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# Improvement of the total station 3D adjustment by using precise geoid model

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**Abstract:** A method of the improvement of the total station observations 3D adjustment by using precise geoid model is presented. The novel concept of using the plumb line direction obtained from the precise geoid model in combined GPS/total station data adjustment is applied. It is concluded that results of the adjustment can be improved if data on plumb line direction is used. Theoretical background shown in the paper was proved with an experiment based on the total station and GPS measurements referred to GRS80 geocentric reference system and with the use of GUGIK2001 geoid model for Poland.

**Keywords:** total station observations adjustment, robust estimation, deflection of the vertical, geoid model

# 1. Introduction

Three-dimensional models of terrestrial objects are created on the basis of a set of points with X, Y, Z coordinates obtained using direct (total station observations, laser scanning) and indirect methods (close range photogrammetry).

The accuracy of created 3D model depends on accuracy of coordinates obtained for object individual points. In the case of direct surveying methods the accuracy of surveyed points depends on good knowledge of the six total station or laser scanner external orientation parameters:

- $X_S, Y_S, Z_S$  geocentric GRS80 coordinates of the origin S of the total station or scanner measuring frame (x, y, z) (Fig. 1 and Fig. 2),
- orientation angles  $\Sigma$ ,  $\xi$ ,  $\eta$  of the measuring frame (x, y, z) with respect to the ETRF89 external reference frame (X, Y, Z) (Fig. 2).

The orientation angles  $\xi$ ,  $\eta$  are the components of the deflection of the total station or laser scanner vertical axis from normal to the GRS80 ellipsoid (Fig. 2). The directional horizontal angle  $\Sigma$  is called an instrument orientation constant.

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The description of the adjustment of the total station observations according to such assumptions is given e.g. in the manual (Osada, 2002). The observations being adjusted there are x,y,z coordinates in the total station or laser scanner measuring frame (Fig. 1). In this article, the observational system is modified in such a way that the directly measured spatial distances s, horizontal angles  $\alpha$  and vertical angles  $\beta$  are to be adjusted (Fig. 1). On the testing basis of five measured GPS points  $P_1, P_2, P_3, P_4, S$  (Fig. 1) the adjustment accuracy is analysed with the assumptions that the position of the total station point S is known or unknown. The effect of the deflection of the vertical components  $\xi$ ,  $\eta$  obtained from the actually official GUGIK2001 geoid model for Poland (Pażus et al., 2002) (Fig. 3) is examined.

Hence, the principal objective of the current research is to prove the hypothesis that the deflection of the vertical components  $\xi$ ,  $\eta$  obtained from the geoid model can substantially improve the effects of the spatial 3D adjustment of the total station observations.

According to the producers of total stations, the direction of the vertical axis of the total station is consistent with the plumb line direction to the level of 1 arcsec. Tests of the GUGIK2001 geoid model conducted by GUGiK (Head Office of Geodesy and Cartography, Warsaw) show that the components of the deflection of the vertical calculated from that geoid model coincide with the measured components with accuracy of 0.5 arcsec (Krynski and Lyszkowicz, 2006). It substantiates the use of the deflection of the vertical components calculated from the geoid model instead of measured components with an average error about 1 second. This prevents accidental arrangement of the total station axis in the 3D adjustment, and ultimately leads to good results.

#### 2. Data used in numerical experiment

#### 2.1. The total station and GPS measurements

The four reference points  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$  and the control point Q were surveyed using a total station set on the point S (Fig. 1). The standard deviations of the spatial distance s, horizontal angle  $\alpha$ , vertical angle  $\beta$ , reflector height j and total station height above ground point i are given in Figure 1.

Coordinates X, Y, Z of the reference points  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$  and the total station position point S were also surveyed using GPS receivers with standard deviations shown in Figure 1.

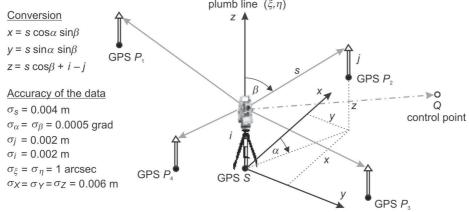


Fig. 1. Total station  $(s, \alpha, \beta)$ , GPS (X, Y, Z) and plumb line  $(\xi, \eta)$  data

### 2.2. Computation of external orientation parameters of the total station

Knowledge of six external orientation parameters  $\Sigma$ ,  $\xi$ ,  $\eta$ ,  $X_S$ ,  $Y_S$ ,  $Z_S$  of total station or laser scanner is needed for conversion of coordinates from the measuring frame (x, y, z) to the external reference frame (X, Y, Z) (e.g. Osada, 2002)

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_S \\ Y_S \\ Z_S \end{pmatrix} + \left[ \mathbf{R} (\Sigma) \cdot \mathbf{Q} (\xi, \eta, \varphi_S) \cdot \mathbf{P} (\varphi_S, \lambda_S) \right]^{\mathrm{T}} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$
(1)

where P, Q, R are rotation matrixes in three dimensional space

$$\mathbf{P}(\varphi_S, \lambda_S) = \begin{pmatrix} -\sin(\varphi_S) \cdot \cos(\lambda_S) & -\sin(\varphi_S) \cdot \sin(\lambda_S) & \cos(\varphi_S) \\ -\sin(\lambda_S) & \cos(\lambda_S) & 0 \\ \cos(\varphi_S) \cdot \cos(\lambda_S) & \cos(\varphi_S) \cdot \sin(\lambda_S) & \sin(\varphi_S) \end{pmatrix}$$
(2)

$$\mathbf{Q}(\xi, \eta, \varphi_S) = \begin{pmatrix} 1 & -\eta \cdot \tan(\varphi_S) & -\xi \\ \eta \cdot \tan(\varphi_S) & 1 & -\eta \\ \xi & \eta & 1 \end{pmatrix}$$
(3)

$$\mathbf{R}(\Sigma) = \begin{pmatrix} \cos(\Sigma) & \sin(\Sigma) & 0 \\ -\sin(\Sigma) & \cos(\Sigma) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
(4)

Approximate values of northern  $\xi$ , and eastern  $\eta$  components of deflection of the vertical at position S of the total station (Fig. 2) can be evaluated from the geoid model

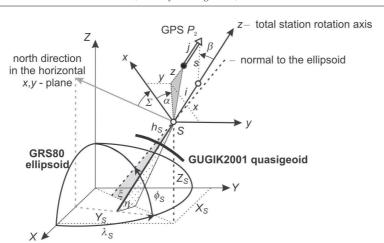


Fig. 2. Six parameters of the total station external orientation:  $\Sigma$ ,  $\xi$ ,  $\eta$ ,  $X_S$ ,  $Y_S$ ,  $Z_S$ 

GUGIK2001 for Poland using the *geoid PL GUGIK2001* computer program (written by E. Osada) (Fig. 3)

$$\xi = 6.01'', \sigma_{\xi} = 1'' \text{ and } \eta = 6.80'', \sigma_{\eta} = 1''$$
 (5)

The height of the point *S* is only about 120 m above geoid. For the biggest heights one should add corrections due to curvature of the plumb line (e.g. Osada, 2002).

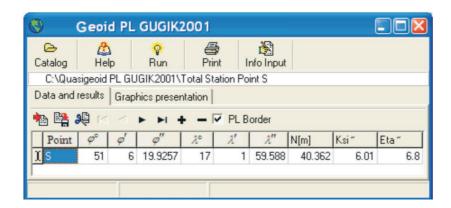


Fig. 3. The geoid PL GUGIK2001 computer program

Approximate value of the total station orientation constant  $\Sigma$  can be obtained by solving the set of transformation equations (1) for the surveyed GPS points. Using the well known Levenberg–Marquardt method of conjugate gradients (Nocedal and Wright, 2006) one obtains  $\Sigma$  386.0907 grad.



### 3. The total station spatial 3D adjustment algorithm

In the examined task of the spatial adjustments of the total station observations in relation to the GPS points and normal plumb line direction, the Gauss-Markov observational model

$$\mathbf{v} = \mathbf{A}\mathbf{x} - \mathbf{l}; \quad \mathbf{P} = \mathbf{C}_{\mathbf{l}}^{-1} \tag{6}$$

is determined on the basis of equations:

measured spatial distances s, horizontal  $\alpha$  and vertical  $\beta$  angles (Fig. 1 and Fig. 2)

$$\begin{pmatrix} v_{s} \\ v_{\alpha} \\ v_{\beta} \end{pmatrix} = \begin{pmatrix} \cos(\alpha) \cdot \sin(\beta) & \sin(\alpha) \cdot \sin(\beta) & \cos(\beta) \\ -\frac{\sin(\alpha)}{s \cdot \sin(\beta)} & \frac{\cos(\alpha)}{s \cdot \sin(\beta)} & 0 \\ \frac{\cos(\alpha) \cdot \cos(\beta)}{s} & \frac{\sin(\alpha) \cdot \cos(\beta)}{s} & -\frac{\sin(\beta)}{s} \end{pmatrix} \cdot \begin{pmatrix} v_{x} \\ v_{y} \\ v_{z} \end{pmatrix}$$
 (7)

where

$$v_{x} = (-\cos(\Sigma) \cdot \sin(\varphi_{s}) \cdot \cos(\lambda_{s}) - \sin(\Sigma) \cdot \sin(\lambda_{s})) \cdot (dX - dX_{s})$$

$$+ (-\cos(\Sigma) \cdot \sin(\varphi_{s}) \cdot \sin(\lambda_{s}) + \sin(\Sigma) \cdot \cos(\lambda_{s})) \cdot (dY - dY_{s})$$

$$+ (\cos(\Sigma) \cdot \cos(\varphi_{s})) \cdot (dZ - dZ_{s}) - \cos(\Sigma) \cdot z_{g} \cdot (\xi + d\xi)$$

$$+ \left(\sin(\Sigma) \cdot \tan(\varphi_{s}) \cdot x_{g} - \cos(\Sigma) \cdot \tan(\varphi_{s}) \cdot y_{g} - \sin(\Sigma) \cdot z_{g}\right) \cdot (\eta + d\eta)$$

$$+ \left(-\sin(\Sigma) \cdot x_{g} + \cos(\Sigma) \cdot y_{g}\right) \cdot d\Sigma + \cos(\Sigma) \cdot x_{g} + \sin(\Sigma) \cdot y_{g} - x$$

$$(8)$$

$$v_{y} = (\sin(\Sigma) \cdot \sin(\varphi_{s}) \cdot \cos(\lambda_{s}) - \cos(\Sigma) \cdot \sin(\lambda_{s})) \cdot (dX - dX_{s})$$

$$+ (\sin(\Sigma) \cdot \sin(\varphi_{s}) \cdot \sin(\lambda_{s}) + \cos(\Sigma) \cdot \cos(\lambda_{s})) \cdot (dY - dY_{s})$$

$$+ (-\sin(\Sigma) \cdot \cos(\varphi_{s})) \cdot (dZ - dZ_{s}) + \sin(\Sigma) \cdot z_{g} \cdot (\xi + d\xi)$$

$$+ (\cos(\Sigma) \cdot \tan(\varphi_{s}) \cdot x_{g} + \sin(\Sigma) \cdot \tan(\varphi_{s}) \cdot y_{g} - \cos(\Sigma) \cdot z_{g}) \cdot (\eta + d\eta)$$

$$+ (-\cos(\Sigma) \cdot x_{g} - \sin(\Sigma) \cdot y_{g}) \cdot d\Sigma - \sin(\Sigma) \cdot x_{g} + \cos(\Sigma) \cdot y_{g} - y$$

$$(9)$$

$$v_z = \cos(\varphi_s) \cdot \cos(\lambda_s) \cdot (dX - dX_s) + \cos(\varphi_s) \cdot \sin(\lambda_s) \cdot (dY - dY_s) + \sin(\varphi_s) \cdot (dZ - dZ_s) + x_g \cdot (\xi + d\xi) + y_g \cdot (\eta + d\eta) + z_g - z$$
(10)

and

$$\begin{pmatrix} x_g \\ y_g \\ z_g \end{pmatrix} = \begin{pmatrix} -\sin(\varphi_S) \cdot \cos(\lambda_S) & -\sin(\varphi_S) \cdot \sin(\lambda_S) & \cos(\varphi_S) \\ -\sin(\lambda_S) & \cos(\lambda_S) & 0 \\ \cos(\varphi_S) \cdot \cos(\lambda_S) & \cos(\varphi_S) \cdot \sin(\lambda_S) & \sin(\varphi_S) \end{pmatrix} \cdot \begin{pmatrix} X - X_S \\ Y - Y_S \\ Z - Z_S \end{pmatrix}$$
(11)

measured coordinates X, Y, Z of the GPS points (Fig. 1 and Fig. 2):

$$v_X = dX$$

$$v_Y = dY$$

$$v_Z = dZ$$
(12)

components  $\xi$ ,  $\eta$  of the deflection of the vertical (5) obtained from geoid model (Fig. 2 and Fig. 3)

$$v_{\xi} = d\xi$$

$$v_{\eta} = d\eta$$
(13)

The observational equations of x, y, z coordinates in the total station coordinate system (6)-(8) are obtained on the basis of differential transformation equations  $(X, Y, Z) \rightarrow (x, y, z)$  inverse to (1) (e.g. Osada, 2002). The observational equations of directly measured values s,  $\alpha \beta$  as functions of residuals  $v_x$ ,  $v_y$ ,  $v_z$  (5) are obtained on the basis of differential transformation

$$\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cos(\alpha) \cdot \sin(\beta) & -s \cdot \sin(\alpha) \cdot \sin(\beta) & s \cdot \cos(\alpha) \cdot \cos(\beta) \\ \sin(\alpha) \cdot \sin(\beta) & s \cdot \cos(\alpha) \cdot \sin(\beta) & s \cdot \sin(\alpha) \cdot \cos(\beta) \\ \cos(\beta) & 0 & -s \cdot \sin(\beta) \end{pmatrix} \cdot \begin{pmatrix} ds \\ d\alpha \\ d\beta \end{pmatrix}$$
(14)

respectively for the known conversion  $(s, \alpha, \beta) \rightarrow (x, y, z)$  (Fig. 1).

The Gauss-Markov observational model (6), for the complete experiment data (Fig. 1) is defined as follows:

x – vector of unknown corrections:

$$d\xi, d\eta, d\Sigma, dX_1, dY_1, dZ_1, ..., dX_4, dY_4, dZ_4, dX_5, dY_5, dZ_5;$$

v – vector of unknown observational error residuals:

$$v_{s_1}, v_{\alpha_1}, v_{\beta_1}, \ ..., v_{s_4}, v_{\alpha_4}, v_{\beta_4}, v_{X_1}, v_{Y_1}, v_{Z_1}, \ ..., v_{X_4}, v_{Y_4}, v_{Z_4}, v_{X_S}, v_{Y_S}, v_{Z_S}, v_{X_4}, v_{\xi}, v_{\eta};$$

 $\mathbf{A}$  – known  $n \times k$  design matrix;

**I** – known observational vector:  $l_{s_1}, l_{\alpha_1}, l_{\beta_1}, ..., l_{s_4}, l_{\alpha_4}, l_{\beta_1}, 0, ..., 0$ ;

 $\mathbf{C}_l$  – covariance matrix of observations, composed of standard deviations (Fig. 1):

$$\sigma_{s_1}, \sigma_{\alpha_1}, \sigma_{\beta_1}, \dots, \sigma_{s_4}, \sigma_{\alpha_4}, \sigma_{\beta_4}, \sigma_{X_1}, \sigma_{Y_1}, \sigma_{Z_1}, \dots, \sigma_{X_4}, \sigma_{Y_4}, \sigma_{Z_4}, \sigma_{X_5}, \sigma_{Y_5}, \sigma_{Z_5}, \sigma_{\xi}, \sigma_{\eta}$$

The solution of the Gauss-Markov model by least squares method is given by

$$\mathbf{x} = (\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{I} \tag{15}$$

with covariance matrix  $C_x = (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1}$  and testing parameter

$$m_0 = \sqrt{\frac{\mathbf{v}^{\mathrm{T}} \cdot \mathbf{P} \cdot \mathbf{v}}{n - k}} \cong 1 \tag{16}$$



It is well known that if the result of adjustment is not positive  $(m_0 \cong 1)$  due to the low precision or movements of reference GPS points:  $|v_X| > \sigma_X$ ,  $|v_Y| > \sigma_Y$ ,  $|v_Z| > \sigma_Z$ , k = 2, 3 then the calculations can be continued iteratively. The weights p of coordinates with large corrections v that were obtained in a previous step are corrected in every step of the iteration process  $p \leftarrow pf(v)$ , where f(v) is a damping function, for example *Huber function* (Walter and Pronzato, 1997):

$$f(v) = \begin{cases} 1 & |v| \le a \\ \frac{a}{|v|} & |v| > a \end{cases}$$
 (17)

where a is a parameter determined empirically.

In this case the experimental data were measured with high accuracy (Fig. 1) and the problem with an outlying data does not occur.

### 4. The numerical experiments

Data observed with the total station were adjusted according to the algorithm presented in section 3, in 11 experiments I - XI:

In the numerical experiment I (Table 1) all possible directional GPS points  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$  (Fig. 1) were taken to the adjustment in combination with 4 modes: known or unknown  $X_S$ ,  $Y_S$ ,  $Z_S$  coordinates of total station point S and known or unknown  $\xi$ ,  $\eta$  components of the deflection of the vertical.

In the numerical experiments II, III, IV, V (Table 2) 4 combinations of 3 out of four GPS directional points  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$  were taken to the adjustment in combination with 4 above mentioned modes concerning knowledge of  $X_S$ ,  $Y_S$ ,  $Z_S$  coordinates of total station point S as well as  $\xi$ ,  $\eta$  components of the deflection of the vertical.

In the numerical experiments VI, VII, VIII, IX, X, XI (Table 3) 6 combinations of 2 out of four GPS directional points  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$  with assumed known  $X_S$ ,  $Y_S$ ,  $Z_S$  coordinates of total station point S were taken to adjustment in combination with 2 modes: known or unknown  $\xi$ ,  $\eta$  components of the deflection of the vertical.

The adjusted  $\xi$ ,  $\eta$  components of the deflection of the vertical at the total station point S and the horizontal HD and vertical VD displacements of the control point Q (Fig. 1) are given in Tables 1, 2, and 3.

The results of numerical experiments performed are as follows:

- 1) adjusted corrections v to the observations in all 11 experiments are smaller than their double standard deviations  $\sigma$ , i.e.  $|v_i| \le 2\sigma_i$  where i = s,  $\alpha$ ,  $\beta$ , X, Y, Z,  $\xi$ ,  $\eta$ ;
- 2) in all experiments with assumed unknown GPS coordinates  $X_{S.GPS}$ ,  $Y_{S.GPS}$ ,  $Z_{S.GPS}$  of the total station point S the differences between the adjusted coordinates  $X_S$ ,  $Y_S$ ,  $Z_S$  and the respective ones obtained from GPS measurements are smaller than their double standard deviations:  $|X_S X_{S.GPS}| < 2\sigma_X$ ,  $|Y_S Y_{S.GPS}| < 2\sigma_Y$ ,  $|Z_S Z_{S.GPS}| < 2\sigma_Z$ ;
- 3) in the majority of experiments with assumed unknown components of the deflection of the vertical the differences between the computed  $\xi$ ,  $\eta$  components and the



Table 1. Numerical experiments: I

I: P <sub>1</sub> P <sub>2</sub> P <sub>3</sub> P <sub>4</sub>										
$P_1$ $P_2$ $P_3$										
coordinates of GPS and total station point S assumed known										
	yes	no								
$\xi, \eta$ assumed known from geoid model										
	yes no yes no									
adjustment results: $\xi$ , $\eta$ at $S$ , and $HD$ , $VD$ at $Q$										
ξ [arcsec]	6.1	2.9	6.1	2.9						
η [arcsec]	6.7	5.7	6.7	5.7						
HD [mm]	0.0	0.7	2.3	2.9						
VD [mm]	0.0	3.1	0.5	3.2						

Table 2. Numerical experiments: II, III, IV, V

	II:	$P_1$	$P_2$	$P_3$	III	$: P_1$	$P_2$	$P_4$	IV:	$P_2$	$P_3$	$P_4$	V:	$P_3$	$P_4$	$P_1$
	P	1	$P_2$	$P_3$	•	$P_1$	P <sub>2</sub>		$P_2$ $P_3$ $P_4$			S P <sub>3</sub>				
coordinates of GPS and total station point S assumed known																
	yes	yes	no	no	yes	yes	no	no	yes	yes	no	no	yes	yes	no	no
$\xi,\eta$ assumed known from geoid model																
	yes	no	yes	no	yes	no	yes	no	yes	no	yes	no	yes	no	yes	no
adjustment results: $\xi$ , $\eta$ at $S$ , and $HD$ , $VD$ at $Q$																
$\xi$ [arcsec]	6.1	5.6	6.2	6.6	6.0	1.7	6.0	0.9	6.1	3.3	6.1	3.3	6.1	-1.1	6.1	-2.2
$\eta$ [arcsec]	6.8	6.7	6.8	7.0	6.8	8.8	6.8	10.8	6.7	4.1	6.7	4.0	6.7	5.2	6.7	5.1
HD [mm]	0.7	0.6	1.3	1.2	3.9	3.0	2.9	3.7	4.1	1.2	4.1	4.2	1.7	2.1	2.9	4.5
VD [mm]	1.5	1.7	2.1	1.9	-0.1	-1.2	-0.1	-4.6	-0.1	3.6	-0.1	3.4	0.0	6.7	0.0	8.4

respective ones obtained from geoid model  $\xi_{geoid}$ = 6.0",  $\eta_{geoid}$  = 6.8" are much bigger than triple standard deviations of the components of the deflection of the vertical, i.e.  $|-\xi_{geoid}| > 3\sigma_{\xi}$ ,  $|\eta - \eta_{geoid}| > 3\sigma_{\eta}$ , (Tables 1-3);

- 4) in the majority of experiments with assumed known components of the deflection of the vertical  $\xi_{geoid}$ ,  $\eta_{geoid}$  the vertical displacements VD of the control point Q are significantly smaller than in the respective experiments with unknown deflection of the vertical;
- 5) in the numerical experiment X very good results were obtained with assumed known components of the deflection of the vertical  $\xi_{geoid}$ ,  $\eta_{geoid}$  but on the other hand very bad results with unknown components of the deflection of the vertical;



Table 3. Numerical experiments: VI, VII, VIII, IX, X, XI

	VI:	TI: $P_1 P_2 S$ VII: $P_2$		$P_2 P_3 S$	VIII:	$P_3 P_4 S$	IX: <i>P</i> <sub>4</sub> <i>P</i> <sub>1</sub> <i>S</i>		X:	$P_1 P_3 S$	XI: <i>P</i> <sub>2</sub> <i>P</i> <sub>4</sub> <i>S</i>		
	P <sub>1</sub>	$P_2$ $P_2$ $P_3$			$P_4$ $S$ $P_3$		$P_1$ $P_4$ $S$		P <sub>1</sub> S P <sub>3</sub>		P <sub>2</sub> /S		
	$\xi,\eta$ assumed known from geoid model												
	yes yes yes yes		yes	no	yes no		yes	no	yes	no			
adjustment results: $\xi$ , $\eta$ at $S$ , and $HD$ , $VD$ at $Q$													
$\xi$ [arcsec]	6.1	4.1	6.1	4.3	6.1	1.8	6.1	-0.9	6.2	-94.0	6.1	4.6	
$\eta$ [arcsec]	6.8	8.7	6.8	4.9	6.7	4.3	6.8	7.6	6.8	-15.0	6.8	0.5	
HD [mm]	3.3	2.8	2.5	2.1	0.3	1.1	15.2	14.0	1.9	19.3	3.2	3.6	
VD [mm]	1.2	-1.0	1.2	2.9	-0.7	4.8	-0.7	2.7	1.3	76.3	-0.8	7.6	

in those cases the vertical displacements of the control point Q was estimated as VD = 1.3 mm and VD = 76.3 mm, respectively (Table 3).

#### 5. Conclusions

The results of numerical experiments performed show that the presented algorithm makes possible to obtain more accurate results of the adjustment of the total station observations with assumed known components  $\xi_{geoid}$ ,  $\eta_{geoid}$  of the deflection of the vertical, taken from GUGIK2001 geoid model as compared to the adjustment with unknown components of the deflection of the vertical when vertical displacements of the surveyed points can reach up to 10 cm.

The method of improvement of the total station 3D adjustment by using geoid model GUGIK2001 for Poland, proposed in the article can thus be applied to obtain a precise metric 3D model of terrestrial objects in the geocentric coordinate system GRS80.

The quality of available geoid model is critical to ensure accuracy improvement by using geoid model according to the method proposed. The accuracy of components of the deflection of the vertical calculated from that model on the level 0.5 arcsec is required.

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# Podniesienie dokładności wyrównania przestrzennego stanowiska tachimetru przy zastosowaniu precyzyjnego modelu geoidy

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#### Streszczenie

W artykule jest zaproponowana metoda podniesienia dokładności wyrównania przestrzennego stanowiska tachimetru nawiązanego do punktów GPS przy zastosowaniu składowych odchylenia pionu, pozyskanych z precyzyjnego modelu geoidy.

Na podstawie eksperymentalnych pomiarów stwierdzono, że otrzymuje się znacznie bardziej dokładne wyniki wyrównania z uwzględnieniem odchylenia pionu w porównaniu do wyników wyrównania z nieznanym odchyleniem pionu. W drugim przypadku można otrzymać wyniki o małej dokładności, z pionowymi przemieszczeniami mierzonych punktów sięgającymi nawet 10 cm.

Zaproponowana metoda ta może być stosowana do tworzenia precyzyjnych metrycznych 3D modeli naziemnych obiektów w geocentrycznym układzie odniesienia GRS80.