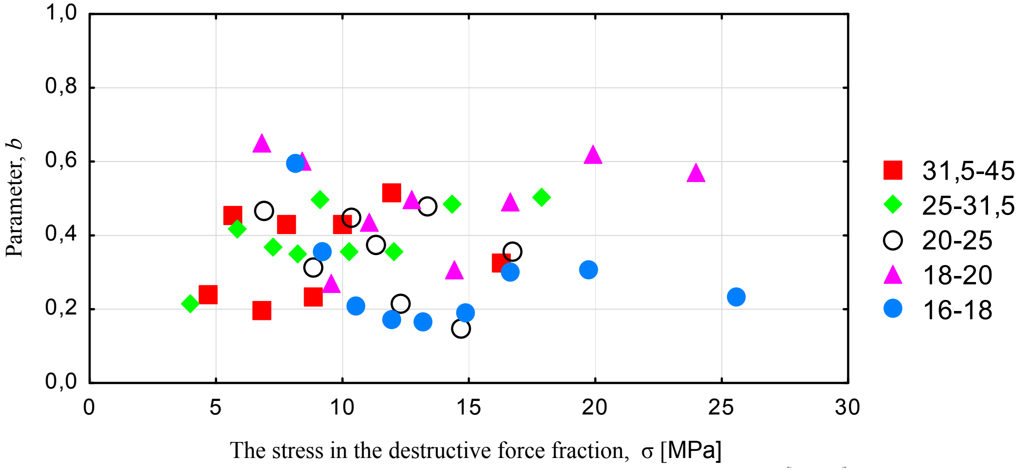

Fig. 4. The dependence of the parameter *b* of a distribution function (9) on the dolomite particle strength

of theoretical model to the experience data is very good, taken into account, that it is cumulative curve and the investigation was performed for irregular particles. This proves the identity of single particle grinding mechanism by slow compression for particles of dolomitic type of ore, regardless of the initial particle size.

The parameter of three-parameters proposed function (9) a increases with increasing strength value of dolomitic particles, whereas the parameter k decreases with strength increasing. As can be seen form Figure 4, parameter b is practically independent of the strength of the particle. Large scatter of results is due to the irregular shape of the particles and the relatively small sample size. Research should be continued.

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