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Review of numerical models of cavitating flows with the use of the homogeneous approach

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Abstract The focus of research works on cavitation has changed since the 1960s; the behaviour of a single bubble is no more the area of interest for most scientists. Its place was taken by the cavitating flow considered as a whole. Many numerical models of cavitating flows came into being within the space of the last fifty years. They can be divided into two groups: multi-fluid and homogeneous (i.e., single-fluid) models. The group of homogenous models contains two subgroups: models based on transport equation and pressure based models. Several works tried to order particular approaches and presented short reviews of selected studies. However, these classifications are too rough to be treated as sufficiently accurate. The aim of this paper is to present the development paths of numerical investigations of cavitating flows with the use of homogeneous approach in order of publication year and with relatively detailed description. Each of the presented model is accompanied by examples of the application area. This review focuses not only on the list of the most significant existing models to predict sheet and cloud cavitation, but also on presenting their advantages and disadvantages. Moreover, it shows the reasons which inspired present authors to look for new ways of more accurate numerical predictions and dimensions of cavitation. The article includes also the division of source terms of presented models based on the transport equation with the use of standardized symbols.

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Nomenclature

A	–	interfacial area concentration, m^2
c	–	speed of sound in the mixture, m/s
c_{wallis}	–	propagation of acoustic waves without mass transfer, m/s
C	–	model constant
C_A	–	additional model constant
$C\mu$	–	empirical turbulent viscosity constant
d	–	body diameter, m
e	–	energy, $(kg\,m^2)/s^2$
f	–	mass fraction
k	–	turbulence kinetic energy, m^2/s^2
k_p	–	scaling constant
k_v	–	scaling constant
L	–	length scale, m
$\dot{m}$	–	mass transfer rates, mass source, $kg/(m^3 s)$
n_o	–	nuclei concentration per unit volume, nuclei/m^3
p	–	local fluid pressure, Pa
$p\mathbf{I}$	–	spherical stress tensor, Pa
p_{sat}	–	saturated vapour pressure, Pa
$\mathbf{q}^m$	–	molecular heat flux, kg/s^3
$\mathbf{q}^R$	–	turbulent heat flux, kg/s^3
r	–	radius of the sphere, m
R	–	bubble radius, m
R_1	–	universal gas constant, J/(mol K)
$\mathbf{s}_b$	–	intensity of the mass forces source, N/m^3
$\mathbf{s}_e$	–	intensity of the energy source, $J/(m^3/s)$
t	–	time, s
t_∞	–	time scale of free stream value, s
T	–	temperature, K
T_{sat}	–	saturation temperature, K
$\mathbf{u}$	–	velocity vector, m/s
$u_{I,n}$	–	normal velocity component to interface, m/s
$u_{I,n}^{Local}$	–	interface flow field local velocity component, m/s
$u_{I,n}^{net}$	–	interface net velocity component, m/s
$u_{v,n}$	–	vapor phase normal velocity component, m/s
U	–	velocity magnitude, m/s
V	–	volume, m^3

Greek symbols

α	–	volume fraction
ε	–	dissipation, m^2/s^3
μ	–	dynamic viscosity, Pa s
μ_{tm}	–	eddy viscosity, Pa s
ρ	–	density, kg/m^3
σ	–	surface tension, N/m
$\boldsymbol{\tau}^m$	–	viscous molecular stress tensor, Pa
$\boldsymbol{\tau}^R$	–	turbulent Reynolds stress tensor, Pa

Subscripts

S	–	Schnerr and Sauer model
d	–	evaporation term
g	–	gas
l	–	liquid
m	–	mixture
p	–	condensation term
v	–	vapor
nuc	–	nucleation site
∞	–	free stream value
Z	–	Zwart model

Boldface lower-case letters refer to vectors while boldface capital letters and Greek lower-case letters refer to matrices.

1 Introduction

The history of cavitation studies dates back to 1894. The first description of the occurrence of vapour bubbles in water appeared in Reynolds' paper, but it should be emphasized that throughout this document there is no such expression as 'cavitation' [1]. Thorneycroft and Barnaby [2] published the work to describe the unknown phenomenon that was responsible for the wear of the surface of a screw propeller and used this expression not until one year later. Twenty two years passed by before Rayleigh [3] published the first mathematical model of cavitation in the incompressible fluid. The awareness of weak points of the first model such as neglecting surface tension and liquid viscosity caused a long scientific discussion that went on for sixty years. The bubble dynamic equation presented by Plesset [4] is until today the well-known form of the mathematical model of bubble dynamics

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 = \frac{p_{sat} - p}{\rho_l} - \frac{2\sigma}{\rho_l R} - 4\frac{\mu_l}{\rho_l R}\frac{dR}{dt} , \tag{1}$$

known as the Rayleigh-Plesset equation.

The Rayleigh-Plesset equation was the basis for pioneering works in the area of numerical investigations, which covered the analyses of behaviour of a single bubble under the influence of the variable pressure of the surrounding liquid [5]. In the course of time, the focus of research works on cavitation has changed. The place of the analyses of the behavior of a single bubble was taken by the cavitating flow considered as a whole. All numerical simulations of the cavitating flow, regardless of the used approach (multi-fluid or homogeneous), require to solve the appropriate set of governing equations, which include mass, momentum or energy equations [6]:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0 , \tag{2}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) = \operatorname{div}\left(-p\mathbf{I} + \boldsymbol{\tau}^m + \boldsymbol{\tau}^R\right) + \rho \mathbf{s}_b , \tag{3}$$

$$\frac{\partial}{\partial t}(\rho e) + \operatorname{div}(\rho e \mathbf{u} + \rho \mathbf{u}) = \operatorname{div}\left[\left(-p\mathbf{I} + \boldsymbol{\tau}^m + \boldsymbol{\tau}^R\right)\mathbf{u} + \mathbf{q}^m + \mathbf{q}^R\right] + \rho \mathbf{s}_e . \tag{4}$$

In the case of multifluid approach, the number of sets of governing equations is dependent on the number of considered phases. In the homogeneous approach, one set of governing equations is solved for all phases. The flow is treated as a mixture of two incompressible phases. One way that enables to take the change of vapour fraction into consideration, is introduction of the additional transport equation that in most of cases is based on the mentioned above Rayleigh-Plesset equation (1). The transport equation, based on the standard formula presented in the Frikha *et al. work* [7], can be expressed in terms of changes in vapour volume:

$$\frac{\partial \alpha_v}{\partial t} + \operatorname{div}(\alpha_v \mathbf{u}) = \alpha , \tag{5}$$

liquid volume

$$\frac{\partial \alpha_l}{\partial t} + \operatorname{div}(\alpha_l \mathbf{u}) = \alpha , \tag{6}$$

vapour mass

$$\frac{\partial \rho_v \alpha_v}{\partial t} + \operatorname{div}(\rho_v \alpha_v \mathbf{u}) = \dot{m}^+ + \dot{m}^- , \tag{7}$$

and liquid mass

$$\frac{\partial \rho_l \alpha_l}{\partial t} + \operatorname{div}(\rho_l \alpha_l \mathbf{u}) = \dot{m}^+ + \dot{m}^- . \tag{8}$$

In this article the phase transformation is presented in the form of liquid mass change Eq. (8). The transport equation is expressed in the form of mass transfer rates, $\dot{m}$, (called source terms) that have different forms for condensation – increase of liquid mass ($\dot{m}^+$), when the local fluid pressure increases above the saturated vapour pressure and evaporation – decrease of liquid mass ($\dot{m}^-$), when the local fluid pressure drops below the saturated vapour pressure:

$$\dot{m} = \begin{cases} \dot{m}^+ & \text{if} \quad p > p_{sat} \, , \\ \dot{m}^- & \text{if} \quad p < p_{sat} \, . \end{cases} \tag{9}$$

The second most popular way to express the source terms, which is also presented in the article, is given by Eq. (5). In this case, when the local fluid pressure increases above the saturated vapour pressure, the vapour volume decreases and when the local fluid pressure drops below the saturated vapour pressure, the vapour volume increases:

$$\alpha = \begin{cases} \alpha^- & \text{if} \quad p > p_{sat} \, , \\ \alpha^+ & \text{if} \quad p < p_{sat} \, . \end{cases} \tag{10}$$

In case using in the original papers of other form of source terms than the chosen variant (Schnerr and Sauer [22], Frobenius *et al.* [25], the both forms of source terms (original and target) are presented.

Until today several works [7–11] tried to order particular approaches and presented short reviews of selected studies. This review summarizes assumptions of all significant homogeneous fluid approaches used to model the sheet and cloud cavitation up to now. The review is not limited to the simple listing of authors and their ideas of cavitation homogeneous models in order of publication year, but also delivers information about the differences in relation to the previous works and the intended application area. By the description of application area the emphasis was put on the number of dimensions of models and character of simulations, steady-state or unsteady. The conclusion includes hints for choice of the appropriate simulation model.

2 The development paths of numerical investigations of cavitating flows with the use of the homogeneous approach

Kubota [12] proposed the first homogeneous model based on transport equation, which gained the international appreciation, in 1992. The model was applied in two dimensional steady-state analysis of the flow around a hydrofoil NACA 0015 at angles of attack of 8° and 20°. The model is called the bubble two-phase flow (BTF) cavity model. Until today, many researches [13,14] use the model as a reference point in their investigations. Kubota formulated the local homogeneous model (LHM) equation of motion

$$\left(1+2\pi r^2 n_0 R\right) R\frac{D^2R}{Dt}+ \left(\frac{3}{2}+4\pi r^2 n_0 R\right)\left(\frac{DR}{Dt}\right)^2+2\pi r^2\frac{Dn_0}{Dt}R^2\frac{DR}{Dt}=\frac{p_{sat}-p}{\rho_l} \tag{11}$$

on the basis of the exact nonlinear Rayleigh-Plesset equation. In the first analyses the initial value of bubble radius R was set to 1×10^{-6} m. Unfortunately, the nonlinear character of the model led to instability.

Many researches tried to identify the weak points of the Kubota model and suggested their own solutions of the pointed problems. Merkle *et al.* [15–17] presented in 1998 their own version of source terms

$$\dot{m}^+=\frac{C_p\rho_v(1-\alpha_l)(p-p_{sat})}{(0.5\rho_l U_\infty^2)t_\infty}, \qquad p>p_{sat}\,, \tag{12}$$

$$\dot{m}^-=-\frac{C_d\rho_l\alpha_l\,(p_{sat}-p)}{(0.5\rho_l U_\infty^2)\,t_\infty}, \qquad p<p_{sat}\,. \tag{13}$$

Unlike the source terms proposed by Kubota, their variant does not refer to the bubble radius, but to the change of the liquid density, which is proportional to the dynamic pressure

$$|p_{sat}-p|=\frac{\kappa\rho_\infty U_\infty^2}{2}\,. \tag{14}$$

The parameter κ has a value between 0.2 and 0.5. Changes of fluid volume caused by changes of its density allow considering the fluid as compressible. Merkle *et al.* took also the characteristic time scale of fluid motion t_∞ into account, through the formulation of source terms. The characteristic time scale of fluid motion

$$t_{\infty} = \frac{d}{U_{\infty}} \tag{15}$$

allowed expressing the time necessary for transition from one phase to the other in the condensation and evaporation source terms. The model can be a useful instrument for many applications. Merkle *et al.* [13] applied the model to numerical simulations of two dimensional steady-state flow over the NACA MOD 66 hydrofoil at the angle of attack of 4°. Ahuja *et al.* [16] tested the model in simulations of sheet cavitation around over a cylindrical head form and a NACA MOD 66 hydrofoil at the angle of attack of 4°. Senocack and Shyy [17] applied the model to two dimensional steady-state flows around the axisymmetric cylindrical body, planar NACA MOD 66 hydrofoil and through a convergent-divergent nozzle.

Kunz *et al.*, in 2000, presented the other solution of source terms [18–20]

$$\dot{m}^{+} = \frac{C_p \rho_v \alpha_l^2 (1 - \alpha_l)}{t_{\infty}}, \qquad p > p_{sat} , \tag{16}$$

$$\dot{m}^{-} = -\frac{C_d \rho_v \alpha_l (p_{sat} - p)}{(0.5 \rho_l U_{\infty}^2)\, t_{\infty}}, \qquad p < p_{sat} . \tag{17}$$

The authors used the approach based on the Ginzburg-Landau potential. Hohenberg and Alperin [21] emphasized usefulness of this theory for dynamic phenomena emphasized in 1977. In the numerical analyses, the k-ε turbulence model was implemented. Kunz *et al.* [18] presented solutions of two sets of numerical analyses. The first of them includes two-dimensional steady-state and unsteady analyses of cavitation on a series of axisymmetric forebodies. In the next set, an extended application for steady-state and unsteady flow over a 1-caliber ogive at angle of attack of 10° is presented.

In 2001 many studies on transport based cavitation models appeared. Schnerr and Sauer [22] presented the first transport based cavitation model which does not need any empirical constants

$$\alpha^{+} = \frac{n_0}{1 + \frac{4}{3} n_0 \pi R^3} 4\pi R^2 \sqrt{\frac{2}{3} \frac{(p - p_{sat})}{\rho_l}}, \qquad p < p_{sat} , \tag{18}$$

$$\alpha^{-} = -\frac{n_0}{1 + \frac{4}{3} n_0 \pi R^3} 4\pi R^2 \sqrt{\frac{2}{3} \frac{(p - p_{sat})}{\rho_l}}, \qquad p > p_{sat} . \tag{19}$$

The new cavitation model was applied in simulations of two-dimensional steady and unsteady cavitating nozzle flows. The transport equation of the model requires only quantitative values of the physical parameters. The model avoids any nonphysical parameter. The initial value of bubble radius, R, was set to 3×10^{-5} m. After transforming Eqs. (18) and (19) to the form of source terms, the equations take the following forms:

$$\dot{m}^{+} = \frac{\rho_v \rho_l}{\rho_m} \alpha_v \left(1 - \alpha_v\right) \frac{3}{R} \sqrt{\frac{2}{3} \frac{\left(p - p_{sat}\right)}{\rho_l}}\,, \qquad p > p_{sat}\,, \tag{20}$$

$$\dot{m}^{-} = -\frac{\rho_v \rho_l}{\rho_m} \alpha_v \left(1 - \alpha_v\right) \frac{3}{R} \sqrt{\frac{2}{3} \frac{\left(p_{sat} - p\right)}{\rho_l}}\,, \qquad p < p_{sat}\,. \tag{21}$$

The source terms – Eqs. (20) and (21) include a part which describes dynamics of the bubble growth. This expression,

$$\dot{R} = \frac{dR}{dt} = \sqrt{\frac{2}{3} \frac{\left(p - p_{sat}\right)}{\rho_l}}\,, \tag{22}$$

follows directly from the simplified form of the original Rayleigh equation which became the basis for many subsequent models of transport equation as well.

Iben [23] based the form of the evaporation and condensation rates of his model on the Rayleigh-Plesset equation:

$$\dot{m}^{+} = C_p \rho_v \frac{6\alpha_v}{2R} \sqrt{\frac{2}{3} \frac{\left(p - p_{sat}\right)}{\rho_l}}\,, \qquad p > p_{sat}\,, \tag{23}$$

$$\dot{m}^{-} = -\rho_l \frac{6\alpha_v}{2R} \sqrt{\frac{2}{3} \frac{\left(p_{sat} - p\right)}{\rho_l}}\,, \qquad p < p_{sat} \tag{24}$$

The main feature that distinguishes the presented formulation from the above listed source terms, is the possibility to use the empirical model constant only in the condensation rate. This procedure is aimed at considering the slower evolution of the condensation process. The initial value of the bubble radius, R, was set to 0.5×10^{-5} m. The model of Iben is intended to simulate cavitation in throttles and nozzles in one- and two-dimensional systems.

In 2002 appeared the first commercially used model called the 'full cavitation model' which was formulated by Singhal *et al.* [24] . The form of the defined source terms

$$\dot{m}^{+} = C_p \frac{\sqrt{k}}{\sigma} \rho_l \rho_l \sqrt{\frac{2}{3} \frac{(p - p_{sat})}{\rho_l}} f_v \,, \qquad p > p_{sat} \,, \tag{25}$$

$$\dot{m}^{-} = -C_d \frac{\sqrt{k}}{\sigma} \rho_l \rho_v \sqrt{\frac{2}{3} \frac{(p_{sat} - p)}{\rho_l}} (1 - f_v - f_g) \,, \qquad p < p_{sat} \tag{26}$$

is similar to the forms that were presented by Schnerr and Sauer [22] or Iben [23]. All mentioned equations (both condensation and evaporation terms) contain the square root of the quotient, where the numerator is the difference between the local pressure and vapor pressure, and the denominator is the liquid density. One feature distinguishes the source terms of Singhal model from all solutions of the time: source terms express not only the changes of bubble dimensions (the Rayleigh-Plesset equation as the starting point through formulating of the transport expressions), but also the local turbulent kinetic energy, $\sqrt{k}$, the surface tension ($\sigma = 0.0717$ N/m) and the content of noncondensible gases (NCG) in the liquid ($f_g \approx 10$ ppm). The vapour mass fraction is calculated as follows:

$$Y_v = \frac{\alpha_v}{\rho_m} \,. \tag{27}$$

The name of the model derives from taking into consideration many factors in the formulation of the source terms. The 'full cavitation model' was validated in steady state two-dimensional numerical analyses by Singhal *et al.* for cavitation on a NACA MOD 66 hydrofoil, in submerged cylindrical bodies and in sharp-edged orifices.

In 2003 other models of cavitating flow based on transport equation appeared, that are worth mentioning and describing. The first of them is the model due to Frobenius [25]. This model's characteristic lies in the specific form of the source terms, which express no more the change of the liquid mass, but the change of the vapour volume fraction, the same as in Schnerr and Sauer model [22]:

$$\alpha^{+} = C_p \frac{n_0}{1 + \frac{4}{3} n_0 \pi R^3} 4\pi R^2 \sqrt{\frac{2}{3} \frac{(p - p_{sat})}{\rho_l}} \,, \qquad p < p_{sat} \,, \tag{28}$$

$$\alpha^{-} = -C_d \frac{n_0}{1 + \frac{4}{3} n_0 \pi R^3} 4\pi R^2 \sqrt{\frac{2}{3} \frac{(p - p_{sat})}{\rho_l}} \,, \qquad p > p_{sat} \,. \tag{29}$$

After transformation to the form of the source terms, the equations have following forms :

$$\dot{m}^{+} = C_p \frac{\rho_v \rho_l}{\rho_m} \alpha_v (1 - \alpha_v) \frac{3}{R} \sqrt{\frac{2}{3} \frac{(p - p_{sat})}{\rho_l}}, \qquad p > p_{sat} , \tag{30}$$

$$\dot{m}^{-} = -C_d \frac{\rho_v \rho_l}{\rho_m} \alpha_v (1 - \alpha_v) \frac{3}{R} \sqrt{\frac{2}{3} \frac{(p_{sat} - p)}{\rho_l}}, \qquad p > p_{sat} . \tag{31}$$

Frobenius *et al.* set the value of nuclei concentration, n_0, to 1×10^8 nuclei/m^3 and the initial value of bubble radius, R, to 30×10^{-6} m. Frobenius *et al.* [25] performed two- and three-dimensional steady-state and unsteady flow simulations around a hydrofoil and steady-state simulations for flow through the centrifugal pump impeller.

The next model that appeared in 2003 was the one due to Saito [26]. The model expresses no more the mass volume change, but the mass surface change, so the unit of mass source, $\dot{m}$, is no longer kg/(m^3 s), but kg/(m^2 s). A new variable A, expressing the interfacial area concentration in the vapour-liquid mixture,

$$A = C_a \alpha_v (1 - \alpha_v) \tag{32}$$

is used as a distinctive feature in this model for both mass transfer rates

$$\dot{m}^{+} = C_p A \alpha_v (1 - \alpha_v) \frac{(p - p_{sat})}{\sqrt{2\pi R_1 T_{sat}}}, \qquad p > p_{sat} , \tag{33}$$

$$\dot{m}^{-} = -C_d A \alpha_v (1 - \alpha_v) \left(\frac{\rho_l}{\rho_v}\right) \frac{(p_{sat} - p)}{\sqrt{2\pi R_1 T_{sat}}}, \qquad p < p_{sat} . \tag{34}$$

The authors do not give the answer about the values of the model constants used. The only hint is a mutual relationship between all constants

$$C = C_p C_a = C_d C_a . \tag{35}$$

The value of the empirical model constant C was set in the numerical simulations of Saito *et al.* to 0.1 m^{-1}. Saito *et al.* assessed the correctness of their model in prediction of cavitation region through unsteady three-dimensional simulations of cavitating flows over a hemisphere and a cylinder geometry. They applied their model in two-dimensional simulations of unsteady flow past the CAV2003 hydrofoil at an angle of attack of 7°.

The source terms of the model proposed by Zwart *et al.* in 2004 [27]

$$\dot{m}^{+} = C_p \frac{3\alpha_v \rho_v}{R} \sqrt{\frac{2}{3} \frac{(p - p_{sat})}{\rho_l}}, \qquad p > p_{sat} , \tag{36}$$

$$\dot{m}^{-} = -C_d \frac{3\rho_v (1-\alpha_v)\,\alpha_{nuc}}{R}\sqrt{\frac{2}{3}\frac{(p_{sat}-p)}{\rho_l}}\,, \qquad p < p_{sat} \tag{37}$$

have a similar form as in the Iben model – Eqs. (23) and (24), but the vapor volume fraction α_v in the condensation rate was replaced by the product of the nucleation site of volume fraction α_{nuc} and the remaining fluid volume fraction $(1-\alpha_v)$. Zwart *et al.* use the standard k-ε model with a modified expression for the eddy viscosity

$$\mu_{tm} = f(\rho)\, C_\mu \frac{k^2}{\varepsilon}\ . \tag{38}$$

The mixture density from the original expression was replaced with the density function

$$f(\rho) = \rho_v + \left(\frac{p_v - \rho_m}{p_v - \rho_l}\right)^n (p_l - \rho_v)\ . \tag{39}$$

The initial value of the bubble radius, R, just as in Kubota model [12], was set to 1×10^{-6} m and the value of the nucleation site of volume fraction, α_{nuc}, to 5×10^{-4}. The Zwart *et al.* model is intended for three-dimensional unsteady cavitating flows. The authors presented three validation examples. The first example is an analysis of the cavitating flow around a hydrofoil. The second example presents results of simulation of the cavitating flow through an inducer and the last one through a venturi.

Senocack and Shyy [17] suggested coupling of source terms with a pressure-based algorithm. The pressure-based algorithm consists of a pressure-velocity-density coupling scheme that uses an upwind density interpolation. The applied source terms

$$\dot{m}^{+} = \frac{(1-\alpha_l)(p-p_{sat})\,\rho_v}{(u_{v,n}-u_{I,n})^2(\rho_l-\rho_v)\,t_\infty}\,, \qquad p > p_{sat}\ , \tag{40}$$

$$\dot{m}^{-} = -\frac{\rho_l \alpha_l (p_{sat}-p)}{(u_{v,n}-u_{I,n})^2(\rho_l-\rho_v)\,t_\infty}\,, \qquad p < p_{sat} \tag{41}$$

approximate in form to the source terms presented by Kunz in 1999. The presented model is called the ‘interfacial dynamics cavitation model’ (IDM). In numerical simulations, the authors applied the k-ε turbulence model. Senocack and Shyy [17] evaluated their model for three most popular applications. To these applications belongs the flow around a cylindrical body and a planar NACA MOD 66 hydrofoil at an angle of attack of 4°, and a flow through a convergent-divergent nozzle. The model is intended for two-dimensional unsteady flow simulations. For unsteady numerical analyses it shows a better agreement with experimental measurements than in the case of steady-state.

The starting point in the development of a new transport equation for Wu *et al.* [28] was the model proposed by Senocack and Shyy in 2004 – Eqs. (40) and (41). Wu *et al.* estimated the interfacial velocity by applying an approximate procedure. Additionally, they used correlations between the net interface velocity, $u_{I,n}^{net}$, and the mass transfer, $\dot{m}$,

$$u_{I,n}^{net} A = \dot{m}^{+} + \dot{m}^{-} \ , \tag{42}$$

and correlation between the net interface velocity $u_{I,n}^{net}$, and the flow field local velocity, $u_{I,n}^{Local}$,

$$(u_{v,n} - u_{I,n})^2 = \left[u_{v,n} - \left(u_{I,n}^{net} + u_{I,n}^{Local}\right)\right]^2 = u_{I,n}^{net2} \tag{43}$$

to achieve the final form of their form of mass transfer rates

$$\dot{m}^{+} = \frac{(1-\alpha_l)\,(p - p_{sat})\,\rho_v}{\left(u_{I,n}^{net}\right)^2 (\rho_l - \rho_v)\, t_\infty} \,, \qquad p > p_{sat} \ , \tag{44}$$

$$\dot{m}^{-} = -\frac{\rho_l \alpha_l\,(p_{sat} - p)}{\left(u_{I,n}^{net}\right)^2 (\rho_l - \rho_v)\, t_\infty} \,, \qquad p < p_{sat} \ . \tag{45}$$

The value of the flow field local velocity, $u_{I,n}^{Local}$, is equal to the value of the vapor phase normal velocity, $u_{v,n}$. Wu *et al.* [28], like Senocack and Shyy [17], intended their model for unsteady two-dimensional flows. They evaluated their model in simulations of cavitating flow around Clark-Y airfoil at two angles of attack, 5° and 8°.

In 2006 Merkle *et al.* [29] presented a new homogeneous model which mass transfer rates are defined as

$$\dot{m}^{+} = k_l \frac{\rho_v \alpha_l}{t_\infty} \left(\frac{p - p_{sat}}{k_p p_v}\right) \,, \qquad p > p_{sat} \ , \tag{46}$$

$$\dot{m}^{-} = -k_v \frac{\rho_v \alpha_l}{t_\infty} \left(\frac{p_{sat} - p}{k_p p_v}\right) \,, \qquad p < p_{sat} \ . \tag{47}$$

The form is distinguished from the all above mentioned models through appearance of two scaling constants k_v, k_l, and k_p – a factor given as small as possible. The values were set in numerical calculations [30] as follow: $k_v = 100.0$, $k_v/k_l = 15.0$ and $k_p = 0.02$. Park *et al.* validated their model using unsteady three-dimensional simulations of cavitating flow over cylinders with 0-, 1/2- and 1-caliber forebody.

Since 2011 researches have been looking for new ways to solve the problem of the insufficient capability of cavitation prediction. Their research area is no more only equations of bubble dynamics or well-known physical relationships between vapour and liquid fraction in fluid. Scientists wish to find the solution of problems with prediction of cavitation through connecting this phenomenon with other physical parameters or complex mathematical formulae.

Huang and Wang in 2011 [31] described a new innovative solution of mass transfer rates

$$\dot{m}^{+} = \chi\left(\rho_m/\rho_l\right)\dot{m}_Z^{+} + \left(1-\chi\left(\rho_m/\rho_l\right)\right)\dot{m}_S^{+}\,, \qquad p > p_{sat}\,, \tag{48}$$

$$\dot{m}^{-} = -\chi\left(\rho_m/\rho_l\right)\dot{m}_Z^{-} + \left(1-\chi\left(\rho_m/\rho_l\right)\right)\dot{m}_S^{-}\,, \qquad p < p_{sat}\,. \tag{49}$$

Their approach to formulate the transport equation consists of using of a blending function $\chi(\rho_m/\rho_l)$

$$\chi\left(\frac{\rho_m}{\rho_l}\right) = 0.5 + \frac{\tanh\left[\frac{C_1\left(\frac{0.6\rho_m}{\rho_l}-C_2\right)}{0.2(1-2C_2)+C_2}\right]}{\left[2\tanh\left(C_1\right)\right]}\,. \tag{50}$$

The authors additionally combined the blending function with the expressions of source terms of Zwart $\dot{m}_Z^{-}$, $\dot{m}_Z^{+}$ [27] and Schnerr and Sauer $\dot{m}_S^{-}$, $\dot{m}_S^{+}$ [22]. The values of the model constants C_1 and C_2 are set to 4 and 0.2. Huang and Wang [31] tested their model for unsteady flows around Clark-Y hydrofoil.

Goncalves presented in 2014 [32] the first version of transport equation which has a form that includes two quantities not used before: the speed of sound, c, and the propagation of acoustic waves without mass transfer, c_{wallis}. These are connected through a following relationship

$$\frac{1}{\rho c_{wallis}^2} = \frac{\alpha}{\rho_v c_v^2} + \frac{1-\alpha}{\rho_l c_l^2}\,. \tag{51}$$

The use of correlation between the sound speed and the thermodynamic equilibrium was an idea with good physical reasoning. good idea, but the proposed form of transport equation required the introduction of changes. In 2014 Goncalves and Charrière showed the modified version of mass transfer rates

$$\dot{m}^+ = \frac{\rho_l \rho_v}{\rho_l - \rho_v}\left(1 - \frac{c^2}{c^2_{wallis}}\right) \operatorname{div}\mathbf{u} - C_p \frac{\rho_v}{\rho_l}\alpha_l \frac{(p - p_{sat})}{0.5\rho_l u^2_\infty}, \qquad p > p_{sat}, \tag{52}$$

$$\dot{m}^- = \frac{\rho_l \rho_v}{\rho_l - \rho_v}\left(1 - \frac{c^2}{c^2_{wallis}}\right) \operatorname{div}\mathbf{u}, \qquad p < p_{sat}. \tag{53}$$

The authors performed computations for two examples of unsteady two-dimensional cavitating flows: underwater explosion with cavitation and flow through a venturi.

In 2015 some scientists revisited the Rayleigh-Plesset equation Eq. (1) and tried to formulate a new equation which on the one hand describes the bubble dynamic with more precision and on the other hand has not any negative influence on the calculations stability. The team of Russian researchers [33] emphasized the relationship between the bubble radius and the Reynolds number, which lead to a new form of the dynamic component of transport equation

$$\frac{dR}{dt} = \tanh\left[1.221\left(\frac{R\sqrt{|p_{sat} - p|\,\rho}}{4\mu}\right)^{0.353}\right]\sqrt{\frac{2}{3}\frac{|p_{sat} - p|}{\rho_l}}\,. \tag{54}$$

Konstantinov *et al.* [33] combined the new dynamic component with the static component from the mass transfer rates proposed by Zwart *et al.* [27] and obtained new source terms

$$\dot{m}^+ = C_p \frac{3\alpha_v \rho_v}{R}\tanh\left[1.221\left(\frac{R\sqrt{(p_{sat} - p)\,\rho}}{4\mu}\right)^{0.35}\right]\sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}, \; p > p_{sat}, \tag{55}$$

$$\dot{m}^- = -C_d \frac{3\rho_v (1 - \alpha_v)\,\alpha_{nuc}}{R}\times \tanh\left[1.221\left(\frac{R\sqrt{(p - p_{sat})\,\rho}}{4\mu}\right)^{0.35}\right]\sqrt{\frac{2}{3}\frac{(p - p_{sat})}{\rho_l}}, \; p < p_{sat} \tag{56}$$

and validated the new model in unsteady flow simulations of a jet element 'pipe-pipe'.

3 Conclusion

This article presents an exhaustive overview of the homogeneous models of cavitating flows since 1992 until today. The focus of research works is still the same – the correct prediction of the occurrence of cavitation. The main starting point for the transport based homogeneous models was the Rayleigh-Plesset equation, which considers dynamics of a single bubble. In numerous cases, by applying of the equation, no account is taken of surface tension and viscosity terms. Current cavitating flow modeling techniques to predict the behaviour of the whole cavitating flow go far beyond the basic equation. They consider, for example, the influence of the speed of sound or introduce a blending function in transport equation. The descriptions of each model are accompanied by essential equations and values of quantities used by the authors in their calculations. The appendix contains the table of empirical coefficients used in the selected transport equation based cavitation models. It should be emphasized that the value of empirical coefficient can change for specific terms and the nuclei concentration n_0 and the initial bubble radius, R, are also not fixed values, but depend on many conditions.

It follows from the presented survey that in the numerical simulations of cavitating flows a trend to carry out unsteady flow simulations and using of three-dimensional models is seen. The progress in this directions is expected to be achieved. On the one hand the predictions of the results have a better accuracy for three-dimensional models and on the other hand there is a better scope for access to more efficient computer resources. Unsteady flow simulations allow monitoring of the development of cavitating region and its changes in time, which is necessary because of the dynamic character of the phenomenon. Hydrofoils predominate in the analysed validation examples of the proposed models. The next place is taken by venturis and cylinders. A summary of the most common application areas of the analysed cavitation models is presented in Tab. 2 of the appendix. It results from these data that the majority of the models can be used in two-dimensional simulations and since 2003 the models are intended for unsteady flow computations. For the analyses of the flow through venturis there are also a few possible models to be used like Schnerr and Sauer or Zwart *et al.* models, for example. For the flow over a cylinder the older models, like Merkle *et al.*, Kunz *et al.* or Singhal *et al.* models, can be recommended.

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Appendix

The appendix contains the table of empirical coefficients used in the chosen transport equation based cavitation models.

Table 1: List of empirical coefficient used in the chosen transport equation based cavitation models.

Lp.	Name of the first authors	C_p	C_d
1.	Kubota (1992)	50	0.01
2.	Merkle (1998)	1	80
3.	Kunz (2000)	0.2	0.2
4.	Schnerr and Sauer (2001)	–	–
5.	Iben (2000)	>1	–
6.	Singhal (2002)	0.02	0.01
7.	Frobenius (2003)	50	0.02
8.	Saito (2003)	Y	Y
10.	Zwart (2004)	50	0.01
9.	Senocack and Shyy (2004)	Y	Y
11.	Wu (2005)	–	–
12.	Merkle (2006)	–	–
13.	Huang and Wang (2011)	–	–
14.	Goncalves (2014)	–	Y
15.	Konstantinov (2015)	50	0.01

Y – no sufficient data

Table 2: Comparison of application areas of the presented cavitation models.

Name of the first author	Two-dimensional	Three-dimensional	Steady-state simulation	Unsteady simulation	Hydrofoil	Venturi	Cylinder
Kubota	X		X		X		
Merkle	X		X		X	X	X
Kunz	X		X	X			X
Schnerr	X		X	X		X	
Iben		X					X
Singhal	X		X		X		X
Frobenius	X	X	X	X	X		
Saito	X	X		X	X		X
Zwart		X		X	X	X	
Senocack	X			X	X	X	X
Wu	X			X	X		
Merkle		X		X	X		
Huang	X			X	X		
Goncalves	X			X		X	
Konstantinov	X			X		X	