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Boundary effects on electrothermal convection in a dielectric fluid layer

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Abstract The instability characteristics of a dielectric fluid layer heated from below under the influence of a uniform vertical alternating current (AC) electric field is analyzed for different types of electric potential (constant electric potential/ electric current), velocity (rigid/free) and temperature boundary conditions (constant temperature/heat flux or a mixed condition at the upper boundary). The resulting eigenvalue problem is solved numerically using the shooting method for various boundary conditions and the solution is also found in a simple closed form when the perturbation heat flux is zero at the boundaries. The possibility of a more precise control of electrothermal convection (ETC) through various boundary conditions is emphasized. The effect of increasing AC electric Rayleigh number is to hasten while that of Biot number is to delay the onset of ETC. The system is more stable for rigid-rigid boundaries when compared to rigid-free and least stable for free-free boundaries. The change of electric potential boundary condition at the upper boundary from constant electric potential to constant electric current is found to instill more stability on the system.

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Besides, increase in the AC electric Rayleigh number and the Biot number is to reduce the size of convection cells.

Keywords: Boundary effect; Dielectric fluid; AC electric field; Thermal convection

Nomenclature

a	–	overall horizontal wave number $(= \sqrt{\ell^2 + m^2})$, m^{-1}
Bi	–	Biot number $(= h\,d/k)$
D	–	differential operator $= d/dz$, m^{-1}
d	–	thickness of the dielectric fluid layer, m
$\vec{E}$	–	electric field
E_0	–	root mean square value of the electric field at $z = 0$, NC^{-1}
$\vec{g}$	–	gravitational acceleration due
h	–	heat transfer coefficient, $\mathrm{Wm}^{-2}\mathrm{K}^{-1}$
k	–	thermal conductivity, $\mathrm{Wm}^{-1}\mathrm{K}^{-1}$
ℓ, m	–	horizontal wave numbers in the x and y directions, m^{-1}
p	–	pressure, Nm^{-2}
Pr	–	Prandtl number $(= \nu/\kappa)$
$\vec{q}$	–	velocity vector $(= (u, v, w))$
Ra_e	–	AC electric Rayleigh number $(= \gamma^2 \varepsilon_0 E_0^2 (\Delta T)^2 d^2/\mu\kappa)$
Ra_t	–	thermal Rayleigh number $(= \alpha\, g\, \Delta T d^3/\nu\kappa)$
t	–	time, s
T	–	temperature, K
T_L	–	temperature of the lower boundary, K
T_U	–	temperature of the upper boundary, K
ΔT	–	constant temperature difference $(= T_L - T_U)$, K
V	–	electric potential, V
u, v, w	–	velocity components
W	–	amplitude of vertical component of perturbed velocity, ms^{-1}
(x, y, z)	–	Cartesian co-ordinates

Greek symbols

α	–	thermal expansion coefficient, K^{-1}
ε	–	dielectric constant, Fm^{-1}
ε_0	–	dielectric constant at reference temperature $T = T_L$, Fm^{-1}
$\gamma\ (> 0)$	–	thermal expansion coefficient of dielectric constant, Fm^{-1}
κ	–	thermal diffusivity, $\mathrm{Wm}^{-1}\mathrm{K}^{-1}$
$\nu = \mu/\rho_0$	–	kinematic viscosity, $\mathrm{m}^2\mathrm{s}^{-1}$
μ	–	fluid viscosity, $\mathrm{Kgm}^{-1}\mathrm{s}^{-2}$
ω	–	growth factor
Φ	–	amplitude of perturbed electric potential, V
ρ	–	fluid density, Kgm^{-3}
ρ_0	–	density at reference temperature $T = T_L$, Kgm^{-3}
ρ_e	–	charge density, Cm^{-3}

Θ	–	amplitude of perturbed temperature, K
σ	–	electrical conductivity, sm^{-1}
$\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$	–	horizontal Laplacian operator
$\nabla^2 = \nabla_h^2 + \partial^2/\partial z^2$	–	Laplacian operator

Subscripts

b	–	basic state
c	–	critical
0	–	reference

1 Introduction

Electrohydrodynamics (EHD) is an interdisciplinary science dealing with the interaction of fluids and electric fields or charges and it has been found applications in many areas such as EHD enhanced heat transfer, micro-electromechanical system (MEMS) and some other industrial processes, EHD pump, electrospray mass spectrometry, electrospray nanotechnology, ink-jetting, drug delivery, design of engineering devices ranging from air-craft and space vehicles to microfluidic devices and some other industrial processes [1–5].

One aspect of EHD encompasses, for instance, the influence of the conductivity and/or dielectric permittivity of the fluids on the instability aspects of the flow. The variation of electrical conductivity of the fluid with temperature produces free charges in the bulk of the fluid. These free charges interacting with applied or induced electric field produce a force that eventually causes fluid motion. On the other hand, when there is variation in dielectric permittivity and the electric field is intense then the polarization force which is induced by the non-uniformity of the dielectric constant causes fluid motion. In either case, convection can occur in a dielectric fluid layer due to dielectrophoretic forces even if the temperature gradient is stabilizing and such an instability produced by an electric field is called electroconvection, which is analogous to Rayleigh-Bénard convection. In addition, if the applied temperature gradient is also destabilizing then such an instability problem is called electrothermal convection (ETC).

Incipient interest in theoretical studies of ETC was limited to convection caused by the dielectrophoretic force due to the variation in the dielectric constant or dielectric permittivity with the non-homogeneous temperature gradient in the bulk flow [6-11]. An exhaustive review on this topic has been given by Jones [12] and Saville [13]. The combined effects of direct current (DC) electric field and volumetric heat source on the onset of convection in a dielectric fluid layer heated from below are investigated by Shivakumara *et al.* [14], while the influences of vertical alternating current (AC) electric field as well as internal heat generation on the onset of ETC in a horizontal dielectric fluid layer is analyzed by Shivakumara *et al.* [15].

It is a well established fact that the boundary conditions dominate the instability characteristics of a fluid dynamical system. It is therefore pragmatic to analyze the influence of various velocity, temperature and electric potential boundary conditions on ETC. Such a study has not been initiated to the best of our knowledge despite the findings help in understanding control of ETC which is important in the contemporary heat transfer research. In the present study, the onset of convection in a horizontal dielectric fluid layer heated from below under the influence of a uniform vertical AC electric field for various boundary conditions on velocity (rigid/free), temperature (constant temperature/heat flux or a mixed condition at the upper boundary) and electric potential (constant electric potential/electric current). The resulting eigenvalue problem is solved numerically using the shooting method with the thermal Rayleigh number as the eigenvalue. The existing results are obtained as particular cases from the present study.

2 Mathematical formulation

We consider a dielectric fluid layer of thickness d with a uniform vertical AC electric field applied across the layer. The lower and upper boundaries of the layer are maintained at uniform, but different temperatures T_L and T_U ($T_U < T_L$) respectively, and thus a constant temperature difference $\Delta T = T_L - T_U$ is maintained between the boundaries. A Cartesian coordinate system $(x,\ y,\ z)$ is chosen with the origin at the bottom of the fluid layer and the z-axis normal to the fluid layer in the gravitational field. The relevant basic equations under the Oberbeck-Boussinesq approximation are [12]:

$$\nabla \cdot \vec{q} = 0\,, \tag{1}$$

$$\begin{aligned}\rho_0\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\,\vec{q}\right] &= \rho_e \vec{E} - \tfrac{1}{2}\vec{E}\cdot\vec{E}\;\nabla\varepsilon \\ &-\nabla\left(p - \tfrac{1}{2}\,\rho\,\frac{\partial\varepsilon}{\partial\rho}\vec{E}\cdot\vec{E}\right) + \rho_0\{1-\alpha(T-T_L)\}\vec{g} + \mu\,\nabla^2\vec{q}\,,\end{aligned} \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \kappa\nabla^2 T\,, \tag{3}$$

$$\nabla \times \vec{E} = 0 \,, \tag{4a}$$

$$\nabla \cdot (\varepsilon \vec{E}) = 0 \,, \tag{4b}$$

where $\vec{q}$ is the velocity vector, T is the temperature, p is the pressure, κ is the thermal diffusivity, μ is the fluid viscosity, $\vec{g}$ is the gravitational acceleration, α is the thermal expansion coefficient, ρ is the fluid density, ρ_0 is the density at reference temperature $T = T_L$, $\vec{E}$ is the electric field, ρ_e is the charge density and ε is the dielectric constant. The first term on the right hand side (Eq. (2)) is the Coulomb force due to a free charge and the second term depends on the gradient of ε. The electrical force will have no effect on the bulk of the dielectric fluid if both the dielectric constant, ε, and the electrical conductivity, σ, are homogeneous. Since ε and σ are functions of temperature, a temperature gradient applied to a dielectric fluid produces a gradient in ε and σ. The application of a DC electric field then results in the accumulation of free charge in the liquid. The free charge increases exponentially in time with a time constant ε/σ, which is known as the electrical relaxation time. If an AC electric field is applied at a frequency much higher than the reciprocal of the electrical relaxation time, the free charge does not have time to accumulate. Moreover, the electrical relaxation times of most dielectric liquids appear to be sufficiently long to prevent the buildup of free charge at standard power line frequencies. At the same time, dielectric loss at these frequencies is so low that it makes no significant contribution to the temperature field [12]. Under the circumstances, only the force induced by non-uniformity of the dielectric constant is considered. Furthermore, since the body force of electrical origin depends on $\vec{E} \cdot \vec{E}$ rather than $\vec{E}$ which varies rapidly, the root mean square value of $\vec{E}$ is assumed as the effective value. In other words, the AC electric field is treated as the DC electric field whose strength is equal to the root mean square value of the AC electric field. Hence, the electric field does not involve time t.

In view of Eq. (4a), $\vec{E}$ can be expressed as

$$\vec{E} = -\nabla V \,, \tag{5}$$

where V is the electric potential. The dielectric constant is assumed to be a linear function of temperature in the form

$$\varepsilon = \varepsilon_0 [1 - \gamma (T - T_L)] \,, \tag{6}$$

where γ (>0) is the thermal expansion coefficient of dielectric constant. The basic state is quiescent and given by

$$T_b - T_L = -DT\,z/d, \qquad E_{bz} = \frac{E_0}{1 + DT\,z/d}$$

or

$$V_b(z) = -\frac{E_0 d}{\gamma DT}\log(1 + \gamma DTz/d)\,, \tag{7}$$

where

$$E_0 = -\frac{V_1\gamma DT/d}{\log(1+\gamma DT)} \tag{8}$$

is the externally applied electric field at $z = 0$ and the subscript b denotes the basic state and $D = \frac{d}{dz}$. To study the stability of the basic state, we superimpose infinitesimally small perturbations $\left(\vec{q}', p', \vec{E}', T', \rho', \varepsilon'\right)$ on the basic state in the form

$$\vec{q} = \vec{q}',\ p = p_b + p',\ \vec{E} = \vec{E}_b + \vec{E}',\ T = T_b + T',\ \rho = \rho_b + \rho', \varepsilon = \varepsilon_b + \varepsilon'\,. \tag{9}$$

Substituting Eq. (9) into Eqs. (1)–(4), linearizing the equations by neglecting the products of primed quantities, eliminating the pressure from the momentum equation by operating curl twice and retaining the vertical component and non-dimensionalizing the resulting equations by scaling (x, y, z) by d, t by d^2/κ, $\vec{q}'$ by κ/d, T' by ΔT, and V' by $\gamma E_0 \Delta T d$, we obtain the stability equations (after neglecting the primes for simplicity) in the form

$$\left(\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2 w = \mathrm{Ra}_t \nabla_h^2 T + \mathrm{Ra}_e \nabla_h^2\left(T - \frac{\partial V}{\partial z}\right)\,, \tag{10}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right) T = w\,, \tag{11}$$

$$\nabla^2 V = \frac{\partial T}{\partial z}\,, \tag{12}$$

where $\mathrm{Ra}_t = \alpha\, g\, \Delta T d^3/\nu\kappa$ is the thermal Rayleigh number, $\mathrm{Ra}_e = \gamma^2 \varepsilon_0 E_0^2$ $(\Delta T)^2\ d^2/\mu\kappa$ is the AC electric Rayleigh number, $\mathrm{Pr} = \nu/\kappa$ is the Prandtl number, $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian operator and $\nabla^2 = \nabla_h^2 + \partial^2/\partial z^2$ is the Laplacian operator. The normal mode analysis procedure is employed in which we look for the solution of the form

$$(w, T, V) = (W, \Theta, \Phi)(z)\exp\left\{i(\ell x + my) + \omega t\right\}\,, \tag{13}$$

where ℓ and m are the horizontal wave numbers in the x and y directions, respectively, and ω is the growth factor. In general, $\omega = \omega_r + i\,\omega_i$, is a complex quantity, where i is the imaginary unit, and the subscript r and i denote real and imaginary part, relatively. Substituting Eq. (13) into Eqs. (10)–(12), and noting that the principle of exchange of stability is valid (see Shivakumara *et al.* [16]) the governing stability equations become

$$(D^2 - a^2)^2 W = \mathrm{Ra}_t\, a^2 \Theta + \mathrm{Ra}_e\, a^2 (\Theta - D\Phi)\ , \tag{14}$$

$$(D^2 - a^2)\,\Theta + W = 0\ , \tag{15}$$

$$(D^2 - a^2)\Phi = D\Theta\ , \tag{16}$$

where $D = d/dz$ and $a = \sqrt{\ell^2 + m^2}$ is the horizontal wave number. The above equations are to be solved subject to four boundary conditions at lower and upper boundaries; two on velocity and one each on the temperature and electric potential. The boundary conditions considered are:

(i) velocity: $W = 0$ (vanishing of normal velocity), $DW = 0$ (vanishing of tangential velocity-rigid boundary) or $D^2W = 0$ (vanishing of shear stress-free boundary).

(ii) temperature: $\Theta = 0$ (isothermal) or $D\Theta = 0$ (perturbation heat flux is zero, i.e. the boundaries are insulating with respect to temperature perturbations) or $D\Theta + \mathrm{Bi}\Theta = 0$ which encompasses the above two types of temperature boundary conditions as particular cases, where $\mathrm{Bi} = h\,d/k$ is the Biot number.

(iii) electric potential: $\Phi = 0$ (constant electric potential) or $D\Phi = 0$ (constant normal electric field).

3 Method of solution

Equations (14)–(16) together with the chosen boundary conditions constitute a Sturm-Liouville's problem with Ra_t or Ra_e as an eigenvalue and other physical parameters as given. The eigenvalue problem is solved numerically using the shooting method which is based on Runge-Kutta-Fehlberg (RKF45) and Newton-Raphson methods. The problem is solved for different velocity, temperature, electric potential boundary conditions and for various values of Ra_e and Bi as an initial value problem with the conditions at $z = 0$ and satisfying the required conditions at $z = 1$. To validate the numerical procedure used, critical thermal Rayleigh number, Ra_{tc}, and the corresponding critical wave number, a_c, computed for different values of Ra_e and Bi are compared with those of Maekawa *et al.* [10] in Tab. 1 for the following set of boundary conditions lower rigid and upper free boundaries:

$$W(0) = 0 = DW(0), \qquad W(1) = 0 = D^2W(1) ,$$

$$\Theta(0) = 0 = D\Theta(1) + Bi\Theta(1), \qquad \Phi(0) = 0 = \Phi(1) .$$

From Tab. 1 it is seen that our numerical results are in excellent agreement with the published ones and thus verifies the accuracy of the method used.

Table 1: Comparison of critical stability parameters with the earlier works for rigid-free boundaries when $\Phi = 0$ at free boundary.

Ra_e	Bi	Maekawa *et al.* [10]		Present study	
		Ra_{tc}	a_c	Ra_{tc}	a_c
0	0	668.9983	2.086	668.999	2.086
	0.1	682.3602	2.116	682.361	2.116
	1	770.5697	2.293	770.570	2.293
	10	989.4917	2.589	989.491	2.589
	100	1085.898	2.672	1085.897	2.672
	1000	1099.124	2.681	1099.120	2.681
	∞	1100.650	2.682	1100.650	2.682
100	0	580.8635	2.090	580.864	2.090
	0.1	594.3658	2.120	594.366	2.120
	1	683.6306	2.297	683.631	2.297
	10	905.9162	2.594	905.916	2.594
	100	1004.069	2.677	1004.070	2.677
	1000	1017.545	2.686	1017.546	2.686
	∞	1019.100	2.687	1019.100	2.687
500	0	228.2693	2.106	228.2696	2.106
	0.1	242.3281	2.136	242.328	2.136
	1	335.7752	2.314	335.775	2.314
	10	571.4484	2.613	571.448	2.613
	100	676.5912	2.696	676.591	2.696
	1000	691.0723	2.706	691.072	2.706
	∞	692.7436	2.707	692.744	2.707

4 Results and discussion

The effect of various types of velocity, temperature and electric potential boundary conditions on the onset of electrothermo convection (ETC) is investigated to understand their influence on control of ETC. The resulting eigenvalue problem is solved using the shooting method with RKF45 and Newton-Raphson methods. The stability parameters extracted for various types of boundary conditions are illustrated graphically and also tabulated in Tab. 2.

Table 2: Values of critical Rayleigh number, Ra_{tc}, and corresponding wave number, a_c, for different values of Biot number, Bi, for different electric boundary conditions and for two values of Ra_e for rigid boundaries.

Ra_e	Bi	$\Phi(0)=D\Phi(1)=0$		$D\Phi(0)=D\Phi(1)=0$		$\Phi(0)=\Phi(1)=0$		$D\Phi(0)=\Phi(1)=0$	
		Ra_{tc}	a_c	Ra_{tc}	a_c	Ra_{tc}	a_c	Ra_{tc}	a_c
100	0	1240.177	2.590	1239.483	2.590	1207.560	2.550	1206.624	2.550
	0.1	1253.476	2.619	1252.776	2.619	1221.597	2.580	1220.666	2.580
	1	1339.884	2.782	1339.151	2.782	1312.291	2.751	1311.391	2.751
	10^1	1544.697	3.053	1543.904	3.053	1525.008	3.033	1524.143	3.032
	10^2	1631.208	3.129	1630.393	3.129	1614.277	3.111	1613.417	3.111
	10^3	1642.931	3.137	1642.113	3.137	1626.354	3.120	1625.494	3.120
	10^4	1644.145	3.138	1643.327	3.138	1627.606	3.121	1626.746	3.121
	10^5	1644.267	3.138	1643.449	3.138	1627.731	3.121	1626.871	3.121
	0	1183.978	2.628	1182.578	2.628	1119.336	2.549	1117.462	2.547
	0.1	1196.833	2.655	1195.422	2.655	1133.642	2.579	1131.779	2.578
	1	1280.811	2.813	1279.337	2.813	1226.064	2.752	1224.264	2.751
200	10^1	1481.982	3.077	1480.394	3.077	1442.883	3.037	1441.155	3.036
	10^2	1567.569	3.151	1565.937	3.151	1533.947	3.116	1532.228	3.115
	10^3	1579.188	3.159	1577.551	3.159	1546.272	3.125	1544.554	3.125
	10^4	1580.392	3.160	1578.754	3.160	1547.549	3.126	1545.831	3.125
	10^5	1580.513	3.160	1578.875	3.160	1547.677	3.126	1545.959	3.126

Figures 1 and 2 respectively exhibit the variation of critical thermal Rayleigh number, Ra_{tc}, and the corresponding critical wave number, a_c, as a function of Biot number, Bi, for different velocity and temperature boundary conditions. The result for Bi = 0 corresponds to zero perturbation heat flux case while for Bi $\to \infty$ corresponds to isothermal case.

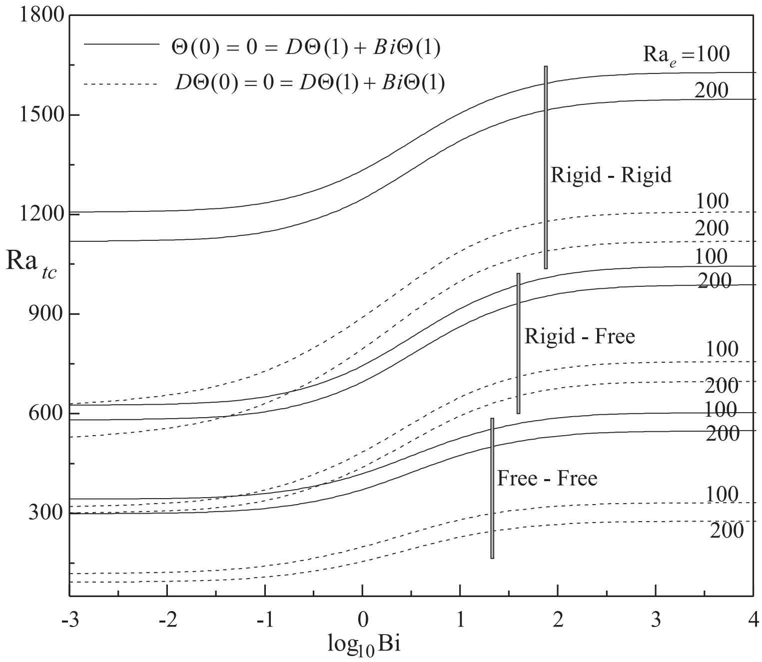

Figure 1: Variation of critical Rayleigh number, Ra_{tc}, as a function of Biot number, Bi, for different values of Ra_e with different boundary conditions.

Figure 1 shows that the values of Ra_{tc} increase steadily with increasing Bi but remain invariant at higher values of Bi. This is because the temperature perturbations are suppressed with an increase in the value of Bi and hence higher values of Ra_{tc} are needed for the onset of ETC. Also, increasing the alternating current electric Rayleigh number amounts to decrease in Ra_{tc} and thereby hastens the onset of convection. This may be attributed to the fact that the destabilizing electric body force induced by the gradient of dielectric constant arising due to variations in temperature under the action of electric field drives an upward fluid motion. In other words, the presence of AC electric field is to augment the heat transfer and to hasten the onset of convection in a dielectric fluid layer heated from below. This is so irrespective of the velocity boundary conditions considered. From the figure it is also evident that the results for different velocity boundary conditions differ only quantitatively and the rigid-rigid boundaries are found to be more stable followed by rigid-free boundaries and the least stable is free-free boundaries. Moreover, the system is more unstable when both boundaries are insulating with respect to temperature perturbations as compared to isothermal ones.

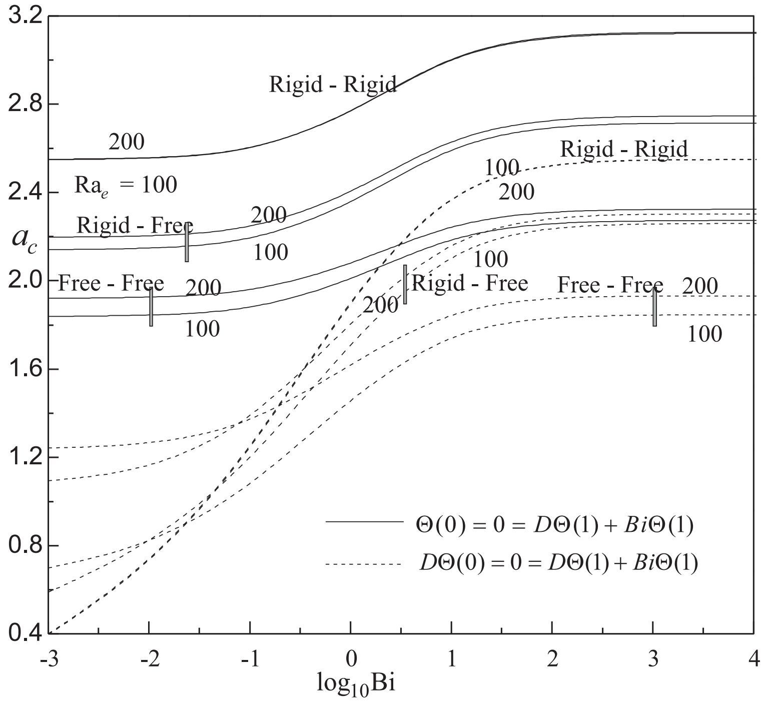

Figure 2: Variation of critical wave number, a_c, as a function of Biot number, Bi, for different values of Ra_e with different boundary conditions.

Figure 2 reveals that increase in Bi is to increase a_c but assumes constant value at higher values of Bi. Thus the effect of Bi is to reduce the size of convection cells. Although the parameter Ra_e shows no traceable effect on a_c in the case of rigid-rigid boundaries, its effect is found to be significant in the case of free-free and lower rigid-upper free boundaries and note that increasing Ra_e is to increase a_c indicating its effect is to diminish the size of convection cells. Also, it is noted that

$$(a_c)_{\mathrm{rigid-rigid}} > (a_c)_{\mathrm{rigid-free}} > (a_c)_{\mathrm{free-free}}$$

in the case of isothermal lower boundary while for adiabatic lower boundary a mixed type of behavior could be seen. At lower values of Bi, say

$\mathrm{Bi} < 10^{-2}$, it is observed that

$$(a_c)_{\text{rigid − rigid}} < (a_c)_{\text{rigid − free}} < (a_c)_{\text{free − free}} \; ,$$

while for values of Bi beyond this range an opposite trend prevails. Further, the critical wave number is higher in the case of isothermal lower boundary as compared to both boundaries insulating with respect to temperature perturbations.

The critical stability parameters, $\mathrm{Ra}_{t\,c}, a_c$, computed for different combinations of electric potential boundary conditions are tabulated in Tab. 2. The results are presented for a representative case of rigid – rigid boundaries with $\Theta(0) = 0 = D\Theta(1) + \mathrm{Bi}\Theta(1)$. From this table we observe that the system is more stabilizing for the case of $\Phi(0) = 0 = D\Phi(1)$ followed by $D\Phi(0) = 0 = D\Phi(1)$, then $\Phi(0) = 0 = \Phi(1)$ and the least stable for the boundary conditions of the type $D\Phi(0) = 0 = \Phi(1)$. It is seen that the deviation in the critical stability parameters between the first two and the last two types of electric potential boundary conditions is not so significant. But the change in the boundary condition from $\Phi(1) = 0$ to $D\Phi(1) = 0$ is to delay the onset of ETC the most. Thus it is possible to control (suppress/augment) ETC by imposing appropriate electric potential boundary conditions.

It is observed that the critical wave number is exceedingly small when the boundaries are insulated with respect to temperature perturbations (Nield [17,18]) and this fact is exploited to obtain an analytic expression for the critical thermal Rayleigh number using regular perturbation technique with wave number as a perturbation parameter. Accordingly, we expand W, Θ, and Φ in powers of a^2 as

$$(W,\, \Theta,\, \Phi) = (W_0,\, \Theta_0,\, \Phi_0) + a^2(W_1,\, \Theta_1,\, \Phi_1) + \ldots \tag{17}$$

Substituting Eq. (17) into Eqs. (14)–(16) and in the boundary conditions and collecting the terms of zeroth order in a^2, we obtain:

$$D^4 W_0 = 0 \; , \tag{18a}$$

$$D^2\Theta_0 = -W_0 \; , \tag{18b}$$

$$D^2\Phi_0 = D\Theta_0 \; . \tag{18c}$$

The boundary conditions considered are:

(i) rigid-rigid boundaries

$$W_0 = DW_0 = D\Theta_0 = \Phi_0 = 0 \quad \text{at} \quad z = 0,\, 1 \; , \tag{19a}$$

(ii) lower rigid-upper free boundaries

$$W_0 = DW_0 = D\Theta_0 = \Phi_0 = 0 \quad \text{at} \quad z = 0\,, \tag{19b}$$

$$W_0 = D^2W_0 = D\Theta_0 = D\Phi_0 = 0 \quad \text{at} \quad z = 1\,, \tag{19c}$$

(iii) free-free boundaries

$$W_0 = D^2W_0 = D\Theta_0 = D\Phi_0 = 0 \quad \text{at} \quad z = 0, 1\,. \tag{19d}$$

The solution to the zeroth order equations are $W_0 = 0$, $\Theta_0 = 1$ and $\Phi_0 = 0$ for rigid-rigid as well a rigid-free boundaries, while $W_0 = 0$, $\Theta_0 = 1$ and $\Phi_0 = 1$ for free-free boundaries case.

The first order equations in a^2 are then:

$$D^4W_1 = \mathrm{Ra}_t + \mathrm{Ra}_e\,, \tag{20a}$$

$$D^2\Theta_1 = 1 - W_1\,, \tag{20b}$$

$$D^2\Phi_1 = D\Theta_1 \tag{20c}$$

for rigid-rigid/lower rigid-upper free boundaries while for free-free boundaries, Eq. (20c) has to be replaced with

$$D^2\Phi_1 = D\Theta_1 + 1\,. \tag{20d}$$

The corresponding boundary conditions are: $W_1 = DW_1 = D\Theta_1 = \Phi_1 = 0$ on the rigid boundary, and $W_1 = D^2W_1 = D\Theta_1 = D\Phi_1 = 0$ on the free boundary.

The general solution of Eq. (20a) is

$$W_1 = \frac{1}{24}\left(\mathrm{Ra}_t + \mathrm{Ra}_e\right)z^4 + c_1 + c_2 z + c_3 z^2 + c_4 z^3\,, \tag{21}$$

where c_1, c_2, c_3, and c_4 are arbitrary constants determined using the boundary conditions $W_1 = DW_1 = D\Theta_1 = \Phi_1 = 0$ on the rigid boundary and $W_1 = D^2W_1 = D\Theta_1 = D\Phi_1 = 0$ on the free boundary. They are given by

(i) rigid-rigid boundaries

$$c_1 = 0,\quad c_2 = 0,\quad c_3 = \frac{1}{24}\left(\mathrm{Ra}_t + \mathrm{Ra}_e\right),\quad c_4 = -\frac{1}{12}\left(\mathrm{Ra}_t + \mathrm{Ra}_e\right)\;; \tag{22}$$

(ii) lower rigid- upper free boundaries

$$c_1 = 0,\quad c_2 = 0,\quad c_3 = \frac{1}{16}\left(\mathrm{Ra}_t + \mathrm{Ra}_e\right),\quad c_4 = -\frac{5}{48}\left(\mathrm{Ra}_t + \mathrm{Ra}_e\right)\;; \tag{23}$$

(iii) free-free boundaries

$$c_1 = 0,\ c_2 = \frac{1}{24}\left(\mathrm{Ra}_t + \mathrm{Ra}_e\right),\ c_3 = 0,\ c_4 = -\frac{1}{12}\left(\mathrm{Ra}_t + \mathrm{Ra}_e\right) . \tag{24}$$

Integrating Eq. (20b) between $z = 0$ and 1, and using the boundary conditions on temperature, it follows that

$$\int_0^1 W_1 dz = 1 . \tag{25}$$

Substituting for W_1 from Eq. (21) into Eq. (25) and carrying out the integration leads to an expression for critical Rayleigh numbers for rigid-rigid, lower rigid-upper free and free-free boundaries, respectively, in the following form:

$$\mathrm{Ra}_{tc} = 720 - \mathrm{Ra}_e , \tag{26}$$

$$\mathrm{Ra}_{tc} = 320 - \mathrm{Ra}_e , \tag{27}$$

$$\mathrm{Ra}_{tc} = 120 - \mathrm{Ra}_e . \tag{28}$$

In case of the critical thermal Rayleigh number is $\mathrm{Ra}_e = 0$, $\mathrm{Ra}_{tc} = 720$, 320, and 120, respectively, which are the known exact values for the ordinary viscous fluid case [19,20].

Table 3: Comparison of critical Rayleigh number, Ra_{tc}, obtained from the shooting method and regular perturbation techniques.

Ra_e	Shooting method			Regular perturbation method		
	rigid-rigid	rigid-free	free-free	rigid-rigid	rigid-free	free-free
	Ra_{tc}	Ra_{tc}	Ra_{tc}	Ra_{tc}	Ra_{tc}	Ra_{tc}
0	720	320	120	720	320	120
10	710	310	110	710	310	110
20	700	300	100	700	300	100
30	690	290	90	690	290	90
40	680	280	80	680	280	80
50	670	270	70	670	270	70

The numerically computed values of Ra_{tc} for different values of Ra_e with $\mathrm{Bi} = 0$ are tabulated in Tab. 3 and note that the results obtained from the simple regular perturbation technique coincide exactly with those obtained from time consuming numerical methods. The study provides a justification for the analytically obtained results for prescribed heat flux condition at the boundaries. In other words, the solutions obtained analytically for the case of insulating boundaries with respect to temperature perturbations are exact. As noticed earlier, an increase in the value of Ra_e is to decrease $\mathrm{Ra}_{t\,c}$ and hence its effect is to hasten the onset of convection.

5 Conclusions

The influence of different types of velocity (rigid or free), temperature (isothermal or constant heat flux or a general thermal condition at the upper boundary) and electric potential (constant electric potential or electric current) boundary conditions on the onset of electrothermal convection in a dielectric fluid layer has been analyzed. The resulting eigenvalue problem is solved numerically using the shooting method and also closed form solution is obtained using a regular perturbation method when the perturbed heat flux is zero at the boundaries. The various boundary conditions considered exhibit significant effects on the instability characteristics of the system and a more precise control of electrothermal convection is found to be possible for a certain choice of boundary conditions. Increase in the strength of vertical alternating current electric field is to facilitate heat transfer and to hasten the onset of convection while increase in the Biot number delays the onset of ETC. The results for different velocity boundary conditions are found to differ only quantitatively and the system is found to be more stable when both boundaries are rigid, while the free boundaries are the least stable. Also, the isothermal boundaries induct more stability when compared to both boundaries insulating with respect to temperature perturbations. The change in the electric potential boundary condition from constant electric potential to constant electric current condition at the upper boundary is to delay the onset of electrothermal convection the most. The effect of increase in the electric Rayleigh and Biot numbers is to decrease the size of convection cells.

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