

## The roadholding ability of the car subjected to the external lateral forces

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Received September 02.2016; accepted September 12.2016

**Summary.** In the article we address matters related to the stability of motion of the car subjected to the external lateral force. We show the possibility to compensate the effect of the external forces by means of the steering wheel angle. We have carried out the analysis for bicycle scheme vehicle and four-wheel model according to linear and nonlinear drift hypothesis. We have considered influence of speed of the car and size of external side force for the period of transition processes of stabilization of the movement.

**Key words:** roadholding ability, turn correction, lateral force, slip angle

### INTRODUCTION

The roadholding ability of the car is one of its most important characteristics concerning its passive safety. The analysis of the effect of both nonlinear characteristics of the lateral force as a function of slip angles and nonlinear functions of stabilizing moment in the contact patch was previously made [1]. In this article we consider the problem of the lateral force compensation by turning the front group steering wheels.

### THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

In real life the lateral force may be generated by the strong side wind in the moment of driving out of the tunnel or passing the roadtrain on the leeward. It's worth noting that Mercedes-Benz company proposed to improve the roadholding ability of its S and GL class cars by means of Crosswind Assist system [2,3] due to skid steering on the windward. The results obtained from linear [4–7] and nonlinear [8–11] models we known. In particular, Litvinov details the aspects of how cars with oversteer and understeer react on external force. The works [8,12], among other issues, consider the car behavior subjected to the external lateral force according to the nonlinear theory of drift. Among foreign researches in this area worth noting are MacAdam C.C. [13,14] and Gillespie [15,16] which consider the stability of the vehicle subjected to the external forces.

In this article we propose to solve the problem of the course deviation by the correction turning of the steering wheel.

### PURPOSE AND OBJECTIVES OF RESEARCH

Justification of the possibility to compensate the effect of the external lateral force on the vehicle by turning the steering wheels (based on the relations obtained for the bicycle model) to return to the linear course of the more full four-wheel vehicle model and the analysis of the stability of the corresponding stationary linear course.

### THE MAIN RESULTS OF THE RESEARCH

In our work we used the results of the dynamic experiments performed as part of “The Measurement and the Analysis of the Parameters Characterizing the Category M1 Vehicles Roadholding” research [17–19]. The prototype for the math model was 2008 front-wheel drive Opel Vectra with the following characteristics: distance from the center of mass to the front axle  $a = 1.273$  m; - to the back axle  $b = 1.427$  m; Curb weight  $m = 1771$  kg; inertia about the center of mass  $J = 600$  kg·m<sup>2</sup>. The tires used were Bridgestone Turanza ER30. These tires have tread that reduces the noise and provides good road grip on both dry and wet surface. The tread and side wall provide reliable steering and high durability.

To measure the parameters of Opel Vectra C roadholding ability we used branded servicing device MDI (Multiple diagnostic interface) [20].

The use of this device made it possible to obtain information on changing the following parameters over time: linear speeds of the car wheels  $V_i$ ; sideward acceleration  $\dot{u}$ ; angular velocity  $\omega$  of the car about the vertical axis passing through the center of mass; steering angle  $\theta$ , the timepoint of the relative wheel slipping in the longitudinal direction – the beginning of the wheels locking up by the brakes. [21]

The equations of the plane motion of the bicycle scheme vehicle with the constant velocity component  $v$  in the longitudinal direction will be as follows:

$$\begin{cases} m(\dot{u} + v\omega) = Y_1 \cos \theta + Y_2 + Q_0 \\ J\dot{\omega} = a \cdot Y_1 \cos \theta - b \cdot Y_2; \end{cases} \quad (1)$$

$$\delta_1 = \theta - \frac{u + a\omega}{v}; \quad \delta_2 = \frac{-u + b\omega}{v}, \quad (2)$$

where:  $u$  is the lateral component of the vehicle mass center velocity;  $\omega$  is the angular speed about the vertical

axis;  $\delta_1, \delta_2$  are slip angles on front and back axis respectively,  $Y_1, Y_2$  are lateral forces between wheels and road surface,  $Q_0$  is the constant external lateral force applied to the center of mass.

It is worth noting that in this case we didn't consider the effect of the stabilizing moments and the longitudinal drag force to make calculations simpler. Values of the lateral slip resistance coefficients according to the experimental data are  $k_1 = 32240$  N/rad,  $k_2 = 27186$  N/rad for front and back axis respectively.

During the primary analysis we integrate the motion equations of a vehicle with absolutely rigid steering module. To simplify the mathematical transformations we introduce the dimensionless analytical expressions by dividing the both parts on  $(m \cdot g)$ : The expressions take the following form:

- back pressure at front and back axis:

$$N_1 = \frac{m \cdot g \cdot a}{(a+b)}, \quad N_2 = \frac{m \cdot g \cdot b}{(a+b)}, \quad (3)$$

- lateral slip resistance coefficients:

$$K_1 = \frac{k_1}{N_1}; \quad K_2 = \frac{k_2}{N_2} \quad (4)$$

- external lateral force:

$$Q = \frac{Q_0}{m \cdot g}, \quad (5)$$

where:  $Q_0$  is the external constant lateral force applied to the center of mass.

- critical speed of the direct motion:

$$V_{kp} = \sqrt{\frac{k_1 k_2 l^2}{m(k_1 a - k_2 b)}}, \quad (6)$$

Steering angle of the steering wheels compensating the effect of the external force according to the linear [4] and nonlinear drift theory respectively [9]:

$$\theta = \frac{N_2 \cdot Q}{k_2} - \frac{N_1 \cdot Q}{k_1}. \quad (7)$$

General form of the correction angle equation:

$$\theta = G(Q), \quad (8)$$

where: the solution of the inverse function is

$$G(Q) = G_2(Q) - G_1(Q), \quad (9)$$

where:  $G_i$  defines the inverse function of  $Y_i(\delta_i)$ , i.e. as a result we have:

$$Q = Y_i(\delta_i) \Rightarrow \delta_i = G_i(Q), \quad (10)$$

Hence,

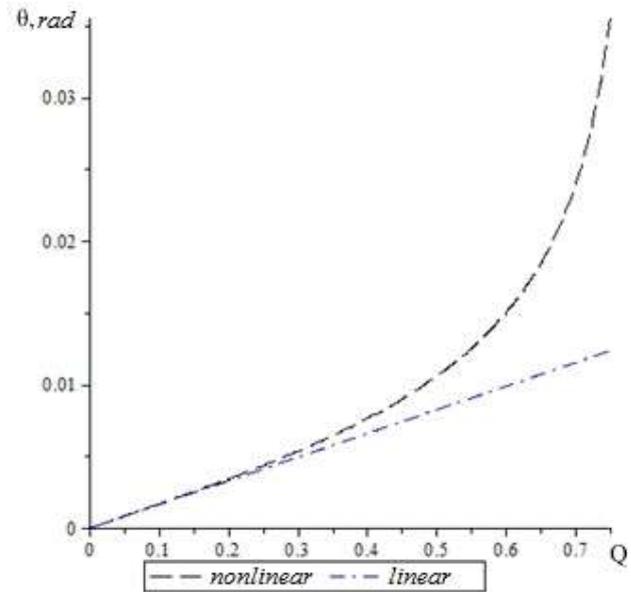
$$G_i = \frac{Q}{K_i \sqrt{1 - \frac{Q^2}{\varphi_i^2}}} \quad (11)$$

$$\theta = \frac{N_2 \cdot Q}{k_2 \sqrt{1 - \frac{Q^2}{\varphi_2^2}}} - \frac{N_1 \cdot Q}{k_1 \sqrt{1 - \frac{Q^2}{\varphi_1^2}}}. \quad (12)$$

Eq. (14) and (8) were obtained for the case of the fractional irrational approximation of the drift forces dependence (9):

$$Y_1 = \frac{k_1 \delta_1}{\sqrt{1 + \left(\frac{k_1 \delta_1}{N_1 \cdot \varphi_1}\right)^2}}; \quad Y_2 = \frac{k_2 \delta_2}{\sqrt{1 + \left(\frac{k_2 \delta_2}{N_2 \cdot \varphi_2}\right)^2}}. \quad (13)$$

Fig 1 shows charts of the steering wheels turn angles compensating the effect of the external lateral force.



**Fig.1.** The external lateral force compensation chart.  $\theta=f(Q)$  acc. to linear and nonlinear drift hypothesis

From Fig. 1 it follows that within small values of  $Q \approx 0,25$  we can use the drift theory to define the  $\theta$ . But with the increase of the external lateral force the discrepancy increases as well (e.g. it equals 15% for  $Q=0,4$ ), and further compensation is better considered within nonlinear theory.

As is seen from the graphs of the integral curves of angular speed and the lateral component of the mass center speed (Fig. 2), the external lateral force leads to a perturbation of the phase variables in the initial period of time ( $t \approx 1.5$  s), and then the parameters of plane parallel unperturbed motion stabilize.

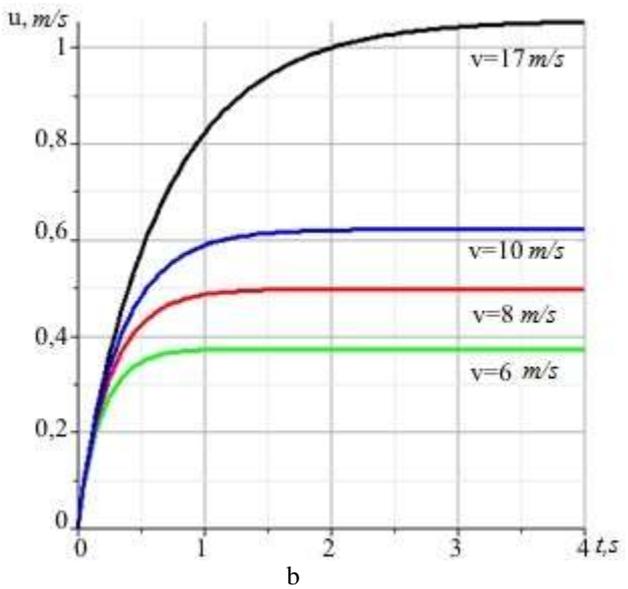
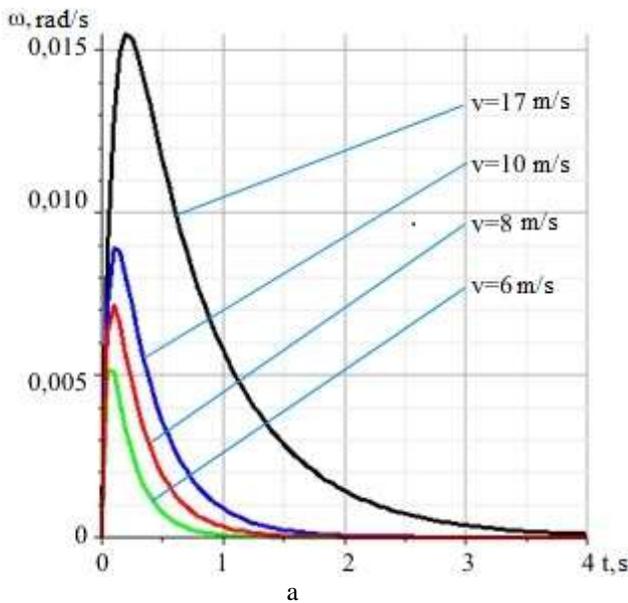


Fig. 2. Integral curves  $\omega(t)$  and  $u(t)$  for different values of speed

Thus, a stable node in the phase plane (with a monotonic decay of the initial perturbations) corresponds to the reestablished linear motion mode.

Furthermore, as follows from the graphs, the motion speed also affects (although moderately) the duration of motion stabilization process. E. g. the stabilization of the angular speed and lateral acceleration takes 4 seconds at  $v=60$  km/h.

Consider the model of the car with rigid steering and taking the wheel track into account (Fig. 3).

Geometric and inertial parameters of the rigid car model:  $a, b$  – distance between the mass center and front (steering) and back axis;  $H$  – wheel track,  $m, J$  – car mass and inertia about the central vertical axis.

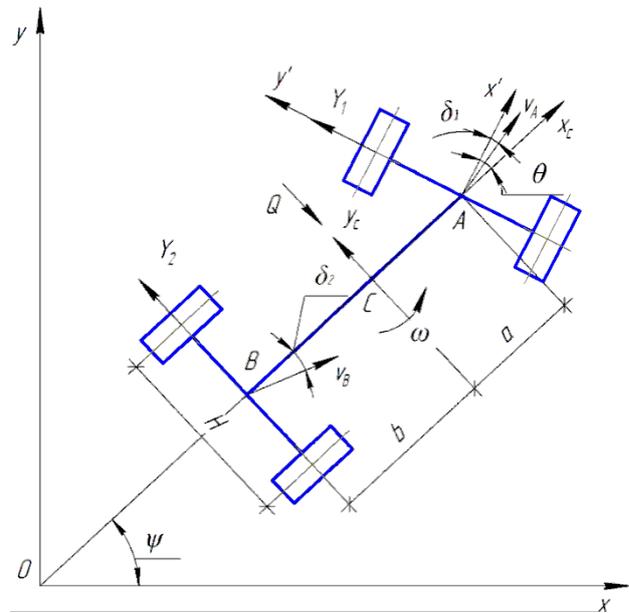


Fig.3. Car model with the “rigid” steering wheel module

Phase variables of a rigid car model:  $v$  – mass center velocity projection longitudinal component,  $u$  – mass center velocity projection lateral component,  $\omega$  – angular speed of a car about the vertical axis; the differential equations system of the motion in the plane of the road ( $u, \omega, \Psi, x, y$ ):  $V$  – derivative of the longitudinal component of the center of mass ( $v$ ),  $u$  – derivative of the lateral component of the center of mass ( $u$ ),  $\Omega$  – derivative of the angular speed ( $\omega$ ) about the vertical axis,  $\Psi$  – course angle;  $x, y$  – coordinates of the center of mass of the car in a flat fixed coordinate system..

The results obtained on the example of the flat bicycle model of the car we can check on the four-wheel model (by taking the wheel track into account). To do this we should correct the longitudinal components of the wheel center speeds. In this case the additional longitudinal speed component ( $H \cdot \omega$ ) will have positive sign for outer wheels and negative sign for inner wheels.

$$\delta_{1,j} = \theta_j - \frac{u + a\omega}{v \pm H\omega}; \quad \delta_{2,j} = \frac{-u + b\omega}{v \pm H\omega}, \quad (14)$$

where:  $i$  is the axis number (1 – front axis, 2 – back axis),  $j$  – number of the wheel on the front and back axis.

The distribution of the resistance to the lateral slip across the wheels will be as follows:

$$Y_{ij} = \frac{k_{ij} \delta_{ij}}{\sqrt{1 + \left( \frac{k_{ij} \delta_{ij}}{N_{ij} \cdot \varphi} \right)^2}}, \quad (15)$$

where:  $\varphi$  is grip coefficient.

$$-m\dot{\omega} + X_1 \sin \theta + Y_{11} \cos \theta + Y_{12} \cos \theta + Y_{21} + Y_{22} + Q \cdot mg = m\ddot{u} \\ (X_1 \sin \theta + Y_{11} \cos \theta + Y_{12} \cos \theta) \cdot a - (Y_{21} + Y_{22}) \cdot b = J\dot{\omega}. \quad (16)$$

The longitudinal drag force  $X_1$  in the Eq. (16) was taken into account to neutralize forces and moments of tractive resistance and to provide the constant speed and moment during the car motion.

After the linearization of the initial equation system in the neighborhood of the nonperturbed linear motion ( $u=u^*$ ,  $\omega=0$ ) we obtain the characteristic equation:

$$\lambda^2 + p\lambda + q = 0, \quad (17)$$

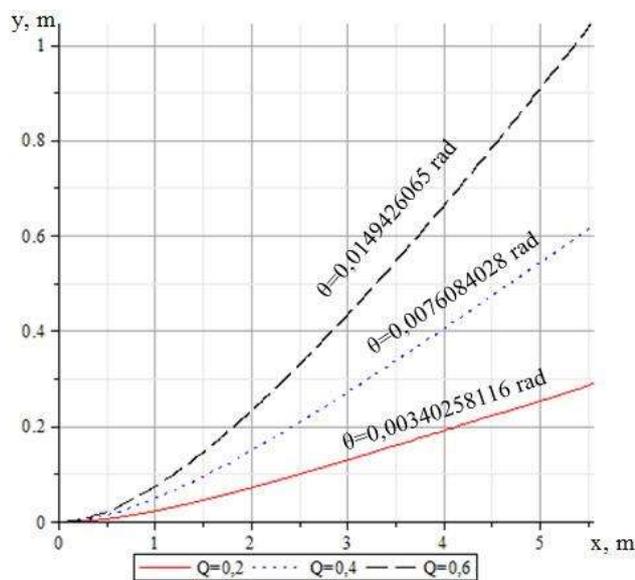
where:  $p=34.83$ ,  $q=157.51$  are defined according to the given data.

Using Maple software and data from the experiment given before, we determine the roots of this characteristic equation:

$$\lambda_1 = -5.34, \lambda_2 = -29.49.$$

Since the real parts of all roots are negative, this stationary state is a stable node.

Fig. 4 shows the motion trajectories of the mass center of the car (for different values of the external lateral force), from which it follows that course angle stabilization is absent in this model.



**Fig.4.** Motion trajectory of the center of mass for different values of  $Q_0$  and corresponding values of  $\theta$  (for the model with wheel track taken into account)

Thus, the problem of returning the vehicle to the nonperturbed course angle is the further development of the lateral force compensation model.

## CONCLUSIONS

The analytical solution of differential equations and its graphical representation allows us to state the following:

1. The correction value of the steering wheel turn angle obtained on the bicycle model was confirmed by checking on the four-wheel model.
2. For large values of the lateral force ( $Q \geq 0,3$ ) the value of the steering wheel turn angle should be considered within the nonlinear drift theory.

3. The duration of the transient processes is almost in proportion to the longitudinal speed of the vehicle.

Thus, we can argue that the effect of the lateral force on the car during the stationary stable motion can be compensated by turning of the steering wheel, while keeping the stability of the linear motion of the vehicle.

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#### КУРСОВАЯ УСТОЙЧИВОСТЬ АВТОМОБИЛЯ ПРИ ВОЗДЕЙСТВИИ ВНЕШНИХ БОКОВЫХ СИЛ

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**Аннотации.** В статье рассмотрен вопрос устойчивости движения легкового автомобиля в условиях воздействия внешней боковой силы. Показана возможность компенсации воздействия внешних сил углом поворота управляемых колес. Анализ проведен для одноколейной и двухколейной моделей автомобиля согласно линейной и нелинейной гипотез увода. Рассмотрено влияние скорости автомобиля и величины внешней боковой силы на время переходных процессов стабилизации движения.

**Ключевые слова:** курсовая устойчивость, коррекция поворота, боковая сила, угол увода.

