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Thermodynamic analysis and analytical simulation of the Rallis modified Stirling cycle

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Abstract A Stirling cycle was developed by Rallis considering the adiabatic behaviour instead of isothermal behaviour of working fluid inside the expansion/compression volume, since the isothermal processes are very difficult to be realised in actual practice due to irreversibilities. In order to increase the performance of Rallis Stirling cycle engine, two modified versions of Rallis Stirling cycle engine model have been proposed and developed, called as Rallis modified Stirling cycle engine (RMSE). In this paper, the thermodynamic analysis of the developed models have been carried out and the simulated results are compared with the Rallis ideal model of Stirling cycle engine, as this model describes more accurately the thermodynamic cycle of practical Stirling machines. The results reveal the fact that the thermal efficiency of RMSE I model is enhanced by 38.06% and that of RMSE II model by 48.42%, whereas the power output is increased by 58.05% and 78.19% in case of RMSE I and RMSE II model respectively, when compared with the Rallis ideal adiabatic model of Stirling engine.

Keywords: Rallis modified Stirling cycle engine; Non-isothermal characteristics; Power output; Thermal efficiency

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Nomenclature

B_N	–	Beale number
C_v	–	specific heat at constant volume, kJ/kgK
k	–	adiabatic index of working fluid
RMSE	–	Rallis modified Stirling cycle engine
m	–	mass of working fluid, kg
N	–	engine speed, rpm
n	–	polytropic index of working fluid
P	–	pressure, kPa
P_i	–	power output of Stirling engine, kW
P_m	–	mean pressure, kPa
Q_S	–	heat supplied, kJ
R	–	gas constant, kJ/kgK
r	–	compression ratio of Rallis Stirling cycle
r_1	–	compression ratio of RMSE I model
r_2	–	compression ratio of RMSE II model
S	–	entropy, J/K
T	–	temperature, K
t	–	temperature ratio
V	–	volume, m^3
V_{ES}	–	expansion space swept volume, m^3
V_{CS}	–	compression space swept volume, m^3
V_{EC}	–	expansion space clearance volume, m^3
V_{CC}	–	compression space clearance volume, m^3
W	–	net work output, kJ

Greek symbols

ε	–	effectiveness of regenerator
η	–	efficiency

Subscripts

E	–	expansion space
C	–	compression space
c	–	Carnot
$poly$	–	polytropic
$adia$	–	adiabatic
Max	–	maximum
Min	–	minimum

1 Introduction

In the last few decades, economic growths around the world bring about a serious concern over the global environment. Also, the gap between the energy supply and energy demand is increasing day-by-day due to rapid depletion and high price of conventional fossil fuel sources. It has been seen that the large amount of energy available around the world in the form of waste heat and geothermal energy remains untapped. The new energy conversion devices and energy sources must be explored and developed to protect the environment as well as in order to fulfill the current energy demands. Stirling engine became a solution for many environmentalists, scientists and policy makers to reduce the gap between the energy supply and its requirements. Stirling engine is highly thermal efficient engine capable of using variety of fuels; biomass, bio gas, fossil fuel, nuclear sources, solar energy, geothermal energy and many more with less harmful emission [1,2]. It is a clean and reliable energy conversion device to produce mechanical/electrical power. Stirling engine is now globally accepted and becomes a key component in the field of renewable energy sector. The acceptability of Stirling engine in the power sectors increases due to its high thermal efficiency, high specific work, high reliability and safe operation compared to any closed regenerative cycle [2–4]. The most commonly cited application of Stirling engine is distributed generation for backup power as these machines are found to be 25–30% efficient for converting heat energy into electrical energy [5]. The other important application of Stirling engine includes solar power generation, Stirling cryocoolers, heat pump, submarine power generation, nuclear power generation, automotive engine, electrical vehicles, air craft engines and low temperature differential engines [6].

The Stirling engine was invented by Dr. Robert Stirling in 1816. It is an external combustion reciprocating engine that works on a closed regenerative cycle with periodic variation of working gas inside the expansion and compression cylinders at different temperature levels [7]. The cycle consists of two isochoric and two isothermal processes as shown in Fig. 1, such that the maximum thermal efficiency of Stirling engine cycle is equivalent to Carnot cycle's efficiency. As the expansion and compression take place in a different cylinder volume, the Stirling engine has been classified as alpha, beta and gamma-type configuration on the basis of the forms of cylinder coupling [8].

After the earlier invention of Stirling cycle engine, various models have been developed and analysed by the researchers for predicting the thermal performance like output power and thermal efficiency. In view of this various thermal models of Stirling engine were developed and classified mainly into three types-empirical, numerical and analytical models [9]. Empirical models are used which correlate Stirling engine performance prediction based on previous experimentation such as the generalized Beale number

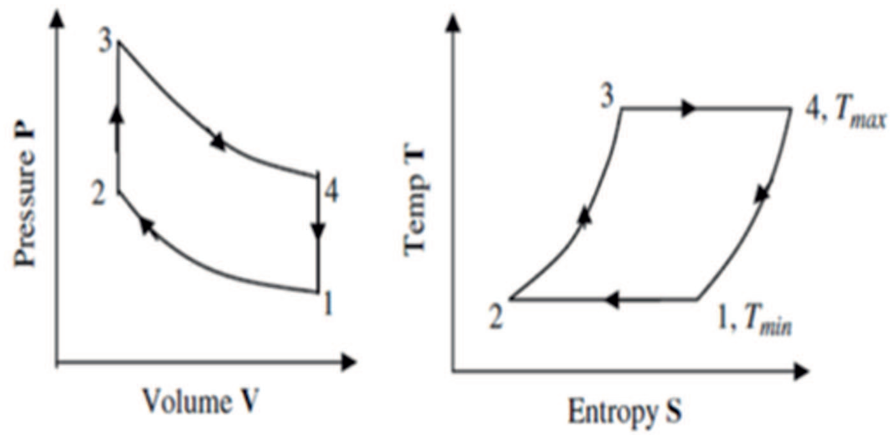

Figure 1: P-V and T-S diagrams for Stirling engine.

derived by J.R. Senft [10]. Numerical models are the second type of thermal models which are used for simulating real engines [11]. The first numerical model was Schmidt's classical thermodynamic analysis of ideal Stirling cycle engine provided by G. Schmidt in 1871, which is also known as isothermal analysis of Stirling cycle engine [12,13] as shown in Fig. 2. However, a simple closed form solution exists for this model but, the isothermal expansion and compression processes are not possible in real Stirling engine as these processes require an infinite heat transfer rate [11]. The isothermal behaviour of Stirling engine can only be realised for better thermal conductivity of walls of the working cylinders or at lower speed of the engine, which is not possible in real Stirling engine. Therefore, the isothermal model is only used for simulating the thermodynamic cycle. It has been seen that, the Stirling engine that runs with a speed more than 200 rpm, adiabatic behaviour appears in the machine [14]. This is due to the fact that, the Stirling engine which runs at the higher speed, there is only very little time for heat transfer through the walls of the cylinder, so that the rate of heat transfer is finite.

Thus, real engines work with an adiabatic change of state instead of isothermal and therefore, the lower expansion work and the higher compression work is observed as shown in the Fig. 3. This deviation of real Stirling cycle engine due to non-isothermal behaviour was first explained by T. Finkelstein in 1960 by an ideal adiabatic model as shown in Fig. 4,

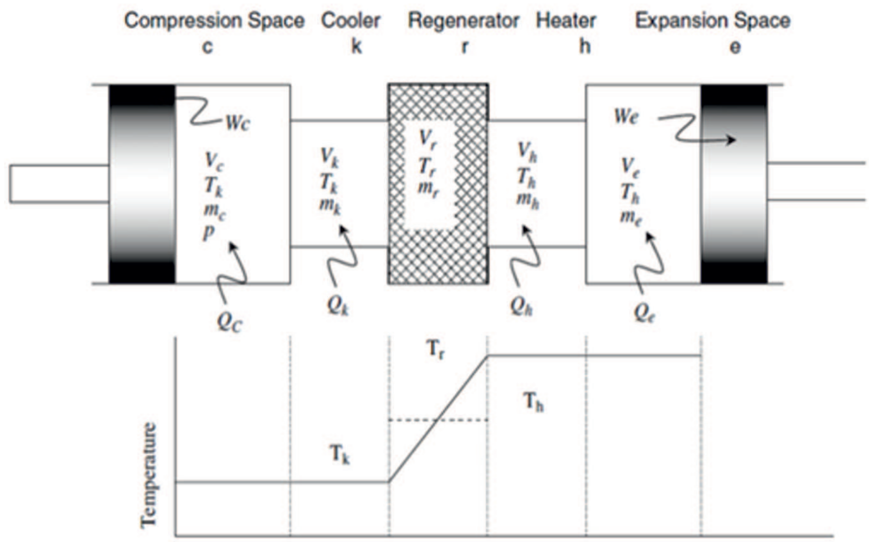

Figure 2: Ideal isothermal Stirling engine model [13].

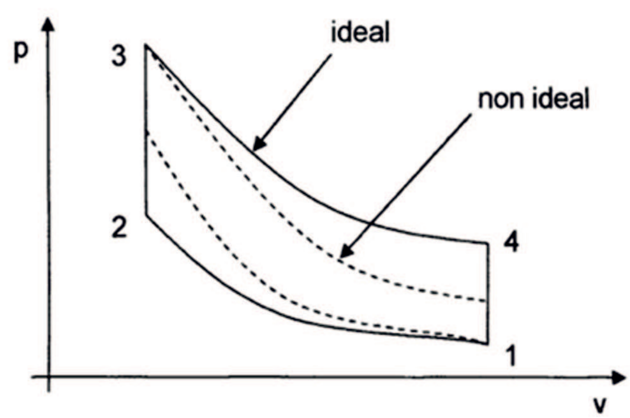

Figure 3: Effects of the adiabatic change of state on the P-V diagram [14].

which is the most significant development at that time [13,15].

He took the Schmidt analysis a stage ahead by analysing the adiabatic behaviour of Stirling engine that runs at higher frequencies on the accounts for irreversible heat transfer between the working fluid and the walls of

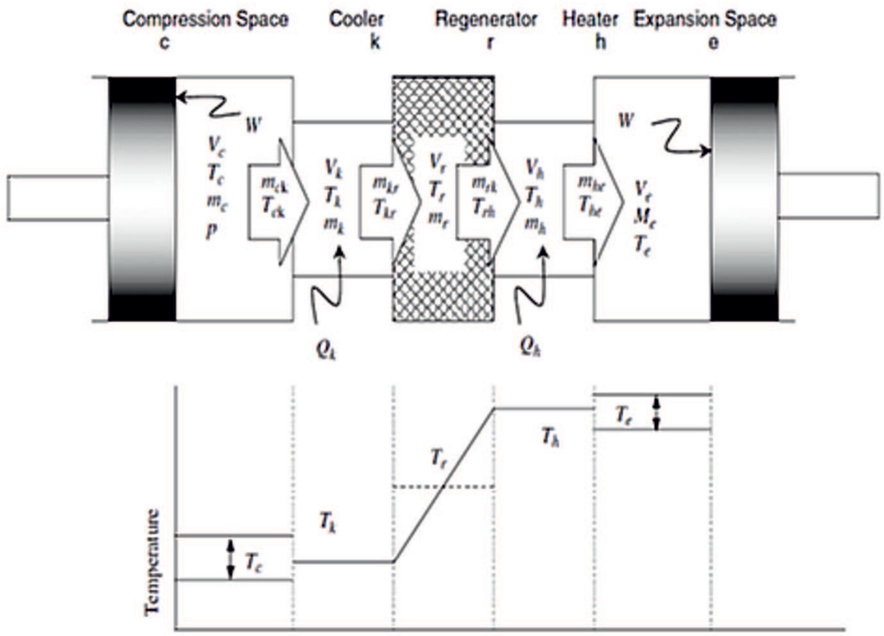

Figure 4: Ideal adiabatic Stirling engine model [13].

expansion and compression cylinder [11,16]. Due to the non-isothermal operational behaviour of the working fluid inside the working volumes of the Stirling engine, it is necessary that the net heat transfer over the cycle must be provided by the heat exchanger and therefore, it is assumed that the heater, regenerator and cooler behave perfectly [11,13]. The adiabatic analysis of Stirling engine leads to 20–30% lower efficiency than that of ideal isothermal model, but this efficiency is still significantly higher than that of a real Stirling engine [17]. After Finkelstein, many researchers have analysed the ideal adiabatic model of Stirling cycle engine. In 1967, Qvale analysed an idealized adiabatic cycle that has no friction or seal leakage [18]. The variation in pressure, volume, mass and piston displacements are all assumed to be sinusoidal. Qvale's adiabatic analysis is more suitable for the engine synthesis than for the performance predictions of a specific engine [19]. Qvale's model was successfully validated with the given test data of the Allison PD-67A experimental Stirling engine. In 1969, Rios analysed the Stirling engine adiabatic model developed by Qvale for non-sinusoidal piston displacements [20], which was further modified by Martini to suit for their Stirling engine model [11].

In 1973, Feurer, a researcher from the Philips Company, described an adiabatic model for non-zero and finite heat transfer coefficients in the cylinders, regenerator and heat exchangers [21]. He found that all the loss mechanisms in Stirling engine are phase angle dependent and the maximum efficiency and the maximum power output do not occur at the same phase angle. In 1980, Lee *et al.* introduced a unique power loss mechanism in Rios model of Stirling engine, called cyclic heat transfer loss that results from periodic heating and cooling of the working gas [22]. A real Stirling engine suffers from four major power losses namely adiabatic, cyclic heat transfer, pressure drop and heat exchanger temperature drop. They have shown in their analysis that cyclic heat transfer loss contributes second most significant loss in Stirling engine, which is about one-third of the total power loss [22]. Shoureshi in 1982 developed two Stirling engine models for low temperature-ratio applications [23]. The first model was based on Rios' adiabatic model in which two important losses mechanical friction and transient heat transfer were introduced and called 'complete model'. The other developed model was 'simplified model' which was based on isothermal analysis with a two-step correction for the net power output and heat input [24].

Further in 1984, Urieli and Berchowitz introduced a quasi steady flow analysis of the Stirling engine by implementing imperfect regeneration, pressure losses in regenerator and heat exchangers into Finkelstein's ideal adiabatic model [25]. This model has been different from Finkelstein's adiabatic model due to the temperature drop between the compression volume and cooler and that of between the heater and the expansion volume as shown in Fig. 5. The quasi steady flow model is a numerical model which provides the better predictions of Stirling engine performance than the Finkelstein's adiabatic model. However, it doesn't accurately account for all losses in the Stirling engine.

Several other remarkable works have been reported by many researchers by considering the non-ideal heat transfer effect and pressure drop in heat exchangers. In 2002, Petrescu *et al.* developed a direct method technique to estimate the thermal efficiency and output power of Stirling engine by considered the effects of frictional pressure loss, finite speed and throttling processes in the regenerator [3]. This method accurately predicted the Stirling engine performance when validated with the real engine. In 2008, Tlili *et al.* considered the effect of shuttle loss, heat loss and pressure drop for

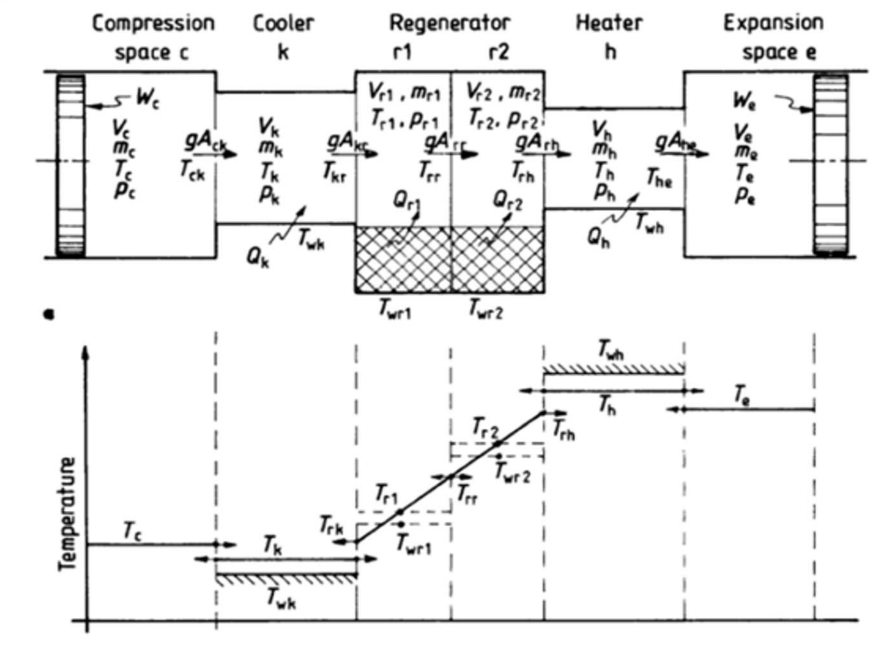

Figure 5: The Quasi steady flow model [25].

evaluating the thermal features of medium-temperature differential Stirling engines [26]. Further, in 2008, Timoumi *et al.* considered the effect of external irreversibilities to analyse the Stirling engine operation and found an appropriate solution as compared with the previous developed models [27]. Abbas *et al.* developed a simple model of Stirling engine in which non-ideal characteristics of regenerator, shuttle loss and heat conduction losses were considered [28]. In another approach, Babaelahi and Sayyaadi in 2014 developed a new thermal model called 'simple-II' model by considering the shuttle effect and gas leakage in original simple adiabatic model, which had better accuracy than the previous model [29]. Further, Hosseinzade and Sayyaadi modified the actual adiabatic model of Stirling cycle engine by considering the effect of finite speed of piston, pressure drop in heat exchangers and piston mechanical friction and developed another combined adiabatic finite speed thermal model (CAFS), which gives better results when simulated with GPU-3 Stirling engine [30]. The accuracy of the developed CAFS model was found superior over other previous models and is more suitable for predicting indicated power compared to other real engines.

The third type of thermal model is analytical model which is used for thermodynamic analysis of the Stirling cycle that approximately simulates a real working engine. They are generally simple in use to simulate Stirling engine cycle as they are free from the complicated and costly numerical solution [9].

From the available literature, it has been found that the various numerical thermal models of Stirling engine have been developed and analysed to predict the performance of real engines. However, these models are associated with the complex, rigorous and costly numerical work for analysis of the Stirling engine. From the above literature, it may also be concluded that the next generation of Stirling cycle engine models need to be developed based on the assumption of non-isothermal expansion and compression processes taking place in the working volume instead of isothermal. The objective of this paper is to develop an analytical thermodynamic model of modified Stirling cycle based on the non-isothermal characteristics of working fluid inside the expansion/compression volume, which could estimate the output power and thermal efficiency of the engine.

2 Methodology

Two modified versions of Stirling cycle have been proposed. Further, a thermodynamic analytical model has been developed for each modified version of newly proposed Stirling cycle engine model based upon the thermodynamic performance of Rallis Stirling cycle engine to evaluate the overall qualitative criteria for performance of modified Stirling cycle engine models. Thermodynamic performance of the two modified Stirling cycle engine models have been analysed under different cases. The design parameters which are used for the Stirling cycle engine models were developed using Beale correlation. The operating parameters have been set by analysing the performance of various Stirling engine as given in literature survey. Finally, results were obtained and compared with Rallis model under different cases based on the analytical simulation of different Stirling cycle engine models.

3 Thermodynamic analysis of Stirling engine models

3.1 Rallis Stirling cycle engine model

From the literature survey, it has been seen that the Stirling cycle model was developed based on the assumption that isothermal expansion and compression processes were unreasonable owing to the poor heat transfer in the working spaces of practical machines. So, it is more reasonable to expect that the expansion and compression processes occur polytropically and in the limit approach adiabatic conditions [31]. Therefore an ideal cycle has been suggested by Prof. C.J. Rallis, in which all heat transfer occurred under constant volume conditions and where the expansion and compression process were assumed to be adiabatic. The cycle proposed by Rallis is generally referred to as the ideal adiabatic Stirling cycle. It has been seen that from the cyclic point of view, the ideal adiabatic Stirling cycle model appears be more accurately describing the thermodynamic cycle of practical machines and the results obtained from this cycle were found in conformity with the natural laws and real engine features [31,32].

The P-V and T-S diagram of ideal adiabatic Stirling cycle is shown in Fig. 6, where isothermal processes (1-2′ and 3-4′) are replaced by isentropic processes (1-2 and 3-4) respectively. Here after the compression process, the working fluid is transferred through the cooler at constant volume where its pressure drops to 2′. The working fluid then passed through the regenerator where its pressure rises to 3′ and finally through the heater at constant volume where the pressure further rises to 3. Similarly, after the isentropic expansion process, the working fluid is transferred through the heater at constant volume where its pressure rises to 4′ and then is passed through the regenerator where its pressure drops to 1′ and finally through the cooler where the pressure further drops to 1. A schematic diagram of the Rallis ideal adiabatic Stirling engine model is also shown in Fig. 7.

However, if it is assumed that expansion and compression processes occur polytropically ($0 < n < k$) due to finite heat transfer instead of adiabatic or isothermal then this cycle will be represented by processes 1-2″-2′-3′-3-4″-4′-1′-1 on P-V and T-S diagram as shown in Fig. 6. This cycle is now referred to as the ideal polytropic Stirling cycle and in the limiting case when index of compression/expansion approaches adiabatically (i.e., $n = k$), it becomes the ideal adiabatic Stirling cycle.

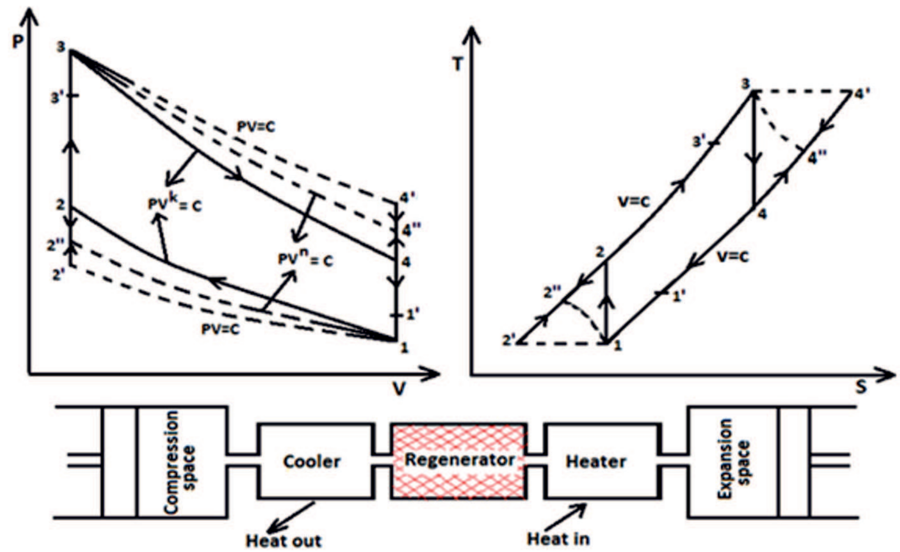

Figure 6: Rallis ideal adiabatic Stirling cycle model [31].

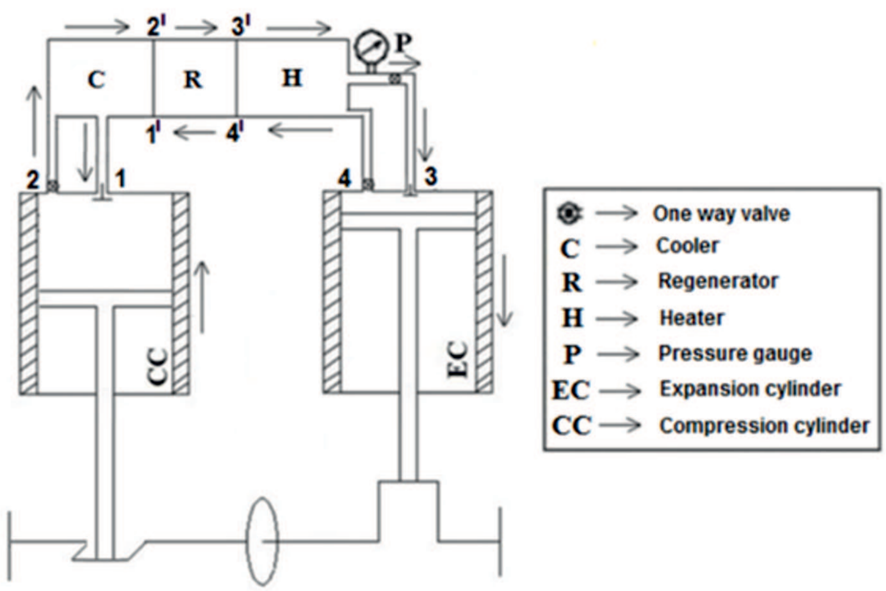

Figure 7: Schematic diagram of Rallis ideal adiabatic Stirling engine model.

The net work output obtained from the ideal polytropic cycle is given as

$$W_{Net} = W_E - W_C = \frac{mR}{n-1}\left[(T_3 - T_{4''}) - (T_{2''} - T_1)\right] . \tag{1}$$

The heat supplied to the cycle is given as

$$\begin{aligned} Q_S &= Q_{3'-3} + Q_{3-4''} + Q_{4''-4'} = \\ &= mC_v\left[(T_3 - T_{3'}) + \frac{k-n}{n-1}(T_3 - T_{4''}) + (T_{4'} - T_{4''})\right] . \end{aligned} \tag{2}$$

Now, the effectiveness of the regenerator is given as

$$\varepsilon = \frac{T_{3'} - T_{2'}}{T_3 - T_{2'}} = \frac{T_{3'} - T_1}{T_3 - T_1} . \tag{3}$$

From Eq. (3), putting the value of $T_{3'} = \varepsilon(T_3 - T_1) + T_1$ and $T_{4'} = T_3$ in Eq. (2), then it becomes

$$Q_S = mC_v\left[(1-\varepsilon)(T_3 - T_1) + \frac{k-1}{n-1}(T_3 - T_{4''})\right] . \tag{4}$$

Therefore, the efficiency of the cycle is given as

$$\begin{aligned} \eta_{poly} = \frac{W_{Net}}{Q_S} &= \frac{\frac{mR}{n-1}\left[(T_3 - T_{4''}) - (T_{2''} - T_1)\right]}{mC_v\left[(1-\varepsilon)(T_3 - T_1) + \frac{k-1}{n-1}(T_3 - T_{4''})\right]} \\ &= \frac{R}{C_v\ (k-1)}\ \frac{\left[(T_3 - T_{4''}) - (T_{2''} - T_1)\right]}{\left[\frac{n-1}{k-1}(1-\varepsilon)(T_3 - T_1) + (T_3 - T_{4''})\right]} . \end{aligned}$$

Now, here the gas constant is written as

$$R = C_p - C_v = C_v\left(\frac{C_p}{C_v} - 1\right) = C_v\,(k-1) ,$$

where $k = \frac{C_p}{C_v}$ is specific heat ratio (or isentropic index) of air ($k = 1.4$) because air is used as working fluid. Hence, the term $\frac{R}{C_v\ (k-1)} = 1$ the above equation, therefore

$$\eta_{poly} = \frac{(T_3 - T_{4''}) - (T_{2''} - T_1)}{(T_3 - T_{4''}) + \frac{n-1}{k-1}(1-\varepsilon)(T_3 - T_1)} . \tag{5}$$

Put $\frac{T_3}{T_{4''}} = \left(\frac{V_{4''}}{V_3}\right)^{(n-1)} = r^{(n-1)} = \frac{T_{2''}}{T_1}$ in the above Eq. (5), where r is the compression ratio for Rallis Stirling cycle

$$\begin{aligned}\eta_{poly} &= \frac{T_{4''}\left(r^{n-1}-1\right) - T_1\left(r^{n-1}-1\right)}{T_{4''}\left(r^{n-1}-1\right) + \frac{n-1}{k-1}\left(1-\varepsilon\right)\frac{(T_3-T_1)}{T_3}T_3} \\ &= \frac{\left(T_{4''}-T_1\right)\left(r^{n-1}-1\right)}{T_3\left[\frac{T_{4''}}{T_3}\left(r^{n-1}-1\right) + \frac{n-1}{k-1}\left(1-\varepsilon\right)\eta_C\right]}\,, \end{aligned} \tag{6}$$

where $\eta_C = \frac{(T_3-T_1)}{T_3}$ is the Carnot efficiency, then

$$\begin{aligned}\eta_{poly} &= \frac{\left(\frac{T_{4''}}{T_3}-\frac{T_1}{T_3}\right)\left(r^{n-1}-1\right)}{\left[\frac{T_{4''}}{T_3}\left(r^{n-1}-1\right) + \frac{n-1}{k-1}\left(1-\varepsilon\right)\eta_C\right]} \\ &= \frac{\left(\frac{1}{r^{n-1}}-t\right)\left(r^{n-1}-1\right)}{\left[\frac{1}{r^{n-1}}\left(r^{n-1}-1\right) + \frac{n-1}{k-1}\left(1-\varepsilon\right)\eta_C\right]}\,, \end{aligned} \tag{7}$$

where $t = \frac{T_1}{T_3}$ is a constant quantity, and finally, the required form for efficiency of ideal polytropic Stirling cycle is obtained

$$\eta_{poly} = \frac{\left(1-tr^{n-1}\right)\left(r^{n-1}-1\right)}{\left[\left(r^{n-1}-1\right) + \frac{n-1}{k-1}\left(1-\varepsilon\right)\eta_C r^{n-1}\right]}\,. \tag{8}$$

In the limiting case, where $n \to k$, Eq.(8) (11) becomes required formula for efficiency of Rallis cycle or ideal adiabatic Stirling cycle

$$\eta_{adia} = \frac{\left(1-tr^{k-1}\right)\left(r^{k-1}-1\right)}{\left[\left(r^{k-1}-1\right) + \left(1-\varepsilon\right)\eta_C r^{k-1}\right]}\,. \tag{9}$$

If the regenerator is ideal (i.e., $\varepsilon = 1$), then Eq. (8) reduces to required form for efficiency of ideal polytropic Stirling cycle under ideal regeneration

$$\eta_{poly} = \left(1-tr^{n-1}\right)\,. \tag{10}$$

In the similar case Eq. (9) reduces for ideal regeneration as

$$\eta_{adia} = \left(1-tr^{k-1}\right)\,. \tag{11}$$

3.2 Rallis modified Stirling cycle engine model I

A new Stirling cycle engine model has been proposed by the authors by introducing reheater and intercooler on expansion and compression side respectively in Rallis Stirling engine model such that compression and expansion process take place in two cylinders rather than one. This new proposed model is called Rallis modified Stirling cycle engine or RMSE I model in which all heat transfer occurred under the constant volume condition. The expansion and compression work are obtained at their optimum value at equal pressure ratio as well as at equal volume ratio on each side. By considering the non-isothermal behaviour of working fluid inside the expansion/compression space, the expansion and compression processes are assumed to be polytropic and in the limiting condition these processes were transformed to be adiabatic. The P-V and T-S diagram of the proposed RMSE I model is shown in Fig. 8.

In Fig. 8, the ideal polytropic Rallis modified Stirling cycle engine (RMSE I) is shown by the process 1-b'-b-$2''$-$2'$-$3'$-3-d'-d-$4''$-$4'$-$1'$-1. Here, the compression of working fluid takes place through first cylinder (1-b') to the pressure $P_{b'}$ at state b' and then through the intercooler (b'-b) at constant volume, where its pressure drops to P_b. The working fluid again compressed through the second cylinder (b-$2''$) up to the pressure $P_{2''}$ and then through the cooler ($2''$-$2'$) where its pressure further drops to the state $2'$. The working fluid then passed through the regenerator ($2'$-$3'$) where its pressure rises to the state $3'$ and finally through the heater at constant volume where the pressure further rises to the state 3. Similar process occurs on the expansion side in which the working fluid expanded through the first cylinder (3-d') where its pressure drops to $P_{d'}$ at the state d' and then passed through the reheater (d'-d) at constant volume where the pressure rises to the state d. The working fluid again expanded through the second cylinder (d-$4''$) such that its pressure drops to the state $4''$ and then passed through the heater ($4''$-$4'$) where its pressure further rises to $P_{4'}$ at the state $4'$. The working fluid then passed through the regenerator ($4'$-$1'$) where its pressure drops to the state $1'$ after rejecting the heat and finally through the cooler ($1'$-1) where pressure further drops to state 1. In this way, the polytropic thermodynamic cycle of RMSE I model is completed. The schematic diagram of ideal adiabatic RMSE I model is shown in Fig. 9.

The adiabatic cycle of RMSE I model has been shown by the line 1-a-b-2-2'-$3'$-3-c-d−4-$4'$-$1'$-1 and that of polytropic cycle of RMSE I model has been shown by the line 1-b'-b-$2''$-$2'$-$3'$-3-d'-d-$4''$-$4'$-$1'$-1 on P-V and T-S di-

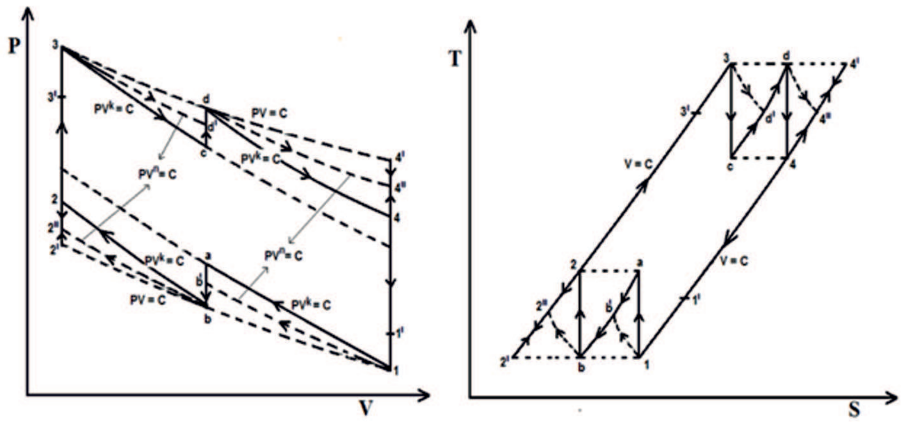

Figure 8: P-V and T-S diagrams of RMSE I model.

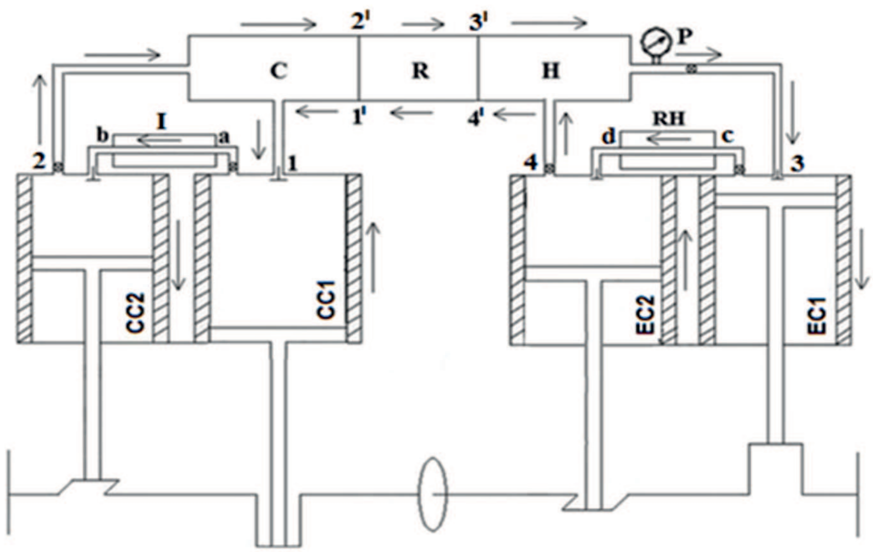

Figure 9: Schematic diagram of ideal adiabatic RMSE I model.

agrams, as shown in Fig. 8. Therefore the net work output obtained from the ideal polytropic cycle of RMSE I model is given as

$$W_{Net} = W_E - W_C = \frac{mR}{n-1}\left[\,(T_3 - T_{d'}) + (T_d - T_{4''}) - (T_{2''} - T_b) - (T_{b'} - T_1)\,\right] . \tag{12}$$

As $T_d = T_3$, $T_{4''} = T_{d'}$, $T_{2''} = T_{b'}$, and $T_b = T_1$ then the above becomes

$$W_{Net} = \frac{2mR}{n-1}\left[(T_3 - T_{d'}) - (T_{b'} - T_1)\right] . \tag{13}$$

The heat supplied to the polytropic cycle is given as

$$\begin{aligned} Q_S &= Q_{3'-3} + Q_{3-d'} + Q_{d'-d} + Q_{d-4''} + Q_{4''-4'} \\ &= mC_v \left[\begin{array}{l} (T_3 - T_{3'}) + \frac{k-n}{n-1}(T_3 - T_{d'}) + (T_d - T_{d'}) \\ +\frac{k-n}{n-1}(T_d - T_{4''}) + (T_{4'} - T_{4''}) \end{array} \right] . \end{aligned} \tag{14}$$

As $T_d = T_3$, $T_{4''} = T_{d'}$, and $T_d = T_{4'} = T_3$ then the above equation becomes

$$\begin{aligned} Q_S &= mC_v \left[(T_3 - T_{3'}) + 2\left(\frac{k-n}{n-1}\right)(T_3 - T_{d'}) + 2(T_3 - T_{d'})\right] \\ &= mC_v \left[(T_3 - T_{3'}) + 2\left(\frac{k-1}{n-1}\right)(T_3 - T_{d'})\right] . \end{aligned} \tag{15}$$

Now, the effectiveness of the regenerator is given as

$$\varepsilon = \frac{T_{3'} - T_{2'}}{T_3 - T_{2'}} = \frac{T_{3'} - T_1}{T_3 - T_1} . \tag{16}$$

From Eq. (16), putting the value of $T_{3'} = \varepsilon(T_3 - T_1) + T_1$ in Eq. (15), then it becomes

$$Q_S = mC_v \left[(1-\varepsilon)(T_3 - T_1) + 2\left(\frac{k-1}{n-1}\right)(T_3 - T_{d'})\right] . \tag{17}$$

Now, the efficiency of the cycle is given as

$$\begin{aligned} \eta_{poly} = \frac{W_{Net}}{Q_S} &= \frac{\frac{2mR}{n-1}\left[(T_3 - T_{d'}) - (T_{b'} - T_1)\right]}{mC_v\left[(1-\varepsilon)(T_3 - T_1) + 2\left(\frac{k-1}{n-1}\right)(T_3 - T_{d'})\right]} \\ &= \frac{(T_3 - T_{d'}) - (T_{b'} - T_1)}{(T_3 - T_{d'}) + \frac{n-1}{2(k-1)}(1-\varepsilon)(T_3 - T_1)} . \end{aligned} \tag{18}$$

Put $\frac{T_3}{T_{d'}} = \left(\frac{V_{d'}}{V_3}\right)^{(n-1)} = {r_1}^{n-1} = \frac{T_{b'}}{T_1}$ in the above equation Eq. (18), where r_1 is the compression ratio for RMSE I model

$$\begin{aligned} \eta_{poly} &= \frac{T_{d'}\left({r_1}^{n-1} - 1\right) - T_1\left({r_1}^{n-1} - 1\right)}{T_{d'}\left({r_1}^{n-1} - 1\right) + \left(\frac{n-1}{k-1}\right)\frac{(1-\varepsilon)}{2}\frac{(T_3 - T_1)}{T_3}T_3} \\ &= \frac{(T_{d'} - T_1)\left({r_1}^{n-1} - 1\right)}{T_3\left[\frac{T_{d'}}{T_3}\left({r_1}^{n-1} - 1\right) + \left(\frac{n-1}{k-1}\right)\frac{(1-\varepsilon)}{2}\eta_C\right]} , \end{aligned} \tag{19}$$

where $\eta_C = \frac{(T_3 - T_1)}{T_3}$ is the Carnot efficiency, then

$$\eta_{poly} = \frac{\left(\frac{T_{d'}}{T_3} - \frac{T_1}{T_3}\right)\left({r_1}^{n-1} - 1\right)}{\left[\frac{T_{d'}}{T_3}\left({r_1}^{n-1} - 1\right) + \left(\frac{n-1}{k-1}\right)\frac{(1-\varepsilon)}{2}\eta_C\right]}$$

$$= \frac{\left(\frac{1}{{r_1}^{n-1}} - t\right)\left({r_1}^{n-1} - 1\right)}{\left[\frac{1}{{r_1}^{n-1}}\left({r_1}^{n-1} - 1\right) + \left(\frac{n-1}{k-1}\right)\frac{(1-\varepsilon)}{2}\eta_C\right]} , \tag{20}$$

where $t = \frac{T_1}{T_3}$ is a constant quantity, and finally, the required formula for efficiency of RMSE I model for polytropic case is obtained

$$\eta_{poly} = \frac{\left(1 - t{r_1}^{n-1}\right)\left({r_1}^{n-1} - 1\right)}{\left[\left({r_1}^{n-1} - 1\right) + \left(\frac{n-1}{k-1}\right)\frac{(1-\varepsilon)}{2}\eta_C {r_1}^{n-1}\right]} . \tag{21}$$

In the limiting case, where $n \to k$ then Eq. (21) becomes the required equation. for efficiency of RMSE I model for adiabatic case

$$\eta_{adia} = \frac{\left(1 - t{r_1}^{k-1}\right)\left({r_1}^{k-1} - 1\right)}{\left[\left({r_1}^{k-1} - 1\right) + \frac{(1-\varepsilon)}{2}\eta_C {r_1}^{k-1}\right]} . \tag{22}$$

If the regenerator is ideal(i.e., $\varepsilon = 1$), then Eq. (22) reduces as

$$\eta_{poly} = 1 - t{r_1}^{n-1} , \tag{23}$$

which is the required form for efficiency of RMSE I model for polytropic case for ideal regeneration.

In the similar case Eq. (22) reduces for ideal regeneration as

$$\eta_{adia} = 1 - t{r_1}^{k-1} , \tag{24}$$

which is the required form for efficiency of ideal adiabatic RMSE I model for ideal regeneration.

3.3 Rallis modified Stirling cycle engine model II

A second model, called RMSE II has been introduced, in which the net work output and efficiency are higher than that of RMSE I as well as Rallis Stirling cycle model. In RMSE II model, expansion as well as compression processes take place in three cylinders and there are two extra heat

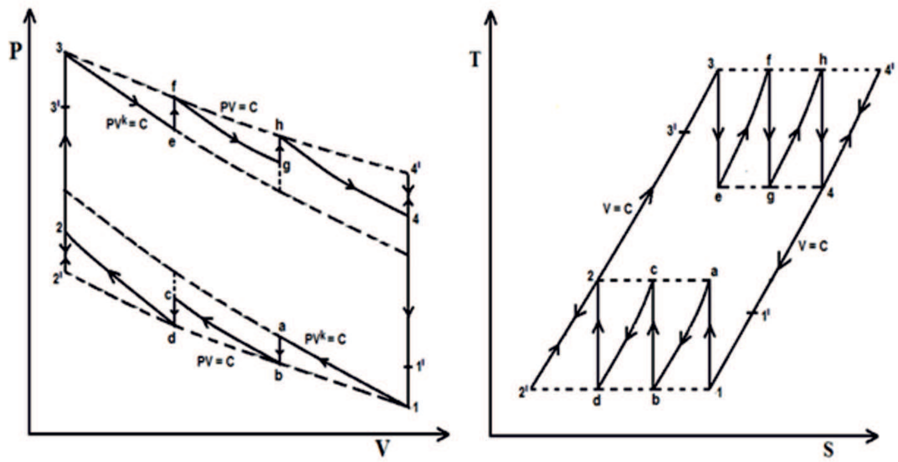

Figure 10: P-V and T-S diagrams of RMSE II model.

exchangers reheater and intercooler on expansion and compression side respectively. The P-V and T-S diagrams of the proposed RMSE II model is shown in Fig. 10.

The net work output and efficiency of the RMSE II model for adiabatic case is given as

$$W_{Net} = \frac{3mR}{k-1}\left[(T_3 - T_e) - (T_a - T_1)\right] \ , \tag{25}$$

$$\eta_{adia} = \frac{\left(1 - t{r_2}^{k-1}\right)\left({r_2}^{k-1} - 1\right)}{\left[({r_2}^{k-1} - 1) + \frac{(1-\varepsilon)}{3}\eta_C {r_2}^{k-1}\right]} \ , \tag{26}$$

where $r_2 = \left(\frac{T_3}{T_e}\right)^{\frac{1}{n-1}} = \left(\frac{T_a}{T_1}\right)^{\frac{1}{n-1}}$ is the compression ratio for RMSE II model. The efficiency of the RMSE II model for ideal regenerator is given as

$$\eta_{adia} = 1 - t{r_2}^{k-1} \ . \tag{27}$$

4 Analytical simulation

In order to analyse the performance of different Stirling cycle engine models, some geometrical and operating parameters must be established accordingly. In the most cases, Stirling engine operates in a typical higher temperature range of 923–1073 K [33]. Here, the operating temperature of 923 K and 308 K is selected for heater and cooler respectively. The normal operating speed of the engine is limited to 1000 rpm, as air is used as working fluid [34]. The mean pressure of the Stirling engine models is taken as 10 bar as it lies between the range of 0.8 MPa to 1 MPa for high temperature differential Alpha-type Stirling engine operating with air as working fluid [35]. The swept volume of the expansion cylinder for 1 kW power output of the Stirling engine is determined by using Beale number [7,36]. Beale number is widely used to predict the preliminary design of Stirling engine, which characterizes the performance of the Stirling engines. It is often used to estimate the power output of a Stirling engine. The Beale number is defined in terms of a Stirling engine's operating parameters as

$$B_N = \frac{P_i}{P_m V_{SE} N} \,, \tag{28}$$

where P_i is the output power, B_N is the Beale number, P_m is the mean pressure, V_{SE} is the swept volume, and N is the operating speed of the Stirling engine. In case of high temperature differential Stirling engines whose heater wall temperature is about 923 K, the Beale number at the engine speed where maximum power output is realized, is about 0.15 [36]. Now, putting all the required parameters in Eq.(28), the swept volume of the expansion cylinder is comes about 0.0004 m^3. The clearance volume in the expansion and compression cylinder and the regenerator volume are taken as 30% of the swept volume of the expansion cylinder.

Now, in order to get maximum net work output from RMSE models, the expansion /compression work must be optimized. Therefore, the work output on expansion side for the polytropic case of the RMSE I model as shown in Fig. 8 is written as

$$W_E = \frac{P_3 V_3 - P_{d'} V_{d'}}{n-1} + \frac{P_d V_d - P_{4''} V_{4''}}{n-1} \,. \tag{29}$$

The above equation is rewritten as

$$W_E = \frac{P_3 V_3}{n-1}\left[1 - \left(\frac{V_3}{V_{d'}}\right)^{n-1}\right] + \frac{P_d V_d}{n-1}\left[1 - \left(\frac{V_d}{V_{4''}}\right)^{n-1}\right] . \tag{30}$$

Now, put $V_{4''} = V_4$ and $V_{d'} = V_d = V_x$, such that $V_3 < V_x < V_4$ in the above equation

$$W_E = \frac{P_3 V_3}{n-1}\left[1 - \left(\frac{V_3}{V_x}\right)^{n-1}\right] + \frac{P_d V_d}{n-1}\left[1 - \left(\frac{V_x}{V_4}\right)^{n-1}\right] . \tag{31}$$

For the isothermal process 3-d, put $P_d V_d = P_3 V_3$, then above equation becomes

$$\begin{aligned} W_E &= \frac{P_3 V_3}{n-1}\left[1-\left(\frac{V_3}{V_x}\right)^{n-1}\right] + \frac{P_3 V_3}{n-1}\left[1-\left(\frac{V_x}{V_4}\right)^{n-1}\right] \\ &= \frac{P_3 V_3}{n-1}\left[2-\left(\frac{V_3}{V_x}\right)^{n-1} - \left(\frac{V_x}{V_4}\right)^{n-1}\right] . \end{aligned} \tag{32}$$

Here for the maximum expansion work, $\frac{dW_E}{dV_x} = 0$, therefore the above equation takes the form

$$\frac{P_3 V_3}{n-1}\left[0 + \frac{(n-1)\,V_3^{n-1}}{V_x^n} - \frac{(n-1)\,V_x^{n-2}}{V_4^{n-1}}\right] = 0 \ ,$$

$$(n-1)\left[\frac{V_3^{n-1}}{V_x^n} - \frac{V_x^{n-2}}{V_4^{n-1}}\right] = 0 \ ,$$

$$\frac{V_3^{n-1}}{V_x^n} - \frac{V_x^{n-2}}{V_4^{n-1}} = 0 \ ,$$

$$V_x^{2(n-1)} = (V_4 V_3)^{n-1}$$

and lastly,

$$V_x = (V_4 V_3)^{\frac{1}{2}} \ . \tag{33a}$$

A similar result is obtained on the compression side for the minimum compression work as

$$V_x = (V_1 V_2)^{\frac{1}{2}} \ . \tag{33b}$$

The result obtained above is valid for any polytropic process, i.e., for $1 < n \leq k$. Further as $V_4 = V_1$ and $V_3 = V_2$ the optimum value of net work output is obtained at $V_x = (V_1 V_2)^{\frac{1}{2}}$ and the maximum value of net work output for RMSE I model is obtained as

$$W_{Net,\ Max} = W_{E,Max} - W_{C,Min} \ . \tag{34}$$

The variation of net work output, expansion work and compression work for RMSE I model with the value of V_x is shown in Fig. 11. The various design and operating parameters of RMSE I and RMSE II models are presented in Tab. 1.

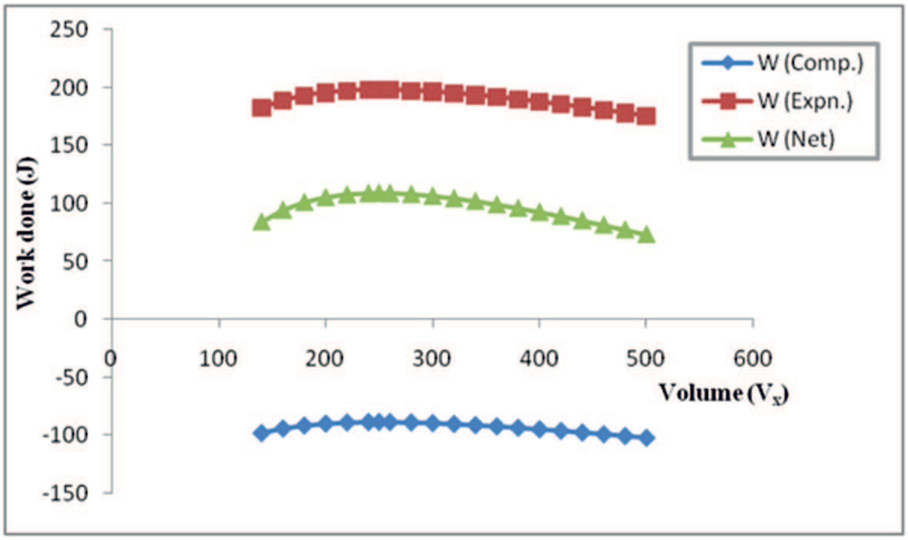

Figure 11: Variation of compression, expansion and net work done of RMSE I model.

The Rallis modified Stirling cycle presented here is based upon the assumption that expansion/compression processes taking place in the cylinders are adiabatic instead of isothermal. However, Hosseinzade *et al.* assumed that, real Stirling engine expansion/compression processes are neither isothermal nor adiabatic, but considered to be polytropic processes as the working fluid inside the cylinders associated with the heat transfer to the surrounding [37]. In view of this, the performance of RMSE models has also been analysed for the polytropic as well as adiabatic processes.

Further, Babaelahi and Sayyaadi considered the effect of the type of working fluid and clearance volume of expansion/compression space on the polytropic index [29]. A methodology was presented by them to find out the polytropic indexes of expansion/compression space as a function of crank angle. They have shown that polytropic index of expansion process and that of compression process are different in two cylinders at the same time and value of polytropic index for air in the compression cylinder lies between $1 < n < k$; and that in expansion cylinder lies for $n > k$ respectively. However, in the present analysis, it is assumed that the value of polytropic index in expansion and compression cylinders is same at the time and it lies between $1 < n \leq k$, in order to make the analysis in its simplest form to predict the thermal performance of the Rallis Modified

Table 1: Design and operating parameters of adiabatic Stirling cycle engine.

Design parameters of adiabatic Stirling cycle engine models				
Stirling engine model	Expansion volume ($\times 10^{-4}$ m^3)		Compression volume ($\times 10^{-4}$ m^3)	
Rallis	$V_{ES} = 400$	$V_{EC} = 120$	$V_{CS} = 400$	$V_{CC} = 120$
RMSE I	$V_{ES1} = 129.8$ $V_{ES2} = 270.2$	$V_{EC1} = 38.94$ $V_{EC2} = 81.06$	$V_{CS1} = 270.2$ $V_{CS2} = 129.8$	$V_{CC1} = 81.06$ $V_{CC2} = 38.94$
RMSE II	$V_{ES1} = 75.6$ $V_{ES2} = 123.32$ $V_{ES3} = 201.08$	$V_{EC1} = 22.68$ $V_{EC2} = 37$ $V_{EC3} = 60.32$	$V_{CS1} = 201.08$ $V_{CS2} = 123.32$ $V_{CS3} = 75.6$	$V_{CC1} = 60.32$ $V_{CC2} = 37$ $V_{CC3} = 22.68$
Operating parameters of Stirling cycle engine models				
Hot side temperature		923.15 K		
Cold side temperature		308.15 K		
Working gas		air		
Mass of working gas		5.88 $\times 10^{-4}$ kg		
Mean pressure of the engine		0.7 MPa		
Speed of the engine		1000 rpm		
Phase angle		90°		

Stirling cycle engine. Simulated results for the net work output, power and efficiency of Rallis model and different RMSE models for different values of polytropic index of air ranging from $1 < n \leq k$ are presented in the next section.

5 Results and discussion

The thermodynamic cycle analysis of Rallis ideal adiabatic model as well as proposed adiabatic RMSE models have been done when air is used as the working fluid. It has been observed that the thermal efficiency of Rallis adiabatic model is found maximum at fairly low compression ratio (i.e., less than 2) under different values of regenerator effectiveness as shown in Fig. 12. The thermal efficiency of Rallis ideal model ($\varepsilon = 1$) against the compression ratio has been shown in Fig. 13 for different value of polytropic indices. Figures 14 and 15 represent the power output of Rallis model against the compression ratio and thermal efficiency respectively for the different value of polytropic indices. The maximum power output of Rallis ideal adiabatic model is found to be 1.16 kW, which occurs at the compression ratio of 4 with thermal efficiency of 42.25%. The variation of volume, pressure and temperature of the working fluid inside the expansion/compression cylinder of the Rallis as well as RMSE I model is also shown in Figs. 16–18 for the adiabatic condition. Further, the performance of RMSE I and RMSE II model has been observed under two different cases:

Case 1: At same compression ratio In this case, all the Stirling engine models operate at the same compression ratio and the results were analysed. The variation of power output with the compression ratio of different models has been shown in Fig. 19. The maximum value of power output has been observed at the compression ratio of 4 for all Stirling engine models. The maximum power output of Rallis, RMSE I and RMSE II model is found to be 1.16 kW, 2.32 kW and 3.48 kW respectively which occur at the thermal efficiency of 42.25% as shown in Fig. 20.

Case 2: At different compression ratio A comparative analytical performance has been performed on Rallis and RMSE models at different compression ratios. Here, volume of RMSE models including swept volume and clearance volume were confined under the volume of Rallis Stirling engine such that the compression ratio differed for every Stirling engine models. On this basis, the compression ratio of Rallis, RMSE I and RMSE II models were found to be 4.33, 2.08, and 1.63, respectively. The performance of Stirling engine models under this condition were analysed for different polytropic indexes.

The results reveal the fact that the thermodynamic performance of Rallis modified Stirling cycle engine models has been always superior over Rallis Stirling cycle engine model. It has been seen from Fig. 21 that, the thermal efficiency of RMSE I and RMSE II model is always enhanced over the Rallis Stirling engine model throughout the polytropic index. Thermal efficiency of RMSE I and RMSE II model is found to be 55.28% and 59.43%, respectively for the adiabatic case, which is 38.06% and 48.42% higher than that of the Rallis Stirling cycle engine model respectively. The net work output of RMSE I and RMSE II model is also enhanced over the Rallis Stirling cycle engine model throughout the polytropic index as shown in Fig. 22. The net work output of RMSE I model is found to be 109.33 J which is 58.05% higher than that of Rallis model. Similarly, the net work output of RMSE II model is 123.36 J which is 78.19% higher than that of Rallis model and only 12.74% higher than that of RMSE I model. The power output of ideal adiabatic Stirling engine model of Rallis, RMSE I and RMSE II under the operating engine speed of 1000 rpm is found to be 1.15 kW, 1.82 kW and 2.05 kW respectively as shown in Fig. 23. The effect of regenerator effectiveness on the thermal efficiency of Stirling engine models for the adiabatic case has been presented in Fig. 24. The efficiency of RMSE I and RMSE II model has been always enhanced over the Rallis model as the regenerator effectiveness increases.

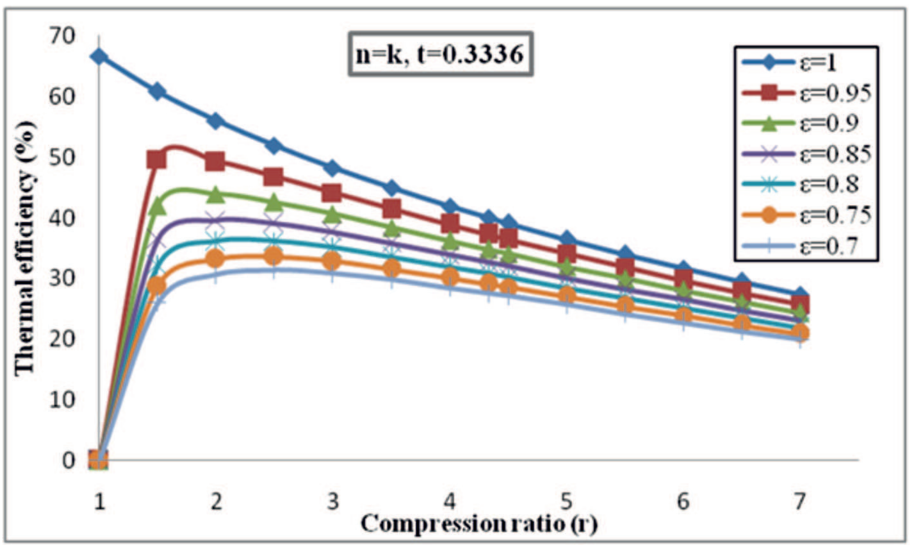

Figure 12: Thermal efficiency of Rallis adiabatic Stirling engine model with compression ratio.

6 Conclusions

The thermodynamic analysis and analytical simulation of Stirling engine models have been carried out for the different cases. The result reveals the fact that, RMSE I and RMSE II models provide better performance over Rallis Stirling engine throughout the polytropic index under both the cases. Further, out of these two working models, RMSE I model is found superior over RMSE II model by its performance, design and on the cost basis. It has been seen from the case 1, the thermodynamic performance of RMSE I model is found superior over Rallis model, when working on same

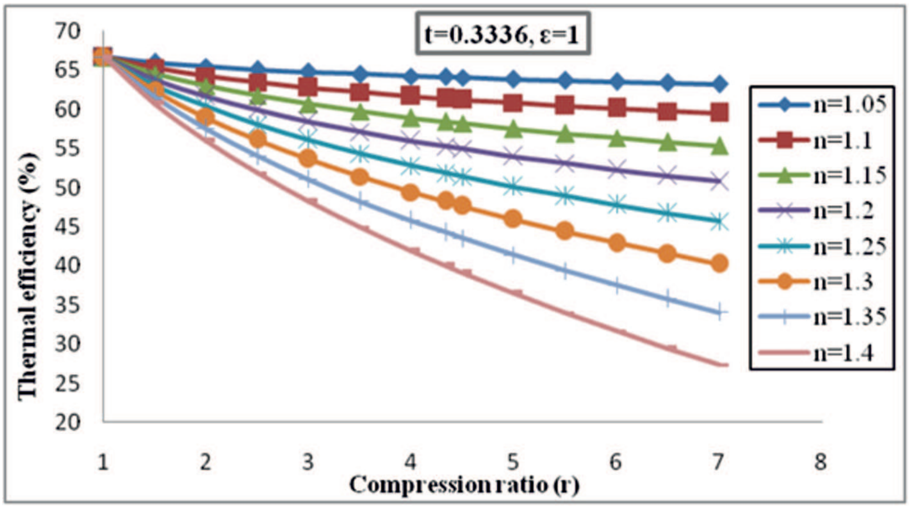

Figure 13: Thermal efficiency of Rallis ideal Stirling engine model with compression ratio.

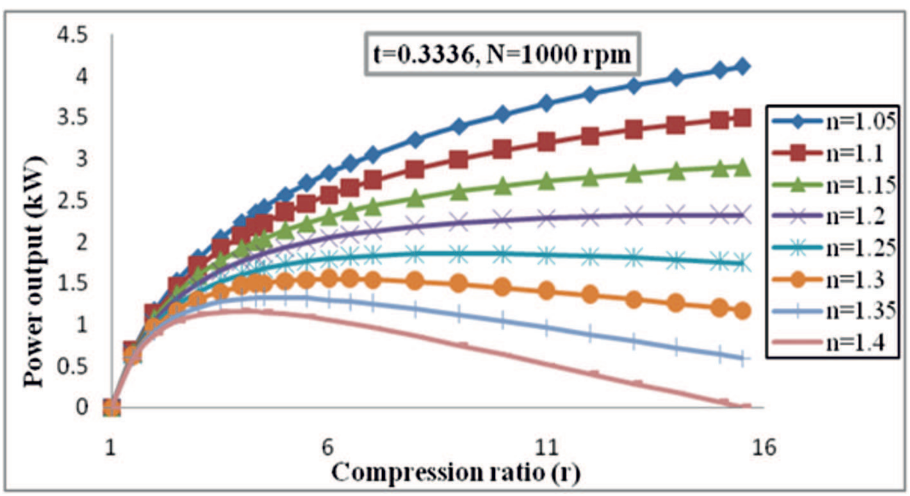

Figure 14: Power output of Rallis Stirling engine model with compression ratio.

compression ratio. However, RMSE II model provide maximum power output, but on design and cost basis its acceptability may be less as compared with RMSE I model. Similarly, in the case 2, the power output of RMSE I and RMSE II model is found to be enhanced over Rallis model by 58.05% and 78.19%, respectively. The results show that the RMSE II model with

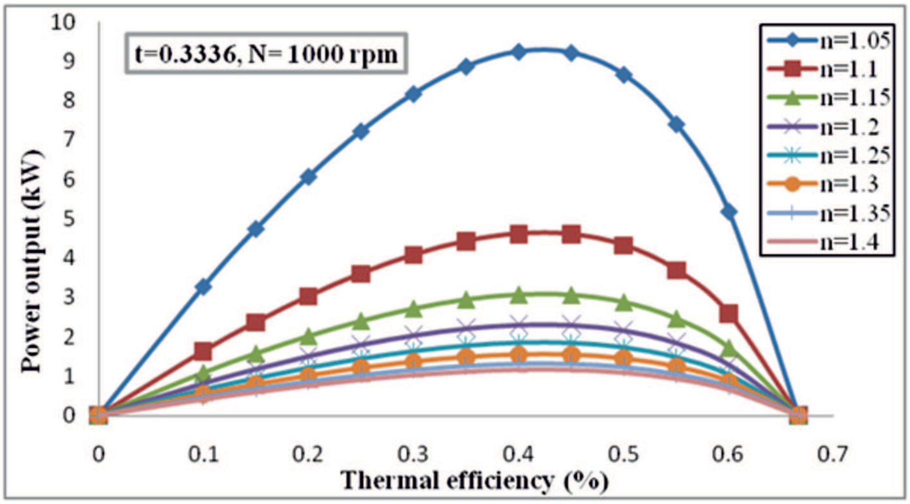

Figure 15: Power output of Rallis Stirling engine model with thermal efficiency.

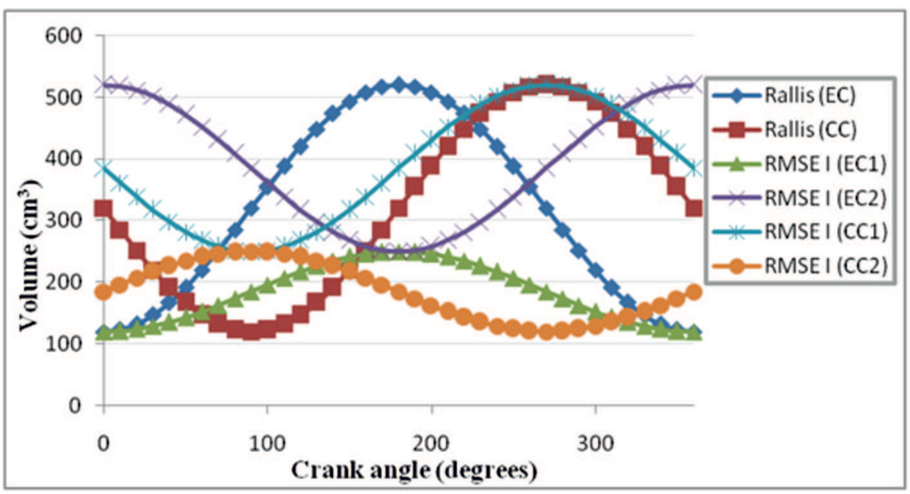

Figure 16: Volume variation of Rallis and RMSE I model.

the three cylinders each on expansion and compression side has 7.5% more thermal efficiency and produced only 12.74% more power than RMSE I model having two cylinders each on expansion and compression side, which may not be economically acceptable with regard to the cost of the extra cylinders. Therefore, techno-economically RMSE I model will be the

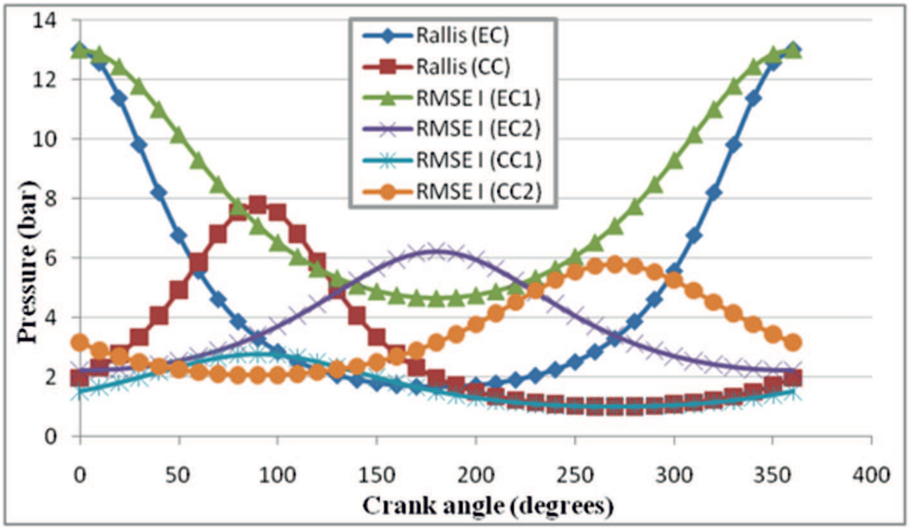

Figure 17: Pressure variation of Rallis and RMSE I model.

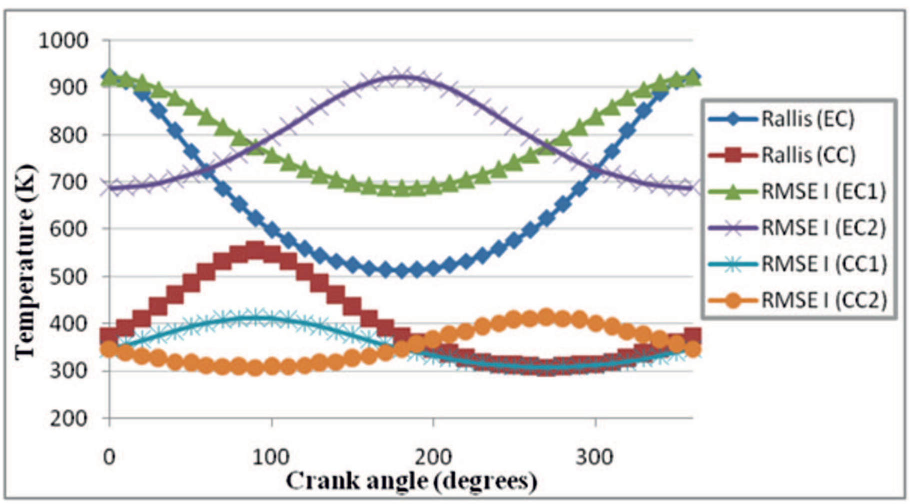

Figure 18: Temperature variation of Rallis and RMSE I model.

better choice by considering its performance as well as its operating and maintenance costs. The other important aspect of RMSE models is that, while working with two or more cylinders on expansion and compression sides it provides better chance for more uniform torque, better mechanical

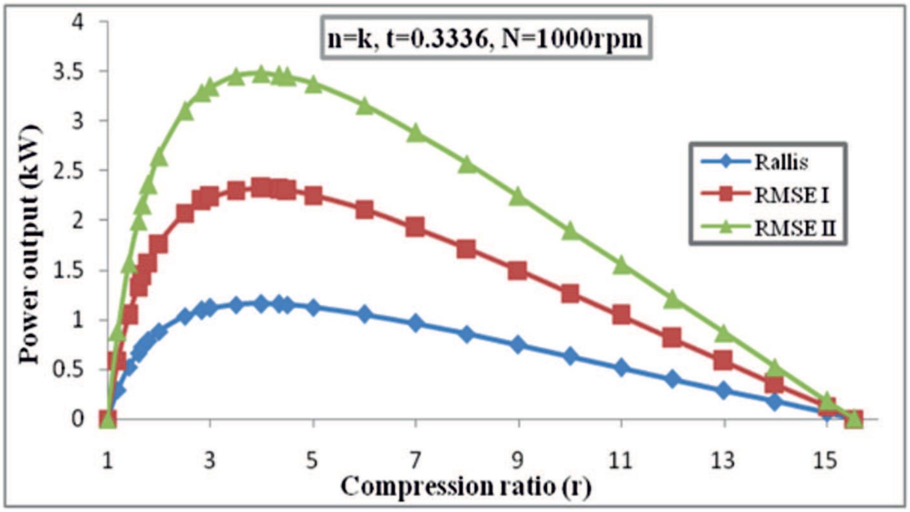

Figure 19: Power output of Stirling engine models with compression ratio.

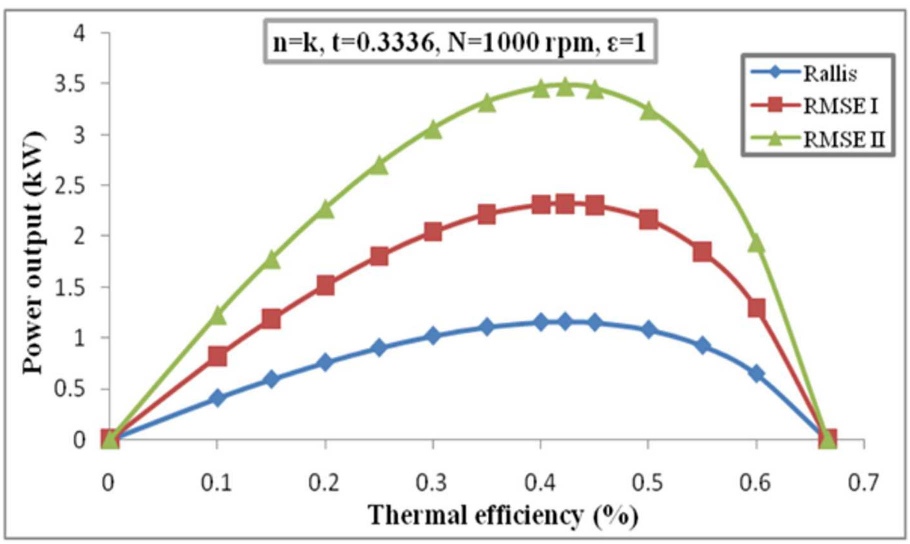

Figure 20: Power output of Stirling engine models with thermal efficiency.

balance between the different components of the engine and reducing the thermal stresses. Further, the leakage of working fluid is a major problem for most operating Stirling engines. If working with RMSE I or RMSE II model, the leakage tendency of working fluid will greatly reduce due to

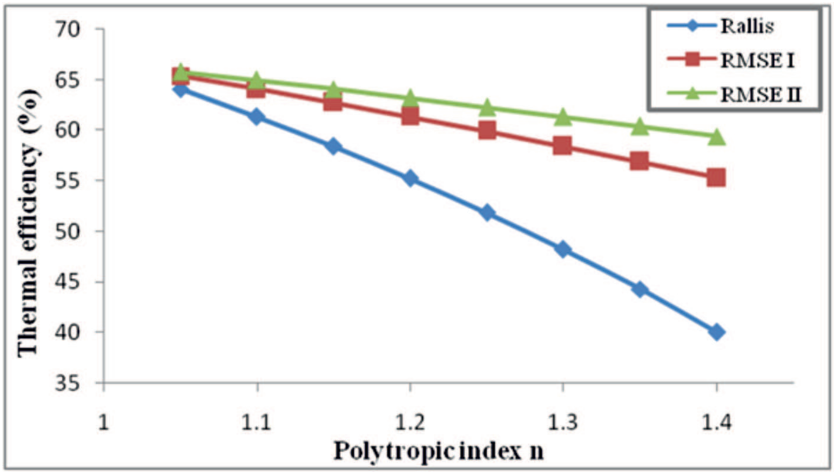

Figure 21: Thermal efficiency of Stirling engine models with polytropic index.

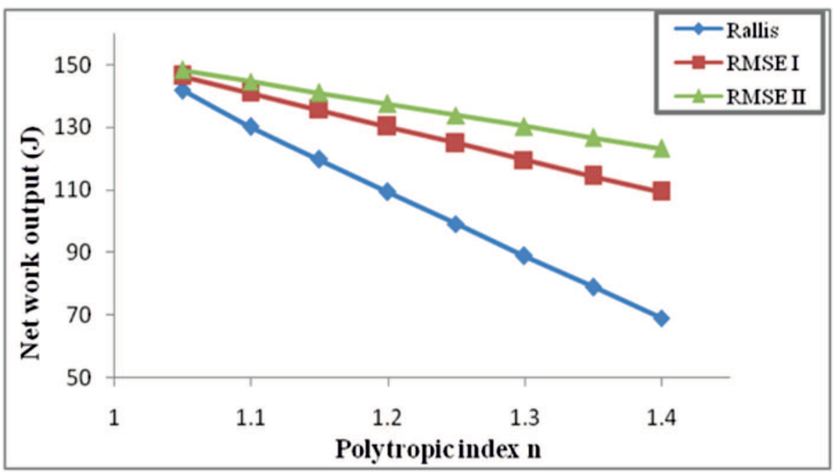

Figure 22: Net work output of Stirling engine models with polytropic index.

low pressure ratio on compression as well as on expansion sides. Generally, RMSE I model is suitable for operating medium temperature differential Stirling engine and RMSE II model is for high temperature differential Stirling engine. Therefore, modification in Rallis Stirling engine presented in this paper provides a new methodology for the better design, better performance and improvement of the practical Stirling engine operating under the actual condition.

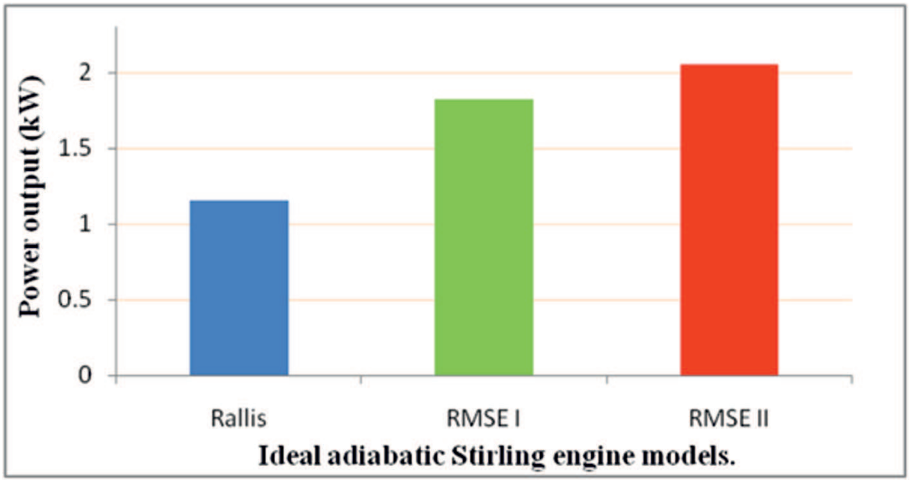

Figure 23: Power output of ideal adiabatic Stirling engine models.

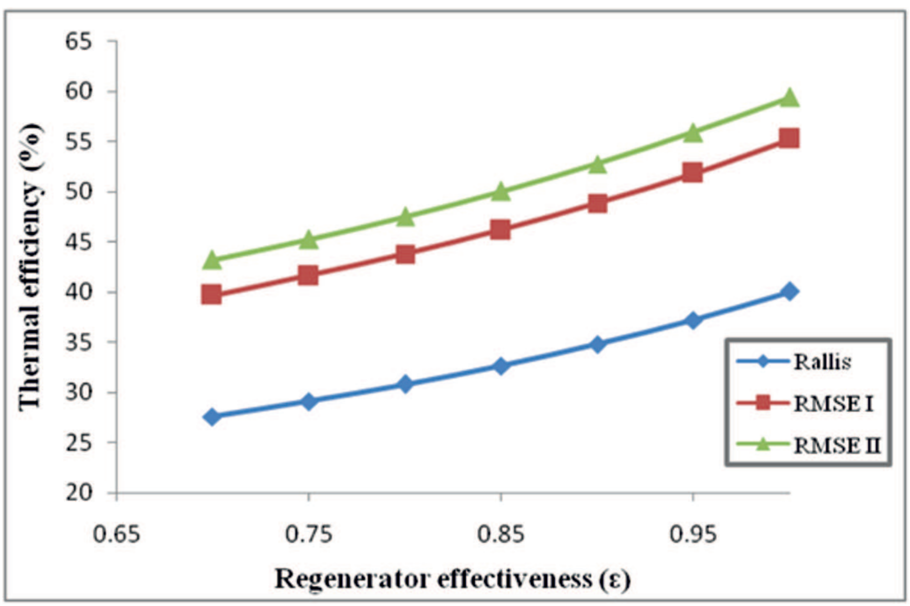

Figure 24: Effect of regenerator effectiveness on efficiency of adiabatic Stirling engine models.

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