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PCA Assisted DTCWT Denoising for Improved DOA Estimation of Closely Spaced and Coherent Signals

Dharmendra Ganage, and Yerram Ravinder

***Abstract*—Performance of standard Direction of Arrival (DOA) estimation techniques degraded under real-time signal conditions. The classical algorithms are Multiple Signal Classification (MUSIC), and Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT). There are many signal conditions hamper on its performance, such as closely spaced and coherent signals caused due to the multipath propagations of signals results in a decrease of the signal to noise ratio (SNR) of the received signal. In this paper, a novel DOA estimation technique named CW-PCA MUSIC is proposed using Principal Component Analysis (PCA) to threshold the nearby correlated wavelet coefficients of Dual-Tree Complex Wavelet transform (DTCWT) for denoising the signals before applying to MUSIC algorithm. The proposed technique improves the detection performance under closely spaced, and coherent signals with relatively low SNR conditions. Also, this method requires fewer snapshots, and less antenna array elements compared with standard MUSIC and wavelet-based DOA estimation algorithms.**

***Keywords*—DOA estimation, signal denoising, closely spaced, coherent signal, adaptive antenna array, antenna elements**

I. Introduction

In recent years, mobile communication networks face many issues related to connectivity, call drops, more demands on their spectrum, and resource handling. The direction of arrival (DOA) estimation plays a vital role in adaptive array processing when multiple signals with similar kind of characteristics impinge on antenna array. The estimation of DOAs of every received signal can increase the capacity and throughputs of the system significantly. The smart antenna technology uses various digital signal processing algorithms with digital signal processor architecture to estimate the location of every incoming signal with the help of different techniques of DOA estimation algorithms [1]. Schmidt [2] developed MUSIC, and Roy [3] developed ESPRIT, both are high-resolution sub-spaced based algorithms which give reasonable performance, and have been developed over the years. However, these algorithms work only in rich environment conditions such as high SNR, a large number of antenna elements and more number of snapshots. These algorithms perform well when array steering vector is known, and the signals are non-coherent. However, in the non-homogeneous environment, the signal becomes coherent and closely spaced because of various multipath signal characteristics results in deteriorating the performance of sub-spaced based DOA estimation algorithms [4,5].

The use of nonlinear wavelet for performance improvement of the MUSIC algorithm under low SNR conditions because of covariance matrix rank degradation while estimating from finite data [6]. However, this technique uses a higher number of antenna elements in the array. X. Mao [7] uses the wavelet operator to improve the performance of MUSIC and ESPRIT algorithm. However, it uses the closely spaced signals only with relatively higher and positive SNR. Jianfeng [8] developed the low computational burden method for DOA estimation with a small number of snapshots using state space-based approach. However, the developed method works on the coherent signal with positive SNR. When the signals are closely spaced and coherent about 3 to 4 degrees resolution between them; with relatively very low SNR of -4 dB and above, DOA estimation methods [4-8] may not suit for real-time signal conditions.

The Dual-Tree Complex Wavelet Transform (DTCWT) is generating the correlated wavelet coefficients after decomposition as standard wavelet transform produces in a small nearby region. If the correlated complex wavelet coefficient has large magnitude at its nearby locations, which results more prone to noise generation. Hence, there is a necessity to design thresholding process to remove such correlated wavelet coefficients. This thresholding process uses nearby complex wavelet coefficients for searching out the correlated wavelet coefficients. The method of thresholding the correlated wavelet coefficients of DTCWT because of the generation of coherent signal results in the rank degradation of array covariance matrix of array manifold vector [9]. This technique denoised the specific type of signals.

The DTCWT proposed by Kingsbury [10] and improved by Selesnick [11] which is widely used in image denoising. The drawback of discrete wavelet transform (DWT), advantages of DTCWT over DWT, different techniques of realization of DTCWT, different types of filters used in DTCWT was discussed in [10,11] in great detail. The detailed overview of various applications of wavelets in wireless communication presented in [12]. The wavelets have numerous features and options that can be used in wireless channel modelling, interference mitigation, and denoising. The use of DTCWT and singular value decomposition (SVD) based technique employed in acoustic noise reduction to avoid leakage in the natural gas pipeline [13]. The different thresholding techniques along with the application of DWT

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and DTCWT with GCV threshold for image denoising presented in [14]. The used DTCWT for biomedical signal denoising to avoid the Gibbs oscillation and severe frequency aliasing due to DWT explored in [15].

DTCWT used for signal denoising and different thresholding techniques using GCV, SVD and PCA applied for noise reduction in leakage pipeline, biomedical signal denoising [16,17] explored in detailed. The techniques [9,13] motivates us to design thresholding technique for DTCWT based signal decomposition which estimates the DOA of the closely spaced coherent signal. Most of the methods [4-8] designed for uncorrelated or coherent signals for DOA estimation. The proposed work has undertaken the signal both in closely spaced and coherent type simultaneously.

In this paper, a novel algorithm namely CW-PCA MUSIC using the DTCWT with PCA is applied for improving the performance of MUSIC algorithm for closely spaced coherent signals simultaneously. DTCWT along with PCA used pre-processing (denoising) stage of MUSIC algorithm. PCA is used as thresholding technique to remove the more significant coefficients in forwarding process of DTCWT. The proposed method uses the minimum number of snapshots with few antenna elements in the antenna array, and low SNR.

The organization of paper: Section 2 introduce the signal model using a uniform linear array. Section 3 describes the pre-processing stage using DWT and DTCWT for MUSIC based DOA estimation algorithms. The proposed method "CW-PCA MUSIC" based on DTCWT and PCA illustrated in Section 4. Section 5 presents experimental verification of proposed algorithm, and comparison with basic MUSIC, DWT and DTCWT based MUSIC algorithms. Section 6 summarises the results with the conclusion.

II. Signal Model

Let Uniform Linear Array (ULA) composed of M isotropic antenna array elements with inter-element spacing d, and K far-field received narrowband signals from different directions $\theta_1,\ldots,\theta_K$ with carrier wavelength λ illustrated in Fig. 1. The final array output calculated by adding the individual antenna array elements. The N number of snapshots/signal samples are denoted by $X(1), X(2),\ldots, X(N)$, and at time 't' the $M\times 1$ array output vector is given by [1],

$$\boldsymbol{x}(t)=\boldsymbol{A}\, s(t)+\boldsymbol{n}(t) \tag{1}$$

Where $\boldsymbol{A}=\left[a(\theta_1),\ldots,a(\theta_K)\right]$ with the $M\times K$ matrix. The antenna array steering vector is,

$$\boldsymbol{a}(\theta)=\left[1, e^{-j\left(\frac{2\pi}{\lambda}\right)d\sin\theta},\ldots,e^{-j\left(\frac{2\pi}{\lambda}\right)d(M-1)\sin\theta}\right]^T \tag{2}$$

and, $\varphi=\dfrac{2\pi d}{\lambda}\sin\theta$

Where $s(t)$ is the $K\times 1$ signal vector, $n(t)$ is the $M\times 1$ noise vector and $(.)^T$ represents the transpose.

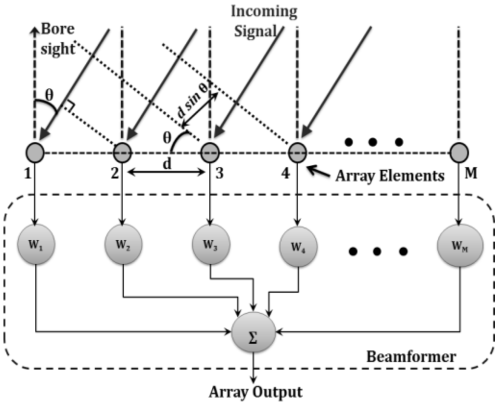

Fig.1. *M* Antenna element uniform linear array

Now, the array covariance matrix R_{XX} is given by,

$$\boldsymbol{R_{XX}}=E\left[X(t)X^H(t)\right]=A\,\boldsymbol{R_{SS}}\,A^H+\boldsymbol{R_{NN}} \tag{3}$$

Where $\boldsymbol{R_{SS}}=E\left[s(t)s^H(t)\right]$ is the source covariance, $\boldsymbol{R_{NN}}$ is the noise covariance matrix, and H represents the Hermitian, i.e., conjugate transpose. To estimate array covariance matrix $\hat{\boldsymbol{R}}$ at signal samples k with array output is given by,

$$\hat{\boldsymbol{R}}=\frac{1}{N}\sum_{k=1}^{N}X(k)X(k)^H \tag{4}$$

III. DOA Estimation Based on DW-MUSIC and CW-MUSIC

Figure 2 shows the conceptual, basic block diagram of denoising techniques using DWT. The DWT act as preprocessing stage of MUSIC algorithm [6,7]. The denoised signal after DWT is processed by MUSIC algorithm to estimate the DOAs of received signals, and the technique is referred as DW-MUSIC.

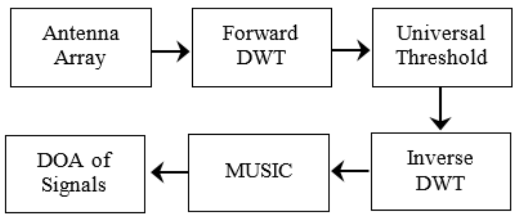

Fig.2. Block diagram of DOA estimation using DW-MUSIC method

Firstly, initialized the number of antenna elements *M*, number of snapshots *N*, and signal-to-noise ratio *SNR*. If a number of received signals impinge on antenna array is less than the number of antenna elements, then array steering vector is calculated and converted into wavelet coefficients by using a forward DWT (decomposition/analysis) filter. The universal thresholding [18] is applied to remove smaller wavelet coefficients by keeping large wavelet coefficients. The generation of the denoised signal is by taking inverse DWT (reconstruction/synthesis) filter. The generated denoised signal is processed by MUSIC algorithm to estimate the DOA of signals [2].

The DW-MUSIC algorithm has been applying for estimating DOA [6,7]. A simulation study has been carried out as part of this research work, to understand the suitability of this algorithm for closely spaced and coherent signals. It has been observed that even when the two signals are about 4 degrees away from each other are not distinguished. Mainly, because of the shift variance, i.e., a small shift of the input signal can cause significant changes and distribution of energy between wavelet coefficients at different scales. Decomposition and reconstruction of DWT produce errors. Overshoot peaks appear on each discontinuity points which leads to oscillations [11]. This drawback overcome by using DTCWT because DTCWT has approximate shift invariance, good directional selectivity, perfect reconstruction using short linear phase filters and limited redundancy [10]. The preprocessing stage is replaced with DTCWT based denoising technique, and the algorithm referred as CW-MUSIC shown in Fig. 3.

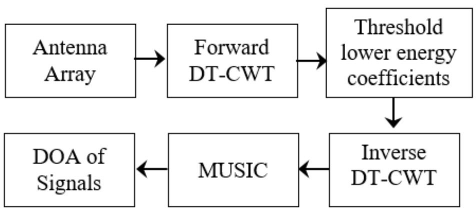

Fig.3. Block diagram of DOA estimation using CW-MUSIC method

DTCWT was proposed by Kingsbury [10] and later on upgraded by Selesnick [11]. DTCWT is an alternative to DWT which not support shift-invariance and directional selectivity for *M*-dimensional signals. Shift-invariance property is essential in the process of closely spaced coherent signals detection because of direction information embedded with the large coefficient plane in the form of magnitude and phase relationship.

DTCWT uses a dual tree of real wavelet filters to generate the real and imaginary parts of complex wavelet coefficients. DTCWT can rely on the observation with a real and approximate DWT by enhancing the sampling rate by twice at every level of the tree which can achieve approximate shift invariance. It accomplishes signal decomposition at analysis part, and reconstruction at synthesis part mainly through two parallel real and approximate wavelets (h tree and g tree). The trees consist of two filters namely: Low Pass Filters (h_0 and g_0), and High Pass Filters (h_1 and g_1). The parallel real wavelet filter banks are designed to get a complex signal, i.e., real and imaginary coefficients. These parallel trees form the Hilbert transform pair approximately to each other, and implementation is carried out by using two mother wavelets [10,11]. Fig. 4 shows the one level DTCWT analysis (decomposition) and synthesis (reconstruction) tree. Two types of filter banks proposed by Kingsbury, odd/even filter bank and Q-shift (quarter-sample) filter bank. Odd/even filter bank exist some problems [10,11]. Q-shift filter bank structure used in this implementation for denoising, and even filter lengths used after level one with no longer strictly linear phase [11].

As the standard wavelet transform, the DTCWT also generates correlated wavelet coefficients with their nearby region during decomposition of the signal at wavelet transform. If the wavelet coefficient has large magnitude will possibly have significant magnitude complex wavelet coefficients in the neighbouring area. These neighbouring coefficients acted as correlated wavelet coefficients and removed by using thresholding process [9].

Principal Component Analysis (PCA) is also referred as Karhunen-Loeve Transform (KLT). PCA is a powerful tool for analyzing data, finding patterns, and removes redundant information. It simplifies the given dataset, by reducing the number of dimensions of data to lower value from a higher range of multi-dimensional datasets [13]. In this paper, the PCA used as thresholding the similar kind of nearby correlated wavelet coefficients after decomposition of DTCWT. The preprocessing stage is now replaced with DTCWT based denoising technique along with PCA based thresholding presented in next section.

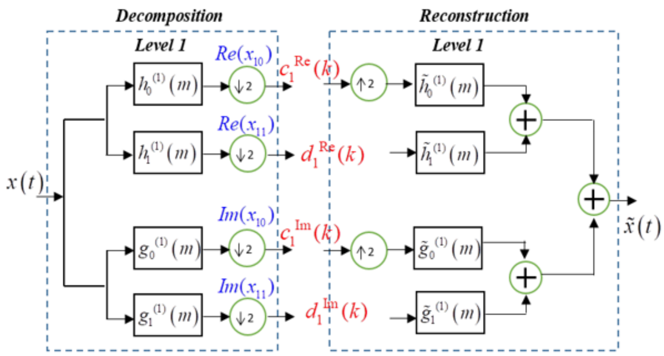

Fig.4. Analysis and synthesis process of a one-level DT-CWT

IV. Proposed DOA Estimation Method Based on DTCWT and PCA Based MUSIC

Figure 5 shows the block diagram of proposed DOA estimation based on DTCWT and PCA based MUSIC algorithm. The DTCWT and PCA are acting as denoising and thresholding stage respectively. The denoised signal is processed by MUSIC algorithm to estimate the DOAs of received signals, and the technique referred to CW-PCA MUSIC.

Firstly, initialized the *M* (number of antenna elements), *N* (number of snapshots), *SNR* and calculated the array steering vector. The array steering vector converted into wavelet coefficients using DTCWT decomposition filter. The decomposition process of signal creates correlated complex wavelet coefficients in the neighbouring region with large magnitude [9]. The decomposition signal consists of detail and approximate signal coefficients. All the detail and approximation signal coefficients are thresholded by using PCA to remove the nearby correlated wavelet coefficients by selecting best principal component [13]. After thresholding, the neighbouring correlated coefficients by using PCA, the inverse DTCWT (reconstruction filter) is used to obtain the denoised signal *ReX* from decomposition signal. Denoised signals processed by using MUSIC algorithm to estimate the DOA of signals.

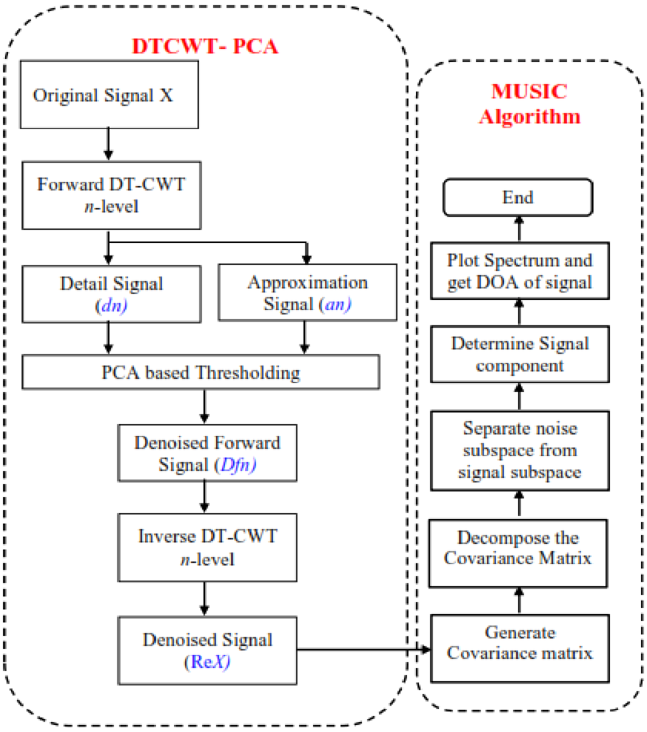

Fig.5. Block diagram of DOA estimation using proposed method

Let noisy input signal $x(t)$ is derived after antenna array, $\psi_h(t)$ represents real / even and $\psi_g(t)$ represents imaginary / odd two wavelets with scaling functions are $\phi_h(t)$ and $\phi_g(t)$ respectively. The complex wavelet signal $\psi_c(t)$ becomes an analytic signal which forms Hilbert transform pair, and it is given by [10,11,13]:

$$\psi_c(t) = \psi_h(t) + i\psi_g(t) \tag{5}$$

Now, real-part of wavelet coefficients and scaling coefficients of the tree calculated by using following formula [13]:

$$\left.\begin{aligned} w_d^{\mathrm{Re}}(n) &= 2^{\frac{d}{2}} \int_{-\infty}^{+\infty} x(t)\,\psi_h\left(2^d t - n\right) dt, \\ s_d^{\mathrm{Re}}(n) &= 2^{\frac{D}{2}} \int_{-\infty}^{+\infty} x(t)\,\phi_h\left(2^D t - n\right) dt \end{aligned}\right\} \tag{6}$$

Where d denotes the decomposition level (d = 1, 2, 3…D), and D represents the maximum value of decomposition level. Similarly, for imaginary-part of wavelet coefficients and scaling coefficient of the tree calculated by using following formula [13]:

$$\left.\begin{aligned} w_d^{\mathrm{Im}}(n) &= 2^{\frac{d}{2}} \int_{-\infty}^{+\infty} x(t)\,\psi_g\left(2^d t - n\right) dt, \\ s_d^{\mathrm{Im}}(n) &= 2^{\frac{D}{2}} \int_{-\infty}^{+\infty} x(t)\,\phi_g\left(2^D t - n\right) dt \end{aligned}\right\} \tag{7}$$

With equations (6) and (7), complex wavelet coefficient and complex scaling coefficient of the tree is denoted as follows:

$$\left.\begin{aligned} w_d^C(n) &= w_d^{\mathrm{Re}}(n) + i w_d^{\mathrm{Im}}(n) \\ s_d^C(n) &= s_d^{\mathrm{Re}}(n) + i s_d^{\mathrm{Im}}(n) \end{aligned}\right\} \tag{8}$$

After the decomposition of the signal, the detail (wavelet) coefficients and scaling (approximation) coefficients taken into PCA to remove the lower order coefficients and to improve the performance of denoising techniques. From Eq.(8) the variance is calculated by considering $w_d^C(n)$ is an actual signal, and μ is the mean [16]:

$$\sigma_Z{}^2 = E\left[\left(w_d{}^C(n) - \mu\right)^2\right] \tag{9}$$

For m = 1 to n number of rows, i.e., the number of antenna elements, the variance is calculated by using following formula:

$$\sigma_{Zm}{}^2 = E\left[\left(w_d{}^C(n)_m - \mu_m\right)^2\right] \tag{10}$$

The vector is determined from variance which has a zero mean, where n indicates number of vectors which has a zero mean,

$$\bar{\boldsymbol{r}} = [\bar{r}_1, \bar{r}_2, \cdots, \bar{r}_n] \tag{11}$$

The variance of the zero mean vector written as,

$$\sigma_r{}^2 = \frac{1}{n} r\, r^T \tag{12}$$

Hence, for m rows, we can determine row vector each of length n

$$\boldsymbol{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \in R^{m \times n} \tag{13}$$

For n samples, Covariance Matrix calculated by the following formula:

$$\boldsymbol{C_x} = \frac{1}{n-1} X\, X^T \in R^{m \times m} \tag{14}$$

By computing, the all diagonal entries are variance & off-diagonal entries are co-variance. This matrix is known as Covariance matrix. Sort the eigenvalues in descending & placed in diagonal. Eigenvalues & eigenvectors are written as,

$$\boldsymbol{S} = X\, X^T \tag{15}$$

The principal components P are eigenvectors of S with the largest eigenvalue by ignoring lowest eigenvalue of data is corresponds to the noise. Repeat above steps from equation (9-15) for scaling function, and generated signal is called denoised forward signal Dfn. Afterward, the denoised forward signal Dfn is extracted to get the complex wavelet coefficients and scaling coefficient by using inverse wavelet transform (reconstruction process) to obtain detail and approximation signal [13]. The detail signals are obtained using reconstruction process,

$$DS_d(t) = 2^{\frac{d-1}{2}} \left\{ \left[\sum_{n=-\infty}^{+\infty} w_d^{\mathrm{Re}}(n)\psi_h\left(2^d t - n\right) \right] + \left[\sum_{n=-\infty}^{+\infty} w_d^{\mathrm{Im}}(n)\psi_g\left(2^d t - n\right) \right] \right\} \quad (16)$$

Similarly, the approximation signals is obtained using reconstruction process,

$$AS_D(t) = 2^{\frac{D-1}{2}} \left\{ \left[\sum_{n=-\infty}^{+\infty} s_D^{\mathrm{Re}}(n)\phi_h\left(2^D t - n\right) \right] + \left[\sum_{n=-\infty}^{+\infty} s_d^{\mathrm{Im}}(n)\phi_g\left(2^D t - n\right) \right] \right\} \quad (17)$$

The reconstructed denoised signal ReX calculated by summing the detail and approximation signals,

$$\mathrm{Re}\, X = AS_D(t) + DS_d(t), \quad d = 1, 2, 3, \cdots, D \quad (18)$$

This reconstructed denoised signal is processed by MUSIC algorithm to estimate the DOA. The MUSIC technique for DOA estimation was proposed by Schmidt [3]. The primary function of the MUSIC algorithm is finding the most relevant signal. The MUSIC algorithm uses matrix equations to decompose received signals into the signal subspace and noise subspace. The main steps of MUSIC algorithm are explored as [1,2]:

1. Compute the signal correlation matrix R_{SS}
2. Execute eigenvalue decomposition on R_{SS} and separate the smallest eigenvalues from larger eigenvalues
3. Eigenvectors corresponding to the lowest eigenvalues from a noise subspace V_n
4. Search through all angles θ in the MUSIC spatial spectrum peak corresponds to DOAs, and it is given by,

$$P(\theta) = \frac{1}{a^H(\theta) V_n V_n{}^H a(\theta)} \quad (19)$$

Therefore, the value of θ corresponding to the N_{th} largest eigenvalue is the DOA of the N_{th} transmission signal. MUSIC algorithm performs well and accurately estimate the DOA of the signal if a number of transmission signals are infinite, the high magnitude of the output signal, and SNR is relatively high.

V. Simulation Results

In this section, simulation results on uncorrelated, coherent and closely spaced coherent signals are presented to show the potential advantages of the proposed CW-PCA MUSIC algorithm compared to CW-MUSIC, DW-MUSIC, and MUSIC algorithm respectively. The simulation has been carried out with Matlab R2014a. Consider the ULA with the distance between antenna elements in the array is 0.5λ. Assume input signals with the same power, with a varying number of snapshots, antenna elements, and SNR conditions.

A. DOA Estimation of uncorrelated and coherent signal

Consider the antenna array of 10 elements, the two distantly spaced coherent signals arriving from 5° and 20°, SNR is assumed to be -2dB, and a number of snapshots to be 100. Fig. 6 shows that the maximum peak with more resolution is achieved at the desired angle using CW-PCA MUSIC algorithm compared to other algorithms. From fig. 6, basic MUSIC algorithm detects the angle with the poor resolution because SNR is very low. DW-MUSIC and CW-MUSIC detect the 6° and 19.9° angle with moderate resolution.

B. DOA Estimation of closely spaced coherent signal with 5° resolution

Consider the antenna array of 10 elements, the two closely spaced coherent signals arriving from 20° and 25°, SNR is assumed to be low at -4dB, and a number of snapshots to be 100. Fig. 7 shows that the sharp peak with better accuracy and resolution achieved at the given angle using CW-PCA MUSIC algorithm compared to other algorithms. Basic MUSIC algorithm cannot distinguish precisely the closely spaced signals. The output SNR of CW-PCA MUSIC algorithm better than DW-MUSIC and CW-MUSIC algorithms. The lower order of magnitude spectrum precisely detected by proposed algorithm compared to other algorithms.

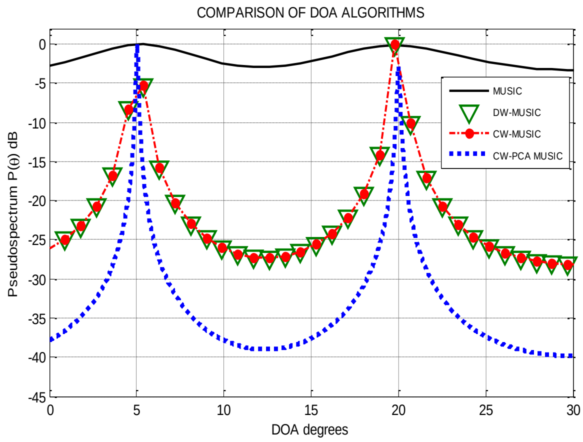

Fig.6. Output spectrum for non-closely coherent signal (SNR=-2dB, N=100, M=10, $\theta_1=5^0$, $\theta_2=20^0$)

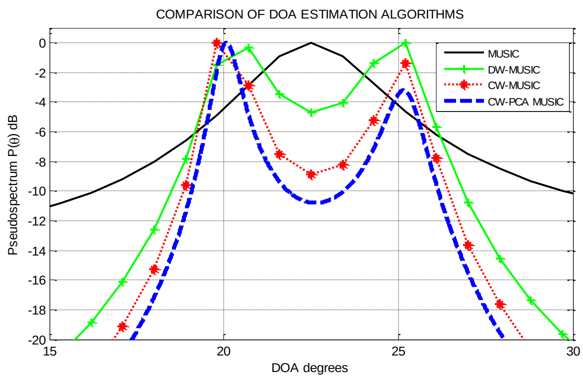

Fig.7. Output spectrum for closely spaced coherent signal (SNR=-4dB, N=100, M=10, $\theta_1=20^0$, $\theta_2=25^0$)

C. DOA Estimation of closely spaced coherent signal with 4^0 resolution

Consider the antenna array of 10 elements, the two closely spaced coherent signals arriving from 20° and 24°, SNR is assumed to be low at -4dB, and a number of snapshots to be

100. Fig. 8 shows that the maximum peak with good resolution achieved at given angle using CW-PCA MUSIC algorithm compared to other algorithms. The CW-MUSIC and DW-MUSIC algorithms fail to detect 24° signal accurately. Also, output SNR of the proposed algorithm is better than other algorithms and resolves the signal with 4° resolution.

Figure 9 shows the results for SNR further reduced to low at -6dB by keeping rest of the parameters same. The DW-MUSIC and CW-MUSIC algorithm fail to detect the given angle and signals not resolved for such low SNR. The proposed CW-PCA MUSIC algorithm detects the given angle with reasonable 4° resolution under a highly noisy environment with high SNR output.

Similarly, Fig. 10 shows the result for SNR = -6dB with 5° resolution of signals keeping rest of the parameters same. It has been observed that DW-MUSIC and CW-MUSIC algorithms fail to detect the given angle even for 5° resolution of the signal. The proposed CW-PCA MUSIC algorithm detects the given angle with a reasonable resolution for such highly noisy environment.

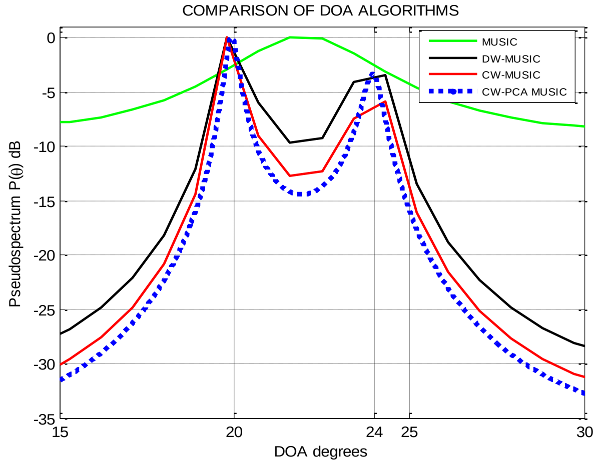

Fig.8. Output spectrum for closely spaced coherent signal (SNR=-4dB, N=100, M=10, $\theta_1=20^0$, $\theta_2=24^0$)

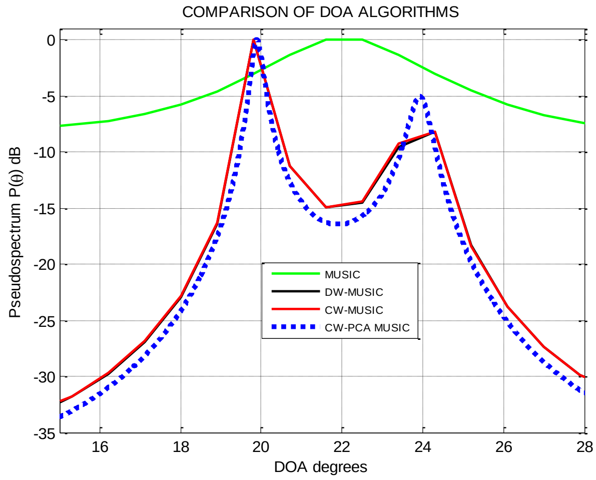

Fig.9. Output spectrum for closely spaced coherent signal (SNR = -6dB, N = 100, M = 10, $\theta_1 = 20^0$, $\theta_2 = 24^0$)

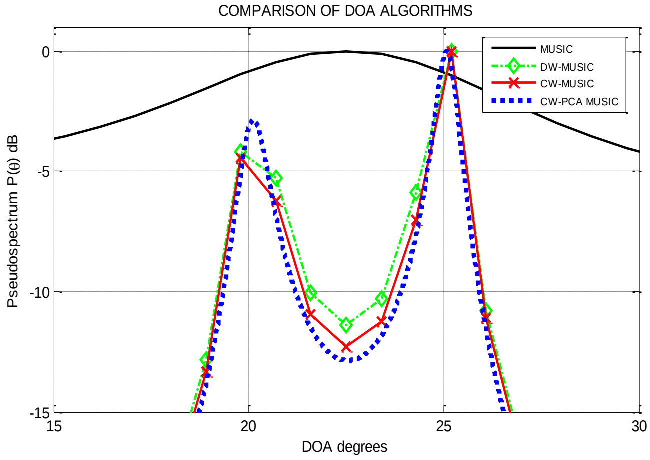

Fig.10. Output spectrum for closely spaced coherent signal (SNR=-6dB, N=100, M=10, $\theta_1=20^0$, $\theta_2=25^0$)

The graph of Root Means Square Error (RMSE) plotted against a number of snapshots and SNR describes the performance of the algorithms in terms of error. The RMSE is given by,

$$RMSE(\deg ree) = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\bar{\theta}_n - \theta_n\right)^2} \qquad (20)$$

Where N is the number of times Monte Carlo Trial taken, $\bar{\theta}_n$ is the estimated angle, and θ_n is the given angle to be estimated. For N = 500, SNR and number of snapshots plotted against the RMSE. Fig. 11 shows the RMSE versus SNR with snapshots set to be 100 and antenna array elements to be 10. Fig. 12 shows the RMSE versus Number of Snapshots with a number of antenna elements set to be 10 and SNR to -4dB.

From Fig. 11 and Fig.12, it has been observed that by increasing the SNR and number of snapshots error starts decreasing. The proposed algorithm gives lower RMSE compared to DW-MUSIC and CW-MUSIC algorithms. The proposed technique resolves the closely spaced coherent signal efficiently. It indicates that proposed method works for poor environment condition, i.e., it has a low SNR, less number of snapshots with moderate antenna array elements.

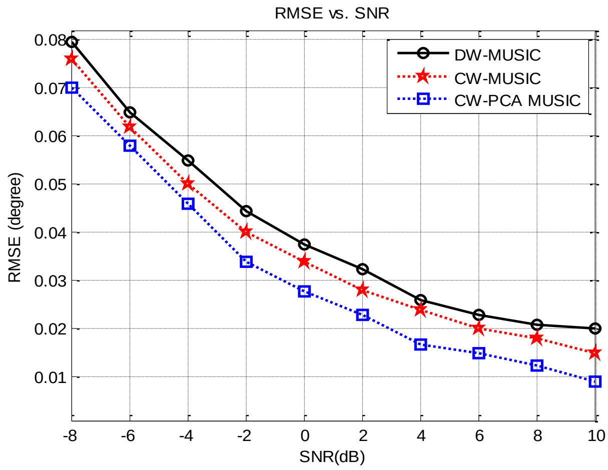

Fig.11. RMSE vs. SNR (Snapshots = 100, Antenna elements = 10)

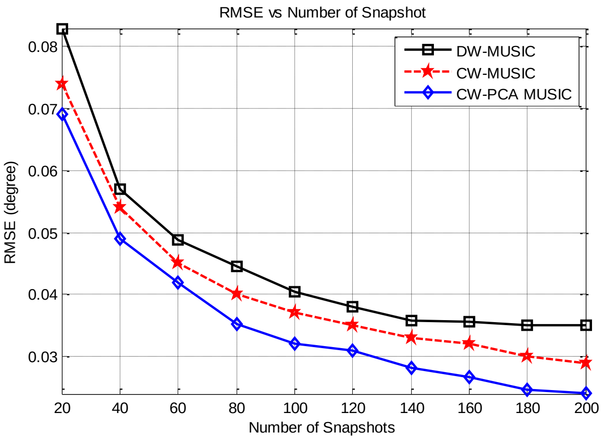

Fig.12. RMSE vs. Number of Snapshots (SNR = -4dB, Antenna elements = 10)

VI. Conclusion

In this paper, the application of DTCWT and PCA for closely spaced coherent signals DOA estimation using MUSIC algorithm is investigated. The DTCWT after decomposition produces the correlated complex wavelet coefficients in the nearby region which are prone to noise generation in the reconstruction process of the original signal. Therefore, the use of PCA for thresholding the correlated complex wavelet coefficients in detail and approximation signal after decomposition of the signal is explored. The DWT based MUSIC (DW-MUSIC), and DTCWT based (CW-MUSIC) algorithms do not work for closely spaced coherent signals with 4^0 resolution for low SNR equal to -6dB. The proposed technique using DTCWT and PCA based MUSIC (CW-PCA MUSIC) algorithm performing better with low SNR, less number of snapshots, and a minimum number of antenna elements. It distinguishes closely spaced coherent signal with a 4^0 resolution correctly in comparison with algorithms mentioned above. DTCWT and PCA based techniques improve the SNR of a noisy signal, which in turn estimated the DOA correctly. Wavelet denoising techniques help in reducing the MSE of the array data covariance matrix and estimate the DOA of desired signals.

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