Metrol. Meas. Syst., Vol. 24 (2017), No. 1, pp. 113–125.

METROLOGY AND MEASUREMENT SYSTEMS

Index 330930, ISSN 0860-8229

www.metrology.pg.gda.pl

NANOSATELLITE ATTITUDE ESTIMATION FROM VECTOR MEASUREMENTS USING SVD-AIDED UKF ALGORITHM

Demet Cilden[1], Halil Ersin Soken[2], Chingiz Hajiyev[1]

1) Istanbul Technical University, Faculty of Aeronautics and Astronautics, Maslak, 34469, Istanbul, Turkey (✉ cilden@itu.edu.tr, +90 212 285 3122*, cingiz@itu.edu.tr)*

2) Japan Aerospace Exploration Agency (JAXA), Institute of Space and Astronautical Science (ISAS), Sagamihara, Japan (ersin_soken@ac.jaxa.jp)

Abstract

The integrated *Singular Value Decomposition* (SVD) and *Unscented Kalman Filter* (UKF) method can recursively estimate the attitude and attitude rates of a nanosatellite. At first, Wahba's loss function is minimized using the SVD and the optimal attitude angles are determined on the basis of the magnetometer and Sun sensor measurements. Then, the UKF makes use of the SVD's attitude estimates as measurement results and provides more accurate attitude information as well as the attitude rate estimates. The elements of "Rotation angle error covariance matrix" calculated for the SVD estimations are used in the UKF as the measurement noise covariance values. The algorithm is compared with the SVD and UKF only methods for estimating the attitude from vector measurements. Possible algorithm switching ideas are discussed especially for the eclipse period, when the Sun sensor measurements are not available.

Keywords: attitude estimation, nanosatellite, UKF, SVD, SVD-Aided UKF.

1. Introduction

Sun sensors and magnetometers are common attitude sensors for nanosatellite missions; they are cheap, simple, light and available as commercial of-the-shelf equipment [1, 2]. However, the overall achievable attitude determination accuracy is limited with these sensors mainly as a result of their inherent limitations and unavailability of the Sun sensor measurements when the satellite is in the eclipse.

Attitude estimation with magnetometer and Sun sensor measurements has been addressed in many research works and various algorithms that intend to improve the estimation accuracy have been proposed. A basic solution is using a Kalman filtering algorithm for integrating the measurements under the propagation model of the satellite dynamics and estimating the satellite attitude possibly along with the sensor biases. For example, in [2] two filtering algorithms are proposed, both based on the multiplicative *Extended Kalman Filter* (EKF). The first algorithm is used for estimation of attitude quaternions, gyro biases and Sun sensor calibration parameters, whereas the second one estimates only the quaternions and gyro biases excluding the Sun sensor calibration parameters. The main drawback of both algorithms is a degradation in the estimation results when the satellite is in its eclipse so the Sun sensor data are not available. A similar phenomenon can be seen in [1] for the *Unscented Kalman Filter* (UKF) estimations. Another approach to the nanosatellite attitude estimation is to determine the attitude using a single-frame attitude estimator. This method is based on computing Sun and magnetic field vectors in the reference frame and measuring the same vectors in the body coordinate system. Then, a deterministic method such as the TRIAD (*two-vector algorithm*) or an optimization method such as the QUEST can be used for the attitude estimation [3]. A drawback of these methods

Article history: received on Mar. 15, 2016; accepted on Sep. 24, 2016; available online on Mar. 10, 2017; DOI: 10.1515/mms-2017-0011.

is that they are based only on measurements; they do not use any knowledge about the satellite dynamics. Attitude estimation methods, which take the advantage of the system's mathematical model, may significantly increase the attitude estimation accuracy. In [4], the Sun-eclipse phases are considered to use both traditional and non-traditional methods depending on whether the Sun sensor is operational or not. In the Sun sensor operational mode, the Gauss-Newton method enables to obtain the quaternion estimates for using in EKF. In the eclipse mode, only the traditional EKF is used. The measurement covariance values in EKF are not provided by the deterministic method to the filter and they are selected. This leads to some jumps in the filter even outside of the eclipse. If the variance values of the first method are used as the measurement noise covariance ones in EKF, the filter will have to compensate these errors.

The traditional approaches to designing a *Kalman Filter* (KF) for the satellite attitude estimation use nonlinear measurements of reference directions (*e.g.* the Sun direction) [1, 5–7]. The measurement models in the filter are based on nonlinear models of the reference directions so the measurements and states are related by nonlinear equations. In the approach based on linear measurements the attitude angles are found first by using the vector measurements and then a suitable single-frame attitude estimation method [3]. Then, these attitude estimates are used as the measurement results within the KF. The filter measurement model is linear in this case, since the single-frame attitude estimator provides directly the states themselves as measurements. We may name such algorithms as "single-frame estimator-aided attitude filtering".

An earlier study on single-frame estimator-aided attitude filtering was carried out in [8]. In this study the authors integrate the algebraic method (TRIAD) and the EKF algorithms to estimate the attitude angles and angular velocities. The magnetometers, Sun sensors, and horizon scanners/sensors are used as measurement devices and three different two-vector algorithms based on the Earth's magnetic field, Sun, and nadir vectors are proposed. An EKF is designed to obtain the satellite's angular motion parameters with the desired accuracy. The measurement inputs for the EKF are the attitude estimates of the two-vector algorithms. Interest in "single-frame estimator-aided attitude filtering" is higher in the more recent literature [9–11]. The attitude determination concept of the Kyushu University mini-satellite QSAT is based on a combination of the *Weighted-Least-Square* (WLS) and KF [9, 10]. The WLS method produces the optimal attitude-angle observations at a single-frame by using the Sun sensor and magnetometer measurements. The KF combines the WLS angular observations with the attitude rate measured by gyros to produce the optimal attitude solution. In [11], an interlaced filtering method is presented for determination of the nanosatellite attitude. In this integrated system, the optimal-REQUEST and UKF algorithms are combined to estimate the attitude quaternion and gyro drifts. The optimal-REQUEST, which cannot estimate gyroscope drifts, is run for the attitude estimation. Then, the UKF is used for the gyro-drift estimation on the basis of linear measurement results obtained as the optimal-REQUEST estimates. There are also similar applications for the UAV attitude estimation. De Marina et al. introduce an attitude heading reference system based on the UKF using the TRIAD algorithm as the observation model in [12].

Here, we may also refer to the studies where a single-frame attitude estimator is used together with an attitude filter but does not provide linear measurements [13, 14]. For linear measurements, it is equivalent to first updating the attitude using the single-frame estimator and subsequently using this updated portion of the state to updating the remainder of the state as if updating the entire state at once. However in [13, 14], the measurement model is a nonlinear one. A nonlinear updating the attitude is obtained by solving the Wahba's problem and subsequently updating the non-attitude states using the optimal gain for the linear measurement case. Therefore, in these studies, the attitude is updated using a single-frame estimator, whereas all remaining non-attitude states are updated using standard nonlinear attitude filters.

In this study we examine an *SVD-aided UKF* (SaUKF) algorithm for the nanosatellite attitude estimation. The nanosatellite has magnetometers and Sun sensors as on-board attitude sensors. In the first phase, Wahba's problem is solved by the *Singular Value Decomposition* (SVD) method and quaternion estimations are obtained for the satellite's attitude. These quaternion estimations are then used as the measurement results for an UKF, which forms the second phase of the algorithm. The SaUKF provides improved attitude knowledge and attitude rate estimates. The whole algorithm runs recursively. The main aim is to propose an easy-to-apply and accurate nanosatellite attitude estimation algorithm, which is also robust against estimation deteriorations when the satellite is in its eclipse. The initial results are presented in [15]. In this study we compare the results with those obtained by an UKF that uses nonlinear measurements. Besides we propose an algorithm that switches between the UKF with nonlinear measurements and the SaUKF to ensure both the accuracy and robustness.

2. Satellite equations of motion and measurement models

In this section we briefly review the satellite equations of motion and the measurement models for magnetometers and Sun sensors.

2.1. Satellite equations of motion

The satellite's kinematics equation of motion derived using the quaternion attitude representation can be presented as [16]:

$$\dot{\boldsymbol{q}}(t) = \frac{1}{2}\Omega(\boldsymbol{\omega}_{BR}(t))\boldsymbol{q}(t). \tag{1}$$

In (1), the quaternion $\boldsymbol{q}$ is composed of four attitude parameters, $\boldsymbol{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T$. Three terms of the quaternion $\boldsymbol{q}$ are vectors, whereas the last term is a scalar. Then, the quaternion can take a form of $\boldsymbol{q} = \begin{bmatrix} \boldsymbol{g}^T & q_4 \end{bmatrix}^T$, $\boldsymbol{g} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T$. Moreover, in (1), $\Omega(\boldsymbol{\omega}_{BR})$ is the skew symmetric matrix as:

$$\Omega(\boldsymbol{\omega}_{BR}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}, \tag{2}$$

where the $\boldsymbol{\omega}_{BR}$ vector is composed of ω_1, ω_2 and ω_3; it indicates the angular velocity of the body frame with respect to the orbit frame. The angular rate vector should be identified because of the sensor usage. Hence, the rate vector in the body frame with respect to the inertial coordinate system can be shown as: $\boldsymbol{\omega}_{BI} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$. $\boldsymbol{\omega}_{BI}$ and $\boldsymbol{\omega}_{BR}$ can be related according to the following equation:

$$\boldsymbol{\omega}_{BR} = \boldsymbol{\omega}_{BI} - A\begin{bmatrix} 0 & -\omega_o & 0 \end{bmatrix}^T. \tag{3}$$

The angular velocity of the satellite on its orbit is specified by ω_o with respect to the inertial reference, found as $\omega_o = \left(\mu / r^3\right)^{1/2}$ for a circular orbit using μ, which is the product of two constants $\left(GM_E\right)$. Here, G is the gravitational constant, M_E – the mass of the Earth and r – the distance between the satellite and Earth centers of masses. In (3) A is a transformation matrix which can be related to the quaternions as follows:

$$A = (q_4^2 - \boldsymbol{g}^2) I_{3\times3} + 2\boldsymbol{g}\boldsymbol{g}^T - 2q_4 [\boldsymbol{g}\times]. \tag{4}$$

The unit matrix I_{3x3} has a dimension of 3×3 and $[\boldsymbol{g}\times]$ is a skew-symmetric matrix as follows:

$$[\boldsymbol{g}\times] = \begin{bmatrix} 0 & -g_3 & g_2 \\ g_3 & 0 & -g_1 \\ -g_2 & g_1 & 0 \end{bmatrix}. \tag{5}$$

The satellite's dynamic equations are necessary to estimate the full state attitude including both the attitude and attitude rates. Based on the Euler's equations the dynamic knowledge can be found by:

$$J\frac{\boldsymbol{\omega}_{BI}}{dt} = \boldsymbol{N}_d - \boldsymbol{\omega}_{BI} \times (J\boldsymbol{\omega}_{BI}), \tag{6}$$

where J is an inertia matrix composed of $J = diag(J_x, J_y, J_z)$ which are the principal moments of inertia. The external torques affecting the satellite can be added in order to find the resulting disturbance torque, $\boldsymbol{N}_d$:

$$\boldsymbol{N}_d = \boldsymbol{N}_{gg} + \boldsymbol{N}_{ad} + \boldsymbol{N}_{sp} + \boldsymbol{N}_{md}, \tag{7}$$

where $\boldsymbol{N}_{gg}$ is the gravity gradient torque, $\boldsymbol{N}_{ad}$ is the aerodynamic disturbance torque, $\boldsymbol{N}_{sp}$ is the solar pressure disturbance torque and $\boldsymbol{N}_{md}$ is the residual magnetic torque caused by the interaction of the satellite's residual dipole and the Earth's magnetic field [16].

2.2. Sensor models

The magnetometer sensor for attitude determination is a very common sensor for small satellite missions. A model of the Earth's magnetic field measurements can be given in (8) (the magnetometers are assumed to be calibrated) [17, 18]:

$$\begin{bmatrix} B_x(\boldsymbol{q},t) \\ B_y(\boldsymbol{q},t) \\ B_z(\boldsymbol{q},t) \end{bmatrix} = A \begin{bmatrix} B_1(t) \\ B_2(t) \\ B_3(t) \end{bmatrix} + \eta_1. \tag{8}$$

The components of the Earth's magnetic field, $B_1(t), B_2(t)$ and $B_3(t)$, in the orbital coordinate frame can be calculated by the common and accurate magnetic field model, *International Geomagnetic Reference Field* (IGRF) [19]. $B_x(\boldsymbol{q},t), B_y(\boldsymbol{q},t)$ and $B_z(\boldsymbol{q},t)$ are the vector components of magnetic field measured by the magnetometers. Therefore, they are presented in the body reference system. Moreover, η_1 is the zero mean Gaussian white noise:

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_m^2\delta_{kj}, \tag{9}$$

where σ_m is the standard deviation and δ_{kj} is the Kronecker symbol.

The Sun direction with respect to the inertial coordinates regarding the Earth center depends only on time referred to Julian Day (T_{TDB}). T_{TDB} can be derived using the satellite's reference epoch and the exact time. The variables are the mean anomaly (M_{Sun}) and the mean longitude ($\lambda_{M_{Sun}}$) of the Sun. Using (10), the ecliptic longitude of the Sun ($\lambda_{ecliptic}$) and its linear model (ε) can be found [20]:

$$M_{Sun} = 357.5277233^0 + 35999.05034T_{TDB}\,, \tag{10a}$$

$$\lambda_{ecliptic} = \lambda_{M_{Sun}} + 1.914666471^0 sin(M_{Sun}) + 0.019994643 sin(2M_{Sun})\,, \tag{10b}$$

$$\lambda_{M_{Sun}} = 280.4606184^\circ + 36000.77005361T_{TDB}\,, \tag{10c}$$

$$\varepsilon = 23.439291^0 - 0.0130042T_{TDB}\,. \tag{10d}$$

From those relations (10), the Sun direction vector ($\boldsymbol{S}_{ECI}$) in the inertial coordinates can be found:

$$\boldsymbol{S}_{ECI} = \begin{bmatrix} \cos\lambda_{ecliptic} \\ \sin\lambda_{ecliptic}\cos\varepsilon \\ \sin\lambda_{ecliptic}\sin\varepsilon \end{bmatrix}. \tag{11}$$

However, since the satellite is rotating along its trajectory, it is necessary to transform the unit Sun direction vector into the orbital frame by using the orbit propagation algorithm. Finally, the (12) shows the relation between the Sun sensor measurement vector and the Sun direction model vector:

$$\boldsymbol{S}_b = A\boldsymbol{S}_o + \eta_2\,, \tag{12}$$

where $\boldsymbol{S}_o$ is the Sun direction vector in the orbit reference system and $\boldsymbol{S}_b$ is the vector of Sun sensor measurements in the body reference system having the zero mean Gaussian white noise η_2 with the characteristic of:

$$E\left[\eta_{2k}\eta_{2j}^T\right] = I_{3x3}\sigma_s^2\delta_{kj}\,, \tag{13}$$

where σ_s is the standard deviation of Sun sensor error.

The satellite's orbital elements and its position on the orbit must be known to model the Earth's magnetic field and Sun vectors in the orbit frame.

3. SVD-aided UKF algorithm

The contents of this section include estimation of the satellite's attitude and the angular velocities during the operational mode of the mission. The estimation process is divided into two stages: SVD and UKF. Firstly, a single frame method SVD minimizes the Wahba's loss function by using two vectors and finds the coarse attitude angles and variance values for each axis. Then, UKF uses the SVD results as the input values in each time step and provides the filtered attitude and attitude rates with a higher accuracy.

3.1. SVD method

As a single-frame method, SVD aims to solve the problem formulated by Grace Wahba [21]. In every single time frame SVD can estimate the coarse attitude only by using the measurement results and the model vectors. In the loss function (see (14)), $\mathbf{b}_i$ and $\mathbf{r}_i$ are sets of unit vectors obtained in two different coordinate systems in every single time interval. From the optimal solution for the orthogonal *A* matrix, the attitude angles can be found [22]:

$$L(A) = \frac{1}{2}\sum_i a_i \left|\mathbf{b}_i - A\mathbf{r}_i\right|^2\,. \tag{14}$$

The unit vectors in the loss function represent the Sun direction and Earth's magnetic field vectors for the orbit frame ($\mathbf{r}_i$) and the body frame ($\mathbf{b}_i$), where a_i is a non-negative weight. The loss can be reduced in (15) as:

$$L(A) = \lambda_0 - \mathrm{tr}\left(AB^T\right), \tag{15}$$

where:

$$\lambda_0 = \sum a_i, \tag{16a}$$

$$B = \sum a_i \mathbf{b}_i \mathbf{r}_i^T . \tag{16b}$$

The SVD method can be used here to maximize the trace function expressed in the (15) by using the most robust algorithm from single-frame methods [22]. B matrix has the singular value decomposition:

$$B = U \Sigma^T V^T = U \mathrm{diag}\left[\Sigma_{11} \quad \Sigma_{22} \quad \Sigma_{33}\right] V^T, \tag{17}$$

where matrices U and V are orthogonal and the singular values hold $\Sigma_{11} \geq \Sigma_{22} \geq \Sigma_{33} \geq 0$. Then, the optimal attitude matrix can be found:

$$U^T A_{opt} V = \mathrm{diag}[1 \quad 1 \quad \det(U)\det(V)], \tag{18}$$

$$A_{opt} = U \mathrm{diag}[1 \quad 1 \quad \det(U)\det(V)] V^T. \tag{19}$$

The covariance analysis is an important process in the integrated filtering technique and the matrix P_{svd} can be obtained by defining secondary singular values $s_1 = \Sigma_{11}$, $s_2 = \Sigma_{22}$, $s_3 = det(U)det(V)\Sigma_{33}$, as follows:

$$P_{svd} = U \mathrm{diag}[(\mathrm{s}_2 + \mathrm{s}_3)^{-1} \quad (\mathrm{s}_3 + \mathrm{s}_1)^{-1} \quad (\mathrm{s}_1 + \mathrm{s}_2)^{-1}] U^T . \tag{20}$$

The method requires measurements at every single moment to accurately provide the attitude angles. Hence, the method fails when either the satellite is in the eclipse or two vectors are parallel.

3.2. Unscented Kalman Filter

The UKF uses an accurate approximation called the Unscented Transform for solving the multidimensional integrals instead of the linear approximation to the nonlinear equations as *Extended Kalman Filter* (EKF) does [23]. The essence is the fact that the approximation of a nonlinear distribution is easier than the approximation of a nonlinear function or transformation. The conventional algorithm for the UKF is not presented here for brevity and the reader may refer to [24], specifically for attitude estimation using the UKF.

When a quaternion in the kinematic modeling of the satellite's motion is used, the UKF in a standard format cannot be implemented straightforwardly. The reason of such a drawback is the constraint of quaternion unity expressed by $\boldsymbol{q}^T \boldsymbol{q} = 1$. If the kinematics (1) is used in the filter directly, than there is no guarantee that the predicted quaternion mean of the UKF will satisfy this constraint.

In the reference [24], the authors overcome this problem by using an unconstrained three-component vector to represent an attitude-error quaternion instead of using all four components of the quaternion vector. They represent the local error-quaternion by the vector of *Generalized Rodrigues Parameters* (GRP). In this paper we use the same method.

Recall that we represent a quaternion with its vector and scalar parts as $\boldsymbol{q} = \begin{bmatrix} \boldsymbol{g}^T & q_4 \end{bmatrix}^T$. After that, when the local error-quaternion is denoted by $\delta\boldsymbol{q} = \begin{bmatrix} \delta\boldsymbol{g}^T & \delta q_4 \end{bmatrix}^T$, the vector of GRP may be given as:

$$\delta\boldsymbol{p} = f\left[\delta\boldsymbol{g} / (a + \delta q_4)\right], \tag{21}$$

where a is a parameter from 0 to1 and f is the scale factor. When $a = 0$ and $f = 1$ then (21) gives the Gibbs vector, whereas when $a = 1$ and $f = 1$ then (21) gives the standard vector of modified Rodrigues parameters. In the paper [24] – as well as in this paper $- f$ is chosen as $f = 2(a+1)$. The inverse transformation from $\delta\boldsymbol{p}$ to $\delta\boldsymbol{q}$ is given by:

$$\delta q_4 = \frac{-a\left\|\delta\boldsymbol{p}\right\|^2 + f\sqrt{f^2 + (1-a^2)\left\|\delta\boldsymbol{p}\right\|^2}}{f^2 + \left\|\delta\boldsymbol{p}\right\|^2}, \tag{22a}$$

$$\delta\boldsymbol{g} = f^{-1}(a + \delta q_4)\delta\boldsymbol{p}. \tag{22b}$$

3.3. Estimation of attitude and attitude rate using SaUKF

Two methods are integrated and the SaUKF algorithm is proposed for the nanosatellite attitude estimation. The main purposes are:

1. As a standalone technique the SVD works well as long as minimum 2 vector measurements are available and not parallel. However, if there is only one vector measurement when the satellite is in the eclipse, the SVD fails to provide any attitude estimate.
2. The SVD method gives attitude estimates as frequent as the sampling rate of the sensor with a lower measurement frequency (if there is no propagation). The SaUKF can provide the attitude estimate with a higher frequency since it makes use of the attitude dynamics.
3. The SVD method does not estimate attitude rates. For most of the cases the satellite attitude rates have to be estimated – especially for control purposes. There are deterministic methods to estimate the satellite's attitude rate from the vector measurement results [25], but usually a filtering-based method gives more accurate estimates.

When the SVD method cannot give any estimation results, the covariance for the SVD estimations – and so the elements of the R matrix – increase. Therefore, the UKF is robust against the failures in the SVD estimations, as we see during the eclipse period.

As the attitude representation, in SVD algorithm there are used quaternions. However, for the SaUKF, the attitude errors regarding GRP are acquired:

$$\delta\boldsymbol{q}_{obs} = \boldsymbol{q}_{mes} \otimes \left[\hat{\boldsymbol{q}}_0(k+1|k)\right]^{-1}, \tag{23}$$

where $\boldsymbol{q}_{mes}$, coming from the SVD method, are quaternion-multiplied with the predicted mean quaternion. Then, regarding $\delta\boldsymbol{q}_{obs} = \begin{bmatrix} \delta\boldsymbol{g}_{obs}^T & \delta q_{4,obs} \end{bmatrix}^T$, the measurement result of the attitude error is calculated as:

$$\delta\boldsymbol{p}_{obs} = f\left[\delta\boldsymbol{g}_{obs} / (a + \delta q_{4,obs})\right]. \tag{24}$$

A scheme of the attitude and rate estimation algorithm of the integrated method is given in Fig. 1.

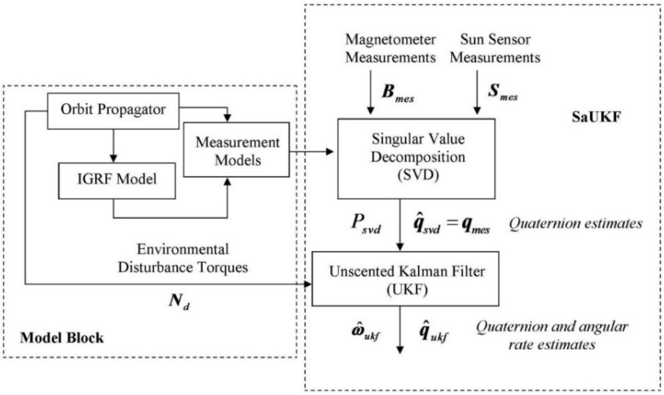

Fig. 1. A scheme of the attitude and attitude rate estimation using the SaUKF.

4. Simulations for nanosatellite

Several simulations were performed in order to evaluate the attitude estimation algorithm. A three-unit cube-sized satellite with about 3 kg mass and $J = diag(0.055\ 0.055\ 0.017)$ kg.m^2 inertia matrix is considered for the estimation scheme. The satellite has an almost circular orbit with an eccentricity of $e = 6.4 \times 10^{-5}$ and $i = 74°$ inclination at 612 km altitude.

All sensors are assumed to be calibrated against biases, scale factors and so on. Therefore, only the sensor noise (zero mean Gaussian white noise) is considered in the algorithm with $\sigma_m = 300nT$ standard deviation for the magnetometer and $\sigma_s = 0.002$ unit for the Sun sensor. The total orbital time is close to 6000 sec. and the time step is taken as 1 sec.

Both SaUKF and UKF use the process noise covariance values of 1×10^{-4} and 1×10^{-9} for attitude and rates, respectively, and have an eclipse period between 2000–4000 sec. In Fig. 2, the estimation error results for SaUKF, SVD and UKF only can be seen and compared. It is clearly seen that the SaUKF estimates the attitude more accurately than both the SVD and UKF only methods, with the exception of the eclipse period. During the eclipse period the SVD method fails because no Sun sensor data are obtained. The quaternion measurements for the SaUKF deteriorate and the values of R, which are coming from the covariance matrix of SVD angle estimation errors (P_{svd}), increase. If the SaUKF gain values become very low (since R values are very high), the correction term of the UKF will become insignificant and the contribution of the propagation model to estimation becomes dominant. That enables the attitude estimation during the eclipse period, even though there is no measurement input to the filter. As it is seen in Fig. 2, the proposed SaUKF method convergences slower than the traditional UKF. This is a drawback of the presented SaUKF method. Therefore, it is recommended to use the proposed method after the convergence of the nontraditional UKF.

The process noise covariance Q is a parameter that enables the filter to base mostly on either the measurements or the dynamics in the filter. In the filter, 1×10^{-4} and 1×10^{-9} pair is used as medium noise. Here, at the end of the eclipse period, before the Sun sensor data arrival, the attitude angle has an error of 10 degrees. If the Q pair is 1×10^{-3} and 1×10^{-7}, which is higher than the selected one, the results are close to the measurement ones and the attitude angles are diverging more during the eclipse period. On the other hand, for lower pair values − such as 1×10^{-9} and 1×10^{-13} – the SaUKF becomes non-agile, i.e. has a smaller convergence rate at the end of the eclipse or the beginning of the orbit (Fig. 3).

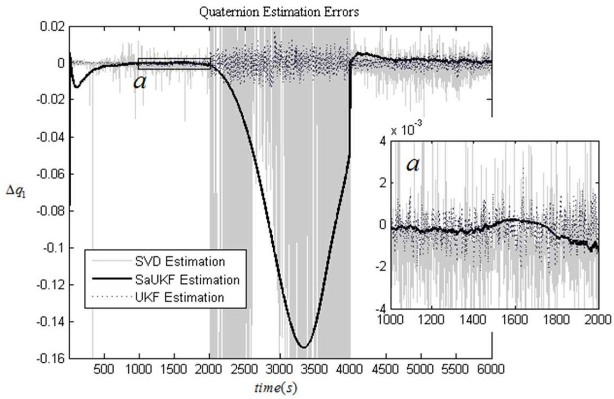

Fig. 2. The estimation error for quaternion q1; comparison of the UKF and SVD only estimations with those of the SaUKF. The subfigure a zooms to the area indicated in the main figure.

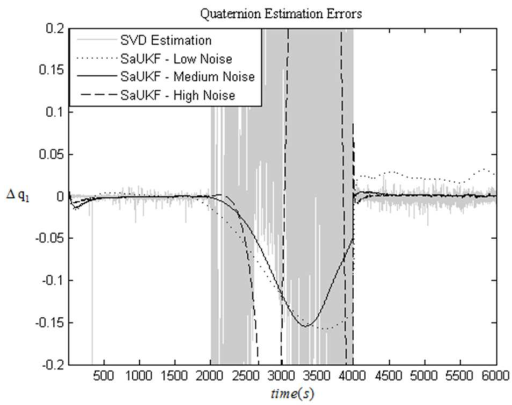

Fig. 3. The quaternion estimation error for the SaUKF with different values of process noise covariance Q.

In the eclipse period, the UKF only method gives the most accurate attitude estimations. During that period it works only with the magnetometer measurements. Since the magnetometers are coarser sensors comparing with the Sun sensors, there is a clear increase in the UKF estimation error in the eclipse but still the estimations are accurate enough for a nanosatellite mission with this sensor configuration (less than 0.1 degrees – see Fig. 4 for the attitude estimation error norms).

The angular velocities of the satellite for each axis can be estimated accurately by using the SaUKF (see Fig. 5). During the eclipse period the attitude rate estimations are not deteriorated as much as the attitude estimates resulting from accurate dynamic knowledge and low process noise for dynamics propagation. The rate estimates obtained by the UKF are similar.

The main disadvantage of the proposed SaUKF method is the requirement of accurate measurements − free of any bias, sensor misalignment and other sorts of errors. The sensors must be calibrated before using their measurement results as an input to the SaUKF. As discussed in several papers [1, 2, 18] particularly for the magnetometers, such a calibration should be performed on-orbit for nanosatellite missions. In addition, as it is clearly demonstrated by the simulation results, the estimation performance of the SaUKF degrades during the eclipse period and the UKF based on the results of nonlinear measurements provides more accurate estimations. Regarding these facts, our suggestion is to use an algorithm which switches between several different filters in accordance with the flight mode. An example is given in Fig. 6.

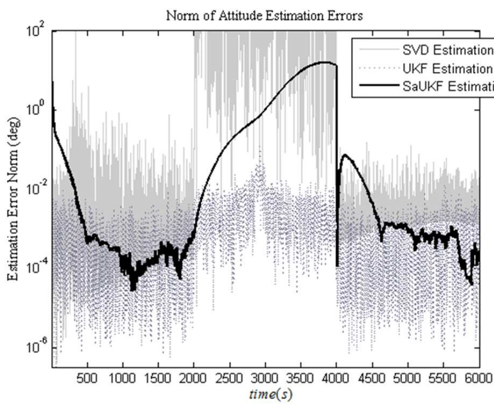

Fig. 4. The norm of attitude estimation errors.

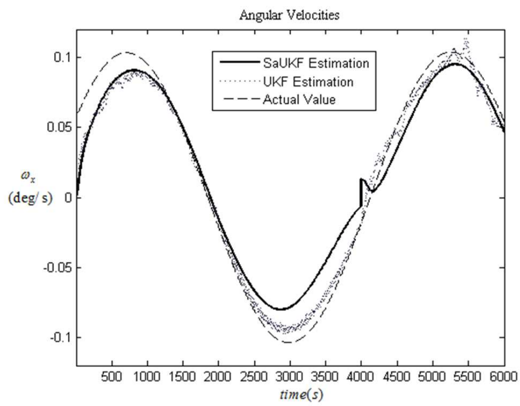

Fig. 5. Estimation of the angular rate along the x axis.

In Fig. 7, two methods are switched in/out of the eclipse period for the more accurate attitude estimation. As mentioned earlier, the SaUKF estimates the attitude more accurately than both SVD and UKF only methods except for the eclipse period; that is why the SaUKF algorithm is used only outside the eclipse. When the satellite is on the dark side of the Earth, the SVD method fails since it is fed with no Sun sensor measurements. The results of the UKF only method are presented in Fig. 7. Also, it should be kept in mind that the switching between the algorithms should be managed after the stabilization of the satellite because right after the eclipse period tumbling may occur.

Certainly, for the nanosatellite application we also need to examine the computational load of each algorithm. Table.1 gives the running times of the algorithms for 6000 sec. simulation, details of which are discussed above. The simulations are performed on a computer with Intel® Core™ i7 @2.93 GHz CPU and 3.49 GB RAM. It shall be noted that all the presented data include the computation time required for simulating the real attitude and measurements. We see that, for the SaUKF algorithm, the SVD is the computationally heavier part and the SaUKF requires a higher load comparing with the UKF based on nonlinear measurements. Yet, the load is not so heavy as to prevent a nanosatellite application, especially if we consider the recent improvements in microprocessors capacity.

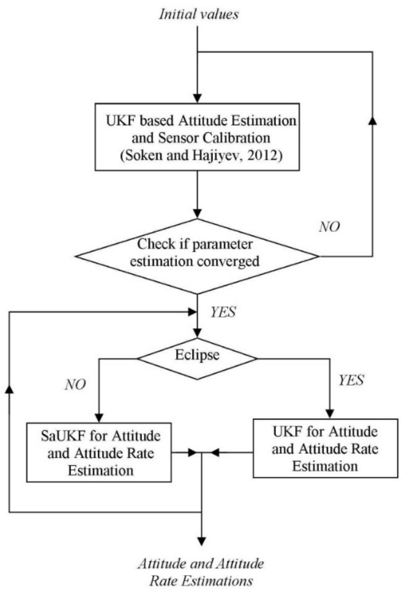

Fig. 6. A block diagram of attitude and attitude rate estimation for the proposed algorithm.

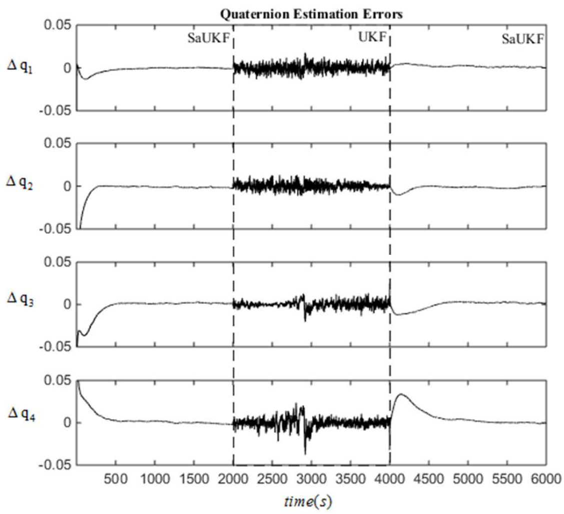

Fig. 7. Estimation of the quaternions by the SaUKF (outside the eclipse period) and UKF (in the eclipse period).

Table 1. The computation times for each algorithm.

Computation time (sec) for 10 Monte Carlo runs	SVD	SaUKF	UKF
	14.30	17.96	10.49

5. Conclusion

In this paper, the *Singular Value Decomposition* (SVD) method and *Unscented Kalman Filter* (UKF) are integrated to determine the attitude and attitude rate for a three-unit cube-sized satellite. The quaternion representation is used to avoid any singularities based on the trigonometric equations. The SVD method fails in the eclipse period because of no Sun observation results. On the other hand, the *SVD-aided UKF* (SaUKF) can estimate the attitude even in the eclipse period, although it is a coarse estimate. The simulation results show that also the UKF with nonlinear vector measurements ensures a reasonable accuracy of the attitude estimation. In the eclipse period the accuracy of the UKF is higher than that of the SaUKF; beyond that period the SaUKF is the most accurate estimation method.

The simulation results show that the proposed SaUKF method convergences slower than the traditional UKF. This is a drawback of the SaUKF method. Therefore, it is recommended to use the proposed method after the convergence of a nontraditional UKF. The ideal algorithm that we suggest for the examined case is composed of the SaUKF and UKF. The SaUKF is used when the Sun sensor measurements are available; in the eclipse period the algorithm switches to the UKF.

Acknowledgments

The work was supported by TUBITAK (The Scientific and Technological Research Council of Turkey), Grant 113E595.

References

[1] Vinther, K., Jensen, K.F., Larsen, J.A., Wisniewski, R. (2011). Inexpensive cubesat attitude estimation using quaternions and unscented Kalman filtering. *Automatic Control in Aerospace*, 4(1).

[2] Springmann, J.C., Cutler, J.W. (2014). Flight results of a low-cost attitude determination systems. *Acta Astronautica*, 99, 201–204.

[3] Markley, F.L. (1999). Attitude determination using two vector measurements. *Flight Mechanics Symposium*, 39–52, Goddard Space Flight Center, Greenbelt, MD.

[4] Cordova-Alarcon, J.R., Mendoza-Barcenas, M.A., Solis-Santome, A. (2015). Attitude Determination System Based on Vector Observations for Satellites Experiencing Sun-Eclipse Phases. *Multibody Mechatronic Systems*, Springer International Publishing, 75–85.

[5] Psiaki, M.L. (1989). Three-axis attitude determination via Kalman filtering of magnetometer data. *Journal of Guidance, Control and Dynamics*, 13(3), 506–514.

[6] Sekhavat, P., Gong, Q., Ross, I.M. (2007). NPSAT I parameter estimation using unscented Kalman filter. *Proc. 2007 American Control Conference*, New York, USA, 4445–451.

[7] Cilden, D., Hajiyev, C. (2015). Error Analysis of the Vector Measurements Based Attitude Determination Methods for Small Satellites. *International Symposium on Space Technology and Science (ISTS)*, Kobe, Hyogo, Japan.

[8] Hajiyev, C., Bahar, M. (2003). Attitude determination and control system design of the ITU-UUBF LEO1 satellite. *Acta Astronautica*, 52(2–6), 493–499.

[9] Mimasu, B.Y., Van der Ha, J.C., Narumi, T. (2008). Attitude determination by magnetometer and gyros during eclipse. *AIAA/AAS Astrodynamics Specialist Conference and Exhibit*, Honolulu, USA.

[10] Mimasu, B.Y., Van der Ha, J.C. (2009). Attitude determination concept for QSAT. *Transactions of the Japan Society for Aeronautical and Space Sciences, Aerospace Technology Japan*, 7, 63–68.

[11] Quan, W., Xu, L., Zhang, H., Fang, J. (2013). Interlaced Optimal-REQUEST and unscented Kalman filtering for attitude determination. *Chinese Journal of Aeronautics*, 26(2), 449–155.

[12] de Marina, H.G., Espinosa, F., Santos, C. (2012). Adaptive UAV Attitude Estimation Employing Unscented Kalman Filter, FOAM and Low-Cost MEMS Sensors. *Sensors*, 12(7), 9566–9585.

[13] Christian, J.A., Lightsey, E.G. (2010). Sequential optimal attitude recursion filter. *Journal of Guidance, Control, and Dynamics*, 33(6), 1787–1800.

[14] Ainscough, T., Zanetti, R. (2014). Q-Method extended Kalman filter. *Journal of Guidance, Control, and Dynamics*.

[15] Cilden, D., Hajiyev, C., Soken, H.E. (2015). Attitude and Attitude Rate Estimation for a Nanosatellite Using SVD and UKF. *Recent Advances in Space Technologies*, Istanbul, Turkey.

[16] Wertz, J.R. (1988). *Spacecraft Attitude Determination and Control*. Dordrecht, Holland: Kluwer Academic Publishers.

[17] Alonso, R., Shuster, M.D. (2002). Complete linear attitude-independent magnetometer calibration. *Journal of Astronautical Sciences*, 50, 477–490.

[18] Soken, H.E., Hajiyev, C. (2012). UKF-Based reconfigurable attitude parameters estimation and magnetometer calibration. *IEEE Transactions on Aerospace and Electronic Systems*, 48(3), 2614–2627.

[19] Finlay, C., Maus, S., Beggan, C.D., Bondar, T.N., *et al.* (2010). International Geomagnetic Reference Field: the eleventh generation. *Geophysical Journal International*, 183, 1216–1230.

[20] Vallado, D.A. (2007). *Fundamentals of Astrodynamics and Applications.* Space Technology Library. USA: Microcosm Press/Springer, 21.

[21] Wahba, G. (1965). Problem 65–1: A Least Squares Estimate of Satellite Attitude. *Society for Industrial and Applied Mathematics Review*, 7(3), 409.

[22] Markley, F.L., Mortari, D. (2000). Quaternion attitude estimation using vector observations. *Journal of the Astronautical Sciences*, 48(2–3), 359–380.

[23] Julier, S.J., Uhlmann, J.K., Durrant-Whyte, H.F. (1995). A new approach for filtering nonlinear systems. *American Control Conference*, Seattle, USA. 1628–1632.

[24] Crassidis, J.L., Markley, F.L. (2003). Unscented filtering for spacecraft attitude estimation. *Journal of Guidance Control and Dynamics*, 26(4), 536–542.

[25] Oshman, Y., Dellus, F. (2003). Spacecraft angular velocity estimation using sequential observations of a single directional vector. *Journal of Spacecraft and Rockets*, 40(2), 234–247.