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A Rough Estimation of Acoustics of the Cuboidal Room with Impedance Walls

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The cuboidal room acoustics field is modelled with the Fourier method. A combination of uniform, impedance boundary conditions imposed on walls is assumed, and they are expressed by absorption coefficient values. The absorption coefficient, in the full range of its values in the discrete form, is considered. With above assumptions, the formula for a rough estimation of the cuboidal room acoustics is derived. This approximate formula expresses the mean sound pressure level as a function of the absorption coefficient, frequency, and volume of the room separately. It is derived based on the least-squares approximation theory and it is a novelty in the cuboidal room acoustics.

Theoretical considerations are illustrated via numerical calculations performed for the 3D acoustic problem. Quantitative results received with the help of the approximate formula may be a point of reference to the numerical calculations.

Keywords: Fourier analysis; room acoustics; absorption coefficient; boundary-value problem; impedance boundary conditions.

1. Introduction

The influence of wall impedances on the acoustic field, e.g., in the cinema, theatre room, is an important and interesting problem from practical and theoretical point of view (Kamisiński, 2012; Kamisiński *et al.*, 2016; Kulowski, 2011). This problem is not simple and for solving it needs applying some special methods. For this purpose several exact and approximate methods have been developed; most of these methods are described in general forms in (Morse, Ingard, 1987; Brański, 2013).

Approximate methods, based on heuristic premises, are used in many papers. They utilise statistical-acoustic methods (Summers, 2012), the diffusion-equation model (Luizard *et al.*, 2014), geometric acoustics methods (Luizard *et al.*, 2014; Lehmann, Johansson, 2008), the combination of radiosity method, geometrical acoustic one (Korany *et al.*, 2001), and so on.

Wave-based approximate methods solve the wave equation in an approximate manner. In the description of the sound field, the most useful methods are finite element method (FEM) (Okuzono *et al.*, 2014), the singular boundary element method (BEM), (Lin *et al.*, 2014; Fu *et al.*, 2014; Chen *et al.*, 2014), the nonsingular BEM (Brański *et al.*, 2012; Brański, Borkowska, 2015), and the finite difference method (FDM) (Lopez *et al.*, 2013). All above methods are numerical ones.

An example of exact methods is the Fourier one which may be used for solving boundary problems of the room acoustics (Blackstock, 2000; Kuttruff, 2000; Brański *et al.*, 2017). It requires to solve the modal equation and an evaluation of eigenvalues of the Helmholtz equation assuming some boundary conditions imposed on the walls. Since the acoustic eigenvalue equation is complicated, the numerical method should be applied to find eigenvalues (Naka *et al.*, 2005; Kuttruff, 2000; Bistafa, Morrissey, 2003), e.g., the Newton method or bisection one. There is no exact general method for finding eigenvalues and consequently room modes for walls with arbitrary impedances. Only for rooms with perfectly flexible or rigid walls this problem is solved exactly (Blackstock, 2000; Kuttruff, 2000), but for some impedance on each pair of parallel walls it is only estimated (Brański *et al.*, 2017). To make a more general analysis of the modal acoustic boundary problem, it is appropriate to refer to the Sturm-Liouville (S-L) eigenvalue problem (Johnson, 2006; Dautray, Lions, 2000). The S-L problems arise throughout applied mathematics and in a standard form, they describe the vibrational modes of various systems, among other the acoustic ones. Then, they arise directly as eigenvalue problems in one-dimensional space (1D). They also commonly arise from linear partial differential equations in multidimensional spaces when the equations are separable in some coordinate system, exactly as in three dimensional acoustic problem.

The benefit of the Fourier method is that it describes the wave nature of the sound field (Meissner, 2009a) and the modal localisation (Meissner, 2009b). Other versions of this method may be found in (Xu, Sommerfeldt, 2010). The Fourier method is difficult to apply for rooms with complex shapes and complex boundary conditions and in more practical cases, it is unusable (Meissner, 2012; 2013a; 2013b).

In this paper, the Fourier method is used for a derivation of the formula which is the basis for rough estimation of room acoustics inside a cuboid; it is the aim of the paper. This formula describes the mean sound pressure level as a function of the absorption coefficient for separate frequencies and for separate volumes of the room. Furthermore, based on this formula, two groups of frequencies are defined. Next, for these groups and separate volumes of the room, approximate graphs of the mean sound pressure level are calculated. These graphs ought to be useful for a rough estimation of the cuboidal room acoustics at first sight. Part of this problem is solved in (Kocan-Krawczyk, 2017).

2. Three dimensional acoustic problem

Let be given the 3D acoustic boundary problem in the cuboid, physical domain Ω with the boundary Γ. To all intents and purposes, the mathematical model is described by the wave equation and Robin and Neumann boundary conditions. In the steady state, it leads to the following boundary problem (Meissner, 2012; 2013a; 2013b; Kocan-Krawczyk, 2017; Brański *et al.*, 2017):

$$D^2U(\mathbf{x}) + k^2U(\mathbf{x}) = F(\mathbf{x}), \qquad \mathbf{x} \in \Omega, \tag{1}$$

where $U(\mathbf{x})$ is acoustic potential; $\mathbf{x} = (x, y, z)$ [m]; $U(\mathbf{x}) = X(x)Y(y)Z(z)$; k [rad/m] is the wave number: $k = \omega/c$, $k^2 = k_x^2 + k_y^2 + k_z^2$; ω [rad/s] is an angular frequency; $D^2U(\mathbf{x}) = \partial^2U/\partial x^2 + \partial^2U/\partial y^2 + \partial^2U/\partial z^2$.

The Robin (R) and Neumann (N) boundary conditions, Fig. 1, are given respectively by

$$D_{\mathbf{n}} \begin{cases} X(x) \\ Y(y) \\ Z(b_z) \end{cases} + z_0(\mathbf{x}) \begin{cases} X(x) \\ Y(y) \\ Z(b_z) \end{cases} = 0, \qquad \mathbf{x} \in \Gamma, \tag{2}$$

$$D_{\mathbf{n}}Z(a_z) = 0, \tag{3}$$

where $\mathbf{n}$ is a unit normal vector to the Γ pointing outward Ω; $D_{\mathbf{n}}U(\mathbf{x}) = \partial U/\partial \mathbf{n}$; $x \in \{a_x, b_x\}$, $y \in \{a_y, b_y\}$, $z \in \{a_z, b_z\}$; $z_0(\mathbf{x}) = (\omega\rho)/z(\mathbf{x})$, $z(\mathbf{x})$ [N·s/m^3] is the specific acoustic impedance: $z(\mathbf{x}) = p(\mathbf{x})/v(\mathbf{x})$, where $p(\mathbf{x}) = i\rho\omega\, U(\mathbf{x})$ [Pa] is the acoustic pressure; $v(\mathbf{x}) = -D_{\mathbf{n}}U(\mathbf{x}) = -\mathrm{grad}\, U(\mathbf{x})$ [m/s] is the particle velocity; ρ [kg/m^3] is the air density.

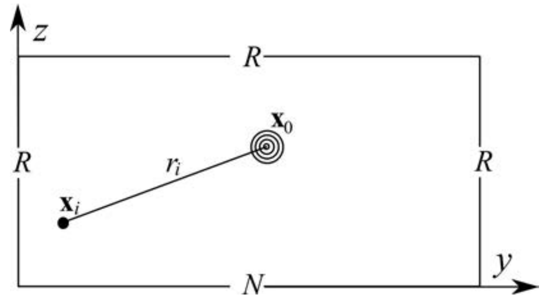

Fig. 1. Cross section geometry of the problem; x_0 – source point, x_i – arbitrary domain point, r_i – distance between x_0 and x_i, R, N – Robin, Neumann boundary conditions.

In practice, the acoustic impedance $z(\mathbf{x})$ is an acoustic impedance of any material and it is wined through the measure of the absorption coefficient $\alpha(\mathbf{x})$, i.e. (Kuttruff, 2000),

$$z(\mathbf{x}) = \rho\, c\, \frac{1 + (1 - \alpha(\mathbf{x}))^{1/2}}{1 - (1 - \alpha(\mathbf{x}))^{1/2}}. \tag{4}$$

The acoustic impedance $z(\mathbf{x})$ has an influence on eigenvalues and eigenfunctions, which translates into Fourier solution of the boundary problem.

3. Eigenvalues and eigenfunctions

The eigenvalues and eigenfunctions in x- and y-directions are pointed out in the case,when the impedance at the left end $z_0(a_x)/z_0(a_y)$ is the same as the impedance at the right end $z_0(b_x)/z_0(b_y)$. Since they are assumed the same in both directions, a solution is given in detail below for the x-direction, while for the y-direction is written by analogy. So, the solution of the homegeneous Eq. (1), in the x-direction is assumed in the form (Kocan-Krawczyk, 2017; Brański *et al.*, 2017):

$$X(x) = C_1 \cos(k_\mu x) + C_2 \sin(k_\mu x), \tag{5}$$

hereunder $k_\mu = k_x$.

Substituting Eq. (5) into Eq. (2) gives $C_1 = (C_2 k_x)/z_0(a_x)$, and after some calculations one obtains the eigenvalues equation

$$\tan(w_x) = \frac{w_x \cdot 2Z_x}{w_x^2 - Z_x^2}, \tag{6}$$

where $w_x = k_\mu(b_x - a_x)$, $Z_x = \{Z_{ax}, Z_{bx}\}$, $Z_{ax} = z_0(a_x)(b_x - a_x)$, $Z_{bx} = z_0(b_x)(b_x - a_x)$; since $z_0(a_x) = z_0(b_x)$, so $Z_{ax} = Z_{bx}$.

The above equation has been solved in (Brański *et al.*, 2017; Kocan-Krawczyk, 2017); selected values of the wave numbers are presented in Table 1.

Table 1. The sets of k_μ, k_ι, k_ν for $f = 2000$ Hz and $\alpha = 0.5$.

k_μ/k_ι	0.5908	1.1821	1.7747	2.3690	2.9650	3.5635	4.1637
k_ν	0.5865	1.7734	2.9647	4.1630	5.3708	6.5870	7.8102

Substituting results, obtained above, into Eq. (5) the general solution and the set of eigenfunctions $\{X_\mu(x)\}$ in x-direction are obtained,

$$X(x) = \sum_\mu C_{2\mu} X_\mu(x) = \sum_\mu C_{2\mu} \left[(k_\mu/z_0(a_x))\cos(k_\mu x) + \sin(k_\mu x)\right], \tag{7}$$

where $X_\mu(x) = [...]$.

By analogy, the set eigenfunctions $\{Y_\iota(y)\}$ in y-direction takes the form

$$Y(y) = \sum_\iota C_{2\iota} Y_\iota(y) = \sum_\iota C_{2\iota} \left[(k_\iota/z_0(a_y))\cos(k_\iota y) + \sin(k_\iota y)\right], \tag{8}$$

where $k_\iota = k_y$ and $Y_\iota(y) = [...]$.

Quite the same, one solves the homogeneous Eq. (1) in the z-direction, with boundary conditions defined by Eqs. (2) and (3). The general solution takes the form, cf. Eq. (5),

$$Z(z) = D_1 \cos(k_\nu z) + D_2 \sin(k_\nu z), \tag{9}$$

where hereunder $k_\nu = k_z$.

Substituting Eq. (10) into Eqs. (2) and (3) gives $D_2 = 0$, and instead of Eq. (6) it is

$$\tan(w_z) = Z_{bz}/w_z, \tag{10}$$

where $w_z = k_\nu(b_z - a_z)$ and $Z_{bz} = z_0(b_z)(b_z - a_z)$; selected values of $\{k_\nu\}$ are also presented in Table 1.

Substituting results obtained above into Eq. (10) leads to the set of eigenfunctions $\{Z_\nu(z)\}$,

$$Z(z) = \sum_\nu D_{1\nu} Z_\nu(z) = \sum_\nu D_{1\nu} \cos(k_\nu z), \tag{11}$$

where $Z_\nu(z) = \cos(k_\nu z)$.

In the end, the solution of the boundary problem is

$$U(\mathbf{x}) = \sum_{\mu\iota\nu} a_{\mu\iota\nu} U_{\mu\iota\nu}(\mathbf{x}) = \sum_{\mu\iota\nu} a_{\mu\iota\nu} X_\mu(x)\, Y_\iota(y)\, Z_\nu(z), \tag{12}$$

where $U_{\mu\iota\nu}(\mathbf{x})$ are $\mu\,\iota\,\nu$-eigenfunctions ($\mu\,\iota\,\nu$-modes).

This way the solution of the inhomogeneous acoustic problem may be achieved. Coefficients $\{a_{\mu\,\iota\,\nu}\}$ remain to be determined.

4. Forced acoustic vibrations

In the steady state, this kind of vibrations is described by inhomogeneous Helmholtz, Eq. (1), where $k = k_f$ and $k_f = \omega_f/c$. For the above problem, the solution is given by the equation $U(\mathbf{x}) = U_1(\mathbf{x}) + U_2(\mathbf{x})$, where $U_1(\mathbf{x})$ are free vibrations given by Eq. (12), $U_2(\mathbf{x})$ are forced vibrations. In the following, just forced vibrations ought to be found; for simplicity of their notation $U_2(\mathbf{x}) = U(\mathbf{x})$.

An acoustic source is represented by the function $F(\mathbf{x})$ in Eq. (1). The $F(\mathbf{x})$ constitutes the solution of the radial part of the Bessel's differential equation in spherical coordinates (McLachlan, 1964; Evans, 2002). Here, the 0-order, spherical Hankel function of the second kind $h_0^{(2)}(k_f r)$ plays the major part; it describes an outward propagating spherical wave.

The solution of Eq. (1) is now formulated as some sum of eigenfunctions, Eq. (12). In a similar manner, the source function $F(\mathbf{x})$ is represented, then

$$F(\mathbf{x}) = \sum_{\mu\iota\nu} b_{\mu\iota\nu} U_{\mu\iota\nu}(\mathbf{x}), \tag{13}$$

where $\{b_{\mu\,\iota\,\nu}\}$ is a set of some coefficients.

Since the function of $F(\mathbf{x})$ is known in advance, then coefficients $\{b_{\mu\,\iota\,\nu}\}$ are first calculated, and they are given by the formula

$$b_{\mu\iota\nu} = (1/\beta_{\mu\iota\nu}) \int_{a_x}^{b_x}\int_{a_y}^{b_y}\int_{a_z}^{b_z} F(\mathbf{x}) X_\mu(x) Y_\iota(y) Z_\nu(y)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z, \tag{14}$$

where

$$\beta_{\mu\iota\nu} = \beta_\mu\, \beta_\iota\, \beta_\nu = \int_{a_x}^{b_x} X_\mu^2(x)\, \mathrm{d}x \int_{a_y}^{b_y} Y_\iota^2(y)\, \mathrm{d}y \int_{a_z}^{b_z} Z_\nu^2(z)\, \mathrm{d}z. \tag{15}$$

Now, coefficients $\{a_{\mu\,\iota\,\nu}\}$ should be computed. To achieve this, Eqs. (12) and (13) are substituted into Eq. (1), hence

$$a_{\mu\iota\nu} = \frac{b_{\mu\iota\nu}}{\left(k_f^2 - k_{\mu\iota\nu}^2\right)}. \tag{16}$$

Finally, forced acoustical vibrations are described by Eq. (12) with above coefficients $\{a_{\mu\,\iota\,\nu}\}$.

In practice, instead of the acoustic potential, the acoustic pressure and secondary acoustic quantities are used. First of all, the acoustic pressure $p(\mathbf{x})$ [Pa] is given by

$$p(\mathbf{x}) = j\, \rho_0\, \omega\, U(\mathbf{x}). \tag{17}$$

Next, to notice quantitative change in the sound pressure level $L(\mathbf{x})$ [dB] in a whole acoustic room, the mean value of it ought to be calculated based on the equation,

$$L_m = 10 \log\left(p_m^2/p_0^2\right), \tag{18}$$

where $p_m = 1/n_i \sum_i p(\mathbf{x}_i)$, $p_0 = 2\cdot 10^{-5}$ Pa, $i = 1, 2, ..., n_i$ the number of calculated points inside the acoustic room is uniform, $n_i = 124 \times 124 \times 124$.

5. A formula to a rough estimation of the cuboidal room acoustics with impedance walls

Based on numerical calculations and the least-squares approximation theory, a simple formula is derived to the rough estimation of the cuboidal room acoustics. This formula expresses the mean sound pressure level L_m as a function of the absorption coefficient, the frequency, and the volume of the room. Below, all numerical calculations are performed assuming the following values: absorption coefficients $\{\alpha\} = \{0.1,\ \text{step } 0.1,\ 0.9\}$, frequencies: $\{f\} = \{250, 500, 1000, 2000, 4000, 8000, 16000\}$, Hz, and two volumes of the room V_1 and V_2.

The following global values and symbols are assumed: $\rho = 1.205$ kg/m^3, $c = 344$ m/s, $a_x = a_y = a_z = 0$ m, $b_x = b_y = 5$ m, $b_z = 2.5$ m, or $b_z = 5$ m. The first geometry data define the volume V_1, but the second one – the volume V_2 of the acoustic room. The point force source is placed at the point $\mathbf{x}_0 = \{x_0, y_0, z_0\} = \{2.5, 2.5, 1.25\}$, m, and $\mathbf{x}_0 = \{2.5, 2.5, 2.5\}$, m, in the V_1 and V_2, respectively. Furthermore, it is assumed that $z_0(a_x) = z_0(b_x) = z_0(a_y) = z_0(b_y) = z_0(b_z)$, since it is the most frequent assumption made in acoustics, where e.g. $z_0(a_x)$ is $z_0(x)$ on the edge a_x, and so on, but, $z_0(a_z) = 0$ as a result of the Neumann boundary condition. Next, $F(x) = A\, h_0^{(2)}(k_f r)$ and, to make the final results real, an intensity of the source A is determined in two ways (they point out to two groups of the numerical calculations):

1) the value of $L_m \approx 75$ dB for $\alpha = 0.1$, volumes V_1 and V_2, and frequencies $\{f\}$ given above; in this case the intensity A is changed,
2) the intensity A is constant, i.e. $A = 5.51\cdot 10^{-3}$ independently of the absorption coefficient, the volume of the room, and the frequency; this value of A is fixed for $L_m \approx 75$ dB for $\alpha = 0.1$, volume V_1 and $f = 250$ Hz.

To omit the singularity of $h_0^{(2)}(k_f r)$ at $r = 0$, the cube space with dimensions $0.2 \times 0.2 \times 0.2$ m around the source is omitted (Brański *et al.*, 2017).

The discrete exact values of L_m, are marked by dot values in all figures below, and they are calculated based on formula derived above; part of them may be found in (Brański *et al.*, 2017; Kocan-Krawczyk, 2017).

The first group of calculations concerns the approximate L_m as a function of α for separate frequencies and separate volumes of the room. An analytical formula describing L_m can be expressed by

$$L_m(\alpha) = a_1 + a_2\alpha^n, \tag{19}$$

where a_1, a_2 are any constants and they are calculated using the least-squares approximation.

The value n is chosen *a priori* basing on the distribution of discrete values of the L_m for particular frequencies. Figures 2 and 3 present discrete values of the L_m (dot values) and lines of the L_m, which are approximated based on Eq. (19) for $b_z = 2.5$ m (volume V_1) and $b_z = 5$ m (volume V_2) respectively. As it can be seen, two ranges of frequencies are distinguished. Namely, for $f = f_{sl} = 2000$ Hz it is a straight line (L_{sl}), however, for frequencies higher than $f = 2000$ Hz ($f_h > f$), the value n is negative and for frequencies lower than $f = 2000$ Hz ($f_l < f$) the value n is positive. For the frequencies pointed out above, the set $\{n\} = \{8, 3, 2, 1, -0.1, -0.2, -0.4\}$.

Next, the calculations concern an approximation of the L_m in two ranges of frequencies, i.e. for $f_h > f$ and for $f_l < f$. As a result, two lines are obtained, namely $L_{m;h}$ and $L_{m;l}$, respectively, see Fig. 4, where the L_{sl} is added. These lines may be used for a rough estimation of the acoustic room and depend on coefficients absorption of walls $\{\alpha\}$ (impedance walls), frequencies $\{f\}$, and volumes of the room $\{V_1, V_2\}$.

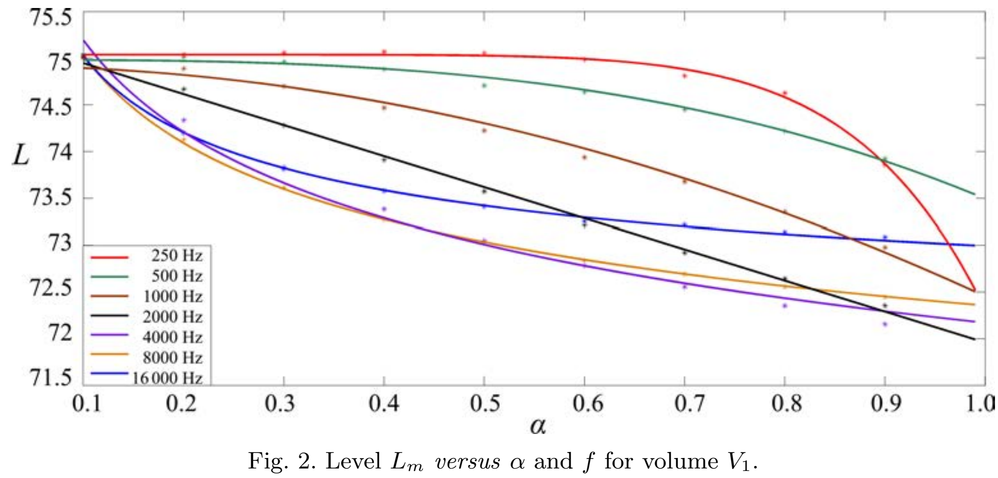

Fig. 2. Level L_m *versus* α and f for volume V_1.

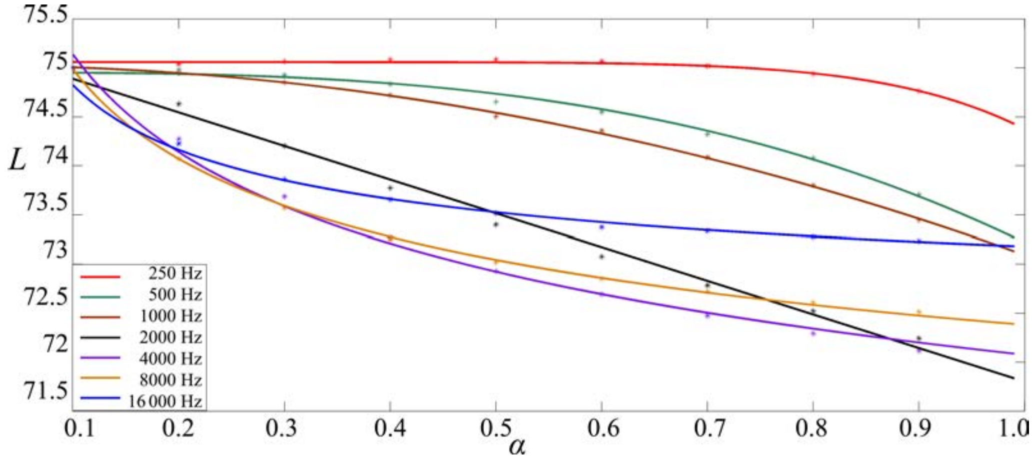

Fig. 3. Level L_m *versus* α and f for volume V_2.

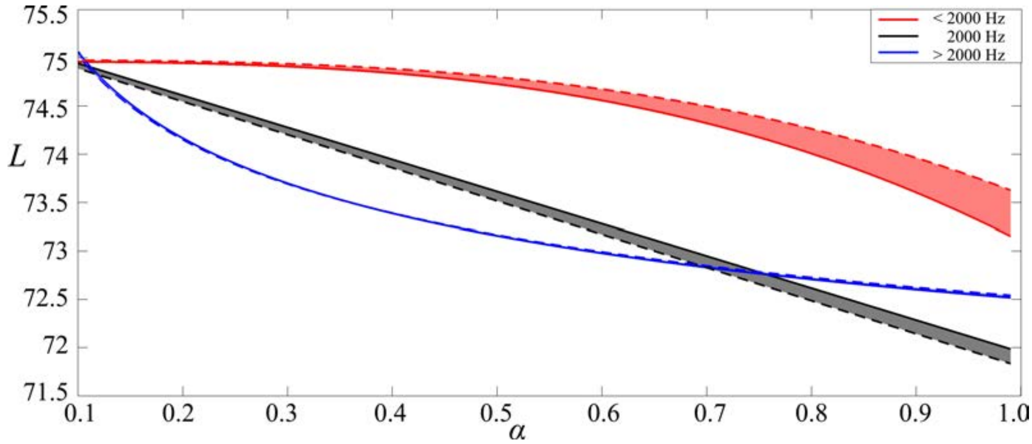

Fig. 4. Levels $L_{m;l}$ (red line) and $L_{m;h}$ (grey line) *versus* $\{\alpha\}$, frequencies groups $\{f_l,\ f_h\}$ and volumes of the room $\{V_1,\ V_2\}$ (solid, dashed lines).

The second group of calculations concerns the approximate L_m as a function of α for separate frequencies and only the volume V_2 of the room, i.e., quite the same assumptions as above, but the intensity of the source A is fixed *a priori* and its value is $A = 5.51 \cdot 10^{-3}$. The results are depicted in Fig. 5 and they are to be compared with the results in Fig. 3.

As it can be noted, for separate frequencies, the L_m graphs in both figures are nearly the same. Furthermore, in both figures the same two groups of fre-

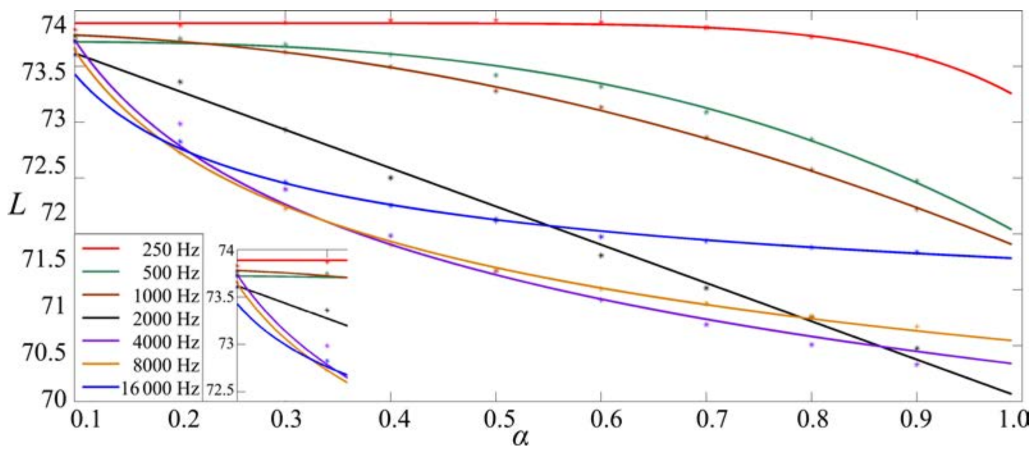

Fig. 5. Level L_m *versus* α, f, for volume V_2 and A (fixed *a priori*).

quencies are distinguished. As could be expected, the graphs for the fixed A are shifted down parallel to the vertical axis; the details are in the additional Fig. 5 where all L_m graphs begin with lower values than in Fig. 3.

6. Conclusions

The formula to the rough evaluation of the cuboidal room acoustics only with impedance walls was achieved. For this purpose, a 3D acoustic problem in the room with rigid floor and impedance walls and celling was considered. In other words, the above solution was aimed at examining the qualitative (not quantitative) response of the acoustic field to the absorption coefficient in a cuboidal room. Calculations were performed for discrete values of the absorption coefficient $\{\alpha\}$ in its full range and for discrete values of frequencies $\{f\}$ of the audible sound.

The main derived formula presents a mean acoustic pressure level L_m as a function of the absorption coefficient α for separate frequencies $\{f\}$ and separate volumes of the room $\{V_1, V_2\}$. The new formula of L_m is assumed in the form of power function where the number exponent is fixed *a priori* for separate frequencies.

Based on the new formula, two groups of frequencies are recognised. It ought to be useful for evaluation of the cuboidal room acoustics at first sight. The theoretical considerations were verified by numerical simulations and basing on them some conclusions can be drawn.

1) It is possible to obtain an approximate formula on the L_m for a rough evaluation of the cuboidal room acoustics as a function of the coefficient absorption of walls for separate frequencies and separate volumes of the room.

2) For $f = f_{sl} = 2000$ Hz the L_m makes up a straight line L_{sl}.

3) Two groups of frequencies are distinguished: $f \equiv f_h > f_{sl}$ and $f \equiv f_l < f_{sl}$.

4) For frequencies f_h, the L_m takes the form polynomials with negative exponents, and for frequencies f_l, the L_m takes the form of polynomials with positive exponents.

5) One can determine the substitute L_m for two groups of frequencies, which can be used for superficial estimation of the cuboidal room acoustics.

6) The change in a room volume slightly affects the obtained L_m values.

7) Quantitative results of the Fourier method can be a point of reference to the results of numerical methods.

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