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VARIABILITY OF EXPLOITATION COEFFICIENT OF KNOTHE THEORY IN RELATION TO ROCK MASS STRATA TYPE

ZMIENNOŚĆ WSPÓŁCZYNNIKA EKSPLOATACJI TEORII S. KNOTHEGO W ZALEŻNOŚCI OD RODZAJU WARSTW SKALNYCH W GÓROTWORZE

Geodesic measurements of mining area deformations indicate that their description fails to be regular, as opposed to what the predictions based on the relationships of the geometric-integral theory suggest. The Knothe theory, most commonly applied in that case, considers such parameters as the exploitation coefficient a and the angle of the main influences range tgβ, describing the geomechanical properties of the medium, as well as the mining conditions. The study shows that the values of the parameters $a = 0.8$ and tg$\beta = 2.0$, most commonly adopted for the prediction of surface deformation, are not entirely adequate in describing each and every mining situation in the analysed rock mass. Therefore, the paper aims to propose methodology for determining the value of exploitation coefficient a, which allows to predict the values of surface subsidence caused by underground coal mining with roof caving, depending on geological and mining conditions. The characteristics of the analysed areas show that the following factors affect surface subsidence: thickness of overburden, type of overburden strata, type of Carboniferous strata, rock mass disturbance and depth of exploitation. These factors may allow to determine the exploitation coefficient a, used in the Knothe theory for surface deformation prediction.

Keywords: underground coal mining with roof caving, surface deformations, subsidence, rock mass type

Pomiary geodezyjne deformacji terenu górniczego wskazują, że ich opis nie jest regularny jak to wynika z prognoz wykonywanych przy użyciu zależności z teorii geometryczno-całkowych. Szeroko stosowana w tym zakresie teoria S. Knothego uwzględnia parametry takie jak współczynnik eksploatacji a oraz kąt zasięgu wpływów głównych tgβ charakteryzujące własności geomechaniczne ośrodka i warunki eksploatacji górniczej. Z wykonanych badań wynika, wartości parametrów $a = 0.8$ i tg$\beta = 2.0$ najczęściej przyjmowane do prognozy deformacji powierzchni terenu nie są w pełni adekwatne do opisu każdej sytuacji górniczej w górotworze. W artykule przedstawiono schemat postępowania konieczny dla określenia wartości współczynnika eksploatacji a umożliwiającego predykcję wartości obniżeń powierzchni terenu

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w zależności od warunków geologiczno-górniczych podczas prowadzenia eksploatacji pokładów węgla z zawałem stropu. Z charakterystyki analizowanych rejonów wynika, że czynnikami wpływającymi na obniżenia powierzchni terenu są: miąższość nadkładu, rodzaj warstw nadkładu, rodzaj warstw karbonu, zruszenie górotworu i głębokość eksploatacji. Wyróżnione czynniki umożliwiają opisanie współczynnika eksploatacji *a* stosowanego w teorii Knothego do prognozowania deformacji powierzchni terenu.

Słowa kluczowe: eksploatacja węgla z zawałem, deformacje powierzchni, obniżenia, rodzaj warstw skalnych

1. Introduction

The prediction of mining-induced surface deformation in Upper Silesian Coal Basin is commonly made with the use of the Knothe theory, describing the course of deformation occurring in the terrain surface. The theory considers such parameters as the exploitation coefficient a and the angle of the main influences range tgβ. The theory parameters embrace the numerical values occurring in the formulae of the rock mass movement theory, characterising the geomechanical properties of the medium and the conditions of mining exploitation (Zych, 1985). The most commonly adopted values of the parameters for the surface deformation prediction are $a = 0.8$ and tg$\beta = 2.0$ (Knothe, 1984). However, some research studies prove that such values are not fully adequate in describing the entire spectrum of mining situations occurring in the rock mass (Białek & Mierzejowska, 2011; Hu et al., 2015; Kruczkowski, 2011; Majcherczyk et al., 2012a; Majcherczyk et al., 2012b; Majcherczyk & Kryzia, 2013; Polanin, 2015). Therefore, it seems crucial to determine the factors affecting the parameters. While analysing the correlation between the results obtained with the use of prediction method and the results of post-mining observation, it may be argued that the values of the exploitation coefficient a and the angle of the main influences range tgβ depend upon the factors related to geological and mining conditions.

The determination of the interdependence between the geological and mining factors, affecting the values of theory parameters still remains unsolved in a definite way, particularly in relation to multi-layer mining in the disturbed rock mass (Hu et al., 2015; Suchowerska et al., 2016). Hence, it seems crucial to determine the relationships of the theory parameters with one or more factors, e.g. overburden thickness of younger strata, number of hitherto extracted seams, or depth of exploitation. The paper presents the methodology for determining the value of exploitation coefficient, which facilitates the prediction of the values of surface subsidence, depending on particular geological and mining conditions during the operation of underground coal mining with roof caving.

2. Areas subject to analysis

Coal mining in the Upper Silesian Coal Basin (USCB) is carried out in several areas. Currently, the highest concentration of mining activity occurs in the areas referred to as Bytomski, Rybnicki, Jastrzębski, Nadwiślański and Katowicki. Each of the areas possesses specific mining and geological conditions affecting the process of rock mass and surface deformation (Jureczka et al., 2005; Doležalová et al., 2009; Jiránková et al., 2013).

In order to determine the relationships describing the influence of rock mass type on the values of surface subsidence caused by underground mining with roof caving, an extensive analysis of coal-mine research studies has been made (Kryzia, 2017). The analysed cases embrace seven various areas of the Upper Silesian Coal Basin, representing a complete geological and mining situation of the entire area of the basin (Fig. 1).

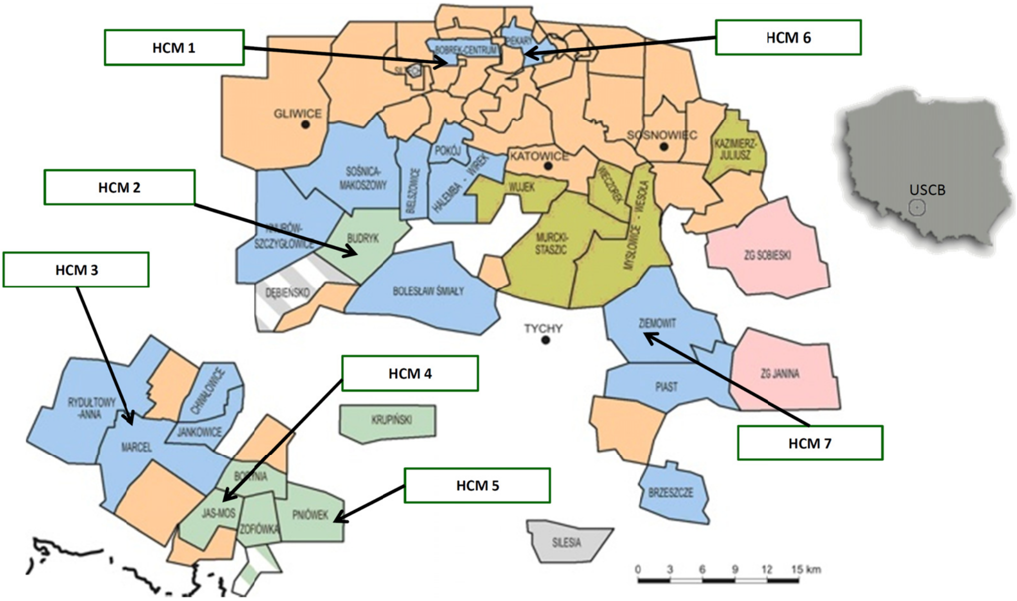

Fig. 1. Observation areas subject to analysis (HCM – Hard Coal Mine)

Among the selected cases, the rock mass consists of both weak and fractured strata, as well as strong and compact rocks. In the analysed areas, underground mining operation was executed at various depths and the overburden of loose rocks is diverse (Fig. 2).

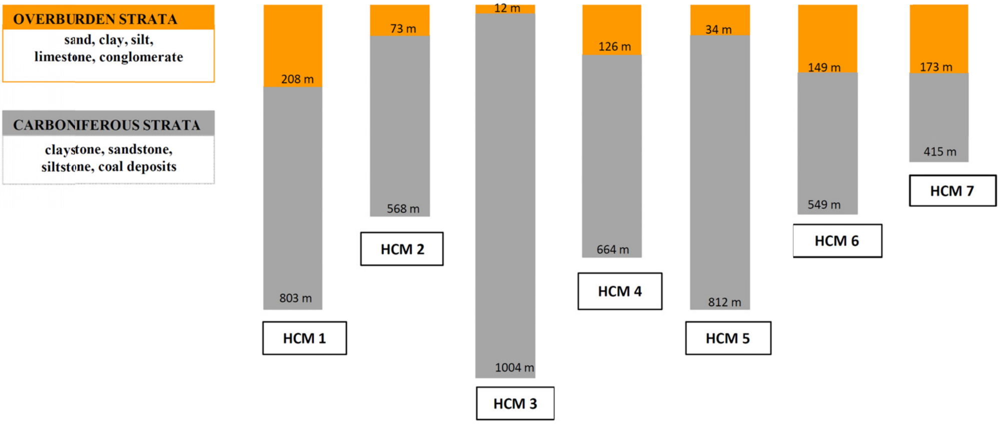

Fig. 2. Overburden and Carboniferous strata structure in the areas of observation

The analysis of numerous geological profiles in the research areas shows that sandstone, claystone or sandy claystone tend to prevail in the rock mass. Coal layers constitute only 2-7 per cent (Fig. 3).

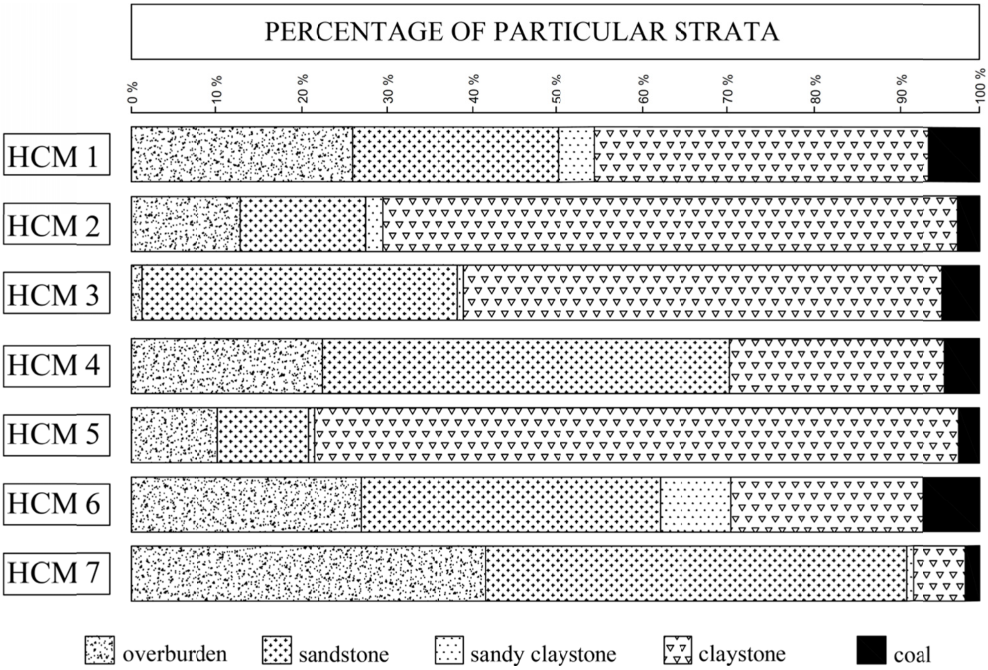

Fig. 3. Percentage of particular strata

3. Determining the values of the theory's parameters

The accuracy of mining-induced surface deformation predictions, carried out with the use of the Knothe theory, particularly depends on the validity of the theoretical model's parameter values, assumed for those predictions, i.e. exploitation coefficient a, rock mass parameter tgβ and proportionality coefficient B. The valid values of the parameters are the values determined on the basis of geodesic observations of the surface above mining exploitation, where the geological and mining conditions are most compatible with the designed exploitation. The parameter values determined on the basis of the analysis of the subsidence profile are used to calculate deformation coefficients (Ostrowski & Malinowska, 2006).

Nowadays, due to multi-layer mining at increasing depths, the obtained results fail to conceive a complete picture of subsidence troughs, which means that the parameter values can be only sporadically determined in a traditional way, by resorting directly to the Knothe theory. In the case of a multi-layer exploitation with a complex geometry, executed at large depths, and with the incomplete observed subsidence troughs, the determination of the impact theory parameters is possible owing to the application of appropriate software (Białek, 2003; Ghabraie et al., 2017; Guo et al., 2014; Mielimąka, 2009).

The determination of the parameter values of the prediction theory (*a*, tgβ) was carried out for the asymptomatic state of deformation, occurring after halting the seam extraction, with the TGB1 software, on the basis of matching the theoretical trough with the trough obtained from the geodesic measurements (Białek, 2003; Mielimąka, 2009). In order to make the comparison analyses for various geological and mining conditions possible, the degree of matching the theoretical results of mining area subsidence with the profile obtained from the geodesic measurements was established. It was accomplished by determining the correlation coefficients (linear, Pearson's *r*), standard deviation σ_D, as well as the so-called variation coefficient M_w (Popiołek, 2009; Białek & Mierzejowska, 2011, 2012). The obtained results are presented in Fig. 4.

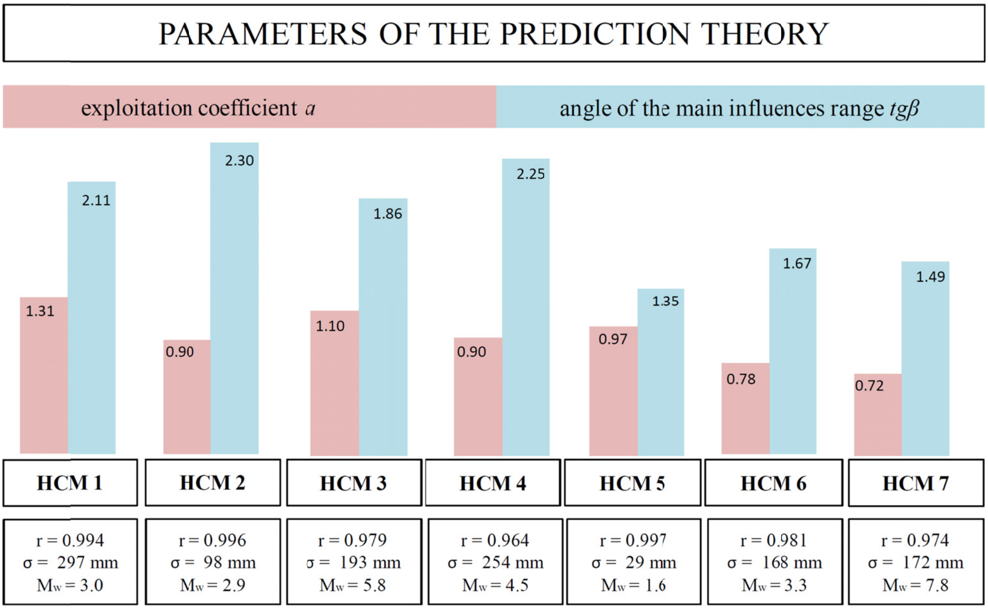

Fig. 4. Parameter values of the calculation model, standard deviation and correlation coefficient

Among the determined values of the parameters *a*, tgβ, affecting the distribution, shape and volume of the predicted subsidence trough, two values of exploitation coefficient *a* above draw a particular attention. According to the existing mining situation in the areas of the Hard Coal Mine 1 (HCM 1) and HCM 3, such high values of the parameter are seriously affected by the rock mass disturbance with an earlier exploitation.

The correlation coefficients describing the link between the calculated and the measured subsidence in particular areas of Upper Silesian Coal Basin were determined in the range from 0.964 (for HCM 4) to 0.997 (for HCM 5). Hence, there is a very strong correlation between the two types of subsidence: the measured ones and the ones obtained from the prediction. The determined standard deviation for the particular subsidence indicates that it ranges between 29.2 mm and 296.7 mm and is affected by the values of subsidence and number of measurement points in the observation lines. The highest value of standard deviation was obtained in the area of HCM 1 (297 mm), which is related to large values of subsidence up to 10 m. The lowest value of 29 mm was obtained for the measurement line in the area of HCM 5, which indicates a slight dispersion of subsidence in the measurement points.

In each of the analysed areas, the volume of the variability indices for subsidence M_w was determined, constituting the relative value of the random dispersion of the process for the comparison of the random dispersion in varied mining and geological conditions, at a varied scale of the analysed phenomenon. The values of the variability indices range between 1.6% and 7.8%. The values obtained from the analysis are larger than the values quoted in the studies of Popiołek (2009) and Kowalski (2007). The increase of the variability indices is likely to be caused predominantly by a multi-layer exploitation.

4. Rock mass model

In order to determine the relationship between the value of the exploitation coefficient and the type of rock strata in the rock mass, the factors useful in characterizing the rock mass in particular areas were introduced. For this purpose, a special classification was utilized (Kryzia, 2017), presented in the form of a scheme (Fig. 5).

In matching particular areas to an adequate type of the rock mass (strong, average and weak), the following factors were assumed: volume and type of overburden, thickness and type of particular Carboniferous strata, number of layers and their situation in the rock mass. In addition, the degree of disturbance by earlier mining was characterised for a better rock mass description. The depth of mining was also taken into consideration. The analysis of characteristic properties of the rock mass in all areas subject to the study is presented in the form of a table (Table 1).

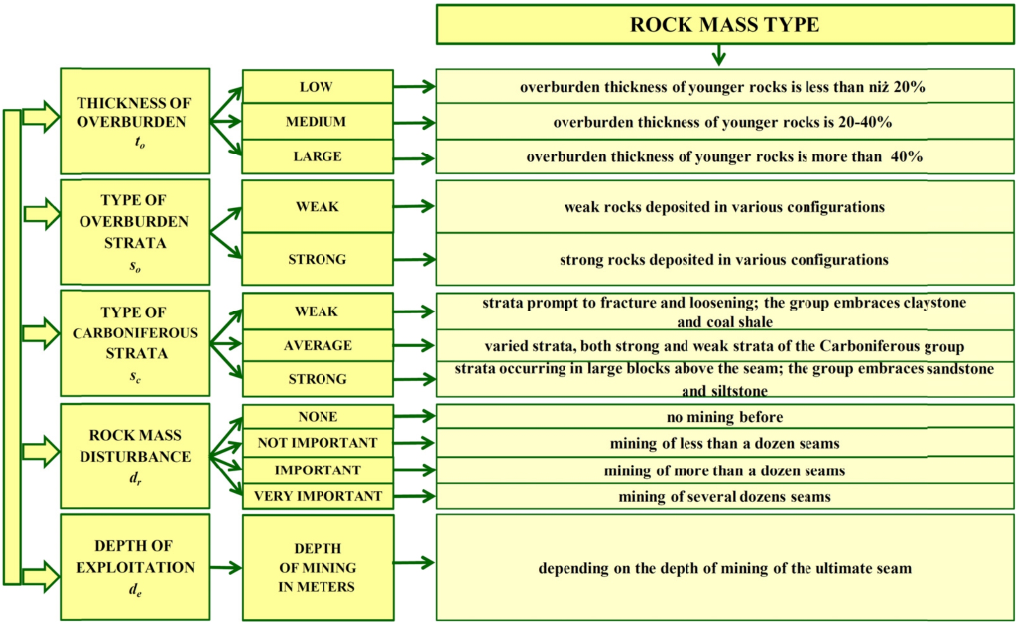

Fig. 5. Factors describing the rock mass (Majcherczyk & Kryzia, 2017)

The following factors have been referred to in the study on the analysed rock mass models: *thickness of overburden* (t_o), *type of overburden strata* (s_o), *type of Carboniferous strata* (s_c), *rock mass disturbance* (d_r) and *depth of exploitation* (d_e).

TABLE 1

Comparison of geological and mining factors in the analysed areas

Factor / Area	Thickness of overburden t_o	Type of overburden strata s_o	Type of Carboniferous strata s_c	Rock mass disturbance d_r	Depth of exploitation d_e, m	Exploitation coefficient a	Rock mass type
HCM 1	medium	strong	weak	very important	803	1.31	weak
HCM 2	low	weak	weak	important	568	0.90	weak
HCM 3	low	weak	average	very important	1004	1.10	weak
HCM 4	low	weak	strong	very important	664	0.90	average
HCM 5	low	weak	weak	important	812	0.97	weak
HCM 6	medium	strong	average	not important	549	0.78	average
HCM 7	large	weak	strong	none	415	0.72	strong

It has been assumed that the above-mentioned factors describing the rock mass type seriously affect the hitherto occurring values of surface subsidence and, in the case of the predicted values of surface subsidence, also the parameters of the prediction theory, including the exploitation coefficient *a*.

5. Determination of exploitation coefficient value related to rock mass type

The estimation of the influence of factors of the assumed rock mass model on the values of the exploitation coefficient *a* has been carried out with the use of a traditional linear regression analysis, where the method of least squares was an estimator.

In order to characterize the exploitation coefficient *a*, the same factors have been used as the ones mentioned in rock mass description. In order to carry out the regression analysis, numerical values have been assigned to particular factors characterising the exploitation coefficient and, at the same time, describing the rock mass in a qualitative way.

The assumed numerical values for particular factors depend upon the impact on the value of subsidence. For instance, for the thickness of overburden: the higher the numerical value, the larger the influence on the value of subsidence.

The pattern scheme for the value determination of particular factors characterising the type and structure of the rock mass is presented in Fig. 6.

After the analysis of various forms of regression equations describing the response variables, the form of a power equation was selected. This can be used for describing the relationships of

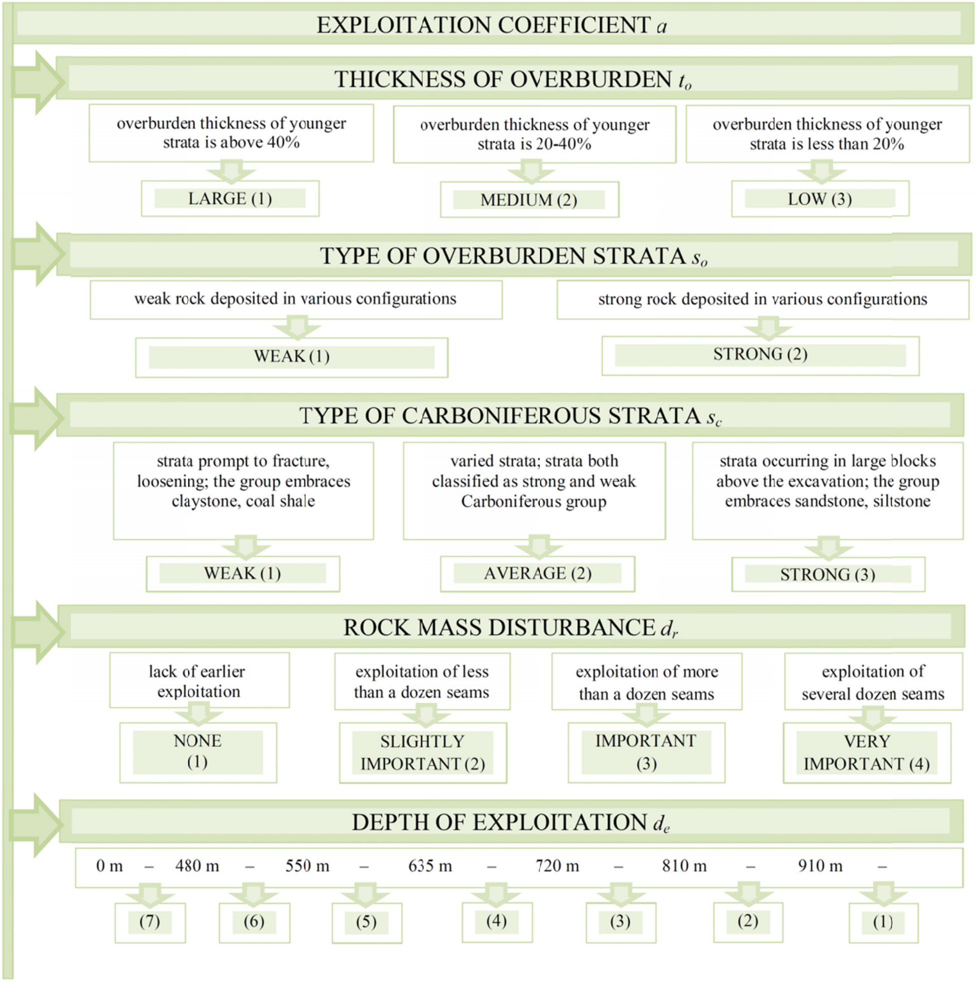

Fig. 6. Pattern scheme for the determination of the point values of particular factors characterising the rock mass for the estimation of the exploitation coefficient value

variables. The power equation in the form of the product X_i^{ai} was utilised to determine the multipliers describing the factors characterising the rock mass.

Due to its mathematical and analytical properties, the power equation is frequently used to analyse the correlations and interdependence between variables (Costa et al., 2015; Wang et al., 2015).

What is characteristic for the power equation is its constant function elasticity in relation to independent variables, which is equal to the power attributed to a given independent variable. The elasticity of power equation expresses the relative (per cent) change of response variable value evoked by a single relative change of the independent variable (Płudowski, 1980).

After attributing the point values to the particular rock mass factors, the database presented in Table 2 was obtained, on the basis of which the unknown values of structural parameters α_i

of linear regression equations have been estimated. The equations, for each of the areas, can be presented in the following form:

$$a = t_o^{\alpha_{to}} \cdot s_o^{\alpha_{so}} \cdot s_c^{\alpha_{sc}} \cdot d_r^{\alpha_{dr}} \cdot d_e^{\alpha_{de}} \tag{1}$$

where:

a — exploitation coefficient, [-]
t_o — thickness of overburden, [-],
α_{t_o} — structural parameter of overburden thickness, [-],
S_o — type of overburden strata, [-],
α_{s_o} — structural parameter of overburden strata type, [-],
S_c — type of Carboniferous strata, [-],
α_{s_c} — structural parameter of Carboniferous strata type, [-],
d_r — rock mass disturbance, [-],
α_{d_r} — structural parameter of rock mass disturbance, [-],
d_e — depth of exploitation, [-],
α_{d_e} — structural parameter of exploitation depth, [-].

TABLE 2

Regression equations describing the rock mass state, i.e. values of independent variables and exploitation coefficient *a* (response variable) in the analysed areas of Upper Silesian Coal Basin

Area		**Linear regression equations**
HCM 1	→	$a_1 = 2^{\alpha_{to}} \cdot 2^{\alpha_{so}} \cdot 1^{\alpha_{sc}} \cdot 4^{\alpha_{dr}} \cdot 3^{\alpha_{de}} = 1.31$
HCM 2	→	$a_2 = 3^{\alpha_{to}} \cdot 1^{\alpha_{so}} \cdot 1^{\alpha_{sc}} \cdot 3^{\alpha_{dr}} \cdot 5^{\alpha_{de}} = 0.90$
HCM 3	→	$a_3 = 3^{\alpha_{to}} \cdot 1^{\alpha_{so}} \cdot 2^{\alpha_{sc}} \cdot 4^{\alpha_{dr}} \cdot 1^{\alpha_{de}} = 1.10$
HCM 4	→	$a_4 = 3^{\alpha_{to}} \cdot 1^{\alpha_{so}} \cdot 3^{\alpha_{sc}} \cdot 4^{\alpha_{dr}} \cdot 4^{\alpha_{de}} = 0.90$
HCM 5	→	$a_5 = 3^{\alpha_{to}} \cdot 1^{\alpha_{so}} \cdot 1^{\alpha_{sc}} \cdot 3^{\alpha_{dr}} \cdot 2^{\alpha_{de}} = 0.97$
HCM 6	→	$a_6 = 2^{\alpha_{to}} \cdot 2^{\alpha_{so}} \cdot 2^{\alpha_{sc}} \cdot 2^{\alpha_{dr}} \cdot 6^{\alpha_{de}} = 0.78$
HCM 7	→	$a_7 = 1^{\alpha_{to}} \cdot 1^{\alpha_{so}} \cdot 3^{\alpha_{sc}} \cdot 1^{\alpha_{dr}} \cdot 7^{\alpha_{de}} = 0.72$

After determining a correlation between the independent variables (t_o, s_o, s_c, d_r, d_e) and the response variable (exploitation coefficient *a*) with the attributed point values, the structure has been changed to the logarithmic form (according to Equations 2 and 3).

The response variable equation, as a product of the independent variables, has a non-linear character but it can be reduced to a linear model by means of the algorithmic treatment of both sides of the equation, e.g. with a natural algorithm.

$$\ln(a) = \ln\left(t_o^{\alpha_{to}} \cdot s_o^{\alpha_{so}} \cdot s_c^{\alpha_{sc}} \cdot d_r^{\alpha_{dr}} \cdot d_e^{\alpha_{de}}\right) \tag{2}$$

$$\ln a = \alpha_{t_o} \cdot \ln t_o + \alpha_{s_o} \cdot \ln s_o + \alpha_{s_c} \cdot \ln s_c + \alpha_{d_r} \cdot \ln d_r + \alpha_{d_e} \cdot \ln d_e \tag{3}$$

where: symbols as in formula (1).

A multiple regression analysis has been made for the formula prepared above. The obtained results indicate that the constructed equation of the linear regression of the exploitation coefficient *a* is correct as it fulfils all the requirements of a traditional linear regression analysis.

6. Analysis of results

The results obtained from the regression analysis indicate that the values of structural parameters α_i of the linear regression equation of the exploitation coefficient *a* with their standard deviation S_{bi} are as follows:

- thickness of overburden –0.465 ± 0.018,
- type of Carboniferous strata –0.136 ± 0.008,
- rock mass disturbance 0.502 ± 0.014,
- depth of exploitation 0.095 ± 0.004.

After the substitution with the determined structural parameters, the equation (1) has the form (4):

$$a = t_o^{-0,465} \cdot s_c^{-0,136} \cdot d_r^{0,502} \cdot d_e^{-0,095} \tag{4}$$

where:

a — exploitation coefficient, [-],
t_o — thickness of overburden, [-],
s_c — type of Carboniferous strata, [-],
d_r — rock mass disturbance, [-],
d_e — depth of exploitation, [-].

The value of the structural parameter of the linear regression equation related to the variable *overburden thickness* t_o may be provided with the following substantial interpretation: If the value of the variable *overburden thickness* t_o increases by 1 per cent, the value of the exploitation coefficient *a* will decrease by 0.465 per cent on average, provided that the other independent variables ((s_c, d_r, d_e) remain unchanged. The value of the structural parameter of the linear regression related to the independent variable *rock mass disturbance* d_r may be interpreted in the following way: If the value of the response variable *rock mass disturbance* increases by 1 per cent, the exploitation coefficient *a* will increase by 0.502 per cent on average, provided that the other independent variables (t_o, s_c, d_e) remain unchanged. The analogical interpretation is possible for all other structural parameters of the linear regression equation.

On the basis of the input data presented in Table 4, the values of particular terms of the regression equation have been calculated for each of the analysed areas. The factor of the regression equation is expressed as the independent variables (factors t_o, s_c, d_r, d_e) to a suitable power of the structural parameter a_i according to the formula (4). The obtained results have been presented in Table 3.

The regression analysis indicates that the value of the exploitation coefficient *a* is seriously affected by four, out of five, factors of the rock mass model (independent variables): *overburden thickness* t_o characterising the percentage of overburden strata thickness in the total thickness of roof strata, *type of Carboniferous strata* s_c, *rock mass disturbance* d_r, and *exploitation depth* d_e.

TABLE 3

Equations presenting the values of particular expressions of regression equation ($X_i^{\alpha_i}$) of exploitation coefficient *a* for the analysed areas

Area		**Linear regression equations**
HCM 1	→	$a_1 = 2^{-0,465} \cdot 1^{-0,136} \cdot 4^{0,502} \cdot 3^{-0,095} = 0.724 \cdot 1.000 \cdot 2.006 \cdot 0.901 = 1.31$
HCM 2	→	$a_2 = 3^{-0,465} \cdot 1^{-0,136} \cdot 3^{0,502} \cdot 5^{-0,095} = 0.599 \cdot 1.000 \cdot 1.736 \cdot 0.858 = 0.89$
HCM 3	→	$a_3 = 3^{-0,465} \cdot 2^{-0,136} \cdot 4^{0,502} \cdot 1^{-0,095} = 0.599 \cdot 0.910 \cdot 2.006 \cdot 1.000 = 1.09$
HCM 4	→	$a_4 = 3^{-0,465} \cdot 3^{-0,136} \cdot 4^{0,502} \cdot 4^{-0,095} = 0.599 \cdot 0.861 \cdot 2.006 \cdot 0.876 = 0.91$
HCM 5	→	$a_5 = 3^{-0,465} \cdot 1^{-0,136} \cdot 3^{0,502} \cdot 2^{-0,095} = 0.599 \cdot 1.000 \cdot 1.736 \cdot 0.936 = 0.97$
HCM 6	→	$a_6 = 2^{-0,465} \cdot 2^{-0,136} \cdot 2^{0,502} \cdot 6^{-0,095} = 0.724 \cdot 0.910 \cdot 1.416 \cdot 0.843 = 0.79$
HCM 7	→	$a_7 = 1^{-0,465} \cdot 3^{-0,136} \cdot 1^{0,502} \cdot 7^{-0,095} = 1.000 \cdot 0.861 \cdot 1.000 \cdot 0.831 = 0.72$

The presented analysis indicates that the values of structural parameters α_i were selected properly. The differences between the exploitation coefficient determined from the regression equation and the one determined with the use of TGB1 software remain in the range of ±0.01 (from –0.007 to +0.007). The value of the correlation coefficient between the exploitation coefficient determined in those two different ways is 0.9996.

Next, the significance of the influence of the *Carboniferous strata* on the value of the exploitation coefficient *a* was analysed. On the basis of the indicated values of equation expressions for the variable *Carboniferous zone* s_c, the parameters of regression equation were determined in order to check the significance of particular types of strata in the rock mass, i.e. *sandstone* (ss), *sandy claystone* (*scs*), *claystone* (*cs*) and *coal* (*c*). The values of the equation expressions for the variable *Carboniferous zone* were calculated for all the areas by assuming the value α_i equal to the confidence limit of the structural parameter of the variable *Carboniferous zone*, i.e. for the lower confidence limit $\alpha_{s_c}^{lower} = -0.1606$, and for the upper confidence limit $\alpha_{s_c}^{upper} = -0.1121$ respectively. Using these data, the values of the expressions $s_c^{\alpha_{s_c}}$ of the equation (1) were calculated. They became the values of the response variable s_c of the equation, whose general form for all the analyzed areas in both variants is as follows:

$$s_c^{\alpha_{s_c}} = \alpha_{ss} \cdot p_{ss} + \alpha_{scs} \cdot p_{scs} + \alpha_{cs} \cdot p_{cs} + \alpha_c \cdot p_c \tag{5}$$

However, for the lower confidence limits $\alpha_{s_c}^{lower}$, the maximal regression equation assumes the following form:

$$s_c^{\alpha_{s_c}^{lower}} = \alpha_{ss}^{lower} \cdot p_{ss} + \alpha_{scs}^{lower} \cdot p_{scs} + \alpha_{cs}^{lower} \cdot p_{cs} + \alpha_c^{lower} \cdot p_c \tag{6}$$

For the upper confidence limits $\alpha_{s_c}^{upper}$, the equation assumes the following form:

$$s_c^{\alpha_{s_c}^{upper}} = \alpha_{ss}^{upper} \cdot p_{ss} + \alpha_{scs}^{upper} \cdot p_{scs} + \alpha_{cs}^{upper} \cdot p_{cs} + \alpha_c^{upper} \cdot p_c \tag{7}$$

where:

$s_c^{\alpha_{s_c}}$ — response variable,

α_{ss} — structural parameter for response variable p_{ss},
p_{ss} — percentage of sandstone in *Carboniferous zone,*
α_{scs} — structural parameter for independent variable p_{scs},
p_{scs} — percentage of sandy claystone in *Carboniferous zone,*
α_{cs} — structural parameter for independent variable p_{cs},
p_{cs} — percentage of claystone in *Carboniferous zone,*
α_{c} — structural parameter for independent variable p_{c},
p_{c} — percentage of coal in *Carboniferous zone.*

The determined response variables for the equations of the lower confidence limit (the values of equation expression $s_c^{\alpha_{s_c}}$ calculated for $\alpha_{s_c}^{lower} = -0.1606$), as well as of the upper confidence limit (the values of equation expression $s_c^{\alpha_{s_c}}$ calculated for $\alpha_{s_c}^{upper} = -0.1121$) are presented in Table 4. They are indicated as $s_c^{\alpha_{s_c}^{lower}}$ and $s_c^{\alpha_{s_c}^{upper}}$ respectively. The values marked as 'basis' constitute the values of calculated expressions of the equation $s_c^{\alpha_{s_c}}$ for the Carboniferous zone for the value α_{s_c} of –0.1364.

TABLE 4

Calculated expressions of response variables of regression equation for *Carboniferous zone*

Area	$s_c^{\alpha_{s_c}^{lower}}$	**Basis** $s_c^{\alpha_{s_c}}$	$s_c^{\alpha_{s_c}^{upper}}$
HCM 1	1.0000	1.000	1.0000
HCM 2	1.0000	1.000	1.0000
HCM 3	0.8946	0.910	0.9252
HCM 4	0.8382	0.861	0.8841
HCM 5	1.0000	1.000	1.0000
HCM 6	0.8946	0.910	0.9252
HCM 7	0.8382	0.861	0.8841

On the basis of the investigated geological data, the per cent share of particular strata were determined for each of the analysed areas in the *Carboniferous zone*, which is presented in Fig. 7.

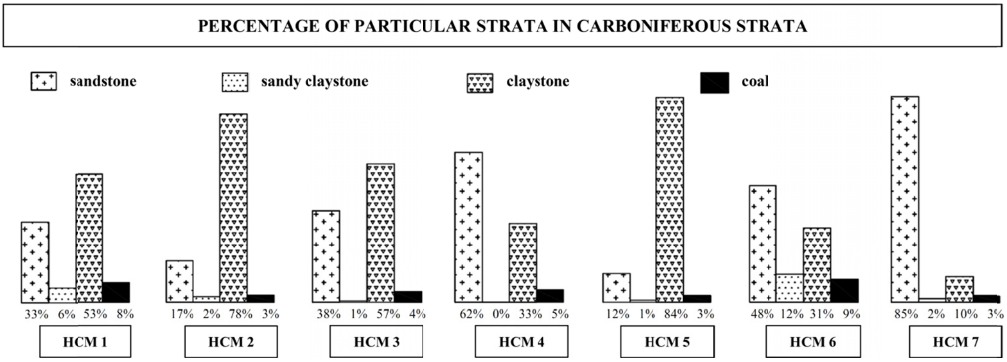

Fig. 7. Per cent share of strata in *Carboniferous zone* – independent variables

The regression analysis was carried out separately for the values of response variables $s_c^{\alpha_{s_c}^{lower}}$ and $s_c^{\alpha_{s_c}^{upper}}$ (Table 4) with the use of independent variables describing the per cent share of particular strata in the Carboniferous zone (Fig. 7). The constructed maximal regression equations for the lower and upper confidence limit are presented in Figs. 5 and 6.

TABLE 5

Maximal regression equations for the lower confidence limit

Area	**Linear regression equations**
HCM 1	→ $1.0000 = \alpha_{ss}^{lower} \cdot 0.330 + \alpha_{scs}^{lower} \cdot 0.058 + \alpha_{cs}^{lower} \cdot 0.533 + \alpha_{c}^{lower} \cdot 0.079$
HCM 2	→ $1.0000 = \alpha_{ss}^{lower} \cdot 0.170 + \alpha_{scs}^{lower} \cdot 0.022 + \alpha_{cs}^{lower} \cdot 0.780 + \alpha_{c}^{lower} \cdot 0.028$
HCM 3	→ $0.8946 = \alpha_{ss}^{lower} \cdot 0.380 + \alpha_{scs}^{lower} \cdot 0.005 + \alpha_{cs}^{lower} \cdot 0.572 + \alpha_{c}^{lower} \cdot 0.043$
HCM 4	→ $0.8382 = \alpha_{ss}^{lower} \cdot 0.621 + \alpha_{scs}^{lower} \cdot 0.000 + \alpha_{cs}^{lower} \cdot 0.327 + \alpha_{c}^{lower} \cdot 0.052$
HCM 5	→ $1.0000 = \alpha_{ss}^{lower} \cdot 0.117 + \alpha_{scs}^{lower} \cdot 0.008 + \alpha_{cs}^{lower} \cdot 0.848 + \alpha_{c}^{lower} \cdot 0.027$
HCM 6	→ $0.8946 = \alpha_{ss}^{lower} \cdot 0.483 + \alpha_{scs}^{lower} \cdot 0.115 + \alpha_{cs}^{lower} \cdot 0.309 + \alpha_{c}^{lower} \cdot 0.093$
HCM 7	→ $0.8382 = \alpha_{ss}^{lower} \cdot 0.852 + \alpha_{scs}^{lower} \cdot 0.015 + \alpha_{cs}^{lower} \cdot 0.106 + \alpha_{c}^{lower} \cdot 0.027$

TABLE 6

Maximal regression equations for the upper confidence limit

Area	**Linear regression equations**
HCM 1	→ $1.0000 = \alpha_{ss}^{upper} \cdot 0.330 + \alpha_{scs}^{upper} \cdot 0.058 + \alpha_{cs}^{upper} \cdot 0.533 + \alpha_{c}^{upper} \cdot 0.079$
HCM 2	→ $1.0000 = \alpha_{ss}^{upper} \cdot 0.170 + \alpha_{scs}^{upper} \cdot 0.022 + \alpha_{cs}^{upper} \cdot 0.780 + \alpha_{c}^{upper} \cdot 0.028$
HCM 3	→ $0.9252 = \alpha_{ss}^{upper} \cdot 0.380 + \alpha_{scs}^{upper} \cdot 0.005 + \alpha_{cs}^{upper} \cdot 0.572 + \alpha_{c}^{upper} \cdot 0.043$
HCM 4	→ $0.8841 = \alpha_{ss}^{upper} \cdot 0.621 + \alpha_{scs}^{upper} \cdot 0.000 + \alpha_{cs}^{upper} \cdot 0.327 + \alpha_{c}^{upper} \cdot 0.052$
HCM 5	→ $1.0000 = \alpha_{ss}^{upper} \cdot 0.117 + \alpha_{scs}^{upper} \cdot 0.008 + \alpha_{cs}^{upper} \cdot 0.848 + \alpha_{c}^{upper} \cdot 0.027$
HCM 6	→ $0.9252 = \alpha_{ss}^{upper} \cdot 0.483 + \alpha_{scs}^{upper} \cdot 0.115 + \alpha_{cs}^{upper} \cdot 0.309 + \alpha_{c}^{upper} \cdot 0.093$
HCM 7	→ $0.8841 = \alpha_{ss}^{upper} \cdot 0.852 + \alpha_{scs}^{upper} \cdot 0.015 + \alpha_{cs}^{upper} \cdot 0.106 + \alpha_{c}^{upper} \cdot 0.027$

For the data prepared in such a way a multi-way regression analysis was carried out. After eliminating the variable *coal*, the obtained regression equations are properly constructed. As a result, the equations described by the independent variables (*sandstone, sandy claystone, claystone*) were obtained.

As the next step in the investigation, the influence of particular strata (*sandstone, sandy claystone, claystone*) on the *Carboniferous zone*, and thus also on the value of the exploitation coefficient *a*, was thoroughly considered. Numerous analyses helped to obtain the confidence limits for the values of structural parameters of the regression equation for the variables *sandstone, sandy claystone* and *claystone.* The ranges of parameter values for particular types of rocks for the variable *Carboniferous zone* was presented in Fig. 8.

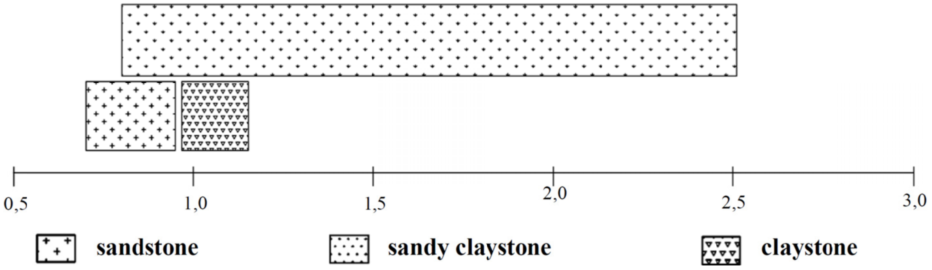

Fig. 8. Segments of parameter values characteristic for particular type of strata for the variable *Carboniferous zone*

The investigation showed that the structural parameter values for sandstone (α_{ss}) ranged from 0.6976 to 0.9554; for sandy claystone (α_{scs}): from 0.8049 to 2.5249; for claystone (α_{cs}): from 0.9663 to 1.1513. The obtained relations can be interpreted in the following way: the increase of sandstone share in the entire *Carboniferous zone* by 10% will cause the value increase of the expression ($s_c^{\alpha_{s_c}}$) from the regression equation (1) by the value ranging from 0.06976 to 0.09554, which in the same time will cause the increase of the value of the exploitation coefficient a. An analogical interpretation can be made for sandy claystone and claystone.

The analysis also proves that the ranges of structural parameter values for sandstone (α_{ss}) and claystone (α_{cs}) are separate. Therefore, the influence of sandstone strata on the value of the exploitation coefficient a is different from the influence of claystone on the value of the parameter. Such a conclusion can be drawn with the probability of 90.25% - 95% from the linear regression equation, for the variable *Carboniferous zone*, for the lower ($\alpha_{s_c}^{lower}$) and upper ($\alpha_{s_c}^{upper}$) confidence limit.

The obtained results pertain to three strata: *sandstone, sandy claystone* and *claystone*. However, the share of *sandy claystone* in the *Carboniferous zone* is relatively low (maximum 11,5% but usually below 3%). Hence, the obtained results for this type of rock are of little importance from a practical point of view. In addition, the analysis did not embrace *coal* as its share in particular areas of the *Carboniferous zone* is low (maximum 9.2% but usually below 5.5%). Hence, eliminating coal from the analysis does not affect results in any significant way.

7. Conclusions

Matching the subsidence curvatures obtained from the theoretical formulae with the measured subsidence proves that the values of the exploitation coefficient range from 0.72 in the area of HCM 7 to 1.31 in the area of HCM 1. The values of the exploitation coefficient higher than one result from the activation of the seams excavated earlier, i.e. in the conditions of consolidating the disturbed and fractured rock strata, as well as from the positioning of measurement lines. Surface deformation prediction should be based on the value of the parameter a, assumed from matching the theoretical curvature of subsidence with the subsidence curvature obtained from the measurements, or from the areas of similar geological structure and mining situation.

Owing to a diverse lithology and varied degree of disturbance of the analysed rocks, a distinction can be made among strong, average and weak rock mass. In order to describe each of the distinguished rock mass models, such characteristic parameters as *thickness of overburden, type of overburden strata, type of Carboniferous strata, rock mass disturbance* and *depth of exploitation* were thoroughly analysed.

The procedure discussed in this study allows for a proper description of the exploitation coefficient, used in the theory of surface deformation prediction. The most important finding of the analysis is that *rock mass disturbance* exerts a crucial impact and hence it should be taken under consideration in predictions pertaining to planned mining exploitation.

References

Białek J., 2003. *Algorithms and computer programs for the prediction of mining ground deformation* (in Polish) *Algorytmy i programy komputerowe do prognozowania deformacji terenu górniczego*. Wydawnictwo Politechniki Śląskiej, Gliwice.

Białek J., Mierzejowska A., 2011. *Influence of the number of measurement points and the mining depth on the error in determining the values of selected parameters according to the impacts theory* (in Polish) *Wpływ liczby punktów pomiarowych oraz głębokości eksploatacji na błąd wyznaczenia wartości wybranych parametrów teorii wpływów*. Bezpieczeństwo Pracy i Ochrona Środowiska w Górnictwie **2**, 3-8.

Białek J., Mierzejowska A., 2012. *Estimation of error values of parameters tgβ, A_{obr}, a determined on the basis of measurements illustrating incomplete subsidence troughs* (in Polish) *Oszacowanie dokładności parametrów tgβ, Aobr, a, wyznaczonych na podstawie pomiarów niepełnych niecek obniżeniowych*. Przegląd Górniczy **8**, 180-184.

Costa M. A., Lopes A.L.M., Matos G.B.B.P., 2015. *Statistical evaluation of data envelopment analysis versus COLS Cobb-Douglas benchmarking models for the 2011 Brazilian tariff revision*. Socio Econ. Plan. Sci. **49**, 47-60.

Doležalová H., Kajzar V., Souček K., Staš L., 2009. *Evaluation of mining subsidence using GPS data.* Acta Geodynamica et Geomaterialia **6**, 3, (155), 359-367.

Ghabraie B., Ren G., Smith J.V., 2017. *Characterising the multi-seam subsidence due to varying mining configuration, insights from physical modelling*. International Journal of Rock Mechanics & Mining Sciences **93**, 269-279.

Guo G., Zhu X., Zha J., Wang Q., 2014. *Subsidence prediction method based on equivalent mining height theory for solid backfilling mining*. Trans. Nonferrous Met. Soc. China **24**, 3302-3308.

Jiránková E., Staš L., Kajzar V., Doležalová H., 2013. *Mechanism of rigid overlaying of carboniferous strata failure in face mining in the case of a multiseams deposit*. Acta Geodynamica et Geomaterialia **10**, 2 (170), 189-195.

Jureczka J., Dopita M., Gałka M., Krieger W., Kwarciński J., Martinec P., 2005. *Geological atlas of coal deposits of the Polish and Czech parts of the Upper Silesian Coal Basin – Atlas geologiczno-złożowy polskiej i czeskiej części Górnośląskiego Zagłębia Węglowego*. Państwowy Instytut Geologiczny, Ministerstwo Środowiska, Warszawa.

Hu Q., Deng X., Feng R., Li Ch., Wang X., Jiang T., 2015. *Model for calculating the parameter of the Knothe time function based on angle of full subsidence.* International Journal of Rock Mechanics and Mining Sciences **78**, 19-26.

Knothe S., 1984. *Prognozowanie wpływów eksploatacji górniczej*. Wydawnictwo ”Śląsk”, Katowice. s. 160.

Kowalski A., 2007. *Specyfika deformacji powierzchni dla dzisiejszego polskiego górnictwa węgla kamiennego*. Górnictwo i Geoinżynieria **31**, 3/1, 269-277.

Kruczkowski M., 2011. *Wyznaczenie wartości parametrów teorii prognozowania wpływów w przypadku eksploatacji górniczej prowadzonej w dwóch pokładach, The identification of the parameters of prediction theory for underground mining effect in case of mining extraction of two coal seams* (in Polish). Zeszyty Naukowe Politechniki Śląskiej Seria: Górnictwo i Geologia **6**, 11, 149-157.

Kryzia K., 2017. *The impact of roof strata type on surface subsidence caused by coal mining with caving* (in Polish) *Wpływ rodzaju warstw stropowych na obniżenia powierzchni terenu spowodowane eksploatacją pokładów węgla z zawałem stropu.* Ph.D. thesis Rozprawa doktorska, AGH University of Science and Technology Akademia Górniczo-Hutnicza w Krakowie, Kraków.

Majcherczyk T., Kryzia K., Majchrzak J., 2012a. *Analysis of the ground surface deformations of the district Moszczenica city ”Jastrzębie-Zdrój” in aspect the influence of mining exploitation of Jas-Mos coal mine* (in Polish)*Analiza* *deformacji powierzchni terenu w dzielnicy Moszczenica miasta Jastrzębie-Zdrój w aspekcie wpływów eksploatacji górniczej Kopalni Węgla Kamiennego „JAS-MOS"*. Ochrona obiektów na terenach górniczych: praca zbiorowa, IV konferencja z cyklu „Bezpieczeństwo i ochrona obiektów budowlanych na terenach górniczych, Rytro, 17-19 października 2012, pod red. Andrzeja Kowalskiego, 190-201.

Majcherczyk T., Niedbalski Z., Kryzia K., 2012b. *Changes to the range of exploitation impact when mining the next coal deposit on the basis of geodetic measurements*. AGH Journal of Mining and Geoengineering, 219-230.

Majcherczyk T., Kryzia K., 2013. *Analysis of measured and predicted land surface subsidences caused by retreat mining.* Studia Geotechnica et Mechanica, 143-156.

Majcherczyk T., Kryzia K., 2017. *Exploitation coefficient and type of rock mass* (in Polish) *Współczynnik eksploatacji a rodzaj górotworu.* Bezpieczeństwo Pracy i Ochrona Środowiska w Górnictwie **12**, 9-14.

Mielimąka R., 2009. *Wpływ kolejności i kierunku eksploatacji prowadzonej frontami ścianowymi na deformacje terenu górniczego*. Wydawnictwo Politechniki Śląskiej, Gliwice.

Ostrowski J., Malinowska A., 2006. *Parametry procesu górniczych wpływów wyznaczone w oparciu o pomierzone obniżenia jako podstawa prognoz deformacji powierzchni*. Miesięcznik WUG **2**, 8-16.

Polanin P., 2015. *Application of two parameter groups of the Knothe-Budryk theory in subsidence prediction*. Journal of Sustainable Mining **14**, 67-75.

Popiołek E., 2009. *Ochrona terenów górniczych*. Wydawnictwa AGH, Kraków.

Płudowski H., 1980. *Badanie współzmienności cech przy pomocy funkcji potęgowej*. Annales Universitatis Mariae Curie-Skłodowska. Sectio H **13-14**, Lublin, 87-93.

Suchowerska Iwanec A.M., Carter J.P., Hambleton J.P., 2016. *Geomechanics of subsidence above single and multi-seam coal mining*. Journal of Rock Mechanics and Geotechnical Engineering **8**, 304-313.

Wang E., Cruse R.M., Zhao Y., Chen X., 2015. *Quantifying soil physical condition based on soil solid, liquid and gaseous phases*. Soil Tillage Res. **146**, 4-9.

Zych J., 1985. *Zmienność parametrów teorii S. Knothego i T. Kochmańskiego w świetle badań geodezyjnych*. Zeszyty Naukowe Politechniki Śląskiej Seria: Górnictwo **134**, 169-182.