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Perfect reduced-order unknown-input observer for standard systems

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Abstract. The problem of the design of a perfect reduced-order unknown-input observer for standard systems is formulated and solved. The procedure of designing the observer using well-known canonical form is proposed and illustrated with a numerical example. Necessary and sufficient conditions for the solvability of the procedure are given.

Keywords: perfect observer, unknown-input observer, simulation.

1. Introduction

The problem of observer design for standard systems with unknown inputs has received considerable attention in the last two decades [1–6]. This problem is of great importance in theory and practice, since there are many situations where part of inputs or disturbances are inaccessible.

Recently, a great deal of work has been devoted to the observer design for descriptor systems [7–9], but only in few works the problem of the design of unknown-input observer for descriptor systems [10,11] was considered. Many practical systems can be described by descriptor models, and the fault diagnosis of these systems may be based on the unknown input observer design. Descriptor systems give many not obvious opportunities, one of which is a recently developed new concept of perfect observers [12]. The idea has been extended for standard linear systems [13], singular 2-D linear systems [14] and functional observers [15]. Recently [16] the problem of perfect unknown-input observer for singular systems has been formulated and solved.

In this paper the concept of a perfect reduced-order unknown-input observer is extended for standard systems or in other words the concept of a perfect observer for standard linear systems [17] is extended for unknown inputs.

2. Problem formulation

Let $\mathbb{R}^{n\times m}$ be the set of $n \times m$ real matrices and $\mathbb{R}^n = \mathbb{R}^{n\times 1}$.

Consider the continuous-time linear system

$$\begin{aligned} \dot{x} &= Ax + Bu + Dv \\ y &= Cx \end{aligned} \tag{1}$$

where $\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t}$, $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^q$ — input vector, $v \in \mathbb{R}^m$ — unknown input (disturbance) vector, $y \in \mathbb{R}^p$ — output vector and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times q}$, $D \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$. The initial condition for (1) is given by x_0.

It is assumed that $\operatorname{rank} C = p < n$. It is not a strong assumption, because we can always eliminate linearly dependent outputs or in case $\operatorname{rank} C = n$ find x from y using $x = C^{-1}y$. For the same reasons it is assumed $\operatorname{rank} D = m$.

We are looking for an r order observer of the form

$$\begin{aligned} E_1\dot{z} &= Fz + Gu + Hy \\ \hat{x} &= Pz + Qy \end{aligned} \tag{2}$$

that for $t > 0$ reconstructs exactly semi-state vector x without knowledge of v, where $z \in \mathbb{R}^r$ is observer state vector, $\hat{x}$ is the estimate of x, $E_1, F \in \mathbb{R}^{r\times r}$, $\det E_1 = 0$, $G \in \mathbb{R}^{r\times q}$, $H \in \mathbb{R}^{r\times p}$, $P \in \mathbb{R}^{n\times r}$ and $Q \in \mathbb{R}^{n\times p}$. The initial condition for (2) is given by $\bar{x}_0$ and in general is different from x_0.

Let $e \in \mathbb{R}^r$ be the observer error and

$$e = z - Tx \tag{3}$$

where $T \in \mathbb{R}^{r\times n}$. Differentiating (3) with respect to time and using (1) and (2) we get

$$\begin{aligned} E_1\dot{e} &= E_1\dot{z} - E_1T\dot{x} \\ &= Fz + Gu + HCx - E_1TAx - E_1TBu - E_1TDv \\ &= Fz - FTx + FTx + HCx + Gu - E_1TAx \\ &\qquad - E_1TBu - E_1TDv \\ &= F(z - Tx) + (FT - E_1TA + HC)x + \\ &\qquad + (G - E_1TB)u - E_1TDv \end{aligned}$$

If

$$E_1TB = G \tag{4}$$

$$FT - E_1TA + HC = 0 \tag{5}$$

$$E_1TD = 0 \tag{6}$$

then

$$E_1\dot{e} = Fe. \tag{7}$$

Note that

$$\begin{aligned} \hat{x} - x &= Pz + QCx - x \\ &= Pz + QCx + PTx - PTx - x \end{aligned}$$

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$$= P(z - Tx) + (QC + PT - I_n x$$
$$= P(z - Tx) = Pe$$

if

$$PT + QC = [\,P \quad Q\,]\begin{bmatrix} T \\ C \end{bmatrix} = I_n. \tag{8}$$

It is possible to show [17], that if

$$\det(E_1 s - F) = \alpha \neq 0, \tag{9}$$

where α does not depend on s, then error e is equal to zero for all $t > 0$.

Problem. Given matrices A, B, C, D. Find E_1, F, G, H, T, P, Q such, that (4), (5), (6), (7) and (8) are satisfied.

3. The main result

The condition (5) can be rewritten as

$$[\,F \quad H\,]\begin{bmatrix} T \\ C \end{bmatrix} = E_1 T A.$$

If $\operatorname{rank} F = r$ then from Sylvester inequality we get $r + n - (r + p) \leqslant rank E_1 T A$, and because $\det E_1 = 0$ the conclusion is $r > n - p$.

Due to the fact, that $\operatorname{rank} E_1 T D = 0$ we get $\operatorname{rank} E_1 T$ $+ m - n \leqslant 0$ and $\operatorname{rank} E_1 T \leqslant n - m$. From this we get $\operatorname{rank} E_1 \leqslant n - m$. Hence we have $p \geqslant m$.

Lemma 1. There exists pair (L, R) of nonsingular matrices that allow us to transform the matrices of the given system (1) to the forms

$$LAR = \tilde{A} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \tag{10}$$
$$CR = \tilde{C} = [\,0 \quad I_p\,],$$
$$LD = \tilde{D} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix},$$

where $A_1 \in \mathbb{R}^{(n-p)\times(n-p)}$, $A_2 \in \mathbb{R}^{(n-p)\times p}$, $A_3 \in \mathbb{R}^{p\times(n-p)}$, $A_4 \in \mathbb{R}^{p\times p}$, $D_1 = [\,I_{n-p} \ 0\,] \in \mathbb{R}^{(n-p)\times m}$, $D_2 = \begin{bmatrix} 0 & I_{m+p-n} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{p\times m}$ if and only if $\operatorname{rank} C = p$ and $\operatorname{rank} D = m$ and $p \leqslant m$.

Proof. There exists a nonsingular matrix R_1, such that $CR_1 = [\,C_1 \quad C_2\,]$, where $C_2 \in \mathbb{R}^{p\times p}$ and $\operatorname{rank} C_2 = p$, if and only if $\operatorname{rank} C = p$. Then

$$CR = CR_1R_2 = [\,C_1 \quad C_2\,]\begin{bmatrix} I_{n-p} & 0 \\ -C_2^{-1}C_1 & C_2^{-1} \end{bmatrix}$$
$$= [\,0 \quad I_p\,].$$

Using similar method with

$$L_2 = \begin{bmatrix} \hat{D}_1^{-1} & 0 \\ -\hat{D}_1^{-1}\hat{D}_2 & I_{n-m} \end{bmatrix}$$

and proper division into blocks we get the forms of D_1 and D_2. The form of $\tilde{A}$ is the consequence of use of above transformations.

The state vector of the system in the canonical form (10) is given by $\tilde{x} = R^{-1}x$.

Using the fact that $p < n$ we can conclude that $\operatorname{rank} D_2$ is not full.

Let $r = 2n - m - p$. Let us choose E_1 and F in forms

$$E_1 = \begin{bmatrix} I_{n-p} & 0 \\ 0 & 0_{n-m} \end{bmatrix}, \quad F = \begin{bmatrix} 0 & I_{n-p} \\ \alpha I_{n-m} & 0 \end{bmatrix}. \tag{11}$$

It is easy to check [17] that such choice satisfies the condition (9).

Let

$$T = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix}$$

where $T_1 \in \mathbb{R}^{(n-m)\times(n-p)}$, $T_2 \in \mathbb{R}^{(n-m)\times(p)}$, $T_3 \in \mathbb{R}^{(n-p)\times(n-p)}$, $T_4 \in \mathbb{R}^{(n-p)\times(p)}$ and

$$X = FT - E_1 T\tilde{A}. \tag{12}$$

It is possible to find H from the equation $HC = -X$ if and only if

$$\operatorname{rank} \tilde{C} = \operatorname{rank}\begin{bmatrix} X \\ \tilde{C} \end{bmatrix}. \tag{13}$$

From (13) it follows that all entries of the first $n - p$ columns of X must be equal to 0.

Let $T = [t_{ij}]$; $i = 1, \ldots, r$; $j = 1, \ldots, n$ and $\tilde{A} = [a_{ij}]$; $i = 1, \ldots, n$; $j = 1, \ldots, n$. Using (11) and (12) we get

$$X = \begin{bmatrix} 0 & I_{n-p} \\ \alpha I_{n-m} & 0 \end{bmatrix}\begin{bmatrix} t_{11} & \ldots & t_{1,n} \\ \vdots & \ddots & \vdots \\ t_{r,1} & \ldots & t_{r,n} \end{bmatrix}$$
$$- \begin{bmatrix} t_{11} & \ldots & t_{1,n} \\ \vdots & \ddots & \vdots \\ t_{n-p,1} & \ldots & t_{n-p,n} \\ 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{bmatrix}\begin{bmatrix} a_{1,1} & \ldots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \ldots & a_{n,n} \end{bmatrix}$$
$$= \begin{bmatrix} t_{n-m+1,1} & \ldots & t_{n-m+1,n} \\ \vdots & \ddots & \vdots \\ t_{2n-m-p,1} & \ldots & t_{2n-m-p,n} \\ \alpha t_{11} & \ldots & \alpha t_{1,n} \\ \vdots & \ddots & \vdots \\ \alpha t_{n-m,1} & \ldots & \alpha t_{n-m,n} \end{bmatrix}$$
$$- \begin{bmatrix} c_{1,1} & \ldots & c_{1,n} \\ \vdots & \ddots & \vdots \\ c_{n-p,1} & \ldots & c_{n-p,n} \\ 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{bmatrix} \tag{14}$$

where $c_{i,j} = \sum_{k=1}^{n} t_{i,k}a_{k,j}$.

Because $\alpha \neq 0$, to satisfy (13) and the condition, that first $n - p$ columns of X must be equal to 0, we need $t_{i,j} = 0$ for $i = 1, \ldots, n - m$ and $j = 1, \ldots, n - p$, which is equal to $T_1 = 0$, what implies $\operatorname{rank} D_2 < m$ (what is satisfied due to the canonical form).

From the equation (8) it comes that

$$\operatorname{rank}\begin{bmatrix} T \\ C \end{bmatrix} = \operatorname{rank}\begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \\ 0 & I_p \end{bmatrix} = n. \tag{15}$$

If $T_1 = 0$ then we need $\operatorname{rank} T_3 = n - p$.

Let

$$\bar{T}_2 = \begin{bmatrix} t_{1,n-p+1} & \dots & t_{1,n} \\ \vdots & \ddots & \vdots \\ t_{n-p,n-p+1} & \dots & t_{n-p,n} \end{bmatrix} \in \mathbb{R}^{(n-p)\times p},$$

so it is T_2 without last $p-m$ rows. What we need is $t_{n-m+i,j} = c_{i,j} = \sum_{l=1}^{n} t_{i,l}a_{l,j} = \sum_{l=n-p+1}^{n} t_{i,l}a_{l,j}$ for $i,j = 1,\dots,n-p$ which is equivalent to

$$T_3 = \bar{T}_2 A_3. \tag{16}$$

If, as we show above, $T_1 = 0$ and we have to satisfy the condition (15) then we need rank $T_3 = n - p$. If $p < n - p$ then this cannot be satisfied. If, in opposite case, $p \geqslant n - p$, then because $\bar{T}_2$ was build on the basis of the kernels of D and its rows are linearly independent, then $\bar{T}_2$ has full rank equal to $n - p$ and what we need is rank $A_3 = n - p$.

In this moment it becomes obvious why we have to set $r = 2n - m - p$: it is needed to have $\operatorname{rank} T_3 = n - p$.

Let X_1 be a matrix constructed from column number $n-p+1,\dots,2n-m-p$ of X. Because $HC = H\,[0 \quad I_m] = X = [0 \quad X_1]$ we get

$$H = X_1. \tag{17}$$

From (7) we have

$$R = [P \quad Q]\begin{bmatrix} T \\ C \end{bmatrix} R = [P \quad Q]\begin{bmatrix} TR \\ \bar{C} \end{bmatrix}.$$

Due to the fact that R is a nonsingular matrix it does not change the rank of the matrices it multiplies. So we get

$$[P \quad Q] = R\begin{bmatrix} TR \\ \bar{C} \end{bmatrix}^{+} \tag{18}$$

where $^{+}$ stands for Moore-Penrose's inverse.

From the above considerations we have:

PROCEDURE.

1. Find nonsingular matrices L and R transforming the system (1) to the form (10).
2. Choose E_1 and F according to (11).
3. Choose $T_1 = 0$ and T_2 of $\operatorname{rank} n - m$.
4. Using $t_{i,j}$ found in step 3 and (16) find $t_{i,j}$ ($i = n-m+1,\dots,2n-m-p; j = 1,\dots,n-p$).
5. Take any values as $t_{i,j}$ ($i = n-m+1,\dots,2n-m-p$; $j = n-p+1,\dots,n$) and find G using (4) and H from (17).
6. Find P and Q from the formula (18).

Using (10) we get

$$\begin{bmatrix} L & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} Is - A & D \\ C & 0 \end{bmatrix}\begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} LRs - A_1 & -A_2 & D_1 \\ -A_3 & LRs - A_4 & D_2 \\ 0 & I_p & 0 \end{bmatrix}. \tag{19}$$

At the beginning we assumed $\operatorname{rank} D = m$. Because of the canonical forms of D_1 and D_2, using elementary operations we can eliminate entries depending on s from $LRs - A_1$ using D_1 and from $Is - A_4$ using I_p. Hence

$$\operatorname{rank}\begin{bmatrix} Is - A & D \\ C & 0 \end{bmatrix} = n + m \text{ for all } s \in \mathbb{C}$$

if and only if $\operatorname{rank} A_3 = n - p$.

Another conclusion coming from (19) is that assumption $p \geqslant n - p$ needed for the Procedure is satisfied if $p \geqslant m$ because $p + m \geqslant n$.

Therefore we have proved the following theorem:

THEOREM . The observer (2) may be constructed using the Procedure if and only if the conditions

a) $p \geqslant m$,

b) $\operatorname{rank}\begin{bmatrix} Is - A & D \\ C & 0 \end{bmatrix} = n + m$ for all $s \in \mathbb{C}$,

are satisfied.

4. Example

Find the perfect reduced-order unknown-input observer for the system of the form (1) with

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 1 & 0 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The given system is in the canonical form. It is easy to check that it satisfies the conditions of the Theorem. Number of disturbances is $m = 2$ and number of outputs is $p = 3$ so from (11) we get

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \alpha & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 \end{bmatrix}.$$

According to the third step of the Procedure we choose

$$[T_1 \quad T_2] = \begin{bmatrix} 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 1 \end{bmatrix}.$$

Using (16) we get all other entries of matrix T

$$T = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \tag{20}$$

Using (20) and (12) we obtain

$$X = \begin{bmatrix} 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & 0 & -\alpha \end{bmatrix}. \tag{21}$$

Using (17) and (21) we get

$$H = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -4 & 0 \\ -\alpha & 0 & 0 \\ 0 & -\alpha & 0 \\ 0 & -\alpha & 0 \end{bmatrix}.$$

Using (4) and (20) we obtain

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Using (18) and (20) we get

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \end{bmatrix} \text{ and } Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}.$$

5. Simulations

Simulations were prepared in Matlab and Simulink. Although they are the most sophisticated software available, they cannot deal with simulations of all singular systems [18]. However, the choice of the form of the observer (11) allows us to transform the system (2) into

$$\begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_{n-p} \end{bmatrix} = \begin{bmatrix} z_{n-m+1} \\ \vdots \\ z_{2n-p-m} \end{bmatrix} + G_1 u + H_1 y$$

$$0 = \hat{F} \begin{bmatrix} z_1 \\ \vdots \\ z_{n-m} \end{bmatrix} + G_2 u + H_2 y$$

where $\hat{F} = \alpha I_{n-m}$, $G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$, $G_1 \in \mathbb{R}^{(n-p)\times q}$ and $H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$, $H_1 \in \mathbb{R}^{(n-p)\times p}$. Because $\hat{F}$ is square and nonsingular, we can find $z_1, \ldots, z_{n-m}$ from

$$\begin{bmatrix} z_1 \\ \vdots \\ z_{n-m} \end{bmatrix} = -\hat{F}^{-1}(G_2 u + H_2 y)$$

and using the derivative of just found $z_1, \ldots, z_{n-m}$ find the rest of the vector z with

$$\begin{bmatrix} z_{n-m+1} \\ \vdots \\ z_{2n-p-m} \end{bmatrix} = \begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_{n-p} \end{bmatrix} - G_1 u - H_1 y.$$

The realization of ideal derivative mentioned above is the only problem with simulations in Matlab, but by reducing the solver step size we can make this error negligible.

Then, for the observer constructed in Section 5, with initial conditions $x_0 = [1 \quad 2 \quad 3 \quad -1 \quad -2]^T$ and for $\alpha = 10$, we get:

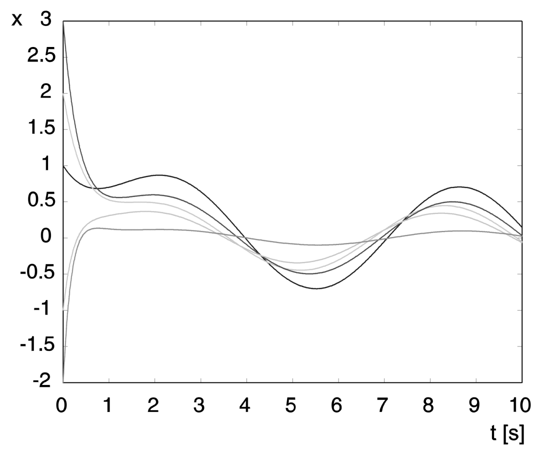

Fig. 1. The state vector of the given system

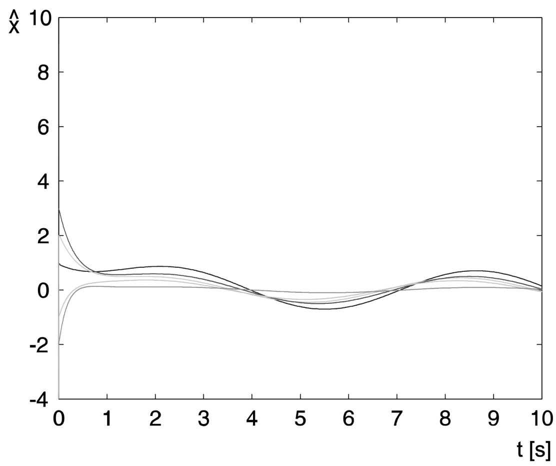

Fig. 2. The estimates (the outputs of the observer)

The Figs. 2–4 show that at $t = 0$, the state variables of the observer change their values impulsively due to the difference between the initial conditions of the system and the observer. This is also the reason for the huge error of the estimation at $t = 0$. But, as it is for perfect observers, for $t > 0$ we have error equal almost 0 (due to the numerical realization of the ideal derivative) — as it can be seen on the Fig. 5.

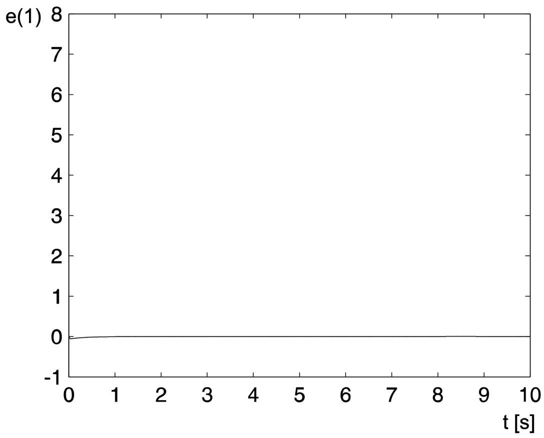

Fig. 3. The error of the estimation of the first state variable

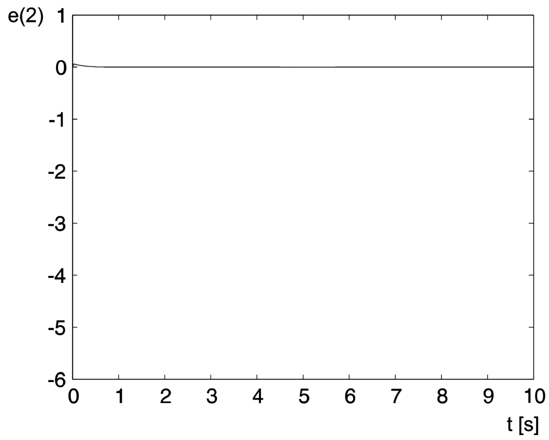

Fig. 4. The error of the estimation of the second state variable

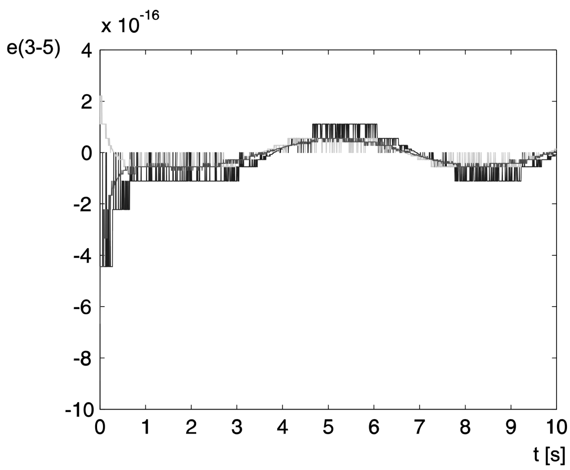

Fig. 5. The error of the estimation of the state variable number 3-5

It is important to notice that choice of α does not influence the result.

6. Conclusions and open problems

The problem of the design of a perfect reduced-order unknown-input observer for standard systems has been formulated and solved. The procedure of designing the observer using well-known canonical form has been proposed. Necessary and sufficient conditions for the solvability of the procedure were given. The method was illustrated by a numerical example and by the plots of the system states, observer outputs and errors.

Necessary and sufficient conditions for the existence of the observer are the most important open problem. Other open problems are extensions of the considerations for 2D systems and for the perfect functional reduced-order unknown-input observers.

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