QUALIMETRICAL MEASUREMENTS: METHODOLOGY BASED ON RELATIONSHIP BETWEEN QUALIMETRY AND METROLOGY

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Abstract

The paper is focused on the main problems of modern metrology in the context of the Fourth Industrial Revolution “Industry 4.0”. The dominant issues of the methodology of qualimetrical measurement, as the interrelation between metrology and qualimetry, are considered. The following questions are raised and analysed: determination of the measurand in the qualimetrical measurement, creation of the virtual product quality pattern, determination of the product quality level by using the theory of multidimensional scaling, assurance of the metrological traceability of the qualimetrical measurement results. A procedure of performing the qualimetrical measurement is described.

Keywords: qualimetry, metrology, qualimetrical measurement, virtual product quality measure, product quality level, multidimensional scaling.

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1. Introduction

In the framework of the Fourth Industrial Revolution Industry – “Industry 4.0”, metrology becomes an integral part of the production process. Measurement, testing and quality control are no longer considered as parallel procedures, but rather as integral parts of a single self-regulated system with insignificant or no human intervention in the production process [1]. Accordingly, a substantial expansion of the functions of metrology and the scope of their research can be traced, in such areas of human activity as psychology, medicine, trade, industry, education, sociology, etc., as reflected in the Recommendations of the International Committee for Weights and Measure, concerning the establishment of new tasks in metrology [2].

One of the progressive areas in modern metrology is qualimetry – a scientific branch, that has appeared as a result of the requirement of scientific and technological progress and innovative technologies in the sphere of knowledge of physical phenomena and processes [3, 4]. According to the generally accepted definition, qualimetry is a discipline which studies the methods and problems of quantitative evaluation of quality of any object – abstract and concrete, material and ideal, living or inanimate, products of labour and products of nature, goods and services, mechanisms and processes. Nowadays in qualimetry, because of the specificity of the researched
object – production, there is a number of unresolved theoretical and methodological issues. Mainly, these are the problems of ensuring the uniformity and metrological traceability of the results of product quality assessment and analysis of accuracy and reliability of the obtained estimates of product quality.

Solving these problems, as reported by the authors, is only possible in a complex combination of the metrology and qualimetry methodologies. Metrology, as a science of measurement and its application, is a strong scientific, practical and legal instrument, which enables to carry out research in any field of cognition.

Thereby, the subject of the paper is a careful analysis of the ways of tackling the stated issues.

2. Concept of “Qualimetical measurement” and principal tasks of research

Conforming with the authors’ idea, one of the ways of how to combine the methodologies of qualimetry and metrology, is the application of the concept *qualimetical measurement* in theory and practice, as a type of measurement and determination of its essence and purpose [5].

The term *qualimetry*, is formed from the Latin *quails* – *of what kind* and from the ancient Greek μέτρον – *measure* [4], literally it means the *measurement of quality*. However, in practice, the literal meaning of *qualimetry* cannot be used, since it is impossible to measure the quality, namely to display it numerically, according to the generally accepted definition of a measurand in VIM3 [6]: *measurand* – is a quantity intended to be measured and *quantity* – is a property of a phenomenon, body or substance, where the property has a magnitude that can be expressed as a number and a reference.

By the authors’ definition, *qualimetical measurement* is an indirect measurement of the product quality level, the value of which is obtained by processing the results of measurement applying the multidimensional scaling methodology.

In the qualimetical measurement, the quality level of a product is taken as the measurand. It is accomplished comparing the values of the quality indices with the basic values of relevant indicators [5], and is expressed numerically in the range 0...1 or 0...100%. This definition completely corresponds to the notion of the *measurand* in modern interpretation of this term, in agreement with the requirements of VIM3 [6].

Generally, the qualimetical measurement, as any other one, consists of two principal stages:

– constructing a measuring experiment, in process of which different characteristics (mechanical, dimensional, electric, magnetic, thermal properties, chemical composition *etc.* ) of the studied product are measured;
– processing the results of experiment, during which the value of quality level of the studied product $Q$ is being obtained. It is considered to be the qualimetical measurement result.

The purpose of the paper is to develop a methodology for performing the procedure of qualimetical measurement, in order to find the level of product quality and its practical implementation.

To achieve the stated objective, the following tasks have been identified:

– defining a virtual measurement standard of the product quality;
– developing a procedure of qualimetical measurement for finding the level of product quality;
– creating a one-dimensional qualimetical scale of the product quality level.
3. Methodology of defining virtual measurement standard of product quality

3.1. Concept of “Virtual measurement standard of product quality”

The basis of any measurement is the comparison of a measured quantity with a measure (measurement standard), that stores and reproduces the unit of the quantity. The specificity of the qualimetrical measurement is the absence, in most cases, of a specific material measure of the product quality. Basic samples do not always correspond to the metrological requirements, which are applied to the measurement standards. Moreover, it is not always methodologically possible to compare the studied product with such basic samples. In fact, this is the main problem of the implementation of qualimetrical measurement.

In the paper, to develop the full procedure of qualimetrical measurement, a virtual measurement standard of the product quality is used. It is the theoretical analogue of the corresponding physical standard of the product quality, that is the basic (reference) sample of the studied product.

To define the virtual standard of product quality, the main technological provisions of virtual measuring instruments and theory of sets, as the corresponding section of mathematics, were used. The essence of technology of the virtual measuring instruments consists in the computer-programmed simulation of real functions of measuring instruments and measuring systems. The virtuality, in this case, is expressed in a sense of the virtual simulation of certain functions of instrument in a mathematical and software way. Consequently, the virtual measurement standard of product quality is the reflection of a real physical standard of product quality, expressed in a mathematical and software way.

On the other hand, since the quality of a product is determined by a set of its properties of different nature, the virtual measurement standard of product quality is a certain set of some arbitrary objects (elements), united according to some common properties. Such objects (elements) in qualimetry are single absolute product quality indexes $P_i$ and single relative product quality indexes $K_i$, $i = 1, \ldots, n$ (here $n$ is the number of indexes, which is equal to the number of properties of the studied product $p_i$). They characterize the quality of the product during its development, manufacturing, operation and consumption [4, 7].

In qualimetry, the above set of some arbitrary objects (elements) is called a product quality profile $\Pi$, which is a set of quantitative unit quality indexes [8]. Product quality profiles $\Pi$ may be formed of single absolute product quality indexes $P_i$, $i = 1, \ldots, n$, denoted as $\Pi_P$, and of single relative product quality indexes $K_i$, $i = 1, \ldots, n$, denoted as $\Pi_K$:

$$\Pi_P = \{P_1; P_2; \ldots; P_n\}, \quad \Pi_K = \{K_1; K_2; \ldots; K_n\}. \quad (1)$$

It should be noticed that between single product quality indexes, in most cases, there is no functional connection. It helps in deriving a product quality profile $\Pi$ from the mathematical model of product quality, which functionally links the quality of a product with its individual properties.

Consequently, the product quality profile is a separate complex characteristic of its quality that can be used to construct a virtual measurement standard of the product quality.

3.2. Types of product quality indexes in qualimetical measurement

Assessing the product quality, indexes are divided into estimated, basic, weighted estimated and weighted basic.

An estimated quality index is a product quality index, the value of which is found in an experimental or calculated way during a comparative evaluation of its quality [7]. In the procedure
of qualimetric measurement, single absolute estimated product quality indexes $P_{e,i}$, $i = 1, \ldots, n$ product quality indexes, single relative estimated product quality indexes $K_{e,i}$ and single weighted relative estimated product quality indexes $K_{w,e,i}$ are used.

A basic quality index is a product quality index, the value of which is taken as the basis for a comparative evaluation of its quality. In the procedure of qualimetric measurement, single absolute basic product quality indexes $P_{b,i}$, $i = 1, \ldots, n$, single relative basic product quality indexes $K_{b,i}$ and single weighted relative basic product quality indexes $K_{w,b,i}$ are used.

Depending on the nature of the impact of the product quality indexes on the value of product quality level $Q$, all $n$ indexes are divided into two groups. The product quality indexes used in the procedure for determining the product quality level $Q$ are given in Table 1.

Table 1. Types of product quality indexes used in the procedure for determining the product quality level $Q$.

<table>
<thead>
<tr>
<th>First group of the product quality indexes</th>
<th>Second group of the product quality indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single absolute basic product quality index of the first group $P_{b,i,1, \ldots, l}$</td>
<td>Single absolute basic product quality index of the second group $P_{b,j, l+1, \ldots, n}$</td>
</tr>
<tr>
<td>Single absolute estimated product quality index of the first group $P_{e,i,1, \ldots, l}$</td>
<td>Single absolute estimated product quality index of the second group $P_{e,j, l+1, \ldots, n}$</td>
</tr>
<tr>
<td>Single relative estimated product quality index of the first group $K_{e,i,1, \ldots, l}$</td>
<td>Single relative estimated product quality index of the second group $K_{e,j, l+1, \ldots, n}$</td>
</tr>
<tr>
<td>Single weighted relative estimated product quality index of the first group $K_{w,e,i,1, \ldots, l}$</td>
<td>Single weighted relative estimated product quality index of the second group $K_{w,e,j, l+1, \ldots, n}$</td>
</tr>
<tr>
<td>Maximum value of the single absolute product quality index of the first group $P_{\max,i,1, \ldots, l}$</td>
<td>Permissible value of the single absolute product quality index of the second group $P_{\text{pem},j, l+1, \ldots, n}$</td>
</tr>
<tr>
<td>Minimum value of the single absolute product quality index of the first group $P_{\min,i,1, \ldots, l}$</td>
<td>Minimum value of the single absolute product quality index of the second group $P_{\min,j, l+1, \ldots, n}$</td>
</tr>
</tbody>
</table>

$n$ – the number of all product quality indexes; $l$ – the number of product quality indexes of the first group; $n - l$ – the number of product quality indexes of the second group.

In the first group, an increase of the single absolute estimated product quality indexes $P_{e,i,1, \ldots, l}$ and, accordingly, an increase of the single relative estimated product quality indexes $K_{e,i,1, \ldots, l}$ lead to an increase of the product quality level $Q$ and vice versa. In the second group, an increase of the single absolute estimated product quality indexes $P_{e,j, l+1, \ldots, n}$ and, accordingly, an increase of the single relative estimated product quality indexes $K_{e,j, l+1, \ldots, n}$ lead to a decrease of the product quality level $Q$ and vice versa.

In the first group of product quality indexes, the minimum $P_{\text{min},i,1}$ and maximum $P_{\text{max},i,1}$ values of the single absolute product quality indexes are normalized. Provided that $P_{e,i,1} \leq P_{\text{min},i,1}$, the product is considered to be defective and unsuitable for use. The maximum value of single absolute product quality index $P_{\text{max},i,1}$ is the maximum possible value of this index. It can be achieved in accordance with the current level of technology production and it corresponds to the highest value of its quality level $Q$. Therefore, the maximum value of single absolute product quality index $P_{\text{max},i,1}$ is taken as the basic value of this index $P_{b,i,1}$, i.e. $P_{b,i,1} = P_{\text{max},i,1}$. Consequently, in the first group of product quality indexes, the values of single relative estimated product quality indexes $K_{e,i,1, \ldots, l}$ are calculated by the formula:

$$K_{e,i,1} = \frac{P_{e,i,1}}{P_{\text{max},i,1}} = \frac{P_{e,i,1}}{P_{b,i,1}}, \quad P_{e,i,1} \leq P_{b,i,1}, \quad i = 1, \ldots, l. \quad (2)$$

The value of single relative basic product quality index $K_{b,i,1}$, as the limit value of single relative estimated product quality index $K_{e,i,1}$, equals 1.
In the second group of product quality indexes, the minimum \( P_{\text{min},j,2} \), \( j = l + 1, \ldots, n \) and permissible \( P_{\text{per},j,2} \), \( j = l + 1, \ldots, n \) values of single absolute product quality indexes are normalized. The Minimum value of single absolute product quality index \( P_{\text{min},j,2} \) is the minimum possible value of this index. It can be achieved in accordance with the current level of technology production and it corresponds to the highest value of its quality level \( Q \) (it is desirable that \( P_{\text{min},j,2} \Rightarrow 0 \)). Therefore, the minimum value of single absolute product quality index \( P_{\text{min},j,2} \) is taken as the basic value of this index \( P_{b,j,2} \), i.e. \( P_{b,j,2} = P_{\text{min},j,2} \), \( j = l + 1, \ldots, n \). Consequently, in the second group of product quality indexes, the values of single relative estimated product quality indexes \( K_{e,j,2} \), \( j = l + 1, \ldots, n \), are calculated by the formula:

\[
K_{e,j,2} = \frac{P_{e,j,2} - P_{\text{min},j,2}}{P_{\text{per},j,2} - P_{\text{min},j,2}} = \frac{P_{e,j,2} - P_{b,j,2}}{P_{\text{per},j,2} - P_{b,j,2}}, \quad P_{e,j,2} \geq P_{b,j,2}, \quad j = l + 1, \ldots, n, \tag{3}
\]

where \( P_{\text{per},j,2} \) – the permissible value of single estimated product quality index. Provided that \( P_{e,j,2} \geq P_{\text{per},j,2} \), the product is considered to be defective and unsuitable for use.

The values of single relative basic product quality indexes \( K_{b,j,2} \), as the limit values of single relative estimated product quality indexes \( K_{e,j,2} \), are equal to 0.

The nomenclature of product quality indexes, for determining the product quality level, is established on the basis of the current normative documents (ND), as well as on the literature sources and the results of experimental studies of a specific product. The requirements of the product quality level are regulated.

Example 1: In the procedure for assessing the quality of natural gas as an energy source, 8 of its main properties, which have the greatest impact on the calorific value of gas, are established [9]. They are divided into two groups. The first group of gas quality indexes includes: gas specific heat \( H_{H} \), MJ/m³; Wobbe index \( B \), MJ/m³ and gas density \( \rho \), kg/m³. The second group of gas quality indexes includes: gas humidity \( W \), g/m³; non-combustible components (nitrogen \( C_{N_{2}} \), \% and carbon dioxide \( C_{CO_{2}} \), \%) and harmful components (hydrogen sulphide \( C_{H_{2}S} \), g/m³ and mercuric sulphur \( C_{CH_{4}S} \), g/m³) contained in the gas mixture.

### 3.3. Establishment of material standard (material measure) of product quality

The problem of standard is important for any quantity to be measured, regardless of its nature. As for the qualimetric measurement, the basic certified reference material of a given product can serve as the material measure of the product quality. This is a sample of the substance (material) with the values established by the metrological attestation of one or more qualities that characterize the properties or composition of the substance (material) [6].

On the contrary, as it is shown above, the basic certified reference material in numerical terms can be defined as a set of numerical values of the single basic absolute product quality indexes \( P_{b,i} \), \( i = 1, \ldots, n \) (here \( n \) is the number of indexes which is equal to the number of properties of the studied product \( p_{i} \)), divided into two groups. These product quality indexes form the basic quality profile \( \Pi_{P,b} \):

\[
\Pi_{P,b} = \{ P_{b,1}; P_{b,2}; \ldots; P_{b,n} \} = \sum_{i=1}^{l} P_{b,i,1} + \sum_{j=l+1}^{n} P_{b,j,2} = \\
\{ P_{b,1,1}; \ldots; P_{b,l,1} \} + \{ P_{b,l+1,2}; \ldots; P_{b,n,2} \}, \tag{4}
\]

where \( P_{b,i,1} \) are single basic absolute product quality indexes of the first group, \( i = 1, \ldots, l; l \) is the number of single product quality indexes of the first group; \( P_{b,j,2} \) are single basic absolute
product quality indexes of the second group, \( j = l + 1, \ldots, n \); \( n - l \) is the number of single product quality indexes of the second group; \( n \) is the total number of single product quality indexes.

The numerical value of each element \( P_{b,i,1} \) and \( P_{b,j,2} \) of the basic quality profile \( \Pi_{b,p} \) is established according to ISO 5725-1:1994 [10] as an accepted reference value – the value used as an agreed standard for comparison with:

a) the theoretical or established value, based on the scientific principles;

b) the attributed or certified value, based on the experimental data of some national or international organizations;

c) the agreed (based on consensus) or attested value, based on the joint experimental work conducted by a scientific or engineering team;

d) the mathematical expectation of the measured value, that is the average value of the set of measurements – only in the case where a), b) and c) are not available.

It is significant to point out that conforming with the procedure of qualimetical measurement, there is no need to make a basic standard sample as the material measure of the product quality. For the implementation of the qualimetical measurement, a virtual measurement standard of the product quality is used, which is shown below.

### 3.4. Constructing virtual measurement standard of product quality

Constructing a virtual measurement standard of the product quality in the qualimetical measurement is carried out as follows.

Grounded on the basic quality profile \( \Pi_{b,p} \), which was created in accordance with (4), the basic quality profile \( \Pi_{b,b} \) is constructed from the corresponding single basic relative product quality indexes \( K_{b,i,1}, i = 1, \ldots, l \) of the first group and \( K_{b,j,2}, j = l + 1, \ldots, n \) of the second group:

\[
\Pi_{b,b} = \sum_{i=1}^{l} K_{b,i,1} + \sum_{j=l+1}^{n} K_{b,j,1} = \{ K_{b,1,1}; \ldots; K_{b,l,1} \} + \{ K_{b,l+1,2}; \ldots; K_{b,n,2} \}. \tag{5}
\]

Then, the corresponding weighed basic quality profile of the product quality \( \Pi_{b,w} \) is created:

\[
\Pi_{b,w} = \sum_{i=1}^{l} K_{w,b,i,1} + \sum_{j=l+1}^{n} K_{w,b,j,1} = \{ K_{w,b,1,1}; \ldots; K_{w,b,l,1} \} + \{ K_{w,b,l+1,2}; \ldots; K_{w,b,n,2} \}, \tag{6}
\]

where \( K_{w,b,i,1} \) and \( K_{w,b,j,2} \) are the corresponding single weighed basic relative product quality indexes of the first and second groups, which are the main elements in the procedure of qualimetical measurement.

The values of indexes \( K_{w,b,i,1} \) and \( K_{w,b,j,2} \) are calculated by the formulae:

\[
K_{w,b,i,1} = K_{b,i,1} \cdot m_{i,1}, \quad i = 1, \ldots, l; \quad K_{w,b,j,2} = K_{b,j,2} \cdot m_{j,2}, \quad j = l + 1, \ldots, n, \tag{7}
\]

where \( m_{i,1} \) and \( m_{j,2} \) are normalized weighted coefficients of the single product quality indexes of the first and second groups, the values of which should satisfy the condition of their normalization:

\[
\sum_{i=1}^{l} m_{i,1} + \sum_{j=l+1}^{n} m_{j,2} = 1. \tag{8}
\]
The numerical values of the basic relative product quality indexes in the first group are, in general, \( K_{b,i,1} \leq 1, \ i = 1, \ldots, l \), in the limit version \( K_{b,i,1} = 1 \), and in the second group – \( K_{b,j,2} \geq 0, \ j = l + 1, \ l + 2, \ldots, n \), in the limit version \( K_{b,j,2} = 0 \).

The weighted basic quality profile of the product quality \( \Pi_{K,wb} \), created from the corresponding single basic relative product quality indexes of the first group \( K_{b,i,1}, \ i = 1, \ldots, l \) and the second group \( K_{b,j,2}, \ j = l + 1, \ldots, n \) and which reflects the basic certified reference material of a given product, is a virtual measurement standard of the product quality. It is used in the procedure of qualimetric measurement to determine the quality level \( Q \) of the studied product as a reference standard of the measured value.

### 3.5. Constructing estimated quality profile of studied product

By analogy with the construction of the basic quality profile \( \Pi_{P,b} \), the estimated quality profile \( \Pi_{P,e} \) of the studied product is created from the single estimated absolute quality indexes \( P_{e,i}, \ i = 1, \ldots, n \) (\( P_{e,i,1}, \ i = 1, \ldots, l \) of the first group and \( P_{e,j,2}, \ j = l + 1, \ldots, n \) of the second group):

\[
\Pi_{P,e} = \sum_{i=1}^{l} P_{e,i,1} + \sum_{j=l+1}^{n} P_{e,j,2} = \{ P_{e,1,1}; \ldots; P_{e,l,1} \} + \{ P_{e,l+1,2}; \ldots; P_{e,n,2} \}. \tag{9}
\]

The numerical values of single estimated absolute quality indexes \( P_{e,i}, \ i = 1, \ldots, n \) of the studied product are found experimentally, by measuring the corresponding properties of this product \( p_i, \ i = 1, \ldots, n \). The measurement is carried out in accordance with the procedures, established in the current normative documents, in which the requirements for the properties of a specific product and methods of its measurement are regulated.

Based on the estimated quality profile \( \Pi_{P,e} \), created according to (9), the estimated quality profile \( \Pi_{K,e} \) is derived from the corresponding single estimated relative product quality indexes \( K_{e,i,1}, \ i = 1, \ldots, l \) of the first group and \( K_{e,j,2}, \ j = l + 1, \ldots, n \) of the second group:

\[
\Pi_{K,e} = \sum_{i=1}^{l} K_{e,i,1} + \sum_{j=l+1}^{n} K_{e,j,2} = \{ K_{e,1,1}; \ldots; K_{e,l,1} \} + \{ K_{e,l+1,2}; \ldots; K_{e,n,2} \}. \tag{10}
\]

Then, the corresponding weighed estimated quality profile of the product quality \( \Pi_{K,we} \) is created:

\[
\Pi_{K,we} = \sum_{i=1}^{l} K_{we,i,1} + \sum_{j=l+1}^{n} K_{we,j,2} = \{ K_{we,1,1}; \ldots; K_{we,l,1} \} + \{ K_{we,l+1,2}; \ldots; K_{we,n,2} \}. \tag{11}
\]

where: \( K_{we,i,1} \) and \( K_{we,j,2} \) are the corresponding single weighed estimated relative product quality indexes of the first and second groups.

The values of indexes \( K_{we,i,1} \) and \( K_{we,j,2} \) are calculated by the formulae:

\[
K_{we,i,1} = K_{e,i,1} \cdot m_{i,1}, \quad i = 1, \ldots, l; \quad K_{we,j,2} = K_{e,j,2} \cdot m_{j,2}, \quad j = l + 1, \ldots, n. \tag{12}
\]

The numerical values of the single estimated relative product quality indexes \( K_{e,i,1}, \ i = 1, \ldots, l \) of the first group and \( K_{e,j,2}, \ j = l + 1, \ldots, n \) of the second group are calculated by the formulae (2) and (3).
4. Methodology of implementation of procedure of qualimetrical measurement

4.1. Application of Multidimensional Scaling Methodology to qualimetrical measurement

To measure the quality level $Q$ of the studied product, i.e., for full implementation of the procedure of qualimetrical measurement, it is necessary to compare the weighted estimated quality profile of the product $\Pi_{K,we}$ with the weighted basic quality profile $\Pi_{K,wb}$, that is to say with the virtual measurement standard of the product quality. To compare the quality profiles $\Pi_{K,we}$ and $\Pi_{K,wb}$, the methodology of multidimensional scaling – one of the sections of mathematical statistics, is used [8]. The basis of the multidimensional scaling (MDS) is the idea of a geometric image of objects in the form of a set of points in a multidimensional space, and a degree of difference between objects is the distance between these points. To measure the distances between the points, it is necessary to use a metric $d(x, y)$, which is an ordered, non-negative, single-valued and real function on a set $D$, defined for any pair of points $x$ and $y$ of the set $D(x, y \in D)$. The set $D$ together with the metric $d(x, y)$ is called a metric space $(D, d)$.

In the MDS the Euclidean metric is used, since in this study the distance does not depend on the direction of the coordinate axes and that enables to make any coordinate rotations. A multidimensional space, in which two objects are compared, namely the degree of closeness or difference between these objects is determined, is called the Euclidean space.

This methodology enables to confront the single weighted estimated relative product quality indexes $K_{we,i,1}$ and $K_{we,j,2}$ of the first and second groups with the corresponding single weighted basic quality indexes $K_{wb,i,1}$ and $K_{wb,j,2}$. It also gives an opportunity to bring together various measurement scales of these indexes up to a one-dimensional scale to determine the quality level $Q$ of the studied product.

4.2. Choice of multidimensional scaling model for determination of product quality level

The values of the single estimated absolute product quality indexes $P_{e,i}$, $i = 1, \ldots, n$, which are the result of measurement in the process of research, are generally random variables. Accordingly, the single estimated relative product quality indexes $K_{e,i}$, $i = 1, \ldots, n$ are random variables too.

The choice of a model of the multidimensional scaling is based on the analysis of the presence or absence of correlation between the single estimated absolute product quality indexes $P_{e,i}$, $i = 1, \ldots, n$, and, respectively, between the single estimated relative product quality indexes $K_{e,i}$, $i = 1, \ldots, n$.

If correlation between the single estimated product quality indexes is absent, to compare the weighted estimated quality profile of the product $\Pi_{K,we}$ with the weighted basic quality profile $\Pi_{K,wb}$, in other words with the virtual measurement standard of the product quality, a weighted Euclidean model of individual differences is used. Then, a function of differences $\Delta \Pi$ between the profiles $\Pi_{K,we}$ and $\Pi_{K,wb}$ is defined by the formula:

$$\Delta \Pi = \sqrt{l \sum_{i=1}^{l} (K_{we,i,1} - K_{wb,i,1})^2 + n \sum_{j=l+1}^{n} (K_{we,j,2} - K_{wb,j,2})^2} =$$

$$= \sqrt{\sum_{i=1}^{l} m_{i,1}^2 \cdot (K_{e,i,1} - K_{b,i,1})^2 + \sum_{j=l+1}^{n} m_{j,2}^2 \cdot (K_{e,j,2} - K_{b,j,2})^2}.$$  (13)
When there exists a correlation between the single estimated product quality indexes of the studied product, to determine the level of its quality a three-modal model of the multidimensional scaling is used. It enables to take into account the correlation between the single estimated relative product quality indexes of gas $K_{e,\chi}$ and $K_{e,\xi}$, $\chi \neq \xi$, $\chi = 1, \ldots, n$, and the function of differences $\Delta \Pi$ between the profiles $\Pi_{K,we}$ and $\Pi_{K,wb}$ is defined by the formula:

$$\Delta \Pi = \sqrt{\sum_{i=1}^{l} (K_{we,i,1} - K_{wb,i,1})^2 + \sum_{j=l+1}^{n} (K_{we,j,2} - K_{wb,j,2})^2 + 2 \sum_{\chi=1}^{n-1} \sum_{\xi=\chi+1}^{n} m_\chi m_\xi \cdot (K_{e,\chi} - K_{b,\chi}) \cdot (K_{e,\xi} - K_{b,\xi}) \cdot r_{K_{e,\chi},K_{e,\xi}}}$$

$$= \sqrt{\sum_{i=1}^{l} m_{i,1}^2 \cdot (K_{e,i,1} - K_{b,i,1})^2 + \sum_{j=l+1}^{n} m_{j,2}^2 \cdot (K_{e,j,2} - K_{b,j,2})^2 + 2 \sum_{\chi=1}^{n-1} \sum_{\xi=\chi+1}^{n} m_\chi m_\xi \cdot (K_{e,\chi} - K_{b,\chi}) \cdot (K_{e,\xi} - K_{b,\xi}) \cdot r_{K_{e,\chi},K_{e,\xi}}}$$

(14)

where $m_\chi$ and $m_\xi$, respectively, are normalized weighted coefficients of the single product quality indexes $K_{e,\chi}$ and $K_{e,\xi}$, $\chi \neq \xi$, $\chi = 1, \ldots, n$; $r_{K_{e,\chi},K_{e,\xi}}$ is a correlation coefficient between the single product quality indexes $K_{e,\chi}$ and $K_{e,\xi}$, $\chi \neq \xi$, $\chi = 1, \ldots, n$.

### 4.3. Synthesis of scale of quality level $Q$

To determine the quality level $Q$ of the studied product using the methodology of qualimetical measurement, a one-dimensional qualimetical scale of the product quality level $Q$ must be constructed. In agreement with the generally accepted international definition, a measurement scale (quantity-values scale) is an ordered set of quantity values used in classifying them, according to e.g. magnitude [6].

A measurement scale of the product quality level $Q$, used by the methodology of qualimetical measurement, is created taking into account the value of the function of differences $\Delta \Pi$ between the weighted quality profiles $\Pi_{K,we}$ and $\Pi_{K,wb}$ obtained by the formula (13) or (14) and can be denoted as follows:

$$Q = 1 - \Delta \Pi \quad \text{or} \quad Q = (1 - \Delta \Pi) \cdot 100\%.$$  

(15)

Consequently, a higher quality of a product corresponds to a smaller value of the function of differences and a greater numerical value of the level of its quality $Q$. The measurement range of the quality level $Q$ is from 0 to 1 or from 0 to 100%. It is a convenient and methodologically grounded way to employ in practice for the assessment of product quality. Due to the obtained value of product quality level of $Q$, it is possible to classify the product according to its quality and, consequently, establish an appropriate price of it.

The constructed measurement scale of the product quality level $Q$, conforming with the classification of measurement scales [6], is a scale of intervals, that is to say a metric scale. It helps to find the difference $\Delta Q = Q_1 - Q_2$ between different values of the product quality levels $Q_1$ and $Q_2$. Additionally, to process the results of qualimetical measurement, apart from mod and median, another mathematical statistics measures, such as mathematical expectation, dispersion and standard deviation can be used.
5. Procedure of determination of product quality level $Q$ in accordance with methodology of qualimetical measurement

In order to determine the product quality level $Q$, according to the methodology of qualimetical measurement, it is necessary to perform the following operations:

1. to establish the nomenclature and to make an analysis and systematization of the quality indexes of the studied product;
2. to establish the parameters of the material standard (material measure) of the product quality;
3. to create a basic quality profile $P_{b,i}$ from the single basic absolute product quality indexes, $P_{b,i}, i = 1, \ldots, n$;
4. to create an estimated quality profile $P_{e,i}$ from the single estimated absolute product quality indexes, $P_{e,i}, i = 1, \ldots, n$;
5. to determine the single relative basic $K_{b,i}, i = 1, \ldots, n$ and estimated $K_{e,i}, i = 1, \ldots, n$ product quality indexes;
6. to determine the normalized weighted coefficients $m_i, i = 1, \ldots, n$ of the single product quality indexes;
7. to determine the single relative weighted basic $K_{wb,i}, i = 1, \ldots, n$ and weighted estimated $K_{we,i}, i = 1, \ldots, n$ product quality indexes;
8. to set up a weighted basic quality profile of the product quality $K_{wb}$, particularly a virtual measurement standard of the product quality;
9. to set up a weighted estimated quality profile of the product quality $K_{we}$;
10. to determine the values of correlation coefficients $r_{K_{e,\chi}, K_{e,\xi}}$ between the single product quality indexes $K_{e,\chi}$ and $K_{e,\xi}, \chi \neq \xi, \chi = 1, \ldots, n$;
11. to choose a multidimensional scaling model to determine the product quality level $Q$ and to calculate the function of differences $\Delta II$;
12. to work out the quality level $Q$ of the studied product;
13. to analyse the accuracy of product quality level $Q$ determination, by finding the uncertainty of the $Q$ measurement result [11].

6. Study of Natural Gas Quality as energy source by methodology of qualimetical measurement

The method of complex estimation of the quality of natural gas (NG) as an energy source by means of determining the gas quality level based on the methodology of qualimetical measurement is developed [12]. The composition of gas and its basic physical and chemical properties, given in Example 1, are taken into account (single absolute gas quality indexes $P_{i}, i = 1, \ldots, n, n = 8$).

The input information for defining NG quality as an energy source, obtained with the methodology of qualimetical measurement was taken from the results of experimental studies on samples of natural gas stored in Lviv region, carried out in the laboratory of the Gas Production Division LvivGasVydobuvannya. The gas components were identified with a chromatograph according to ISO 6974-1:2012 [13]. Calculations of the gas specific heat $H_H$, MJ/m$^3$, $\rho$ gas density, kg/m$^3$ and Wobbe index $B$, MJ/m$^3$ were made basing on the gas composition with regard to ISO 6976:2016 [14]. The sulphur compounds – mass concentrations of hydrogen sulphide $C_{H_2S}$, g/m$^3$ and mercuric sulphur $C_{CH_2S}$, g/m$^3$ in the gas mixture were determined according to ISO 19739:2004. The dew point temperature at humidity $\theta_r$, °C was measured by a hygrometer, and absolute gas
humidity $W$, g/m$^3$ depending on the measured dew point of gas at humidity $\Theta_p$, °C was defined according to ISO 6327:1981 [16].

The results of performed analysis of gas samples are provided in Table 2.

Table 2. Results of the experimental defining of natural gas quality as an energy source.

<table>
<thead>
<tr>
<th>No</th>
<th>Title of the gas quality index</th>
<th>Absolute indexes</th>
<th>Relative indexes</th>
<th>Weight coefficients</th>
<th>Relative weighted indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$P_{b,i}$</td>
<td>$P_{e,i}$</td>
<td>$K_{b,i}$</td>
<td>$K_{e,i}$</td>
</tr>
<tr>
<td>1</td>
<td>Gas specific heat $H_H$, MJ/m$^3$</td>
<td>45.00</td>
<td>39.65</td>
<td>1</td>
<td>0.881</td>
</tr>
<tr>
<td>2</td>
<td>Wobbe index $B$, MJ/m$^3$</td>
<td>54.50</td>
<td>48.35</td>
<td>1</td>
<td>0.877</td>
</tr>
<tr>
<td>3</td>
<td>Gas density $\rho$, kg/m$^3$</td>
<td>0.960</td>
<td>0.820</td>
<td>1</td>
<td>0.854</td>
</tr>
<tr>
<td>4</td>
<td>Absolute gas humidity $W$, g/m$^3$</td>
<td>0</td>
<td>0.085</td>
<td>0</td>
<td>0.773</td>
</tr>
<tr>
<td>5</td>
<td>Concentration of nitrogen $C_{N_2}$, %</td>
<td>0</td>
<td>1.134</td>
<td>0</td>
<td>0.810</td>
</tr>
<tr>
<td>6</td>
<td>Concentration of carbon dioxide $C_{CO_2}$, %</td>
<td>0</td>
<td>0.272</td>
<td>0</td>
<td>0.604</td>
</tr>
<tr>
<td>7</td>
<td>Mass concentration of hydrogen sulphide $C_{H_2S}$, g/m$^3$</td>
<td>0</td>
<td>0.002</td>
<td>0</td>
<td>0.100</td>
</tr>
<tr>
<td>8</td>
<td>Mass concentration of mercuric sulphur $C_{CH_4S}$, g/m$^3$</td>
<td>0</td>
<td>0.005</td>
<td>0</td>
<td>0.139</td>
</tr>
</tbody>
</table>

**Note 1.** All symbols of the absolute and relative quality indexes given in the table correspond to the notation of these indexes in the text of the paper.

**Note 2.** Dimensions of values, provided in the table, are taken for standard conditions, namely for $p_c = 0.101325$ MPa, $K_c = 293.15$ K.

The NG quality level $Q$ was determined with the qualimetric measurement method in two stages. First, the function of differences $\Delta \Pi$ between the profiles $\Pi_{K,we}$ and $\Pi_{K,wb}$ was determined by the formula (13) to be equal to $\Delta \Pi = 0.187$ (the correlation between the single estimated gas quality indexes was not examined). After this, according to (15), the level of gas quality $Q$ was determined – $Q = 0.813$ or 81.3%. For this value $Q$ the level of NG quality was estimated as quite high. This was due to the proximity of absolute values of single-quality evaluated gas indexes $P_{e,i}$ and estimated relative individual quality gas indexes $K_{e,i}$, to the basic values of these indexes – $P_{b,i}$ and $K_{b,i}$, respectively.

It is possible to improve the gas quality by applying such technological operations as gas drying and gas mixture cleaning from non-combustible and harmful components. Then, the theoretical values of the last five single absolute estimated gas quality indexes $P_{e,i}$, namely of the second group indexes, and – respectively – the last five single relative estimated gas quality indexes $K_{e,i}$, may be reduced to zero. The value of the function of differences in this case $\Delta \Pi \equiv 0.05$ and the value of the gas quality level $Q = 0.95$ or 95% are close to the base values. However, this augmentation of gas quality could lead to a significant increase in the gas price.

7. Conclusions

1. The usage in theory and practice of qualimetry of the conceptual notation of “qualimetric measurement”, as one of the types of measurement, makes it possible to combine the
methodologies of qualimetry and metrology. It significantly expands the capability of qualimetry onto the objectiveness of the product quality estimates.

2. The application of main provisions of the representative theory of measurement to the analysis of the qualimetical measurement, enables to obtain the condition of uniformity of the measurement. This makes it possible to compare the measurement results of the properties of identical objects, obtained in different laboratories, conditions, methods and means. In this case, the metrological traceability of results of the qualimetical measurement is achieved.

3. Solving the basic problems of the development of qualimetical measurement methodology should be carried out in a complex way, on the basis of theory of the virtual measurement standard of product quality and theory of the multidimensional scaling. It is advisable to use a weighted Euclidean model of individual differences and a three-dimensional model of multidimensional scaling to determine the quality level of the studied product.

4. The method of complex evaluation of natural gas quality based on the methodology of qualimetical measurement enables to perform an objective assessment of natural gas quality, aiming to differentiate gas samples regarding their quality and, further, to compare the quality of gas in various reservoirs and to set an appropriate price.

References


