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Passivity-based control of single-phase multilevel grid connected active rectifiers

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Abstract. The use of the passivity-based control (PBC) properly fits stability problems related to multilevel converters. Two approaches for the PBC design have been proposed and will be reviewed in the present paper. Particularly the second is developed by splitting the system into n subsystems and controlling them independently. The partition of the multilevel converter is done on the basis of energy considerations. The main advantage of the second approach is the separate control of the different DC-links and a flexible loading capability.

Key words: H-bridge, passivity based control, PWM technique.

1. Introduction

The single-phase voltage source converter (VSC) also called H-bridge or full bridge can be used as universal converter due to the possibility to perform dc/dc, dc/ac or ac/dc conversion [1]. Moreover it can be used as basic cell of the cascade multilevel converters [2,3]. The control of this kind of converter has been object of many scientific studies. Among the control theories, the passivity based approach [4] has been extensively investigated both for single-phase and multilevel configurations and will be the object of the present paper [5–7].

The PBC shapes the energy of the system and its variations according to the desired state trajectory [4]. If the controller is designed aiming at obtaining the minimum energy transformation, optimum control is achieved. A stable system can be obtained only by indirectly controlling the dc voltages through the ac current i^* and an internally generated variable x_d. As shown in Fig. 1 the PBC can be summarised in three main points: 1) the calculation of the reference grid current i^* on the basis of a power balance equation developed considering the reference dc-link voltage V_d and the load conductance θ; 2) a differential equation conditioned by a damping term (ODE) that is responsible of the evolution of the internally generated variable x_d; 3) an algebraic equation consisting of a predictive action conditioned by a damping term that allows the calculation of the average switching function of the converter S.

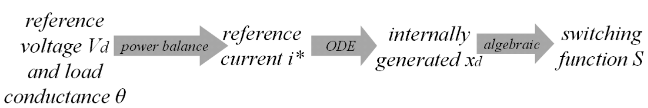

Fig. 1. Principle of operation of a passivity-based controller

The PBC has been successfully applied to the control of a single-phase H-bridge grid connected [5]. The design of the controller is reported in Fig. 2.

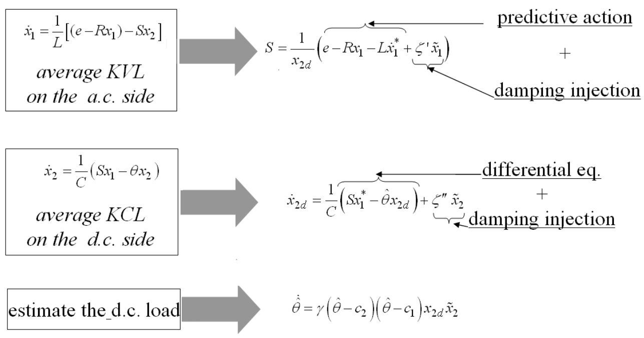

Fig. 2. Principle of operation of a passivity-based controller applied to the control of an H-bridge grid connected

However it is not possible a simple extension of the PBC of an H-bridge active rectifier to an n-H-bridge active rectifier [2–3] (Fig. 3). In fact the PBC of an H-bridge active rectifier needs one algebraic equation and one differential equation. Thus "a simple extension" of this control needs n algebraic equations and n differential equations. However this is not possible since the n H-bridges are connected in series on the grid side and the ac voltage equation results in only one algebraic equation.

Two approaches for the PBC design have been considered: the first is developed using the overall multilevel converter model [6]; the second is developed by splitting the system into n subsystems and controlling them independently [7]. As it regards the second, the partition of the multilevel converter is done on the basis of energy considerations. The main advantage of the second approach is the separate control of the different DC-links and a flexible loading capability.

The two PBC approaches will be denoted as:

– NI-PBC (non-independent control of the H-bridges), based on one algebraic eq. plus n differential eq. (with $x_{2d,1} = \ldots = x_{2d,n}$) [6];
– I-PBC (independent control of the H-bridges), based on n

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algebraic eq. (based on n virtual KVL's) plus n differential eq. (with $x_{2d,1} \neq \ldots \neq x_{2d,n}$) [7].

Both of them are discussed and compared in the present paper.

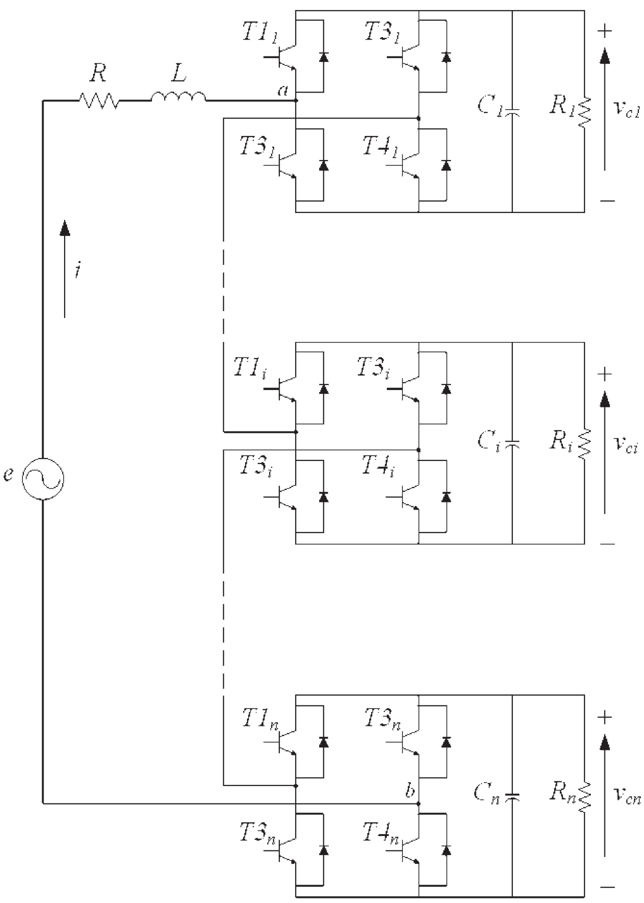


Fig. 3. Scheme of the n-H-bridges multilevel active rectifier

2. Mathematical model of the multilevel rectifier

As shown in Fig. 3 the ac-sides of the H-bridge converters are connected in series, hence the ac voltage applied to the points "a" and "b" is shared between them and the voltage e can be n times higher that the one applicable to each bridge.

The multilevel behaviour can be described by means of a discrete switching function $P_i(i = 1, 2, \ldots, n)$ for each H-bridge

$$P_i = T1_i \cdot T4_i - T2_i \cdot T3_i \tag{1}$$

where TJ_i is the switch state of the transistor $TJ(J = 1, 4)$ for the i-th H-bridge. It is worth noting that the couple of switches of each leg are complementary driven thus

$$P_i = T1_i - T3_i. \tag{2}$$

From (2) it is clear that, while TJ_i can assume the states 0 or 1, P_i can assume the states -1, 0 or 1.

According to the state of the power switches system behaviour can be fully described by the following differential equations for $i = 1, 2, \ldots, n$

$$\dot{x}_1 = \frac{1}{L}\left[(e - Rx_1) - \sum_{i=1}^{n} P_i x_{2,i}\right] \tag{3}$$

$$\dot{x}_{2,i} = \frac{1}{C_i}\left(P_i x_1 - \theta_i x_{2,i}\right) \tag{4}$$

where x_1 is the input inductor current, $x_{2,i}$ are the voltages across the capacitors C_i respectively, L, R are the inductance and the resistance of the coupling inductor and θ_i are the output load conductances for each bridge. Finally $e = E\sin(\omega t)$ is the ac-line source voltage.

3. Passivity-based control

The basic idea of the PBC is to use the energy to describe the state of the system. A straightforward extension leads to view the energy variations as describing the system dynamic behaviour.

Since the main goal of any controller is to feed a dynamic system through a desired evolution as well as to guarantee its steady state behaviour, an energy-based controller shapes the energy of the system and its variations according to the desired state trajectory. If the controller is designed aiming at obtaining the minimum energy transformation, optimum control is achieved. Thus, in the following an energy function is introduced, then it is used to design the controller in view of fulfilling the desired goals.

The main objective of the multilevel active rectifier control strategy is to maintain each capacitor voltage at a specified constant value, even if the working conditions change, in addition a sinusoidal current with unity power factor is highly desired.

It is possible to prove that system stability can only be achieved by indirectly controlling the dc voltages through the ac current [4]. Then, in the proposed system, the ac current is controlled by calculating a suitable ac-side voltage. Thus in order to develop a passivity-based controller both internally generated variable (indicated with subscript "d") and externally fixed reference (indicated with "*") are needed [4].

4. Non independent PBC of multilevel converters

4.1. Indirect control of the overall output voltage. The indirect control of the output voltage is based on the calculation of the current reference amplitude through the power balance. Thus an adaptive law is needed to estimate the output load.

The stable equilibrium can be obtained choosing the inductor current in (3) and (4) such as

$$x_{1d} = x_1^* = I_d \sin(\omega t). \tag{5}$$

The capacitor voltages $x_{2,i}$ are indirectly controlled once the proper value of I_d is calculated as described in [4] considering the steady-state input-output power balance

$$P_{in} = \frac{1}{2}(I_d E - R I_d^2) = P_{out} = V_d^2 \sum_{i=1}^{n} \theta_i. \tag{6}$$

To simplify the analysis, it is possible to neglect in (6) the coupling inductor power losses.

The proper value of I_d is given by

$$I_d = \frac{2V_d^2 \sum_{i=1}^{n} \theta_i}{E} \quad (7)$$

Since in most applications the output load is unknown, it is necessary its estimation. The adaptive laws (one for each load) have been chosen as:

$$\dot{\hat{\theta}}_i = \gamma_i \left(\hat{\theta}_i - c_{2,i}\right)\left(\hat{\theta}_i - c_{1,i}\right) x_{2d,i}\tilde{x}_{2,i}, \quad c_{1,i} \leqslant \hat{\theta}_i(0) \leqslant c_{2,i} \quad (8)$$

where $x_{2d,i}$ is an internally generated variable associated to the capacitor voltage state variable $x_{2,i}$ and $\tilde{x}_{2,i}$ is the error between $x_{2d,i}$ and $x_{2,i}$. Further details can be found in [6].

Moreover $c_{1,i}, c_{2,i} \in \Re_+$ and $\gamma_i \in \Re_+$ are the adaptive gains set such as a good convergence rate to the exact value is achieved. It is important to notice that in order to use the adaptive law (8) the estimate of θ_i has to be bounded inside the range $[c_{1,i}; c_{2,i}]$; moreover (8) can be used only if θ_i is sufficiently > 0, this means that the proposed adaptive law does not work in no-load condition.

4.2. Necessary condition for NI-PBC design. In the design of the first controller the output load conductances θ_i are assumed to be unknown and equal

$$\theta_1 = \theta_2 = \ldots = \theta_n = \theta. \quad (9)$$

Thus from (7) I_d is chosen as

$$I_d = \frac{2nV_d^2\theta}{E}. \quad (10)$$

Moreover, since the load conductances have been assumed equal also the internally generated variables for the DC voltage control are equal $x_{2d,i} = x_{2d}$ and the adaptive law is unique

$$\dot{\hat{\theta}} = \gamma\left(\hat{\theta} - c_2\right)\left(\hat{\theta} - c_1\right) x_{2d}\tilde{x}_2, \quad c_1 \leqslant \hat{\theta}(0) \leqslant c_2. \quad (11)$$

4.3. NI-PBC formulation. The first controller is defined by the following set of $n+2$ equations

$$\begin{cases} \sum_{i=1}^{n} S_i = \dfrac{1}{x_{2d}}\left(e - Rx_1 - L\dot{x}_1^* + \zeta'\,\tilde{x}_1\right) \\ \dot{x}_{2d} = \dfrac{1}{C_i}\left(S_i x_1^* - \hat{\theta}x_{2d}\right) + \zeta_i''\,\tilde{x}_{2,i} \quad \text{for} \quad i = 1, 2, \ldots, n \\ \dot{\hat{\theta}} = \gamma\left(\hat{\theta} - c_2\right)\left(\hat{\theta} - c_1\right) x_{2d}\tilde{x}_2 \end{cases} \quad (12)$$

with $x_{1d} = x_1^* = I_d \sin(\omega t)$ and ζ', ζ_i'' damping coefficients.

5. Independent PBC of multilevel converters

5.1. Mathematical model formulation in subsystems. As shown in Fig. 3 the ac-sides of the H-bridge converters are connected in series and it is possible to schematise this system as shown in Fig. 4. The scheme shown in Fig. 4 is divisible in n separated subsystems in order to control each H-bridge independently.

Considering

$$v_{ai} - v_{a(i+1)} = \beta_i e \quad (13)$$

it is possible to obtain the subsystems of Fig 5.

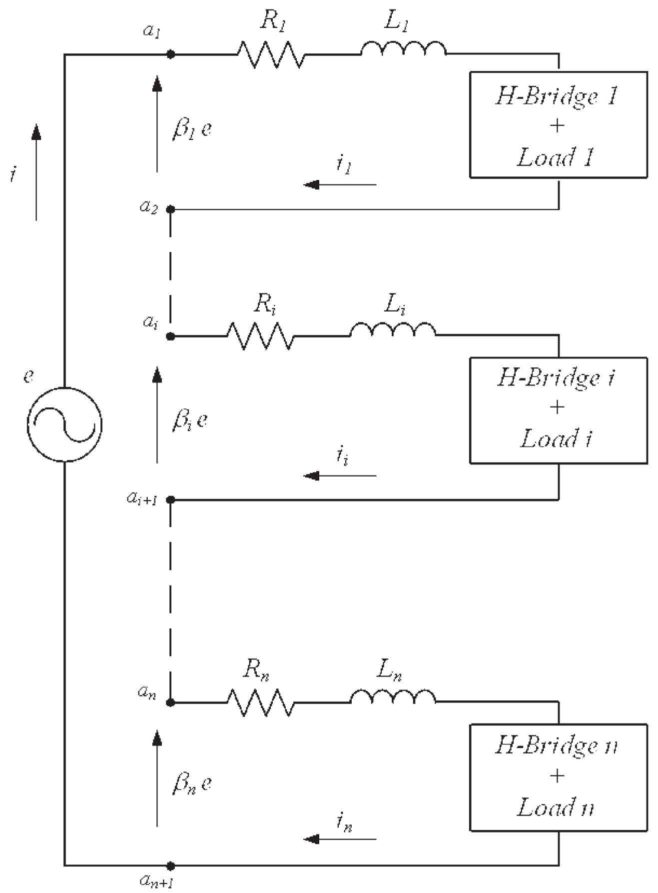


Fig. 4. Simplified scheme of the n-H-bridges multilevel active rectifier

In order to guarantee that the systems shown in Fig. 4 and in Fig. 5 are equivalent it is necessary to impose the following conditions:

$$i_1 = i_2 = \ldots. = i_n \quad (14)$$

$$\sum_{i=1}^{n} \beta_i = 1 \quad (15)$$

where β_i are dynamic parameters responsible for the power balancing among the H-bridges.

According to the state of the power switches, the system behaviour can be fully described by the following differential equations (two for each H-Bridge):

$$\begin{cases} L_i\dot{x}_1 = \beta_i e - R_i x_1 - P_i x_{2,i} \\ C_i\dot{x}_{2,i} = P_i x_1 - \theta_i x_{2,i} \end{cases} \quad (16)$$

where $x_1 = i_1 = i_2 = \ldots = i_n$ is the input current, $x_{2,i}$ are the voltages across the capacitors C_i respectively, $L_i = L/n$ and $R_i = R/n$ are the partial inductance and the partial resistance of the coupling inductor.

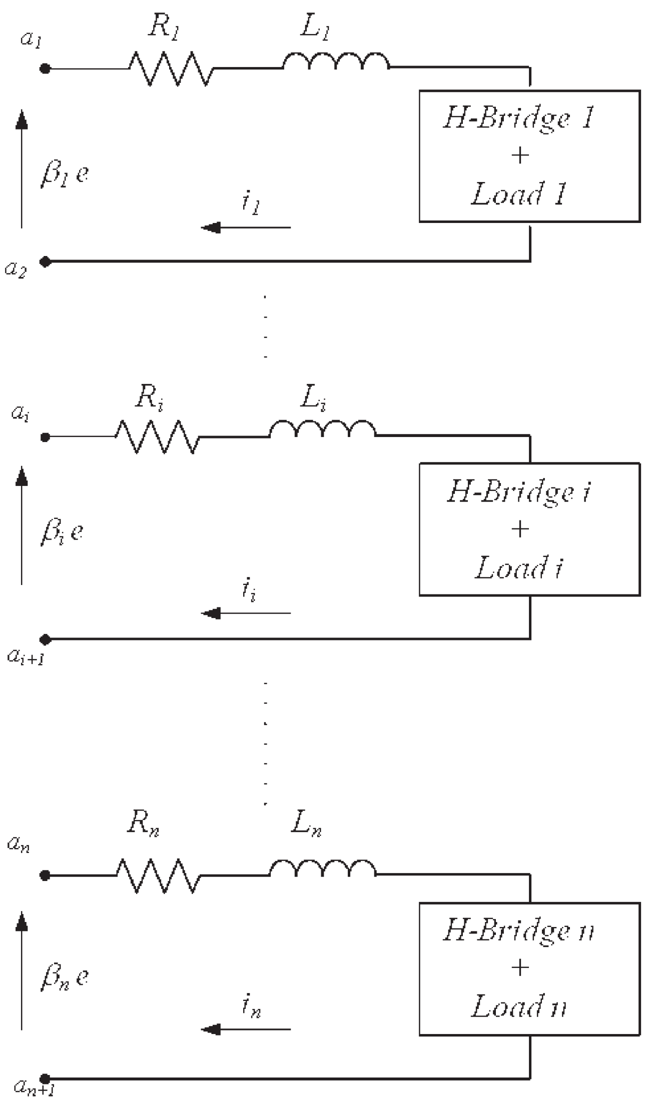


Fig. 5. Scheme of the n-H-bridges multilevel active rectifier divided in subsystems

5.2. Average model of each subsystem. A suitable PWM technique should be able to preserve both the information about each continuous switching function S_i and a multilevel waveform on the ac side. The modulation technique proposed in [3] fulfils the above mentioned requirements and therefore allows a separate control of the H-bridges.

It is possible to prove that the continuous average model of the considered converter is simply carried out by replacing the discrete control inputs with the continuous switching functions.

If the switching frequency is suitable higher than the double of maximum frequency to be controlled, an average model can effectively replace the discrete one expressed by (16) for analysis and control purpose.

From (16) the multilevel rectifier average model can be obtained

$$\begin{cases} L_i\dot{x}_1 = \beta_i e - R_i x_1 - S_i x_{2,i} \\ C_i\dot{x}_{2,i} = S_i x_1 - \theta_i x_{2,i} \end{cases} \tag{17}$$

where the discrete switching functions $P_i \in \{-1,\ 0,\ 1\}$ have been replaced by the continuous ones $S_i \in [-1,\ 1]$.

The passivity-based controllers given by (14), directly balances the DC voltages by separately selecting the continuous switching functions S_i.

The continuous time approximation (17) of the PWM regulated system (16) has been used to transform a discrete-time non-linear control synthesis problem into an equivalent continuous time non linear control problem. Once the continuous time control problem has been solved for the average approximation, i.e. a suitable continuous time expression for the switching functions has been found, then the obtained solution can be made discrete again through the above mentioned modulation technique.

5.3. Indirect control of each output voltage. The control system has been designed using the PBC approach [4]. The main control task is to regulate independently each capacitor voltage obtaining input currents with unity power factor and ensuring system stability.

The multilevel ac-side voltage is the sum of the ac voltages generated by each H-bridge and determined in order to balance the capacitors:

$$v_{a(n+1)} - v_{a1} = \sum_{i=1}^{n} P_i V_{ci}. \tag{18}$$

The stable equilibrium can be obtained choosing the inductor current in (17) such as

$$x_{1d} = x_1^* = I_d \sin(\omega t) \tag{19}$$

The capacitor voltages x_{2i} are indirectly controlled once the proper value of I_d is calculated as described in [7] considering the steady-state input-output power balance of each subsystem

$$P_{in,i} = \frac{1}{2}(I_{d,i}\beta_i E - R_i I_{d,i}^2) \tag{20}$$

$$P_{out,i} = V_{d,i}^2\theta_i \tag{21}$$

$$P_{in,i} = P_{out,i}. \tag{22}$$

Neglecting R_i in (20) it is possible to obtain

$$I_{d,i} = \frac{2V_{d,i}^2\theta_i}{\beta_i E}. \tag{23}$$

Considering that the H-bridges are connected in series

$$I_{d,i} = I_{d,i+1} \tag{24}$$

it is possible to obtain

$$\beta_i = \frac{V_{d,i}^2\theta_i}{V_{d,i+1}^2\theta_{i+1}}\beta_{i+1}. \tag{25}$$

From (15) and (25) it is possible to obtain

$$\beta_i = \frac{V_{d,i}^2\theta_i}{\sum\limits_{i=1}^{n} V_{d,i}^2\theta_i}. \tag{26}$$

Since the output load is unknown the adaptive law (8) has been chosen.

5.4. Control of the errors dynamic. In order to derive an expression for the PBC controllers an auxiliary system can be defined as

$$\begin{cases} \dot{x}_{1d} = \dfrac{1}{L_i}(\beta_i e - R_i x_1 - S_i x_{2d,i} + \zeta_i' \tilde{x}_1) \\ \dot{x}_{2d,i} = \dfrac{1}{C_i}\left(S_i x_{1d} - \hat{\theta}_i x_{2d,i}\right) + \zeta_i'' \tilde{x}_{2,i} \end{cases} \tag{27}$$

where $\hat{\theta}_i$ is an estimate of θ_i and $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$. Thus the error dynamics are given by the following set of equations

$$\begin{cases} \dot{\tilde{x}}_1 = \dfrac{1}{L_i}(-S_i \tilde{x}_{2,i} + \zeta_i' \tilde{x}_1) \\ \dot{\tilde{x}}_{2,i} = \dfrac{1}{C_i}\left[S_i \tilde{x}_1 - \theta_i \tilde{x}_{2,i} + \tilde{\theta}_i x_{2d,i}\right] - \zeta_i'' \tilde{x}_{2,i} \end{cases}. \tag{28}$$

It is possible to consider the (partial) Lyapunov function to prove the stability of the closed-loop system

$$\tilde{H} = \sum_{i=1}^{n} \tilde{H}_i = \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{L_i}{2}\tilde{x}_1^2 + \dfrac{C_i}{2}\tilde{x}_{2i}^2 + \\ +\dfrac{1}{\gamma_i} \log \left| \dfrac{\left(\hat{\theta}_i - c_{2,i}\right)^{\tau_i}}{\left(\hat{\theta}_i - c_{1,i}\right)^{\tau_i+1}} \right| \end{array} \right] \tag{29}$$

where $\tau_i = (\theta_i - c_{2,i}) \,/\, (\theta_i - c_{1,i}) < 0$.

The time derivative is given by

$$\begin{aligned} \dot{\tilde{H}} &= \sum_{i=1}^{n} \dot{\tilde{H}}_1 \\ &= \sum_{i=1}^{n} \left[\begin{array}{l} -\zeta_i' \tilde{x}_1^2 - (\theta_i + C_i \zeta_i'')\, \tilde{x}_{2,i}^2 + \tilde{\theta}_i x_{2d,i} \tilde{x}_{2,i} + \\ -\dfrac{\tilde{\theta}_i \dot{\hat{\theta}}_i}{\gamma_i \left(\hat{\theta}_i - c_{2,i}\right)\left(\hat{\theta}_i - c_{1,i}\right)} \end{array} \right] \end{aligned} \tag{30}$$

since (30) is seminegative definite, the system is Lyapunov asymptotically stable.

5.5. I-PBC formulation. The second passivity based controller is defined by the following set of 3n equations

$$\begin{cases} S_i = \dfrac{1}{x_{2d,i}}\left(\beta_i e - R_i x_1 - L_i \dot{x}_{1d} + \zeta_i' \tilde{x}_1\right) \text{ for } i = 1, 2, \ldots, n \\ \dot{x}_{2d,i} = \dfrac{1}{C_i}\left(S_i x_{1d} - \hat{\theta}_i x_{2d,i}\right) + \zeta_i'' \tilde{x}_{2,i} \text{ for } i = 1, 2, \ldots, n \\ \dot{\hat{\theta}}_i = \gamma_i \left(\hat{\theta}_i - c_{2,i}\right)\left(\hat{\theta}_i - c_{1,i}\right) x_{2d,i} \tilde{x}_{2,i} \text{ for } i = 1, 2, \ldots, n \end{cases} \tag{31}$$

with $x_{1d} = x_1^* = I_d \sin(\omega t)$ and ζ_i', ζ_i'' damping coefficients.

6. Experimental results

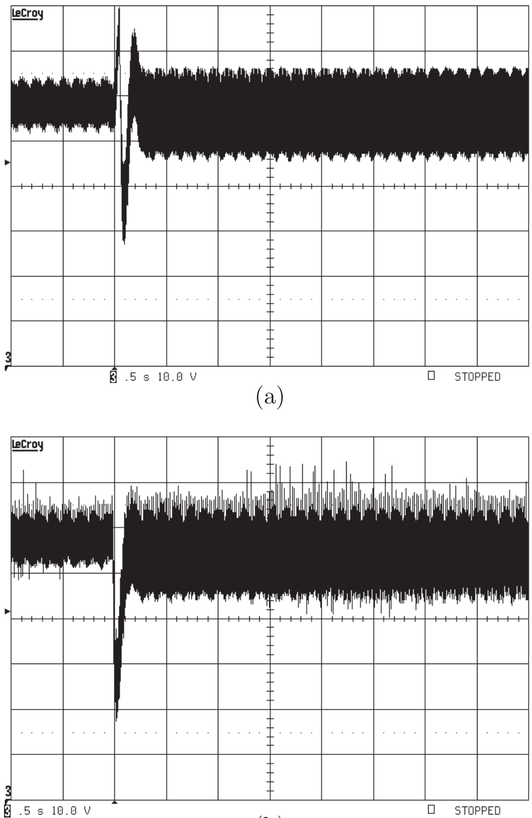
The laboratory set-up (Tab. 1) consists of two three-phase converters (Danfoss VLT® 5006–7.6 KVA) where only two legs of each converter are employed. An interface card (developed by Aalborg University) performs different protections: over temperature, over current, inverter-leg-shoot through, dc over voltage. All the logical functions are implemented on a PLD (Programmable Logic Device). Optical fibers have been used to transfer the switching signals from the controller to the interface card. The use of optic fibers guarantees galvanic isolation between the control unit and the power section and allows high noise immunity. Moreover an acquisition board scales and filters the feedback signals, i.e. the grid voltage and current, as well as the voltages across the capacitors, in order to match the microcontroller requirements.

Table 1
System parameters

Rated rms grid voltage single-phase	230 V @ 50 Hz
ac inductance	10 mH
dc capacitors ($C_1 = C_2$)	2330 µF / 330 µF
Rated load power	1 kW

The complete control algorithm has been implemented using DS1104 R&D Controller Board from dSPACE®. All the reported results have been obtained considering a sampling frequency of 10 kHz and low-pass filters (5 kHz) on the feedback signals. The controller damping parameters have been set to $\zeta' = 66$, $\zeta_1'' = \zeta_2'' = 1$ for NI-PBC and to $\zeta_1' = \zeta_2' = 33$, $\zeta_1'' = \zeta_2'' = 1$ for I-PBC.

Figure 6 allows evaluating the better performance of the I-PBC respect to the NI-PBC in terms of dynamic behaviour due to a load step change on both the dc-links. The NI-PBC suffers a longer transient with higher overshoot due to the fact that the two dc-links dynamics are constrained by the same internally generated voltage variable $(x_{2d,1} = x_{2d,2})$.



(a)

(b)

Fig. 6. Measured dc voltages [10 V/div] consequent to a load step change from half to full load on both the dc-links: (a) NI-PBC, (b) I-PBC (330 µF)

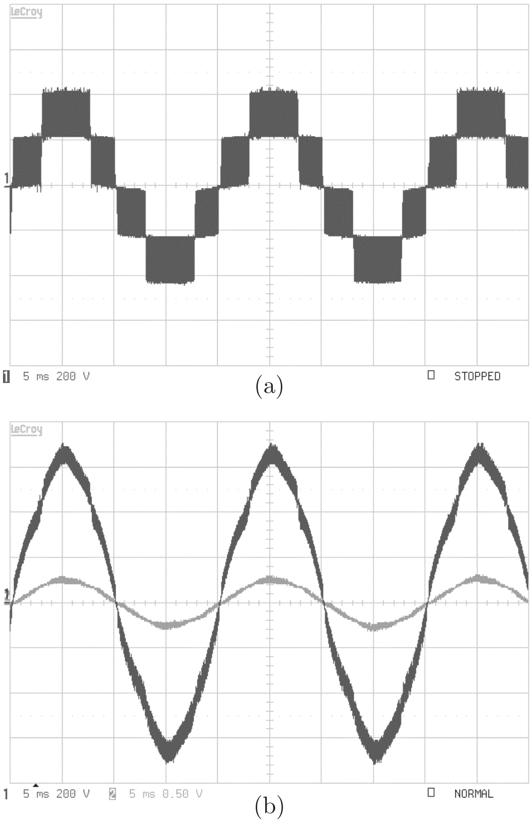


Fig. 7. Steady-state behaviour of I-PBC with full load on dc bus 1 and half load on dc bus 2: (a) multilevel ac voltage [200 V/div], (b) grid voltage [100V/div] grid current [10 A/div]

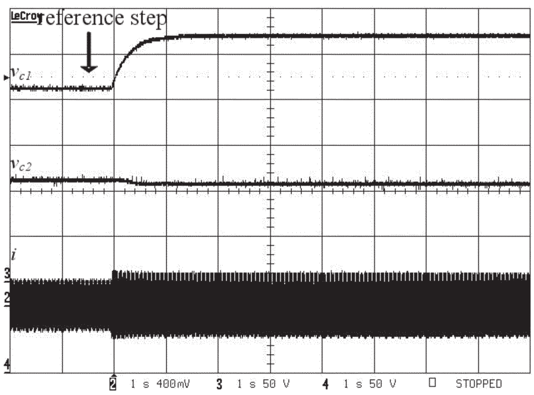


Fig. 8. Measured dc voltages [50 V/div] and grid current [4 A/div]: dc voltage reference step for one of the two H-bridges (I-PBC) (2330 μF)

Once proven that the I-PBC has a better behaviour respect to NI-PBC some tests have been performed in order to fully show its performance.

Figure 7 shows the multilevel ac voltage, grid voltage and grid current for unbalanced loads on the two dc-links.

Figure 8 shows the response to a dc-link reference change for only one H-bridge with the I-PBC since the NI-PBC can not work with two difference references. Moreover Fig. 9 offers a further proof of the complete decoupling of the two H-bridges achieved with I-PBC.

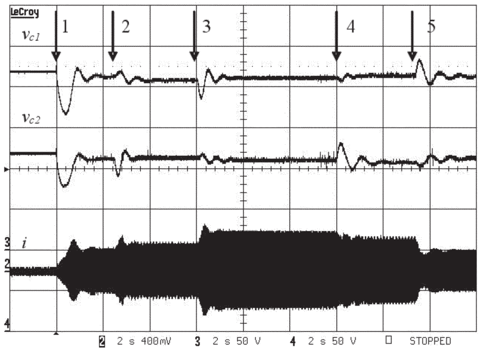


Fig. 9. Measured dc voltages [50 V/div] and grid current [4 A/div]: 1. load step change on both dc-links from no load condition to half load; 2. load step change on the second dc-link from half load to full load; 3. load step change on the first dc-link from half load to full load; 4. load step change on the second dc-link from full load to half load; 5. load step change on the first dc-link from full load to half load; (I-PBC) (2330 μF)

7. Conclusions

In this paper two passivity-based approaches to the control of an H-bridge based active rectifier are discussed. The first results in a constraint on the dc voltages dynamics that does not allow the active rectifier feeding different loads, the second one is based on the partition of the system on the basis of energy considerations and it works well with different dc loads. The two approaches are compared via laboratory results analysis.

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