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Approximation abilities of neuro-fuzzy networks

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Abstract: The paper presents the operation of two neuro-fuzzy systems of an adaptive type, intended for solving problems of the approximation of multi-variable functions in the domain of real numbers. Neuro-fuzzy systems being a combination of the methodology of artificial neural networks and fuzzy sets operate on the basis of a set of fuzzy rules "if-then", generated by means of the self-organization of data grouping and the estimation of relations between fuzzy experiment results.

The article includes a description of neuro-fuzzy systems by Takaga-Sugeno-Kang (TSK) and Wang-Mendel (WM), and in order to complement the problem in question, a hierarchical structural self-organizing method of teaching a fuzzy network. A multi-layer structure of the systems is a structure analogous to the structure of "classic" neural networks. In its final part the article presents selected areas of application of neuro-fuzzy systems in the field of geodesy and surveying engineering. Numerical examples showing how the systems work concerned: the approximation of functions of several variables to be used as algorithms in the Geographic Information Systems (the approximation of a terrain model), the transformation of coordinates, and the prediction of a time series. The accuracy characteristics of the results obtained have been taken into consideration.

Keywords: neural network, neuro fuzzy system, clustering

1. Introduction

Neural networks and neuro-fuzzy systems have been included in the arsenal of strong adaptation procedures because of the possibility of applying them in specialized forms of approximation of functions (classification, auto-association, prediction of time series). Both types of networks have one common feature, which is parallel information processing. Neuro-fuzzy systems are neural networks characterised by the processing of fuzzy sets. An advantage of neuro-fuzzy networks is the possibility of interpreting knowledge contained in the weights of neural connections. "Classic" neural networks perform processing a numerical operation but they lack the so called explaining module, because knowledge represented by values of weights is dispersed and does not have a physical representation in a form understandable to the user (Łęski, 2008). The construction of the neuro-fuzzy systems under discussion makes use of the method of extraction of inference rules on the basis of numerical data about the inputs and outputs of the phenomenon being modelled. The paper also includes a description of the structure of systems and algorithms of linear weights as well as parameters of fuzzy functions of the fuzzificator. The approximation process ends when a state of balance is achieved in a fuzzy network as a result of the minimization of the error function (the objective function), defined in general by means of the Euclidean norm (Osowski, 2006) as

$$E = \frac{1}{2}\sum_{l=1}^{p}[y(x^{(l)}) - d^{(l)}]^2 \tag{1}$$

where: $y(\mathbf{x})$ is the input signal of a neuro-fuzzy system, d is the value assigned corresponding to the input vector $\mathbf{x}$,p is the number of teaching data pairs ($\mathbf{x}$, d).

2. Fuzzy logic system

The basic problem in the process of constructing a model is how to obtain the number of inference rules "if-then" representing a certain local output-input dependence. It is known from subject literature (Jang and Sun, 1993) that a number of methods of extracting rules on the basis of measurement data results from a functional equivalence between neural networks and certain types of fuzzy logic, which leads to the equivalence of results. This idea is reflected in the construction of a neural network taught by means of the gradient method, which realizes the Takaga-Sugeno-Kang system under the name of ANFIS (Adaptive Neuro Fuzzy Inference System).

Fuzzy sets as a generalization of ordinary sets are characterized by a fuzzy membership of components in a particular set, i.e. each component can belong to a particular set "in part". The fuzzy set A in the space X can be characterised as a set of ordered pairs $(x, \mu_A(x))$, where the value of the membership function $\mu_A(x) \in [0, 1]$ expresses the rate of membership of the component x in the fuzzy set A (Zadech, 1965). The most frequently used functions are Gaussian membership functions, bell functions and triangular and trapezoidal functions simple in shape. For example, the Gaussian membership function used for a fuzzy representation of numbers is defined as (Duch et al., 2000; Osowski, 2006)

$$\mu_A(x) = exp\left[-\left(\frac{x-c}{\sigma}\right)^2\right] \tag{2}$$

where the parameters c and σ denote the centre of the fuzzy set (for $x = c$, $\mu_A(x) = 1$), and the width of the fuzzy set (variance of the corresponding set A), respectively, and

$$\int_{-\infty}^{+\infty} \mu_A(x)dx = \sigma\sqrt{\pi} \tag{3}$$

The basic fuzzy logic "if-then" rule called a fuzzy implication (function) in the form "if x is A, then y is B" for the input variables x_j (j= 1, 2, ..., N) and one output y, can be written in a canonical form as follows (Markowska-Kaczmar, 2006):

$$\text{if} \underset{1\le j\le N}{\forall} x_j \text{ is } A_j, \text{ then } y \text{ is } B \tag{4}$$

With the use of the operator in the form of an algebraic product, the random membership function $\nu_A\mathbf{x})$, where $x = [x_1, x_2, ..., x_N]^T$, can be written as

$$\nu_A(x) = \prod_{j=1}^{N} \mu_{A_j}(x_j) \tag{5}$$

A connection between input variables and a base of knowledge consisting of fuzzy rules leads to a fuzzy system in the form of a fuzzificator on the input and a defuzzificator on the output. The fuzzificator transforms input data into a fuzzy set, whereas the defuzzificator transforms the fuzzy set into a unique solution point. The transformation of the N-dimensional vector $\mathbf{x}$ into the fuzzy set A is represented by the membership function $\nu_A(\mathbf{x})$, and the process of choosing an optimum point from the domain of a fuzzy membership function called defuzzification depends on the problem in question. One of the popular defuzzification methods (a sharpening operation) is the centre average method

$$y = \frac{\sum_{k=1}^{M} c_k \mu_A^{(k)}(x^{(k)})}{\sum_{k=1}^{M} \mu_A^{(k)}(x^{(k)})} \tag{6}$$

where c_k denotes the centre of the k^{th} fuzzy rule, and $\mu_A^{(k)}(x^{(k)})$ is the value of the membership function of a fuzzy set corresponding to the k^{th} rule.

3. Takaga-Sugeno-Kang and Wang-Mendel neuro-fuzzy systems

The knowledge base of the Takaga-Sugeno-Kang (TSK) system consists of a specified number M of inference rules "if-then" with a linear function in the conclusion of the k^{th} rule, namely

$$M^{(k)} = \text{ if } \underset{1\le j\le N}{\forall} x_j \text{ is } A_j^{(k)}, \text{ then } y = f_k(x) \text{ for } k = 1, 2, \ldots, M \tag{7}$$

Bearing in mind a fuzzy TSK system of the first rank the function $f_k(x)$ assumes the form

$$f_k(x) = p_{k0} + \sum_{j=1}^{N} p_{kj} x_j \tag{8}$$

of a first order polynomial, whose parameters p_{k0}, ..., p_{kN} are weights with values characterizing the ratios of the linear function $f_k(x)$.

Applying a bell fuzzifying function (a rational function) in the relation

$$\mu_{A_j}(x_j) = \left[1 + \left(\frac{x_j - c_j}{\sigma_j}\right)^{2b_j}\right]^{-1} \tag{9}$$

and the aggregation of the predecessor in the form of an algebraic product for the k^{th} inference rule

$$\nu_A^{(k)}(\mathbf{x}) = \prod_{j=1}^{N}\left[1 + \left(\frac{x_j - c_j^{(k)}}{\sigma_j^{(k)}}\right)^{2b_j^{(k)}}\right]^{-1} \tag{10}$$

one obtains a numerical output value of the system

$$y(\mathbf{x}) = \frac{\sum_{k=1}^{M} \nu_A^{(k)}(x)\, f_k(x)}{\sum_{k=1}^{M} \nu_A^{(k)}(x)} \tag{11}$$

The parameters $c_j^{(k)}$, $\sigma_j^{(k)}, b_j^{(k)}$ of the aggregated value $\nu_A^{(k)}(x)$ undergo adaptation in the process of teaching a TKS neuro-fuzzy network, whose architecture presented in Figure 1 results from the description of the fuzzy system in question.

A simplified version of the Takaga-Sugeno-Kang (TSK) system is the Wang-Mendel (WM) system, in which the linear functions $f(\mathbf{x})$ have been restricted to the form of a zero order polynomial. The WM system is a relatively simple method of obtaining knowledge on the basis of numerical data, and the activation of the k^{th} rule is carried out analogously to the TKS system. A precisely determined output value of the system is expressed by the formula

$$y(\mathbf{x}) = \frac{\sum_{k=1}^{M} u^{(k)} \nu_A^{(k)}(x)}{\sum_{k=1}^{M} \nu_A^{(k)}(x)} \tag{12}$$

where $u^{(k)} = p_{k0}$ (the constant component – cf. formula (6)). A fuzzificator characterised by the generalised Gaussian membership function has been used in this paper

$$\mu_{A_j}(x_j) = exp\left[-\left(\frac{x_j - c_j}{\sigma_j}\right)^{2b_j}\right] \tag{13}$$

where the parameters of the fuzzification function c_j, σ_j, b_j (centre, width and shape (exponent)) are adjusted by means of the gradient method in the process of learning. Both the TSK network and the WM network make it possible to solve of the problem of

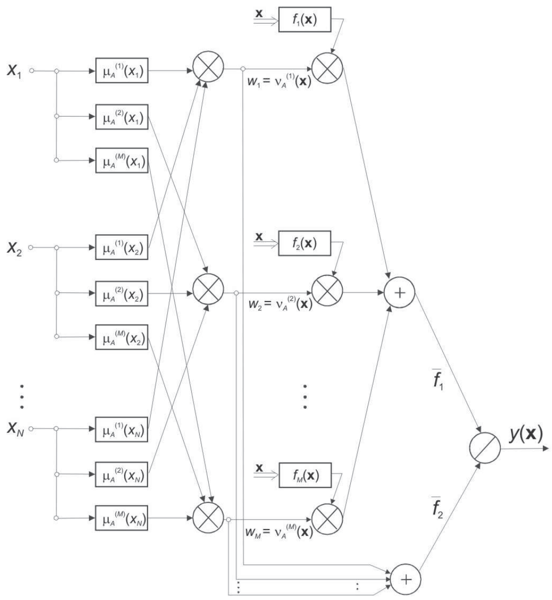

Fig. 1. The layout of a TSK neuro-fuzzy system

approximation in the form of the representation of the value assigned d corresponding to the input vector $\mathbf{x}$, on the basis of teaching data pairs ($\mathbf{x}$, d).

4. Selected self-organisation algorithms in neuro-fuzzy systems

Self-organization included in localised areas consists in the decomposition of the data vector $\mathbf{x}$ into subsets according to dominating features, which represent the centre $\mathbf{c}$. The analysis of homogenous features is known as the *analysis of concentrations* or *clusterisation*. In general, the number of possible divisions of p input vectors into c non-empty subsets is described by the formula (Feller, 1980; Łęski, 2008)

$$\frac{1}{c\,!}\sum_{k=1}^{c}\binom{c}{k}(-1)^{(c-k)}k^{p} \tag{14}$$

For example, for p = 38 input vectors, the number of possible divisions into three groups is 3.14×10^{21} (a practically unreal number of divisions).

From among a number of fuzzy methods of data grouping (partial membership), methods frequently used are those effected on the basis of the minimization of a criterion function, namely:

– grouping method c–means,
– grouping method of Gustafson – Kessel.

In order to carry out an initial analysis of centres the method of differential grouping is used, effected on the assumption that each N–dimensional input vector $\mathbf{x}$ is accompanied by an assigned value d. Therefore, when we deal with a teaching data set in the form of pairs (X, d), where $\mathbf{X}$ is the matrix consisting of rows equivalent to consecutive vectors $\mathbf{x}_k$ $(k = 1, 2, \dots, p)$ and $\mathbf{d}$ is the vector of the respective d_k, centres of teaching data set are obtained by extending the matrices $\mathbf{X}$ to the form

$$Y \leftarrow [X, d] \tag{15}$$

It results from the above that the dimension of the centre being determined will be equal to the sum of the dimensions of the matrix $\mathbf{X}$ and vector $\mathbf{d}$. Denoting as $\mathbf{p}$ the matrices $\mathbf{X}$ (the number of components $N \times p$) and as $\mathbf{q}$ the vectors $\mathbf{d}$ one obtains centres corresponding to both the input variables and the output variables, written as

$$c = [p, q] \tag{16}$$

For the denotations assumed, the aggregation of the output result of the Wang-Mendel fuzzy network will be expressed by the formula

$$y(X) = \frac{\sum_{i=1}^{M} q_i exp\left(-\frac{\|X-p_i\|^{2b_i}}{\sigma_i^{2b_i}}\right)}{\sum_{i=1}^{M} exp\left(-\frac{\|X-p_i\|^{2b_i}}{\sigma_i^{2b_i}}\right)}. \tag{17}$$

The geometrical shape of clusters grouped by means of the c-means method is characterised by circular symmetry for the vector $\mathbf{x} \in R^2$ (for $\mathbf{x} \in R^N$ the reception area has a spherical shape). It is suggested that the condition

$$\underset{1 \le j \le N}{\forall} \sum_{k=1}^{M} \mu_{kj} = 1 \tag{18}$$

should be met for the sum of membership of the vector $\mathbf{x}_j$ in all the clusters represented by centres $\mathbf{c}_k$.

The solution of the task consists in minimising the non-linear function

$$E = \sum_{k=1}^{M} \sum_{j=1}^{p} \mu_{kj}^{m} \|\mathbf{c}_k - x_j\|^2 \tag{19}$$

where p denotes the number of input vectors, and the weight ratio is $m \in [1, +\infty]$.

The problem in question leads to a search for an optimum point of the Lagrange function (Jang et al., 1997)

$$L(\mu, \mathbf{c}, \lambda) = \sum_{k=1}^{M} \sum_{j=1}^{N} \mu_{kj}^{m} \|\mathbf{c}_k - x_j\|^2 + \sum_{j=1}^{p} \lambda_j (\sum_{k=1}^{M} \mu_{kj} - 1) \tag{20}$$

which is represented by

$$\mathbf{c}_k = \frac{\sum\limits_{j=1}^{p} \mu_{kj}^m x_j}{\sum\limits_{j=1}^{p} \mu_{kj}^m} \tag{21}$$

and

$$\mu_{kj} = \frac{1}{\sum\limits_{i=1}^{M} \left(\frac{d_{kj}}{d_{ij}}\right)^{\frac{2}{m-1}}} \tag{22}$$

where $d_{kj} = \|\mathbf{c}_k - x_j\|$ (Osowski, 2006). The iterative process of searching for an optimum point should be preceded by an preliminary initialisation of centres by means of one of the abovementioned methods.

An improvement in the quality of grouping in comparison with the c-means method can be achieved by means of the Gustafson-Kessel algorithm (Gustafson and Kessel, 1976). The clusterisation of data is effected on the basis of the minimization of the criterion function

$$E = \sum_{k=1}^{M} \sum_{j=1}^{p} \mu_{kj}^m d^2\left(x_j,\ \mathbf{c}_k\right) \tag{23}$$

by means of the iterative method. The symbols included in formula (21) denote: parameter $m \in [1, +\infty]$, $d^2(x_j, c_k) = (x_j - c_k)^T G(x_j - c_k)$ is the square of the distance between the vector x_j and the centre c_k, $\mathbf{G}$ is the transformation matrix (positively definite). The ratio of the membership of the vectors $\mathbf{x}_j$ ($j = 1, 2, \ldots, p$) in the centres $\mathbf{c}_k$ ($k = 1, 2, \ldots, M$) is defined by the formula

$$\mu_{kj} = \frac{1}{\sum\limits_{i=1}^{M} \left[\frac{d^2(\mathbf{x}_j,\mathbf{c}_k)}{d^2(\mathbf{x}_j,\mathbf{c}_i)}\right]^{\frac{2}{m-1}}} \tag{24}$$

and the position of the centres is determined on the basis of formula (19). For each centre a fuzzy covariance matrix

$$S_k = \frac{\sum\limits_{j=1}^{p} \mu_{kj}^m (x_j - c_k)(x_j - c_k)^T}{\sum\limits_{j=1}^{p} \mu_{kj}^m} \tag{25}$$

is generated, which is used in the iterative process for specifying the transformation matrix

$$A_k = \sqrt[N]{det(S_k)} S_k^{-1} \tag{26}$$

where N is the dimension of the input vector. For $\mathbf{G} = \mathbf{I}$ clusters will have a circular symmetry, and for a random positively definite matrix $\mathbf{G}$, clusters in Euklidean space have an elliptical shape, and in general hyper ellipsoids are preferred, which give optimum shapes to the grouping results.

5. Algorithm for determinig an optimum number of inference rules

An important component in the construction of fuzzy systems is the determination of the number of inference rules, whose activity corresponds to a group of data contained in the cluster. Criteria for the quality of input data grouping, which can be used for restricting the number of clusters (the elimination of empty clusters) are as follows (Babuska and Verbruggen, 1997; Osowski, 2006):

- *fuzzy volume of the cluster*

$$V_h = \sum_{k=1}^{M} \sqrt{det(S_k)} \tag{27}$$

- *average density of the partition*

$$D_A = \frac{1}{M} \sum_{k=1}^{M} \frac{U_k}{\sqrt{det(S_k)}} \tag{28}$$

where $U_k = \sum_j u_{kj}$for j satisfying the condition $(x_j - c_k)^T S_k^{-1}(x_j - c_k)$, i.e. the vectors x_j must belong to a hyper ellipsoid in the general sense of the word,

- *average internal distance* D_w between the data in the cluster and its centre $\mathbf{c}_k$

$$D_w = \frac{1}{M} \sum_{k=1}^{M} \frac{\sum_{j=1}^{p} \mu_{kj}^m d_{kj}^2}{\sum_{j=1}^{p} \mu_{kj}^m} \tag{29}$$

- *average flattening of the cluster*

$$t_A = \frac{1}{M} \sum_{k=1}^{M} t_k \tag{30}$$

where t_k is the ratio of the smallest eigenvalue of the covariance matrix $\mathbf{S}_k$ to its greatest eigenvalue. Small values of the coefficients V_h and t_A and great values of the coefficients D_A and D_w indicate a good quality of the division into clusters. These conditions cannot be simultaneously satisfied. It is possible to obtain a sub-optimum number of clusters on the basis of the value of the *global static measure*

$$\alpha = a_1 V_h - a_2 D_A - a_3 D_w + a_4 t_A \tag{31}$$

where $a_i \in [0, 1]$ (i= 1, 2, 3, 4) are scalar coefficients, whose values have randomly been chosen in this paper.

6. Numerical examples

The working quality of neuro-fuzzy systems in a particular branch of geodesy has been illustrated by the examples below.

Example 1. The approximation of a non-linear function of several variables

In this case the problem consists in identifying a model of reality. The approximation has been carried out by the TSK system and the WM system. In order to show the approximation abilities of both systems an approximation of a complex non-linear function of two variables $x = [x_1, x_2]$ having the form

$$f_1(x) = 0.1 + (1.0 + sin(2x_1 + 3x_2))/(3.5 + sin(x_1 - x_2)) \tag{32}$$

graphically illustrated in Figure 2.

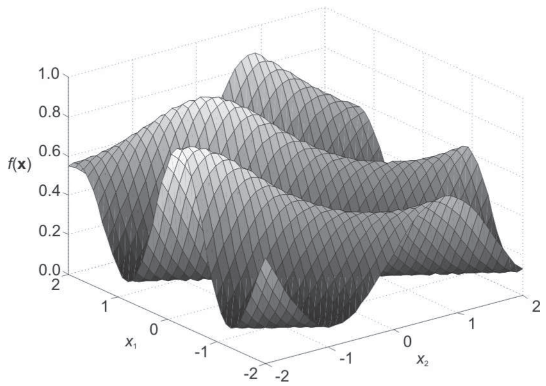

Fig. 2. The form of function (32) being tested

The accuracy of the approximation for the test set with 1600 points carried out by means of the TSK system and the WM system with a specified sub-optimum number of 25 inference rules expressed in the form the root mean square error *RMSE* was: for the TSK system equal to 0.0223, for the WM system equal to 0.0412.

Real data in the form of spatial coordinates $(x_1, x_2, f(\mathbf{x}))$ of 483 points situated in an area of 3 km^2 were used for identifying the terrain model by means of the systems under discussion. The efficiency of the approximation was evaluated on the basis of the classification of testing errors in the form of a divergent multi-level series. The classification of the absolute values of testing errors was carried out on the basis of an optimum choice of the length of class ranges containing maximum information, according to formula (Brillouin, 1969; Feller, 1980)

$$s = \left[t + \ln \frac{tT}{s\,(s-1)} \right] \tag{33}$$

where T is the range of the feature under research, t is the length of the class range, s is the value of the error classified. The solution of this equation is the number $k \approx t/s$, whose value expresses an optimum numerical relation between the length of the class range and the value of the variable classified. Results of the representation of the terrain model have been illustrated in Figure 3.

Evaluations of the quality of approximation by means of the abovementioned procedures was carried out on the basis of *RMSE*, defined by the formula

$$RMSE = \sqrt{\sum_{j=1}^{p} [d_j - f(x_j)]^2}. \tag{34}$$

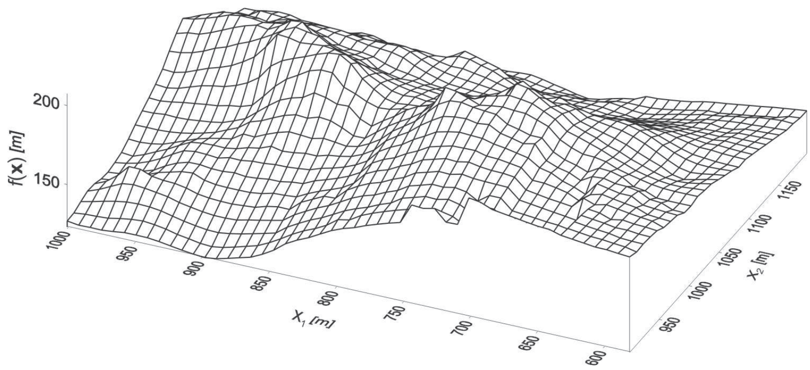

Fig. 3. Result of the representation of a terrain model by means of the TSK system

The working quality of the TSK and WM systems in the case of the problem of approximating a terrain model expressed in the form of a mean square error has been presented in Table 1.

Table 1. Errors of TSK and WM neuro-fuzzy approximations

The fuzzy systems	The approximation errors *RMSE* [m]	
	The learninig set *RMSE*(*L*)	The testing set *RMSE*(*T*)
Takagi – Sugeno – Kanga fuzzy system	0.11	0.19
Wang – Mendel fuzzy system	0.12	0.21

It results from the above data concerning the approximation under way that the evaluated accuracy of the task carried out by means of both systems is almost identical. The evaluation method of the accuracy of the approximation by means of the *RMSE* is most often mentioned in literature and preferred in practice.

Example 2. The transformation of coordinates

Transformation understood as re-calculating coordinates from the primary system onto the secondary system in the case of a two dimensional task consists in the realization of the function $f : R^2 \rightarrow R^2$ i.e. $X = f^1(x, y)$, $Y = f^2(x, y)$. The most frequently used method in the numerical realization of this problem is the Helmert transformation (imperfection of the method – lack of immunity to gross errors). The numerical experiment of the transformation of coordinates from a local system onto the system "2000" that is an official spatial reference system in Poland, was carried out on a teaching set (adaptation points) consisting of 5 points and on a testing set of 25 points (Fig. 4)

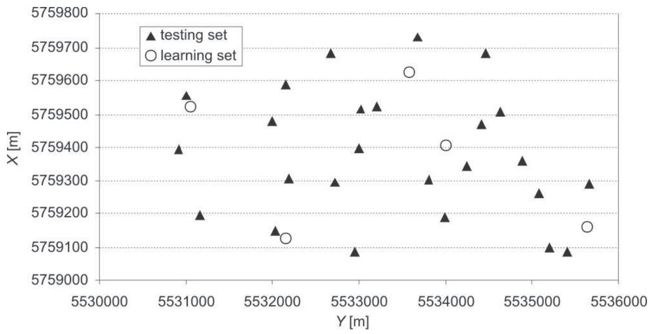

Fig. 4. Position of points of the learing set and the testing set

Calculations were carried out in two stages. The first stage consisted in specifying good values of approximate transformation results by means of neural networks including the Levenberg-Marquardt gradient optimisation method. At the second stage input variables (results of the operations carried out at the first stage) were transformed into values of output variables by means of the TSK system, which are the final transformation results.

The evaluation of how the algorithm works is optimistic, because the value of the *RMSE* turned out to be 0.0071, and when the professional software C-GEO was used the *RMSE* was equal to 0.0069. Let us notice that neural networks are explicitly non-linear, and the functions $f_k(\mathbf{x})$ (formula (6)) in the conclusion of the k^{th} rule "if-then" are most often linear functions. Therefore, it is possible to conclude that the TSK system makes it possible to model complicated relations between the input and output of a system. The problem was discussed more profoundly in the monthly journal "Przegląd Geodezyjny" (Mrówczyńska, 2009).

Example 3. The prediction of a time series

A time series is a series of specified values, registered at fixed time intervals. The problem of prediction consists in estimating future values of a time series on the basis of future values of the components of the series. From information available about the variable **x** at past moments as the set $\{x(k-1), x(k-2), \ldots, x(k-p)\}$, a neuro-fuzzy system determines the value $y(k)$ at the moment k.

Research into the application of a neuro-fuzzy system for predicting a time series was carried out on GPS-RTK data, representing changes in the module of the "zero" vector between the base station and a "moving" receiver (Szpunar et al., 2003). The number of changes in the module of the vector $\mathbf{x}(k)$ undergoing prediction was reduced to k= 2500, from which the first 1250 were adopted as a teaching part of the data set. The time series, the 6-step prediction carried out with a neuro-fuzzy system and the prediction error have been presented in Figures 5, 6 and 7. A thick vertical line separates the teaching part and the testing part, and errors for both parts are also presented.

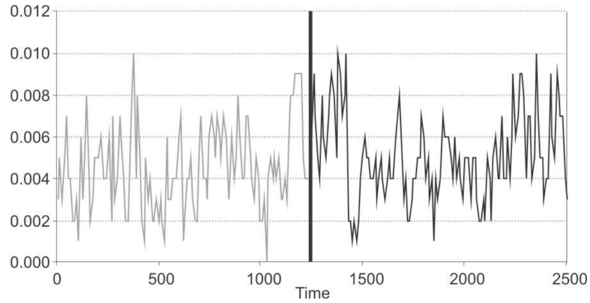

Fig. 5. Time series

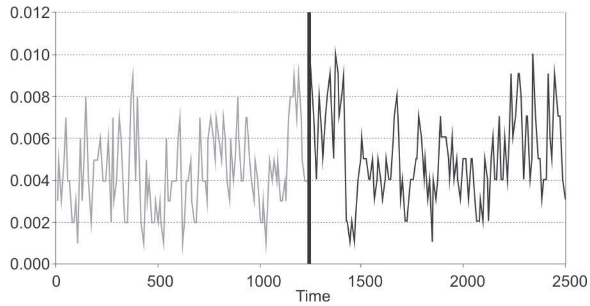

Fig. 6. 6-step prediction by means of a TSK neuro-fuzzy system

The values of prediction estimated are presented as a function called the predicator of series p. The predicator which results from the application of a neuro-fuzzy system is a non-linear predicator.

Apart from the abovementioned examples of the application of neuro-fuzzy systems in the field of geodesy, the author has also attempted to use the systems for assessing the state of deformation of an object resulting from use, and she has applied the adaptation algorithm functioning in the Wang-Mendel neuro-fuzzy system for the approximation of a single variable function expressed by the equation

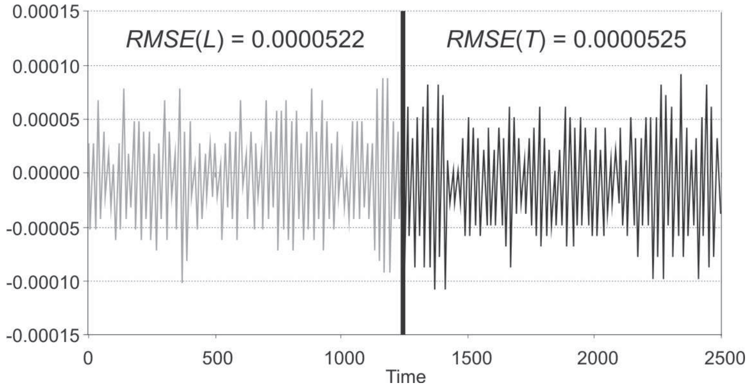

Fig. 7. Prediction error (*RMSE* = 0.0000525)

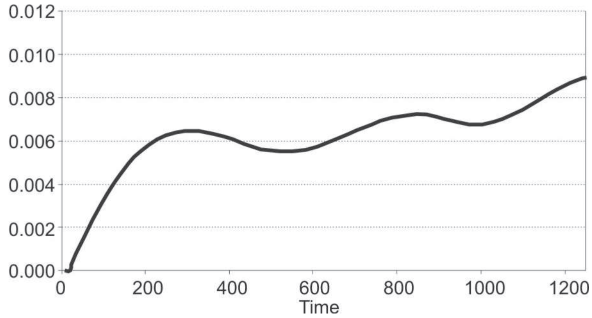

Fig. 8. GPS signal smoothing by means of the Henderson moving average

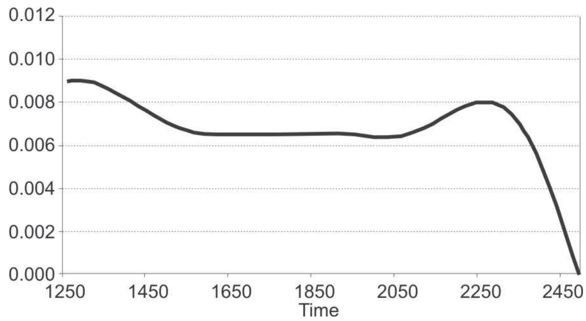

Fig. 9. Smoothing of the predicted GPS signal by means of the Henderson moving average

$$f_3(x) = 0.1\sin(0.2\pi x) + 0.2\sin(0.3\pi x) + 0.4\sin(0.1\pi x) + 0.9\sin(1.9\pi x) + 1.9\sin(5.1\pi x) \quad (35)$$

Neuro-fuzzy systems can also be used to obtain less distorted information from a signal registered with interference noise and disturbances.

7. Conclusions

Fuzzy systems have exceptional approximation abilities for extremely complex non-linear functions of several variables, which is indicated by the results of the tests carried out in this research. Basic attributes of the systems presented in this paper are the assumptions adopted for the Gaussian type of membership function and the method of choosing the number of inference rules. The structure of neuro-fuzzy systems created by a set of fuzzy rules makes it possible to deduce a cause and effect relation between the input and the output of fuzzy systems, which is impossible to obtain by means of ”classic” neural networks. The numerical experiments carried out by the author indicate that a high quality of approximation by means of the TSK and Wang-Mendel systems mostly depends on the sub-optimum number of inference rules determined on the basis of an analysis of the value of the global statistical measure.

The numerical procedures presented in the article resulted from implementation carried out by the author herself, with the exception of the procedure concerning the prediction of a time series, available in the software Fuzzy Logic in the MATLAB environment.

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Zdolności aproksymacyjne systemów neuronowo rozmytych

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Streszczenie

W pracy przedstawiono działanie dwóch systemów neuronowo rozmytych typu adaptacyjnego, przeznaczonych do rozwiązywania zagadnienia aproksymacji funkcji wielu zmiennych w dziedzinie liczb rzeczywistych. Systemy neuronowo rozmyte jako połączenie metodologii sztucznych sieci neuronowych i zbiorów rozmytych funkcjonują na podstawie zbioru reguł rozmytych „jeżeli-to”, generowanych z zastosowaniem samoorganizacji grupowania danych oraz estymacji relacji rozmytych wyników eksperymentu.

Artykuł zawiera opis systemów neuronowo rozmytych Takagi-Sugeno-Kanga (TSK), Wanga-Mendela (WM) oraz celem uzupełnienia rozpatrywanego zagadnienia hierarchiczną strukturalną samoorganizującą się metodę uczenia sieci rozmytej. Struktura wielowarstwowa systemów stanowi strukturę analogiczną do struktury „klasycznych” sieci neuronowych. W końcowej części artykułu zostały zaprezentowane wybrane obszary aplikacji systemów neuronowo rozmytych w dziedzinie geodezji. Przykłady numeryczne działania systemów dotyczyły: aproksymacji funkcji wielu zmiennych w aspekcie ich wykorzystania jako algorytmów uzupełniających w Systemach Informacji Przestrzennej (aproksymacja rzeźby terenu), transformacji współrzędnych oraz predykcji szeregu czasowego. Uwzględniono charakterystykę dokładności uzyskanych wyników.