

Innovativeness of Banks as a Driver of Social Welfare

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Abstract

The general aim of the paper is to indicate the key role played by banks in Schumpeterian innovative evolution. It includes formal modelling of innovative evolution of financial subsystem of an economy which goes beyond routine behavior of banks deprived of innovative, financial strategies. Moreover this paper studies an impact of innovations in financial sphere of modified Debreu monetary economy on its real sector and, specifically, on social welfare. Thus the paper main aims are to prove that pro-innovative banks may intensify innovative evolution and to specify the sufficient conditions to guarantee the preservation of the rules of circular flow for a consumption sector within the innovative changes in banks behavior.

Keywords: financial innovations, Arrow-Debreu model, Schumpeterian approach

JEL Classification: D11, D50, C6, O10, O30, O31

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1 Introduction

In general, the aim of the paper is to combine two previous partial results (Ciałowicz, Malawski, 2011, 2013) by expanding them and indicating the key role played by banks in Schumpeterian innovative evolution. In the former setting this role is very “Schumpeterian”, but more auxiliary in its nature, since banks select innovative projects of firms and finance some of them to shape innovative evolution of the real sector of an economy. This way, banks by extending credits to firms can make some groups of agents involved in this evolutionary process better off. However, the innovativeness within a banking sector itself has been neglected there. On the other hand, in the latter contribution some hints have been given on how to model the circular flow of a banking sector in a rigorous way as well as an innovative metric has been suggested to measure intensity of innovative changes in the whole economy. In this context, the present elaboration, making use of the previous results, is aimed at modeling innovative evolution of financial/banking subsystem of an economy which goes beyond routine behavior of banks deprived of innovative, financial strategies, and studying of its impact on its real sector and, specifically, social welfare. At the same time, it is inscribed in the research program of studying Schumpeterian vision on economic development in the dynamic, formal apparatus of modern general equilibrium theory in the Arrow-Debreu setup, which has been initiated in Malawski (2004) and developed in Malawski (2005), Malawski and Woerter (2006), Ciałowicz and Malawski (2007, 2011, 2013), *Innovative Economy* (2013), Ciałowicz (2015). For better understanding of this framework, let us expand the above premises of the present research in some detail.

First, the idea to treat the banking sector of an economy analogously to the production system being a component of a Debreu economy with the private ownership has been suggested in Ciałowicz and Malawski (2011) and developed in Ciałowicz and Malawski (2013), however financial plans of banks were reduced there to monetary instruments such as deposits and credits, only. Now, they will be completed by others like bonds, options, etc. Consequently, interpreting them as components of ℓ_F dimensional financial plans we will abstract from their temporal structure, the analysis of which requires stochastic conceptual apparatus (cf. Duffie, 1988), and price such “portfolios” using their “spot” value. What is more, the innovative bank (bank-innovator) behaves like the innovator in the real sector of an economy, meaning that it gains an excess innovative profit, which like bank deposits of consumers, is essential source of credits extended to firms and households.

Second, the idea that Schumpeterian evolution may improve upon positions of some groups of economic agents involved in the evolutionary process is rooted in the current discussion on mechanisms to explain its course, that takes place within Neo-Schumpeterian research program (Andersen, 2007; Hanusch and Pyka, 2007; Kitchel, 2016). It suggests, that the price mechanism typical for routine behavior of agents (Lipieta and Malawski, 2016) should be replaced by a qualitative one to take into account the structural changes of an economy based on innovative processes

as drivers of economic evolution. Now, we use the idea that qualitative mechanism should improve positions of some groups of economic agents and apply it to a Debreu monetary economy, where the financial sector of banks is also modeled.

The analysis of the idea sketched above is divided into five parts. In the first section, the model of a Debreu monetary economy is presented in the form of multi-range relational system, which includes the production, the consumption and the financial systems. In the next section modifications in the formal model of a banking sector are introduced to reconstruct the model as a combination of real sphere E and financial sector F and define innovative changes in financial one. Next section is devoted to analysis of pro-innovative extension in a financial sphere which generates innovative, technological changes in a real sphere. Finally the main results are presented. First of all, it is proved that innovative changes in financial sphere affect innovative changes in a real sphere and in a whole economic system.

2 Model of a Debreu monetary economy

Let be given a Debreu monetary economy $E_m = (\mathbb{R}^{\ell+2}, P_m, C_m, F, \theta, \varpi, \mu)$ (Ciałowicz and Malawski 2007) in the form of multi-range relational system, which includes:

1. the production system with money denoted: $P_m = (B, \mathbb{R}^{\ell+2}; Ch_{P_m})$, where $Ch_{P_m} = (y, p, \eta, \pi)$ is a characteristic of the system P_m ,
2. the consumption system with money denoted: $C_m = (A, \mathbb{R}^{\ell+2}, \mathcal{P}; Ch_{C_m})$, where $Ch_{C_m} = (\chi, e, \varepsilon, p, \beta, \varphi)$ is a characteristic of the system C_m ,
3. the financial system denoted: $F = (M, \mathbb{R}^{\ell+2}; Ch_F)$, where $Ch_F = (g, p, \gamma, \zeta)$ is the characteristic of the system F .

It is assumed that in the given model $\mathbb{R}^{\ell+2}$ is an $\ell + 2$ -dimensional commodity-price space, where the last two coordinates are assigned to deposits and credits, $p \in \mathbb{R}^{\ell+2}$ is a price system and three groups of agents are operating in the space $\mathbb{R}^{\ell+2}$:

$A = \{a : a = 1, \dots, m\}$ is a finite set of the consumers,

$B = \{b : b = 1, \dots, n\}$ is a finite set of the producers,

$M = \{r : r = 1, \dots, k\}$ is a finite set of banks.

The formal model of a financial system F has a form of a two-range relational system $F = (M, \mathbb{R}^{\ell+2}; g, p, \gamma, \zeta)$, where:

$g \subset M \times P_0(\mathbb{R}^{\ell+2})$ is the correspondence of financial sets which to every bank $r \in M$ assigns a non-empty set of feasible financial plans: $g(r) = F_r \subset \mathbb{R}^{\ell+2}$ such that $f_r = (0, \dots, 0, s_r, c_r) \in F_r \iff c_r = \lambda s_r$ and $s_r = \sum_{a \in A} s_{ar}$, $s_{ar} \leq 0$

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denotes the savings of the consumer a in the bank r , $c_r = \sum_{b \in B} c_{rb} + \sum_{a \in A} c_{ra}$, $c_{rb} \geq 0$ denotes the credit extended to the producer b by bank r , $c_{ra} \geq 0$ denotes credit of consumer a given by a bank r . According to the rule of money creation with the multiplier $\lambda < 0$: $c_r = \lambda s_r$ (see: Sharafeddine, 2015).

$p = (p_1, \dots, p_\ell, i_s, i_c) \in \mathbb{R}^{\ell+2}$ is a price system, where i_s, i_c denote interest rates of savings and credits, respectively. Assuming that interest rates of consumer's and producer's credits are the same and $i_s < i_c$, the difference $i_c - i_s$ is the source of the bank profits.

$\gamma \subset M \times P_0(\mathbb{R}^{\ell+2})$ is a correspondence of optimal financial plans (a money supply correspondence) which to every bank r assigns a non-empty set of financial plans maximizing its profit with the given interest rates, where the profit of bank r of financial plan f_r in a price system p , is defined as follows:

$$z_r(p, f_r) := p \cdot f_r = i_c \left(\sum_{b \in B} c_{rb} + \sum_{a \in A} c_{ra} \right) + i_s \left(\sum_{a \in A} s_{ar} \right).$$

Hence $\gamma(r) := \gamma_r(p) := \{f'_r \in F_r : z_r(p, f'_r) = \max_{f_r \in F_r} z_r(p, f_r)\}$ for each $r \in M$. However, in real-world economies, some of banks often do not maximize their profit being satisfied with, e.g., gaining non-negative profit. But throughout our analysis, we will make the technical assumptions that financial sets F_r are non-empty and closed to make it more likely that optimal financial plans exist.

$\zeta : M \rightarrow \mathbb{R}$ is a maximal profit function which measures the value of maximum profit, it means: $\zeta(r) := \zeta_r(p) := \max_{f_r \in F_r} z_r(p, f_r)$ for each $r \in M$.

The formal model of a production system with money has a form of two-range relational system $P_m = (B, \mathbb{R}^{\ell+2}; y, p, \eta, \pi)$ (Ciałowicz and Malawski, 2007, 2011; Innovative Economy, 2013), where:

$y \subset B \times P_0(\mathbb{R}^{\ell+2})$ is a correspondence of production sets which to every producer $b \in B$ assigns a production set $y(b) = Y_b \subset \mathbb{R}^{\ell+2}$ being a non-empty subset of the commodity space and representing the producer's feasible production technology,

$$y_b = (y_{b1}, \dots, y_{b\ell}, 0, -c_b) \in Y_b, \quad c_b = \sum_{r \in M} c_{rb}.$$

$\eta \subset B \times P_0(\mathbb{R}^{\ell+2})$ is a correspondence of supply which to every producer $b \in B$ assigns a set $\eta(b)$ of the production plans maximizing his profit $p \cdot y_b$ in a price system p ; that is to say: $\eta(b) := \eta_b(p) := \{y_b \in Y_b : p \cdot y_b = \max_{y_b \in Y_b} (p \cdot y_b)\}$. Similarly to financial system we assume that production sets Y_b are non-empty and closed so optimal production plans exist.

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$\pi : B \rightarrow \mathbb{R}$ is a maximal profit function which measures the maximum profit value in the set of plans $\eta(b)$, i.e., for $b \in B$:
 $\pi(b) := \pi_b(p) := \max_{y_b \in \eta(b)} (p \cdot y_b)$.

In this system each producer $b \in B$, operating on an $\ell + 2$ -dimensional commodity-price space $\mathbb{R}^{\ell+2}$ tries to choose the production plans maximizing his profit in a given price system p .

Similarly, the formal model of a consumption system with money has a form of three-range relational system (Ciałowicz and Malawski, 2011; Innovative Economy, 2013) $C_m = (A, \mathbb{R}^{\ell+2}, \mathcal{P}, \chi, e, \varepsilon, p, \beta, \varphi)$, where:

$\mathcal{P} \subset \mathbb{R}^{\ell+2} \times \mathbb{R}^{\ell+2}$ is the family of all preference relations defined on the commodity space $\mathbb{R}^{\ell+2}$,

$\chi \subset A \times P_0(\mathbb{R}^{\ell+2})$ is a correspondence of consumption sets which to every consumer $a \in A$ assigns a consumption set $\chi(a) = X_a \subset \mathbb{R}^{\ell+2}$ being a non-empty subset of the commodity space and representing the consumer's feasible consumption plans with respect to his psychophysical structure, $x_a = (x_{a1}, \dots, x_{a\ell}, s_a, c_a) \in X_a \iff s_a = \sum_{r \in M} s_{ar}$, $c_a = \sum_{r \in M} c_{ra}$, for $p \cdot x_a - w_a = s_a \leq 0$,

$e \subset A \times \mathbb{R}^{\ell+2}$ is an initial endowment mapping which to every consumer $a \in A$ assigns some initial endowment vector $e(a) := e_a = (e_{a1}, \dots, e_{a\ell}, s_a, c_a) \in X_a$,

$\varepsilon \subset A \times (\mathbb{R}^{\ell+2} \times \mathbb{R}^{\ell+2})$ is a correspondence which to every consumer $a \in A$ assigns a preference relation $\varepsilon(a) := \varepsilon_a := \preccurlyeq_a \in \mathcal{P}$ restricted to the consumption set X_a ,

$\beta \subset A \times P(\mathbb{R}^{\ell+2})$ is a correspondence of budget sets which to every consumer $a \in A$ assigns his set of budget constraints $\beta(a) \subset \chi(a)$ with the price system p and the initial endowment $e(a)$, i.e., for every $a \in A$:

$$\beta(a) := \left\{ x_a \in \chi(a) : px_a \leq \text{proj}_{\mathbb{R}^\ell}(p) \cdot \text{proj}_{\mathbb{R}^\ell}(e(a)) + \sum_{b \in B} \theta_{ab} \pi_b(p) + \sum_{r \in M} \mu_{ar} \zeta_r(p) + s_a + c_a \right\},$$

where μ_{ar} describes the consumer a share of the bank r profit,

$\varphi \subset A \times P_0(\mathbb{R}^\ell)$ is a demand correspondence which to every consumer $a \in A$ assigns his consumption plans maximizing preferences on the budget set $\beta(a)$, i.e., for every $a \in A$: $\varphi(a) := \varphi_a(\varepsilon_a, p, e_a) := \{x \in \beta_a(p, e_a) : \forall x' \in \beta_a(p, e_a) x' \preccurlyeq_a x\}$.

The role of consumer is to choose and perform the consumption plans maximizing its preference relation in the budget set $\beta(a)$.

The Debreu monetary economy $E_m = (\mathbb{R}^{\ell+2}, P_m, C_m, F, \theta, \varpi, \mu)$ is a combination of a production system P_m , a consumption system C_m and a financial system F

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such that the consumers share in the producers' and banks' profits. The shares are measured by mappings:

$\theta \subset (A \times B) \times [0, 1]$, i.e. for every $(a, b) \in A \times B$ the number $\theta(a, b) := \theta_{ab} \in [0, 1]$ describes the consumer a share in the producer b profit, and there is, for every $b \in B$, $\sum_{a \in A} \theta_{ab} = 1$,

$\mu : A \times M \rightarrow [0, 1]$ i.e. for every $(a, r) \in A \times M$ the number $\mu(a, r) = \mu_{ar} \in [0, 1]$ describes the consumer a share in the bank r profit, and there is, for every $r \in M$, $\sum_{a \in A} \mu_{ar} = 1$.

Moreover, some fixed (initial) total resource $\varpi = (\varpi_1, \dots, \varpi_\ell, s, c) \in \mathbb{R}^{\ell+2}$, $s = \sum_{a \in A} s_a$, $c = \sum_{a \in A} c_a + \sum_{b \in B} c_b$ of the economy E_m is the consumers' property, i.e. $\varpi := \sum_{a \in A} e(a)$.

In the economic system E_m described above the role of each market participant is to select and implement the optimal plan in a given price system and individual constraints. Specifically, for given vector of prices and interest rates, the role of each producer is to choose and perform the production plans maximizing his profit in a given price system p and technologies, each consumer chooses and performs the consumption plans maximizing his preference relation on his budget set. Similarly to producers each bank chooses and perform the financial plans maximizing his profit. But it is worth to remember that, each consumer decides, if he should allocate a part of his endowments for savings or to take consumer's credit to enhance his purchasing power and perform an optimal consumption plan better than one without such possibility in the real sphere (according to his individual preference relation). At the same time producers can get a credit from a bank, to introduce innovations by modifying feasible technologies.

3 Modifications in the formal model of a banking sector

In the formal model of monetary economy described in Section 2 financial plans of banks were reduced to monetary instruments such as deposits and credits, only. Now, they will be completed by others financial instruments like bonds, options, derivatives etc. In this paper financial instruments are understood as monetary contracts between investors (banks) and producers or consumers. They can also be seen as packages of capital that may be traded.

To this aim modifications in the formal model of a banking sector are introduced to reconstruct the given model as a combination of real sphere E and financial sector F and then define innovative changes in financial one. This idea has been suggested by Schumpeter who divided economic analysis into two categories: real and monetary analysis and wrote: "...*Real analysis proceeds from the principle that all essential*

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phenomenon of economic life are capable of being described in terms of goods and services, of decisions about them, and of relations between them..." (Schumpeter 2006, p. 302). *"It is true that goods and not money are needed to produce in the technological sense ..."* but *"...it is impossible to pierce the money veil in order to get premiums on concrete goods..."* (Schumpeter 1961, p.184).

In a commodity space \mathbb{R}^ℓ of the given model all human things and human activities have their own money-values with regard to the given vector of prices and can be categorized as monetary or real products. Thus let a commodity space \mathbb{R}^ℓ be given in a form $\mathbb{R}^\ell = \mathbb{R}^{\ell_R} \times \mathbb{R}^{\ell_F} = \mathbb{R}^{\ell_R + \ell_F}$, where:

\mathbb{R}^{ℓ_R} is a space of real commodities,

\mathbb{R}^{ℓ_F} is a space of financial instruments (deposits, credits, bonds, options, etc.).

Consequently, interpreting them as components of ℓ_F dimensional financial plans we will abstract from their internal-time structure, the analysis of which requires stochastic conceptual apparatus (cf. Duffie, 1988), and price such "portfolios" using their "spot" value.

In the basic model of a Debreu monetary economy, described in Chapter 2 we assumed that in the commodity space there is only one kind of savings s_r and one kind of credits c_r with the same interest rates of consumer's and producer's credits. Now, for the purpose of this research, in the modified commodity space there are a number of different kinds of savings (s_r), loans (c_r) and also monetary derivatives (d_r) with different interest rates. Moreover monetary derivatives can be interpreted as outputs for banks and inputs for producers and consumers, so $d_r \geq 0$ for every $r \in M$. As a result demand correspondence, a correspondence of optimal financial plans and supply correspondence are modified.

Hence financial plan has a form:

$$f_r = (0, \dots, 0, (s_r), (c_r), (d_r)) \in F_r \iff \sum c_r = \lambda \sum s_r,$$

where (d_r) denotes all (monetary) derivatives and $\text{proj}_{\mathbb{R}^{\ell_R}}(f_r) = \mathbf{0} = (0, 0, \dots, 0)$; $\text{proj}_{\mathbb{R}^{\ell_F}}(f_r) = ((s_r), (c_r), (d_r))$.

Moreover each production plan has a form:

$$y_b = (y_{b1}, \dots, y_{b\ell}, 0, \dots, 0, (-c_b), (d_b)),$$

each consumption plan has a form:

$$x_a = (x_{a1}, \dots, x_{a\ell}, (s_a), (c_a), (-d_a))$$

and total resource:

$$\varpi = (\varpi_1, \dots, \varpi_\ell, (s), (c), (d)).$$

Thus modified financial system has a form $F = (M, \mathbb{R}^{\ell_R + \ell_F}, f, p, \gamma, \zeta)$, and

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the Debreu monetary economy $E_m = (E_R, E_F)$ is a combination of real sphere $E = E_R = \text{proj}_{\mathbb{R}^{\ell_R}}(E_m)$ (standard Debreu economy, cf. Debreu, 1959) and financial sphere $E_F = \text{proj}_{\mathbb{R}^{\ell_F}}(E_m)$. Similarly $P = \text{proj}_{\mathbb{R}^{\ell_R}}(P_m)$ is a production system (without financial instruments) and $C = \text{proj}_{\mathbb{R}^{\ell_R}}(C_m)$ is a consumption system (without financial instruments).

4 Innovative changes in a financial sphere

In his works Schumpeter questioned the received concept of a dichotomy of the economy in a real and a monetary sphere, where the monetary sphere influences the real sphere through rationing credits. In this perspective money can be removed from a system without much effect. But it should be noted that Schumpeter's approach leaves many problems unsolved as regard to the innovative changes in the banks system (see e.g. Andersen, 2007; Hanusch and Pyka, 2007; Caiani, Godin and Lucarelli, 2014; Malerba and Orsenigo, 1995).

The most important fact is that a financial sector (banks) plays two different roles in innovative development:

1. banks as auxiliary agents have a capability to effect economic development through rationing credits and funding innovative ventures, what means that banks shape the economic growth of the whole economy (Ciałowicz and Malawski, 2011),
2. banks as central unit agents operate like producers and can generate innovative changes in a real sphere by pro-innovative activity to improve social welfare.

To analyze the second role of banks a sequence of definitions will be introduced.

Definition 4.1 (Ciałowicz, 2016; cf. Malawski, 2004): A production system $P' = (B', \mathbb{R}^{\ell'}, y', p', \eta', \pi')$ is called an innovative extension of a system $P = (B, \mathbb{R}^{\ell}, y, p, \eta, \pi)$, in short $P \subset_i P'$, iff:

1. $\ell \leq \ell'$
2. $\exists b' \in B' \forall b \in B$
 - 2.1. $\text{proj}_{\mathbb{R}^{\ell}}(Y'_{b'}) \not\subset Y_b$
 - 2.2. $\text{proj}_{\mathbb{R}^{\ell}}(\eta'_{b'}(p')) \not\subset \eta_b(p)$
 - 2.3. $\pi_b(p) < \pi'_{b'}(p')$.

According to the given definition in the set of producers we may distinguish the producer-innovator $b' \in B'$ who satisfies Conditions (2.1) – (2.3). The set of all innovators will be denoted by B'_{in} . Moreover among all the new production plans of the innovator b' innovative production plans $y'_{b'} \in \text{proj}_{\mathbb{R}^{\ell}}(Y'_{b'})/Y_b$ which satisfy Condition 2.2 can be distinguished. It means that they maximize the profit of

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innovator that is greater than the one any of the producers in a system P can make. Let $Y_b^{i'}$ is a set of all innovative production plans of a producer b' .

The same idea can be applied to a production system with money, what we suggest below.

Definition 4.2: A production system with money P'_m is called an innovative extension of a system P_m , in short $P_m \subset_i P'_m$, iff: $\text{proj}_{\mathbb{R}^{\ell_R}}(P_m) \subset_i \text{proj}_{\mathbb{R}^{\ell'_R}}(P'_m)$.

According to the above definition innovative extension of production system with money means that only innovative changes in a real sphere are important.

Now, to perform our task, innovative extension of both standard Debreu economy E and a Debreu monetary economy E_m will be defined. But it should be noticed that according to Schumpeter's theory, changes in the production sphere determine changes in the whole economy. Moreover, the below list of definitions allows us to define an innovative extension of a financial sector, extension of a Debreu monetary economy with innovative changes in a financial sector and, finally, a pro-innovative extension of financial sphere.

Definition 4.3 (Innovative Economy, 2013): An economic system $E' = (\mathbb{R}^{\ell'}, P', C', \theta', \varpi')$ is called an innovative extension of a system $E = (\mathbb{R}^{\ell}, P, C, \theta, \varpi)$, in short $E \subset_i E'$, iff $P \subset_i P'$.

Definition 4.4: A Debreu monetary economy $E'_m = (\mathbb{R}^{\ell'}, P'_m, C'_m, F', \theta', \varpi', \mu')$ is called an innovative extension of an economy $E_m = (\mathbb{R}^{\ell}, P_m, C_m, F, \theta, \varpi, \mu)$ (in short $E_m \subset_i E'_m$), iff $P_m \subset_i P'_m$ (Definition 4.2).

It means that the concept just defined can be reduced to the previous one of the innovative extension of a production system with money.

Definition 4.5: Let two Debreu monetary economies E_m and E'_m be given for which $E_m \subset_i E'_m$. A bank $\bar{r} \in M'$ is called pro-innovative iff there exists a financial plan $f'_{\bar{r}} = (0, \dots, 0, (s_{\bar{r}}), (c_{\bar{r}}), (d_{\bar{r}})) \in F'^{i'}$, there is an innovative production plan $y_{\bar{b}} \in Y_{\bar{b}}^{i'}$ and there is a financial commodity $k \in \{\ell'_R + 1, \dots, \ell'_F\}$ such that $f'^k_{\bar{r}} = c^k_{\bar{r}} = c^k_{\bar{b}\bar{r}} = y^k_{\bar{b}}$.

The above definition shows a close relationship between the monetary and the real dimensions of the economic system because specifically, as financial capital acts as a condition for starting new production process. In our context pro-innovative banks play an important role in realization of innovative projects what means that innovative production plan $y_{\bar{b}}$ is financed by credit $c_{\bar{b}\bar{r}}$ extended by a bank \bar{r} , i.e. formally: k -th component of a financial plan $f'_{\bar{r}}$ of bank \bar{r} is equal to the k -th component of the innovative production plan $y_{\bar{b}}$. It is coherent with the Schumpeter's viewpoint that the realization of an innovative production plan requires entrepreneurs to have access to financial means in a form of credit extended by a bank. The bank which provides producer-innovator with a new purchasing power in the form of credit is called pro-innovative.

Among all innovative changes in a space of commodities $\mathbb{R}^{\ell} = \mathbb{R}^{\ell_R} \times \mathbb{R}^{\ell_F}$ for innovative extension of a Debreu monetary economy $E_m \subset_i E'_m$ we may distinguish:

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1. products innovations (innovations modeled by innovative changes of real space) (Ciałowicz and Malawski, 2011) if $\mathbb{R}^\ell = \mathbb{R}^{\ell_R} \times \mathbb{R}^{\ell_F}$ changes into $\mathbb{R}^{\ell'} = \mathbb{R}^{\ell'_R} \times \mathbb{R}^{\ell'_F}$,
2. financial innovations (new financial instruments; modelled by innovative extension of a financial sector) if $\mathbb{R}^\ell = \mathbb{R}^{\ell_R} \times \mathbb{R}^{\ell_F}$ changes into $\mathbb{R}^{\ell'} = \mathbb{R}^{\ell_R} \times \mathbb{R}^{\ell'_F}$,
3. pro-innovative extension of a financial sphere if $\mathbb{R}^\ell = \mathbb{R}^{\ell_R} \times \mathbb{R}^{\ell_F}$ changes into $\mathbb{R}^{\ell'} = \mathbb{R}^{\ell'_R} \times \mathbb{R}^{\ell'_F}$.

Moreover in the Schumpeterian vision of economic evolution (Schumpeter, 1961) banks can be interpreted as producers operating on the money markets, so innovative extension of a financial system will be defined similarly to an innovative extension of a production system.

Definition 4.6: A financial system $F' = (M', \mathbb{R}^{\ell'}, f', p', \gamma', \zeta')$ is called an innovative extension of a system $F = (M, \mathbb{R}^\ell, f, p, \gamma, \zeta)$, in short $F \subset_i F'$, iff:

1. $\ell_F \leq \ell'_F$
2. $\exists r' \in M' \forall r \in M$
 - 2.1. $\text{proj}_{\mathbb{R}^\ell}(F'_{r'}) \not\subset F_r$
 - 2.2. $\text{proj}_{\mathbb{R}^\ell}(\gamma'_{r'}(p')) \not\subset \gamma_r$
 - 2.3. $\zeta_r(p) < \zeta'_{r'}(p')$.

The above definition covered all types of financial innovations defined as the acts of creating and then popularizing new financial instruments as well as new financial technologies, institutions and markets. It includes institutional, product and process innovations. Institutional innovations relate to the creation of new types of financial firms (such as specialist credit card firms or discount broking firms, internet banks and so on). Product innovation relates to new products such as derivatives, securitized assets, foreign currency mortgages and so on. Process innovations relate to new ways of doing financial business including online banking, phone banking and new ways of implementing information technology and so on. Moreover, in a set of banks we may distinguish the innovative bank r' who satisfies Conditions (2.1) – (2.3) and among all the new financial plans of this innovative bank innovative financial plans $f_{r'} \in F'_{r'}$ such that $\text{proj}_{\mathbb{R}^\ell}(f_{r'}) \in \text{proj}_{\mathbb{R}^\ell}((F'_{r'})/F_r)$ which satisfy Condition (2.2) can be distinguished.

Definition 4.7: A Debreu monetary economy E'_m is called:

1. an extension of an economy E_m with innovative changes in a financial sphere (in short $E_m \subset_{F_i} E'_m$), iff $F \subset_i F'$,
2. a financial innovative extension of an economy E_m (in short $E_m \subset_{if} E'_m$), iff $F \subset_i F'$ and $\mathbb{R}^{\ell_R} = \mathbb{R}^{\ell'_R}$.

Definition 4.8: Let two Debreu monetary economies E_m and E'_m be given for which $E_m \subset_{F_i} E'_m$. A financial system F' is called a pro-innovative extension of a system F , in short $F \subset_{pi} F'$, iff: $[F \subset_i F' \implies P_m \subset_i P'_m]$.

The above definition says that changes in a financial sector are pro-innovative when they generate (technological) innovative changes in a real sphere but it should be noted that innovative extension of a financial sector doesn't mean existing of pro-innovative bank. On the one hand it is possible that there are not innovative changes in a financial sector but at the same time there is a pro-innovative bank which finances innovative projects. On the other hand innovative extension of financial sphere, i.e. innovative financial instruments like new kinds of options is not connected with activity of pro-innovative banks.

5 The role of banks in an innovative evolution

Innovative changes in a financial sector are based on possibility of extension by banks growing amount of credits. It gives producers opportunity to realize innovative production plans and to initiate changes in the whole system. It means that the financial system is an important determinant of innovative evolution. This idea can be formalized in the next definition.

Definition 5.1: A Debreu monetary economy E'_m is an innovative extension of an economy E_m under a bank control, in short $E_m \subset_{ib} E'_m$, iff:

1. $E_m \subset_i E'_m$
2. In an economy E'_m at least one bank $r \in M'$ is pro-innovative.

According to this definition in an innovative extension under a bank control banks have a capability to effect economic development through rationing credits and funding innovative ventures. It should be emphasized (see: Ciałowicz and Malawski, 2011) that innovative production plans are sometimes financed not by banks but by other investor or internal cash flow in the firm. But when there is a pro-innovative bank which has a capability to fund innovative project, the same bank can ration credit by making decisions about extending credits and about the amount of the credit and it is the role of the bank to evaluate the given project, and to eliminate those that are not profitable or too risky. In this case innovative development of the whole system is under a bank control.

Remark: $E_m \subset_{ib} E'_m \implies E_m \subset_i E'_m$ (an innovative extension under a bank control implies an innovative extension of an economy E_m).

Theorem 5.1 If 1) $E_m \subset_{if} E'_m$, 2) $\text{proj}_{\mathbb{R}^\ell}(p') = p$, 3) at least one bank $r \in M'$ is pro-innovative then $E_m \subset_i E'_m$.

Proof: If $E_m \subset_{if} E'_m$ then $\ell_F \leq \ell'_F$, $\ell_R + \ell'_R$ and $\exists r' \in M' \forall r \in M$

1. $\text{proj}_{\mathbb{R}^\ell}(F'_{r'}) \not\subset F_r$

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2. $\text{proj}_{\mathbb{R}^\ell}(\gamma'_{r'}(p')) \not\subset \gamma_r$
3. $\zeta_r(p) < \zeta'_{r'}(p')$.

Part1: If $\ell_F \leq \ell'_F$ and $\ell_R = \ell'_R$ then $\ell = \ell_R + \ell_F \leq \ell' = \ell'_R + \ell'_F$ (condition 1 of Definition 4.1 is fulfilled).

Part 2: From the assumption 3) and definition 4.4 if there exist pro-innovative bank $r \in M'$ then there exists innovative production plan and producer-innovator in an economy E'_m , so condition 2 of Definition 4.1 is fulfilled and $P \subset_i P'$.

According to Parts 1-2 we have $E_m \subset_i E'_m$.

According to the above theorem innovative changes in a financial sphere determine innovative changes in a real sphere through activity of pro-innovative bank.

Moreover innovative changes in a financial system may improve consumers situation in a cumulative sense. To prove this fact we need to give a sequence of definitions.

Definition 5.2 (c.f. Ciałowicz and Malawski, 2011): A consumption system $C' = (A', \mathbb{R}^{\ell'}, Pref', Ch'_{C'})$ is said to be a cumulative extension of a consumption system $C = (A, \mathbb{R}^\ell, Pref, Ch_C)$, in short $C \subset_c C'$ if:

1. $\ell \leq \ell'$ and $A \subset A'$
2. $\forall a \in A$
 - (a) $X_a \subset \text{proj}_{\mathbb{R}^\ell}(X'_a)$ so that $(x_{1a}, \dots, x_{\ell a}, 0, \dots, 0) \in X'_a$ for every $(x_{1a}, \dots, x_{\ell a}) \in X_a$
 - (b) $e_a \leq \text{proj}_{\mathbb{R}^\ell}(e'_a)$
 - (c) $\varepsilon_a \subset \text{proj}_{\mathbb{R}^{2\ell}}(\varepsilon'_a) \iff \preceq_a = \text{proj}_{X_a^2}(\preceq'_a)$
 - (d) $\beta_a(p, e_a) \subset \text{proj}_{\mathbb{R}^\ell}(\beta'_a(p', e'_a))$
 - (e) $\varphi_a(\varepsilon_a, p, e_a) \subset \text{proj}_{\mathbb{R}^\ell}(\varphi'_a(\varepsilon'_a, p', e'_a)) \iff \forall x_a^* \in \varphi_a(\varepsilon_a, p, e_a) \quad \forall x_a^{*'} \in \varphi'_a(\varepsilon'_a, p', e'_a) \quad x_a^* \preceq_a \text{proj}_{\mathbb{R}^\ell}(x_a^{*'})$.

According to the definition, in a cumulative extension of a consumption system the psychophysical structure of all individuals does not grow worse and each consumer is able to ignore new goods (Condition 2.1), the initial resources do not decrease (Conditions 2.2), the budget constraints of individuals are relaxed (Condition 2.4), and their wants are satisfied at least at the same level of utility (Condition 2.5). Moreover with the assumption that for each consumer value of financial components of initial endowments is not decreasing it is possible to define cumulative extension of a consumption system with money.

Definition 5.3: A consumption system with money C'_m is said to be a cumulative extension of a system C_m , in short $C_m \subset_c C'_m$, iff:

1. $\text{proj}_{\mathbb{R}^{\ell_R}}(C_m) \subset_c \text{proj}_{\mathbb{R}^{\ell'_R}}(C'_m)$,

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$$2. \forall a \in A \text{ proj}_{\mathbb{R}^{\ell_F}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_F}}(e_a) \leq \text{proj}_{\mathbb{R}^{\ell'_F}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell'_F}}(e'_a)$$

According to the given definitions it is possible to give specific conditions in which innovative changes in a financial sphere of Debreu monetary economy may improve consumers situation in cumulative sense.

Theorem 5.2: Let two Debreu monetary economies E_m and E'_m be given, for which $A = A'$, $B = B'$, $M = M'$. Assume that:

1. $F \subset_{pi} F'$
2. $\forall \hat{b} \in B \setminus B_{in} \quad \pi_{\hat{b}}(p) \leq \pi'_{\hat{b}}(p')$
3. $\forall a \in A$
 - (a) $X_a \subset \text{proj}_{\mathbb{R}^{\ell_R}}(X'_a)$ so that $(x_{1a}, \dots, x_{\ell a}, 0, \dots, 0) \in X'_a$ for every $(x_{1a}, \dots, x_{\ell a}) \in X_a$
 - (b) $e_a \leq \text{proj}_{\mathbb{R}^{\ell}}(e'_a)$
 - (c) $\varepsilon_a \subset \text{proj}_{\mathbb{R}^{\ell}}(\varepsilon'_a) \iff (\preceq'_a / X_a^2) = \preceq_a$
 - (d) $0 \leq \text{proj}_{\mathbb{R}^{\ell_F}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_F}}(e_a) \leq \text{proj}_{\mathbb{R}^{\ell'_F}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell'_F}}(e'_a)$
 - (e) $\forall b \in B \theta_{ab} = \theta'_{ab}$
 - (f) $\forall r \in M \mu_{ar} = \mu'_{ar}$
4. $\text{proj}_{\mathbb{R}^{\ell_R}}(p') = p$ then $C_m \subset_c C'_m$.

Proof. According to Definition 4.8 and Definition 4.4 $F \subset_{pi} F' \implies P_m \subset_i P'_m \implies \implies E_m \subset_i E'_m$.

Part 1: $\ell < \ell'$. For each $a \in A$ we have to prove that its budget constraints are relaxed (with respect to the previous space of commodity \mathbb{R}^{ℓ}):

$$\beta_a(p, e_a) \subset \text{proj}_{\mathbb{R}^{\ell}}[\beta'_a(p', e'_a)] \quad (1)$$

and its wants are satisfied at least at the same level of utility (with respect to the previous space of commodity \mathbb{R}^{ℓ}):

$$\varphi_a(\varepsilon_a, p, e_a) \subset \text{proj}_{\mathbb{R}^{\ell}}[\varphi'_a(\varepsilon'_a, p', e'_a)]. \quad (2)$$

Proof of (1). According to the definition of a budget set we have:

$$\beta_a(p, e_a) = \left\{ x_a \in X_a : p \cdot x_a \leq w_a = \text{proj}_{\mathbb{R}^{\ell_R}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_R}}(e_a) + \sum_{b \in B} \theta_{ab} \pi_b(p) + \sum_{r \in M} \mu_{ar} \zeta_r(p) + \text{proj}_{\mathbb{R}^{\ell_F}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_F}}(e_a) \right\},$$

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$$\beta'_a(p', e'_a) = \left\{ x'_a \in X'_a : p' \cdot x'_a \leq w'_a = \text{proj}_{\mathbb{R}^{\ell'_R}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell'_R}}(e'_a) + \sum_{b' \in B'} \theta'_{ab'} \pi'_{b'}(p') + \sum_{r' \in M'} \mu'_{ar'} \zeta'_{r'}(p') + \text{proj}_{\mathbb{R}^{\ell'_F}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell'_F}}(e'_a) \right\},$$

and $\text{proj}_{\mathbb{R}^{\ell}}[\beta'_a(p', e'_a)] = \{\text{proj}_{\mathbb{R}^{\ell}}(x'_a) : x'_a \in \beta'_a(p', e'_a)\}$.

Let $x_a \in \beta_a(p, e_a)$, i.e., $x_a \in X_a$ and $p \cdot x_a \leq w_a$. According to Assumption (3.1), for each $x_a \in X_a$ there is $x'_a = (x_{1a}, \dots, x_{\ell a}, 0, \dots, 0) \in X'_a$ such that $x_a = \text{proj}_{\mathbb{R}^{\ell_R}}(x'_a)$. The Assumption 1) $F \subset_{pi} F'$ implies $P_m \subset_i P'_m$, so $\text{proj}_{\mathbb{R}^{\ell_R}}(P_m) \subset_i \text{proj}_{\mathbb{R}^{\ell'_R}}(P'_m)$.

Thus if $\text{proj}_{\mathbb{R}^{\ell_R}}(p') = p$ (Assumption 4) then

$$\begin{aligned} p \cdot x_a &= \text{proj}_{\mathbb{R}^{\ell_R}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_R}}(x_a) + \text{proj}_{\mathbb{R}^{\ell_F}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_F}}(x_a) \\ &\quad \text{proj}_{\mathbb{R}^{\ell_R}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell_R}}(x_a) + \text{proj}_{\mathbb{R}^{\ell_F}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_F}}(x_a) \\ &\geq \text{proj}_{\mathbb{R}^{\ell}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell}}(x'_a) \end{aligned}$$

By Assumption 2) for each producer non-innovator $\hat{b} \in B \setminus B_{in}$ we have $\pi_b(p) \leq \pi'_b(p')$ and $\pi_{b'}(p) < \pi'_{b'}(p')$ for each producer-innovator $b' \in B_i$.

If $\theta_{ab} = \theta'_{ab}$ (Assumption 3.5) then $\sum_{b \in B} \theta_{ab} \pi_b(p) \leq \sum_{b \in B} \theta'_{ab} \pi'_b(p')$ for each $a \in A$.

Similarly $F \subset_{pi} F'$ means that for each bank $r \in M = M'$ $\zeta_r(p) < \zeta'_{r'}(p')$.

If $\mu_{ar} = \mu'_{ar}$ (Assumption 3.6) then $\sum_{r \in M} \mu_{ar} \zeta_r(p) \leq \sum_{r \in M} \mu'_{ar} \zeta'_{r'}(p')$.

Moreover, by Assumptions (3.2), (3.4) and 4), we have: $\text{proj}_{\mathbb{R}^{\ell_R}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_R}}(e_a) + \text{proj}_{\mathbb{R}^{\ell_F}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_F}}(e_a) \leq \text{proj}_{\mathbb{R}^{\ell'_R}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell'_R}}(e'_a) + \text{proj}_{\mathbb{R}^{\ell'_F}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell'_F}}(e'_a)$ which implies

$$\begin{aligned} w_a &= \text{proj}_{\mathbb{R}^{\ell_R}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_R}}(e_a) + \sum_{b \in B} \theta_{ab} \pi_b(p) + \sum_{r \in M} \mu_{ar} \zeta_r(p) + \text{proj}_{\mathbb{R}^{\ell_F}}(p) \cdot \text{proj}_{\mathbb{R}^{\ell_F}}(e_a) \\ &\leq \text{proj}_{\mathbb{R}^{\ell'_R}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell'_R}}(e'_a) + \sum_{b' \in B'} \theta'_{ab'} \pi'_{b'}(p') + \sum_{r' \in M'} \mu'_{ar'} \zeta'_{r'}(p') + \\ &\quad + \text{proj}_{\mathbb{R}^{\ell'_F}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell'_F}}(e'_a) = w'_a. \end{aligned}$$

As a result $\text{proj}_{\mathbb{R}^{\ell}}(p') \cdot \text{proj}_{\mathbb{R}^{\ell}}(x'_a) \leq p \cdot x_a \leq w_a \leq w'_a$ and $x'_a \in \text{proj}_{\mathbb{R}^{\ell}}[\beta'_a(p', e'_a)]$, i.e. for $x_a = \text{proj}_{\mathbb{R}^{\ell}}(x'_a) \in \text{proj}_{\mathbb{R}^{\ell}}[\beta'_a(p', e'_a)]$, so $\beta_a(p, e_a) \subset \text{proj}_{\mathbb{R}^{\ell}}[\beta'_a(p', e'_a)]$.

Proof of (2). Assume by contradiction that there exists a consumer $a \in A$ for whom $\varphi_a(\varepsilon_a, p, e_a) \not\subset \text{proj}_{\mathbb{R}^{\ell}}[\varphi'_a(\varepsilon'_a, p', e'_a)]$ i.e., there is $x_a \in \varphi_a(\varepsilon_a, p, e_a)$ and $x'_a \in \varphi'_a(\varepsilon'_a, p', e'_a)$ such that

$$\text{proj}_{\mathbb{R}^{\ell}}(x'_a) \prec_a x_a \quad (3)$$

Particularly, $x_a \in \beta_a(p, e_a)$ and by Condition (1), which has already been proved, $x_a \in \text{proj}_{\mathbb{R}^{\ell}}[\beta'_a(p', e'_a)]$ and particularly, there exists $x''_a \in \beta'_a(p', e'_a)$ such that $\text{proj}_{\mathbb{R}^{\ell}}(x''_a) = x_a$. But, for each $x'_a \in \varphi'_a(\varepsilon'_a, p', e'_a)$, $x'_a \in \beta'_a(p', e'_a)$. It gives $x''_a \prec_a x'_a$. Therefore, by Assumption (3.3) $x_a = \text{proj}_{\mathbb{R}^{\ell}}(x''_a) \prec_a \text{proj}_{\mathbb{R}^{\ell}}(x'_a)$ contrary to

(3).

Part 2. For $\ell = \ell'$, by replacing projections by suitable identity mappings, the proof is analogous, i.e. the above result holds for a weak technological extension.

Theorem proved above specified some conditions under which there are no losers in the evolutionary game household to play. Consequently, their position times out to be not worse off implying that social welfare also at least is not worsening.

6 Conclusions and future research directions

This work is coherent with research path which emphasizes the active role of financial sphere in innovation processes and with the idea that economic evolution is an immensely complex process. It analysis the feed-back effects between innovative changes in financial and real sides of the economic system.

The main findings of the present article are that:

1. Banks innovativeness can be define in the formal apparatus of the modern Arrow-Debreu theory of general equilibrium.
2. The financial system is an important determinant of innovative evolution what was proved in adequate theorems.

The conclusions drawn from the analysis of banks innovativeness provide the ideas for a future study, i.e. to analyze relationships between financial innovations and economic evolution in the real models based on sample data.

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