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# A two dimensional problem on laser pulse heating in thermoelastic microelongated solid

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**Abstract** In the present discussion, the plane strain deformation due to laser pulse heating in a thermoelastic microelongated solid has been discussed. The analytic expressions for displacement component, force stress, temperature distribution and micro-elongation have been derived. The effect of pulse rise time and micro-elongation on the derived components have been depicted graphically.

**Keywords:** Laser pulse; Thermoelasticity; Normal mode; Microelongation

## 1 Introduction

In modern engineering and science, laser heating has become a very prominent aspect of surface modification. Laser finds a wide application in material deformation and geological treatments of particles. Consequently, the laser is an exceptionally flexible device for carrying out the change in the surfaces of materials, with the depth of material which is affected may

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range from a few nanometers to several millimeters. When the intensity is very high, laser interacts with the surface of solid and absorption takes place at the surface of solid due to which internal energy increase in the material and heat is released from the irradiated region. This process is very fast, due to which temperature gradients increase in the region. To modify the material as thin films, the microscopic two-step models, namely parabolic and hyperbolic are very useful. When a laser pulse heats a metal film, a thermoelastic wave is generated due to thermal expansion near the surface. Sun *et al.* investigated laser-induced vibrations of microbeams in which he showed that large thermal gradients exist at the boundaries for ultra-short-pulsed laser heating [1]. Youssef and Al-Felali discussed the effect of thermal loading due to laser pulse in generalized thermoelasticity problem [2]. Youssef and El-Bary studied the response due to laser pulse heating in the thermoelastic material [3]. Othman *et al.* discussed thermoelasticity under thermal loading due to laser pulse [4]. Othman and Hilal discussed the influence of temperature dependent properties and gravity on porous thermoelastic solid due to laser pulse heating [5]. Othman and Abd-Elaziz studied the effect of thermal loading due to laser pulse in generalized thermoelastic medium with voids in dual phase lag model [6]. Kumar *et al.* discussed the thermo-mechanical interactions due to laser pulse in the microstretch thermoelastic medium [7]. Othman and Hilal studied the influence of gravity in a magneto-thermoelastic medium with voids under the thermal effect of the laser pulse for Green-Naghdi (G-N) theory [8]. Abbas and Marin investigated the influence of laser heating for a traction-free and thermally insulated thermoelastic solid under Lord-Shulman (L-S) theory [9]. Ailawalia *et al.* investigated laser pulse heating in thermo-microstretch elastic layer overlying thermoelastic half-space [10].

To model the behavior of materials having internal structure, classical theory is not sufficient. Eringen and Suhubi [11,12] developed a nonlinear theory of microelastic solids. Later on, a theory was formulated in which material particles in solids can undergo macro-deformations as well as micro-rotations by Eringen [13–15] and named this theory as 'linear theory of micropolar elasticity'. Then a theory of micropolar elastic solid with stretch was introduced by Eringen in which he included axial stretch [16]. Nowacki [17], Eringen [18], Tauchert *et al.* [19] and Nowacki and Olszak [20] included thermal effects in the micropolar theory. Lord and Shulman is one of two important generalized theories of thermoelasticity and the second one is the theory of temperature-rate-dependent thermoelasticity [21].

Muller in the review of thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations [22]. A generalization of this inequality was proposed by Green and Laws [23]. Green and Lindsay obtained another version of these constitutive equations [24]. Suhubi obtained these equations independently and explicitly that contains two constants which act as relaxation times and transform all the equations of coupled theory [25].

Sherief obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source [26]. Dhaliwal *et al.* investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous isotropic unbounded solid [27]. Chandrasekharaiah and Srinath studied thermoelastic interactions due to a continuous point heat source in a homogeneous and isotropic unbounded body [28]. Sharma and Chauhan discussed mechanical and thermal sources in a generalized thermoelastic half-space [29]. Sarbani and Amitava studied the transient disturbance in half-space due to moving internal heat source under L-S model and obtained the solution for displacements in the transformed domain [30]. Youssef solved the problem on a generalized thermoelastic infinite medium with a spherical cavity subjected to a moving heat source [31]. A microelongated elastic solid possesses four degrees of freedom: three for translation and one for microelongation. In microelongation theory, the material particles can perform only volumetric micro elongation in addition to classical deformation of the medium. The material points of such medium can stretch and contract independently of their translations. Solid-liquid crystals, composite materials reinforced with chopped elastic fibers, porous media with pores filled with non-viscous fluid or gas can be categorized as a microelongated medium. Shaw and Mukhopadhyay discussed the variation of periodical heat source response in a functionally graded microelongated medium [32]. Shaw and Mukhopadhyay studied the thermoelastic interactions in a microelongated, isotropic, homogeneous medium in the presence of a moving heat source [33]. Ailawalia *et al.* investigated internal heat source in thermoelastic microelongated solid at an interface under G-L theory [34].

In the present work, taking into account the microelongation effect and laser pulse heating, we established a model for a thermoelastic microelongated solid by using normal mode analysis technique. The normal displacement, stress component, temperature distribution and microelongation were computed numerically. The resulting quantities are presented graphically to show the effect of micro-elongation, pulse rise time and pulse length.

## 2 Fundamental model

The constitutive equations for a homogeneous, isotropic, microelongated, thermoelastic solid are [33]:

$$\sigma_{kl} = \lambda\delta_{kl}u_{r,r} + \mu(u_{k,l} + u_{l,k}) - \beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T\delta_{kl} + \lambda_0\delta_{kl}\varphi\,, \tag{1}$$

$$m_k = a_0\varphi_{,k}, \tag{2}$$

$$s - \sigma = \lambda_0 u_{k,k} - \beta_1\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T + \lambda_1\varphi\,, \tag{3}$$

$$q_k = \frac{K^*}{T_0}T_{,k}\,. \tag{4}$$

The field equation of motion according to [35,36] and heat conduction equation according to [37] for the displacement, microelongation and temperature changes are

$$(\lambda + \mu)u_{j,ij} + \mu u_{i,jj} - \beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T_{,i} + \lambda_0\varphi_{,i} = \rho\ddot{u}_i\,, \tag{5}$$

$$a_0\varphi_{,ii} + \beta_1\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T + \lambda_1\varphi - \lambda_0 u_{j,j} = \frac{1}{2}\rho j_0\ddot{\varphi}\,, \tag{6}$$

$$K^*T_{,ii} - \rho C^*\left(1 + t_0\delta_{1k}\frac{\partial}{\partial t}\right)\dot{T} - \beta_0 T_0\left(1 + t_0\delta_{1k}\frac{\partial}{\partial t}\right)\dot{u}_{k,k} - \beta_1 T_0\dot{\varphi} + \rho\dot{Q} = 0\,. \tag{7}$$

Here $Q$ is the heat input of the laser pulse that illuminates the plate surface, and is given by

$$Q = \frac{I_0\gamma t}{2\pi r^2 t^{*2}}\exp\left(-\frac{y^2}{r^2} - \frac{t}{t^*}\right)\exp\left(-\gamma x\right)\,,$$

where $I_0$ is the energy absorbed per unit area, $t^*$ is the pulse rise time, $r$ is the beam radius, $y$ is the pulse length, $x$ is the heat deposition due to the laser pulse and is assumed to decay exponentially within the solid and $\beta_0 = (3\lambda + 2\mu)\alpha_{t_1}$, $\beta_1 = (3\lambda + 2\mu)\alpha_{t_2}$, $\sigma = \sigma_{kk}$ microelongational stress tensor, $s = s_{kk}$ component of stress tensor, $\delta_{kl}$ Kronecker delta, $m_k$ component of microstretch vector, $\lambda, \mu$ are lame's elastic constants, $a_0, \lambda_0, \lambda_1$ microelongational constants, $C^*$ is the specific heat at constant strain, $K^*$ is the thermal conductivity, $\alpha_{t_1}$ and $\alpha_{t_2}$ are coefficent of linear thermal expansion, $\rho$ is the density of microelongated medium, $j_0$ is microinertia, $t_0, t_1$ are thermal relaxation times, $T$ is the thermodynamic temperature above reference temperature $T_0$, $\varphi$ is microelongational scalar, $\vec{u} = (u_i)$ is displacement vector and $k = 2$ for Green-Lindsay (G-L) theory.

We consider a rectangular Cartesian coordinate system O$xyz$ having origin on $x$-axis with the $x$-axis pointing vertically downward in to the medium. A homogeneous isotropic, microelongated thermoelastic solid half space occupying the region $0 \le x < \infty$ is considered. The laser pulse $Q$ is applied on the surface $x = 0$ as shown in Fig. 1.

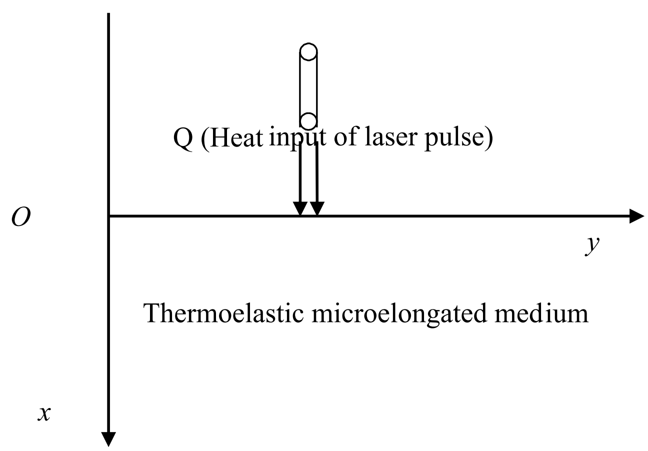


Figure 1: Geometry of the problem.

We have considered two dimensional disturbance of medium parallel to $xy$-plane with all field quantities depending upon $(x, y, t)$. For this we use displacement vector $\vec{u}_i = (u_1, u_2, 0)$. Hence, Eqs. (5)–(7) become:

$$(\lambda + 2\mu)\frac{\partial^2 u_1}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 u_2}{\partial x \partial y} + \mu\frac{\partial^2 u_1}{\partial y^2}$$
$$-\beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} + \lambda_0\frac{\partial \varphi}{\partial x} = \rho\frac{\partial^2 u_1}{\partial t^2}\ , \tag{8}$$

$$\mu\frac{\partial^2 u_2}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 u_1}{\partial x\partial y}+(\lambda+2\mu)\frac{\partial^2 u_2}{\partial y^2}$$
$$-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}+\lambda_0\frac{\partial\varphi}{\partial y}=\rho\frac{\partial^2 u_2}{\partial t^2}\,, \tag{9}$$

$$a_0\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)$$
$$+\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T-\lambda_1\varphi-\lambda_0\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)=\frac{1}{2}\rho j_0\frac{\partial^2\varphi}{\partial t^2}\,, \tag{10}$$

$$K^*\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\rho C^*\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}$$
$$-\beta_0 T_0\left(\frac{\partial}{\partial t}+t_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)-\beta_1 T_0\frac{\partial\varphi}{\partial t}+\rho\frac{\partial Q}{\partial t}=0. \tag{11}$$

The constitutive components of microelongational stress tensor are given by

$$\sigma_{xx}=(\lambda+2\mu)\frac{\partial u_1}{\partial x}+\lambda\frac{\partial u_2}{\partial y}-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_0\varphi\,, \tag{12}$$

$$\sigma_{yy}=\lambda\frac{\partial u_1}{\partial x}+(\lambda+2\mu)\frac{\partial u_2}{\partial y}-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_0\varphi\,, \tag{13}$$

$$\sigma_{xy}=\mu\left(\frac{\partial u_1}{\partial y}+\frac{\partial u_2}{\partial x}\right)\,. \tag{14}$$

To simplify calculations, we use following non-dimensional variables:

$$x'=\frac{\omega^*}{c_1}x,\quad y'=\frac{\omega^*}{c_1}y,\quad u_i'=\frac{\omega^*\rho c_1}{\beta_0 T_0}u_i,\quad t'=\omega^* t, t_0'=\omega^* t_0,\quad t_1'=\omega^* t_1,$$

$$\sigma_{ij}'=\frac{\sigma_{ij}}{\beta_0 T_0},\quad \varphi'=\frac{\lambda_0}{\beta_0 T_0}\varphi,\quad T'=\frac{T}{T_0},\quad Q'=\frac{1}{C^* T_0}Q\,,$$

where

$$\omega^*=\frac{\rho c_1^2 C^*}{K^*}, c_1^2=\frac{\lambda+2\mu}{\rho}.$$

Using the above non dimensional variables in Eqs. (8)–(14), after dropping superscripts we get:

$$\frac{\partial^2 u_1}{\partial x^2}+l_2\frac{\partial^2 u_2}{\partial x\partial y}+l_3\frac{\partial^2 u_1}{\partial y^2}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}+\frac{\partial\varphi}{\partial x}=\frac{\partial^2 u_1}{\partial t^2}\,, \tag{15}$$

$$l_3\frac{\partial^2 u_2}{\partial x^2}+l_2\frac{\partial^2 u_1}{\partial x\partial y}+\frac{\partial^2 u_2}{\partial y^2}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}+\frac{\partial \varphi}{\partial y}=\frac{\partial^2 u_2}{\partial t^2}\,, \tag{16}$$

$$\left(\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}\right)+l_4\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T-l_5\varphi-l_6\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)=l_7\frac{\partial^2 \varphi}{\partial t^2}\,, \tag{17}$$

$$\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-l_8\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}$$
$$-l_9\left(\frac{\partial}{\partial t}+t_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)-l_{10}\frac{\partial \varphi}{\partial t}+l_{11}\frac{\partial Q}{\partial t}=0\,, \tag{18}$$

$$\sigma_{xx}=\frac{\partial u_1}{\partial x}+l_{12}\frac{\partial u_2}{\partial y}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\varphi\,, \tag{19}$$

$$\sigma_{yy}=l_{12}\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\varphi\,, \tag{20}$$

$$\sigma_{xy}=l_3\left(\frac{\partial u_1}{\partial y}+\frac{\partial u_2}{\partial x}\right)\,, \tag{21}$$

where

$$l_2=\frac{(\lambda+\mu)}{\rho c_1^2},\ l_3=\frac{\mu}{\rho c_1^2},\ l_4=\frac{\beta_1\lambda_0 c_1^2}{a_0\omega^*\beta_0},\ l_5=\frac{\lambda_1 c_1^2}{a_0\omega^*},\ l_6=\frac{\lambda_0^2}{\rho a_0\omega^*},\ l_7=\frac{\rho j_0\omega^* c_1^2}{2a_0},$$

$$l_8=\frac{\rho C^* c_1^2}{K^*\omega^*},\ l_9=\frac{\beta_0^2 T_0}{K^*\omega^*\rho},\ l_{10}=\frac{\beta_0\beta_1 T_0 c_1^2}{K^*\omega^*\lambda_0},\ l_{11}=\frac{\rho C^* c_1^2}{K^*\omega^*},\ l_{12}=\frac{\lambda}{\rho c_1^2}.$$

### 2.1 Special case

If we neglect microelongation effect, i.e., $\lambda_0=\beta_1=\lambda_1=a_0=j_0=0$, we obtain the results for thermoelastic solid (TS).

## 3 Analytic solution

Here, we use normal mode analysis technique to decompose the solution of the considered physical variables as

$$(u_i,T,\varphi,\sigma_{ij})(x,y,t)=(u_i^*,T^*,\varphi^*,\sigma_{ij}^*)(x)e^{\omega t+iby} \tag{$**$}$$

where $\omega$ is complex frequency, $b$ is wave number in $y$-direction and $u_i^*(x), T^*(x)$, $\varphi^*(x)$, and $\sigma_{ij}^*(x)$ are the amplitudes of field quantities.

Using normal mode given by (**) in (15)–(21), we get:

$$(D^2 - B_1)u_1^* + ibl_2Du_2^* - B_2DT^* + D\varphi^* = 0\,, \tag{22}$$

$$ibl_2Du_1^* + (l_3D^2 - B_3)u_2^* - ibB_2T^* + ib\varphi^* = 0\,, \tag{23}$$

$$-l_6Du_1^* - ibl_6u_2^* + B_2l_4T^* + (D^2 - B_4)\varphi^* = 0\,, \tag{24}$$

$$-l_9B_6Du_1^* - ibl_9B_6u_2^* + (D^2 - B_7)T^* - l_{10}\omega\varphi^* = Q_1F(y,t)\exp(-\gamma x)\,, \tag{25}$$

$$\sigma_{xx}^* = Du_1^* + ibl_{12}u_2^* - B_2T^* + \varphi^* \tag{26}$$

$$\sigma_{yy}^* = l_{12}Du_1^* + ibu_2^* - B_2T^* + \varphi^*\,, \tag{27}$$

$$\sigma_{xy}^* = l_3\left(ibu_1^* + Du_2^*\right)\,, \tag{28}$$

where

$$D \equiv \frac{d}{dx}, \quad F(y,t) = \left(1 - \frac{t}{t^*}\right)\exp\left(-\frac{y^2}{r^2} - \frac{t}{t^*} - \omega t - iby\right),$$

$$Q_1 = \frac{-l_{11}I_0\gamma}{2\pi r^2 t^{*2}}, \quad B_1 = \omega^2 + l_3b^2, \quad B_2 = (1 + t_1\delta_{2k}\omega),$$

$$B_3 = \omega^2 + b^2, \quad B_4 = b^2 + l_5 + l_7\omega^2, \quad B_5 = (1 + t_0\delta_{1k}\omega),$$

$$B_6 = \omega(1 + t_0\delta_{1k}\omega), \quad B_7 = b^2 + l_8A_5\omega\,.$$

Eliminating $u_2^*(x)$, $T^*(x)$, and $\varphi^*(x)$ from Eqs. (22)–(25), we get the differential equation for $u_1^*(x)$ as

$$(D^8 + AD^6 + BD^4 + CD^2 + E)u_1^*(x) = RF(y,t)\exp(-\gamma x)\,, \tag{29}$$

where:

$$A = \frac{-1}{l_3}\left[l_3(B_4 + B_7) - B_3 + l_3B_1 + l_3l_6 + B_2l_3l_9B_6 + b^2l_2^2\right]$$

$$\begin{aligned}B = \frac{-1}{l_3}\Big[ &- B_2l_4l_{10}l_3\omega + l_3B_4B_7 + B_3(B_4 + B_7) - b^2B_2B_6l_9 + b^2B_6 \\ &- B_1l_3(B_4 + B_7) + B_1B_3 - b^2l_2^2(B_4 + B_7) + l_3l_6l_{10}B_2\omega \\ &- l_3l_9B_2B_4B_6 - l_9B_2B_3B_6 - l_3l_6B_7 - l_3l_4l_9B_2B_6 - B_3l_6\Big]\,,\end{aligned}$$

$$C = \frac{-1}{l_3}\Big[B_2B_3l_4l_{10}\omega + B_3B_4B_7 - b^2l_6l_{10}B_2\omega + b^2l_9B_2B_4B_6 - b^2l_6B_7$$
$$-b^2l_4l_9B_2B_6 + B_1B_2l_3l_4l_{10}\omega^2 - l_3B_1B_4B_7 + B_1B_3(B_4 + B_7)$$
$$+b^2B_1B_2B_6l_9 + b^2l_2^2B_2l_4l_{10}\omega + b^2l_2^2B_4B_7 - 2b^2B_7l_2l_6$$
$$-2b^2B_2B_6l_2l_4l_9 - l_6l_{10}B_2B_3\omega + B_2B_3B_4B_6l_9$$
$$+B_3B_7l_6 + B_2B_3B_6l_4l_9 - b^2l_6B_1\Big] ,$$

$$E = \frac{-1}{l_3}\Big[ - l_4l_{10}B_1B_2B_3\omega - B_1B_3B_4B_7 + b^2l_6l_{10}B_1B_2\omega$$
$$-b^2l_9B_1B_2B_4B_6 + b^2l_6B_1B_7 + b^2l_4l_9B_1B_2B_6\Big] ,$$

$$R = \frac{ibB_2Q_1}{l_3}\Big[-l_3\gamma^5+\{(B_3-b^2l_2)-l_3(l_4-B_4)\}\gamma^3+(l_4-B_4)(B_3-b^2l_2)\gamma\Big] .$$

Equation (29) can be written as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)u_1^*(x) = RF(y,t)\exp(-\gamma x) , \quad (30)$$

where $k_n^2$ $(n = 1,2,3,4)$ are roots of Eq. (29).

The solution of Eq. (30), which is bounded as $x \to \infty$ is given by

$$u_1^*(x) = \sum_{n=1}^{4}[L_n(b,\omega)e^{-k_nx}] + \xi\exp(-\gamma x) . \quad (31)$$

Similarly,

$$u_2^*(x) = \sum_{n=1}^{4}[L_n^{'}(b,\omega)e^{-k_nx}] + \xi_1\exp(-\gamma x) , \quad (32)$$

$$T^*(x) = \sum_{n=1}^{4}[L_n^{''}(b,\omega)e^{-k_nx}] + \xi_2\exp(-\gamma x) , \quad (33)$$

$$\varphi^*(x) = \sum_{n=1}^{4}[L_n^{'''}(b,\omega)e^{-k_nx}] + \xi_3\exp(-\gamma x) , \quad (34)$$

where $L_n(b,\omega)$, $L_n^{'}(b,\omega)$, $L_n^{''}(b,\omega)$, $L_n^{'''}(b,\omega)$ are specific function depending upon $b$, $\omega$.

Using (31)–(34) in Eqs. (22)–(25), we get:

$$L_n^{'}(b,\omega) = R_{1n}L_n(b,\omega) , \quad (35)$$

$$L_n''(b,\omega) = R_{2n}L_n(b,\omega)\ , \tag{36}$$

$$L_n'''(b,\omega) = R_{3n}L_n(b,\omega)\ . \tag{37}$$

Using (35)–(37), the solution of physical quantities in series form can be rewritten as:

$$u_2^*(x) = \sum_{n=1}^{4}\left[R_{1n}L_n(b,\omega)e^{-k_nx}\right] + \xi_1\exp(-\gamma x)\ , \tag{38}$$

$$T^*(x) = \sum_{n=1}^{4}\left[R_{2n}L_n(b,\omega)e^{-k_nx}\right] + \xi_2\exp(-\gamma x)\ , \tag{39}$$

$$\varphi^*(x) = \sum_{n=1}^{4}\left[R_{3n}L_n(b,\omega)e^{-k_nx}\right] + \xi_3\exp(-\gamma x)\ , \tag{40}$$

$$\sigma_{xx}^*(x) = \sum_{n=1}^{4}\left[R_{4n}L_n(b,\omega)e^{-k_nx}\right] + \xi_4\exp(-\gamma x)\ , \tag{41}$$

$$\sigma_{yy}^*(x) = \sum_{n=1}^{4}\left[R_{5n}L_n(b,\omega)e^{-k_nx}\right] + \xi_5\exp(-\gamma x)\ , \tag{42}$$

$$\sigma_{xy}^*(x) = \sum_{n=1}^{4}\left[R_{6n}L_n(b,\omega)e^{-k_nx}\right] + \xi_6\exp(-\gamma x)\ , \tag{43}$$

where

$$R_{1n} = \frac{ib\left[(1-l_2)k_n^2 - B_1\right]}{\left[(B_3 - b^2l_2)k_n - l_3k_n^3\right]}\ ,$$

$$R_{2n} = \frac{[l_3k_n^4 - (B_4l_3 + B_3)k_n^2 + (B_3B_4 - b^2l_6)]R_{1n} - ib[l_2k_n^3 - (l_2B_4 - l_6)k_n]}{ib[B_2(k_n^2 - B_4) + B_2l_4]}\ ,$$

$$R_{3n} = \frac{(k_n^2 - B_1 - ibl_2k_nR_{1n} + B_2k_nR_{2n})}{k_n}\ ,\quad R_{4n} = ibl_{12}R_{1n} - B_2R_{2n} + R_{3n} - k_n\ ,$$

$$R_{5n} = ibR_{1n} - B_2R_{2n} + R_{3n} - l_{12}k_n\ ,\quad R_{6n} = l_3(ib - k_nR_{1n})\ ,$$

$$\xi = \frac{RF(y,t)}{\gamma^8 + A\gamma^6 + B\gamma^4 + C\gamma^2 + E}\ ,\quad \xi_1 = \frac{ib[(1-l_2)\gamma^2 - B_1]\xi}{[(B_3 - b^2l_2)\gamma - l_3\gamma^3]}\ ,$$

$$\xi_2 = \frac{[l_3\gamma^4 - (B_4l_3 + B_3)\gamma^2 + (B_3B_4 - b^2l_6)]\xi_1 - ib[l_2\gamma^3 - (l_2B_4 - l_6)\gamma]\xi}{ib[B_2(\gamma^2 - B_4) + B_2l_4]}\ ,$$

$$\xi_3 = \frac{(\gamma^2 - B_1)\xi - ibl_2\gamma\xi_1 + B_2\gamma\xi_2}{\gamma}\ , \quad \xi_4 = ibl_{12}\xi_1 - B_2\xi_2 + \xi_3 - \gamma\xi\ ,$$

$$\xi_5 = ib\xi_1 - B_2\xi_2 + \xi_3 - l_{12}\gamma\xi\ , \quad \xi_6 = ibl_3\xi - \gamma l_3\xi_1\ .$$

## 4 Boundary conditions

To determine the constants $L_n$ $(n = 1, 2, 3, 4)$, the boundary conditions at the surface $x = 0$ are given by:

i the normal surface is stress-free, $\sigma_{xx} = 0$,

ii the tangential surface is stress-free, $\sigma_{xy} = 0$,

iii condition of the micro-elongation (half-space is free from elongation), $\varphi = 0$,

iv thermal condition (half-space is a thermally insulated boundary), $\frac{\partial T}{\partial x} = 0$.

Using the expressions of $\sigma_{xx}$, $\sigma_{xy}$, $\varphi$ and $T$ into above boundary conditions, we get the following non-homogeneous equations:

$$\sum_{n=1}^{4} R_{4n}L_n = -\xi_4\ , \quad \sum_{n=1}^{4} R_{6n}L_n = -\xi_6\ ,$$

$$\sum_{n=1}^{4} R_{3n}L_n = -\xi_3\ , \quad \sum_{n=1}^{4} k_nR_{2n}L_n = -\gamma\xi_2.$$

Solving the above system of four equations, we get the values of constants $L_1, L_2, L_3, L_4$ and hence obtain the components of normal displacement, normal force stress, temperature distribution and microelongation for microelongated thermoelastic half-space under laser pulse heating.

## 5 Numerical results and discussion

For numerical computations, we consider the values of constants for aluminum epoxy-like material as [33]:
$\lambda = 7.59 \times 10^{10}\ \frac{\mathrm{N}}{\mathrm{m}^2}$ , $\mu = 1.89 \times 10^{10}\ \frac{\mathrm{N}}{\mathrm{m}^2}$ , $a_0 = 0.61 \times 10^{-10}$ N,
$\rho = 2.19 \times 10^3\ \frac{\mathrm{Kg}}{\mathrm{m}^3}$, $\beta_1 = 0.05 \times 10^5\ \frac{\mathrm{N}}{\mathrm{m}^2\mathrm{K}}$, $\beta_0 = 0.05 \times 10^5\ \frac{\mathrm{N}}{\mathrm{m}^2\mathrm{K}}$,
$C_E = 966$ JKg$^{-1}$K$^{-1}$, $T_0 = 293$ K, $j_0 = 0.196 \times 10^{-4}$ m$^2$,
$\lambda_0 = \lambda_1 = 0.37 \times 10^{10}\ \frac{\mathrm{N}}{\mathrm{m}^2}$ , $t_0 = 0.01$, $t_1 = 0.0001$, $K = 252\ \frac{\mathrm{J}}{\mathrm{msK}}$. The

computations are carried out for the value of non-dimensional time $t = 0.2$ in the range $0 \leq y \leq 1.0$ and on the surface $x = 1.0$. The numerical values for normal displacement, normal force stress, temperature distribution and microelongation are shown in Figs. 2–5 for generalized theory (G-L theory) by taking $\delta_{1k} = 0$, $\delta_{2k} = 1$, and $r = 0.1$ m, $I_0 = 10$ J/m$^2$, $\gamma = 50\,\frac{1}{\text{m}}$, $\omega = \omega_0 + \iota\zeta$, $\omega_0 = -0.2$, $\zeta = 0.1$, and $b = 0.7$ for:

(a) thermoelastic microelongated solid (TMS) with pulse rise time $t^* = 0.1$ by solid line with the centered symbol $\diamondsuit$,

(b) thermoelastic microelongated solid (TMS) with pulse rise time $t^* = 0.01$ by dashed line with the centered symbol ■,

(c) thermoelasic solid (TS) with pulse rise time $t^* = 0.1$ by dashed line with the centered symbol ▲,

(d) thermoelasic solid (TS) with pulse rise time $t^* = 0.01$ by dashed line with the centered symbol $\times$.

## 6 Discussion

The variations of normal displacement and normal force stress are similar in nature for TMS and TS in the range $0 \leq y \leq 1.0$, but the variations for TMS and TS are opposite in nature. The values of TMS are more for pulse rise time $t^* = 0.1$ in comparison to pulse rise time $t^* = 0.01$ whereas the values of TS are more for $t^* = 0.01$ in comparison to $t^* = 0.1$, which show that for a fixed pulse rise time, the pulse length has an appreciable effect on both the physical quantities namely normal displacement and normal force stress as depicted in Figs. 2 and 4. The variations of temperature distribution are same for both the medium in the same range $0 \leq y \leq 1.0$ as shown in Fig. 3. The values of microelongation are more for $t^* = 0.01$ in comparison to $t^* = 0.1$ in the range $0 \leq y \leq 0.2$ and approache zero with an increase in pulse length as evident from Fig. 5. All the physical quantities, i.e., normal displacement, temperature distribution, normal force stress, and microelongation approaches zero with an increase in pulse length.

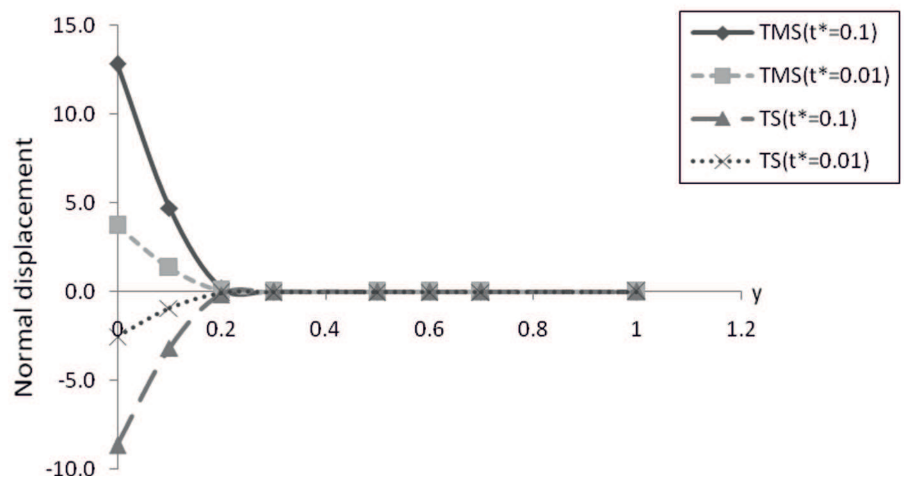


Figure 2: Variation of normal displacement with horizontal distance.

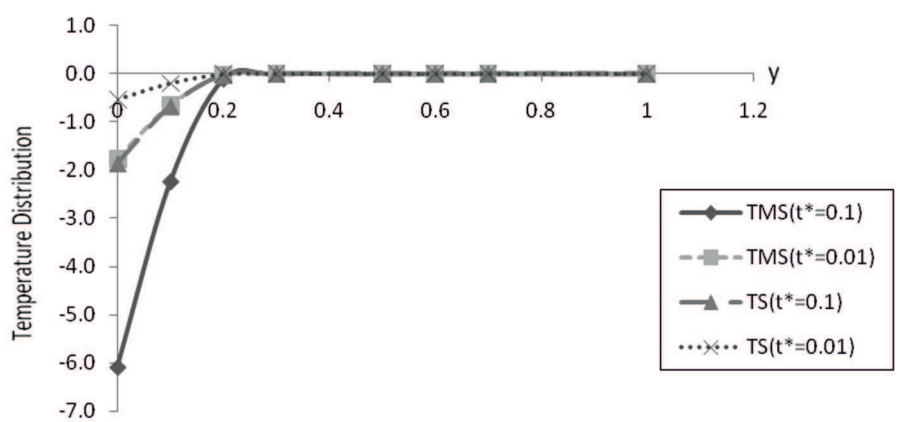


Figure 3: Variation of temperature distribution with horizontal distance.

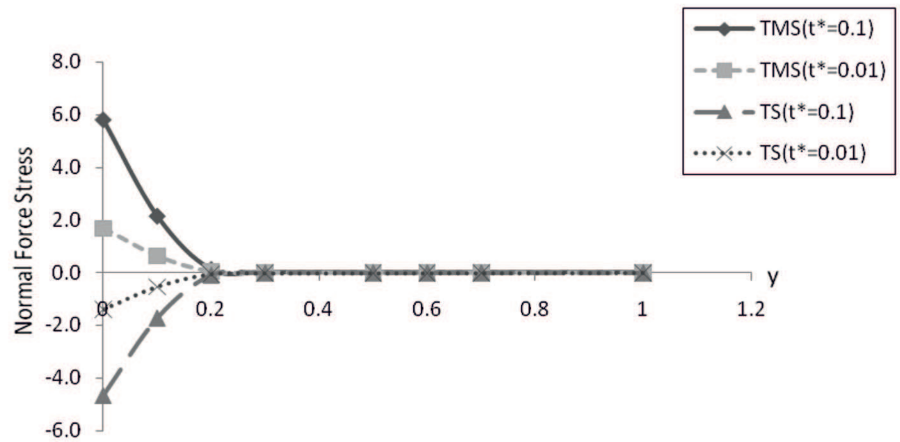


Figure 4: Variation of normal force stress with horizontal distance.

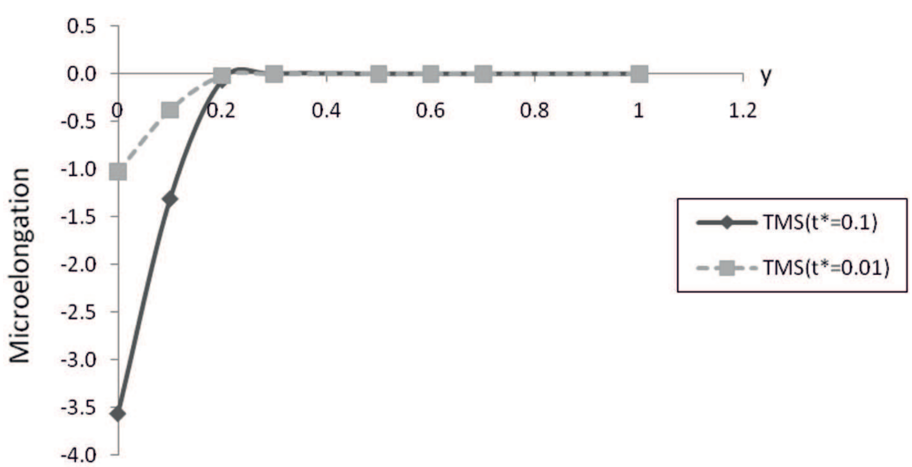


Figure 5: Variation of microelongation with horizontal distance.

## 7 Conclusion

1. A significant effect of laser pulse heating, pulse rise time and pulse length is observed in all the quantities, i.e., normal displacement, temperature distribution, and normal force stress and microelongation.

2. The variations of normal displacement and normal force stress show opposite nature in the presence and absence of microelongation for a fixed value of pulse rise time and increasing pulse length. This proves that microelongation has a significant effect on the considered physical quantities.

3. All the physical quantities, i.e., normal displacement, temperature distribution, normal force stress and microelongation decrease very sharply in the range $0 \leq y \leq 0.2$, which approaches zero with the increase in pulse length.

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