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THE PERFORMANCE EVALUATION OF THE SPARE PARTS MANAGEMENT: CASE STUDY

Bounou Oumaima, Abdellah El Barkany, Ahmed El Biyaali

Mechanical Engineering Laboratory, Faculty of Science and Techniques, Sidi Mohammed Ben Abdellah University, Morocco

Corresponding author:
Bounou Oumaima
Mechanical Engineering Laboratory
Faculty of Science and Techniques
Sidi Mohammed Ben Abdellah University
B.P. 2202 – Route d'Imouzzer – FEZ, Morocco
phone: (+212) 675880673
e-mail: oumaima.bounou@usmba.ac.ma

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Abstract

The supply chain of spare parts is the intersection between the supply chain, the after-sales and the maintenance services. Some authors have tried to define improvement paths in terms of models to satisfy the performance criteria. In addition, other authors are directed towards the integration of risk management in the demand forecasting and the stock management (performance evaluation) through probabilistic models. Among these models, the probabilistic graphical models are the most used, for example, Bayesian networks and petri nets. Performance evaluation is done through performance indicators.

To measure the appreciation of the supply of the spare parts stock, this paper focuses on the performance evaluation of the system by petri nets. This evaluation will be done through an analytical study. The purpose of this study is to evaluate and analyze the performance of the system by proposed indicators. First, we present a literature review on Petri nets which is the essential tool in our modeling. Secondly, we present in the third section the analytical study of the model based on bath deterministic and stochastic petri networks. Finally, we present an analysis of the proposed model compared to the existing ones.

Keywords

spare parts, inventory management, shortage; obsolescence, stochastic petri networks, performances evaluation, supply policies.

Introduction

The supply chain of spare parts is the intersection between the supply chain, after-sales and maintenance services. The decentralized management configuration of spare parts causes the lack of information sharing between technicians, which can generate significant costs and low quality of service. In order to reduce these effects, several maintenance organizations have directed towards centralized management of a spare parts supply chain. For example, some authors have tried to define improvement paths in terms of models to meet the performance criteria. In addition, other authors are directed towards the integration of risk management in forecasting and stock management (performance evaluation) through probabilistic models. Among these models, the probabilistic graphical models are the most used, for example, Bayesian networks and petri nets. The evaluation of performance is done through indicators that are variables of a measure or criterion of appreciation of a phenomenon at a given moment. The indicators can be qualitative or quantitative. Jaulent and Quarès presented in the paper [1] the different types of indicators, for example the risk indicators that need to be identified.

To simulate the solution space in search of a better combination of replenishment order, Ghorbel proposes in [2] a discrete event systems particulate model that is used in replenishment order logic and performance evaluation , built in four stages. This model deals with three replenishment cases for the supply management system (T, s, S) and the calculated parameters are: the stock level, the cumulative storage cost, the cost of parts acquisition, and accumulated duration and periodic cost of out of stock. The model inputs takes as unit costs for each type of cost. For each case, the author has associated indicators that focus on the rate of consumption and the inevitable or expected break. Based on these presented indicators, Ghorbel processed 32 combinations and for each combination 1 to 5 scenarios: 97 scenarios. She excluded 32 scenarios not produced in reality and constructed 12 classes of graphs by analyzing and comparing the remaining 65 scenarios [2].

On the other hand, Lazrak has dealt with the evaluation of management models in a simpler way with indicators on two levels: level of service and level of stock. Indicators are considered to measure the level of service in quantities and periods, the level and inventory value for stock-level evaluation [3]. In order to analyze the performance of logistic supply systems, more generally stochastic discrete event systems with batch behavior, Labadi proposed in [4] a model based on batch deterministic and stochastic petri networks which an extension of other classes of discrete petri nets. Labadi applied analysis techniques (analytical and simulation) in the evaluation of the stock management system (s, S) with continuous revision. The performance of the system is processed according to the parameters related to the demand and the delay. The indicators taken into consideration are: average stock, average storage cost, probability of empty stock, average supply frequency, average ordering cost, average purchase cost, coverage rate [4].

In the analytical analysis, Labadi and al proposed in [5] a sensitivity analysis approach through a disturbance realization on the parameters by modifying the transition matrix. Fattah and al have tried in [6] to combine the batch deterministic and stochastic petri networks with the supply chain operations reference (SCOR) model in the inventory management system. SCOR is a modeling tool that defines the approach, the process, the supply chain indicators. They made the application for a storage system for a production workshop. In another way, Abbou and al discussed [7] the performance analysis of a maintenance workshop for machining machines based on stochastic petri networks in three cases: corrective maintenance, preventive maintenance and finally both maintenances, taking account of the production phase.

Pérès and Grenouilleau used in [8] the traditional optimization technique to minimize the risks related to supply management and the risk of postponing the maintenance task, through a mathematical reformulation of the problem of available budget allocation and the use of a graph. For the resolution, they used dynamic programming. For a better compromise between the unavailability of the function in the absence of spare parts and the cost related to the selected strategy, they developed a method for evaluating the supply of spare parts in the operation phase, based on petri nets for each level of system performance. They found that two policies are close to the performance level. The elements taken into account in the model are: the notion of element priority, the constraint of capacity, the limited time, the flows of operation, information and decisions [8].

Grenouilleau and Pérès propose in [9] a method based on the minimization and the evaluation of the risks of postponement of a maintenance operation (gravity classes) and the initial supply through the risks related to the supply of an insufficient amount. They used the dynamic programming and the risk reduction approach (as low as possible). In order to combine within the same model, the different processes and to make a global assessment of the technical and economic performance of a system in its exploitation phase for the identification of better results, Pérès and al propose in [10] a structured modeling methodology leading to the construction of petri nets capable to take into account the physical aspect of the studied systems and immaterial notions related to the strategies put in place. The petri nets used in this work are associated with Monte-Carlo simulation and a guarantee of durability.

In this paper, we will discuss the performance evaluation of supply management of spare parts inventory. In the next section, we present a literature review on Petri nets which is the essential tool in our modeling. The third section presents the analytical study of the model based on batch deterministic and stochastic petri networks. Finally, we present an analysis of the proposed model compared to the existing ones.

Petri networks

In order to model and validate the behavior, the communication and the synchronization of the discrete systems or different parallel systems, petri nets have known an immediate development in the field of theoretical research and the industrial world. The originality and interest of this tool is due to the many different validation techniques that are simply examined by the graph [11]. From the systems described in the form of a set of communicating subsystems, petri nets are approached by Carl Adam Petri in order to uniquely describe automata associated with subsets and communications between these automata [12]. Then, petri nets can describe a dynamic system with discrete events.

Petri nets provide a graphical presentation with operational semantics. They are composed of two types of objects: places and transitions. The places, expressed by circles, represent the states of the whole-valued system. The transitions, expressed by rectangles, represent all the events whose crossing causes the modification of the system's state. Each state is a marking that usually involves several places [13]. The causal relationships are indicated by the presence of the evaluated arcs connecting between places and transitions [11, 13]. Two vertices of the same type (places or transitions) are never connected directly to each other. To summarise, Petri nets are a 4-tuple $R = (P, T, W^-, W^+)$, where [13, 14]:

- P is a finite and non-empty set of places;
- T is a finite set of transitions;
- W^- is the forward incidence function ($P \times T \rightarrow N$) or the precondition associated with the transition t and the place p. It defines the minimum number of marks in p required to enable the transition t and removed from p in the event of firing;
- W^+ is the domain back incidence function ($P \times T \rightarrow N$) or the post-condition associated with the transition t and the place p. It defines the number of marks made to p by the crossing of t.

An inhibitory arc is an oriented arc starting from a place p to reach a transition t. Its end is marked by a small circle. The inhibitory arc between the p place and the transition t means that the transition t is validated only if the place p contains no mark. The transition's firing consists in removing a mark in each entry place of t except for p, and adding a mark in each output place of t [15]. Then, the dynamic behavior of the graph is described by a third element which is the tokens. They circulate in places according to certain defined rules. The distribution of the tokens in the places is called the marking of the Petri net [12]. The types of petri nets are diverse. Murata presented in [14] some modifications and extensions to petri nets that are useful for applications. Thus, Labadi also presented in its review of the literature in [4] the extensions of petri nets. We present the types and extensions [4, 14]:

- The ordinary Petri network: if all weights are equal to 1;
- The generalized Petri network: if there is at least one weight greater than one. All arcs, whose weight is not explicitly specified, have a weight of 1;
- The synchronized petri network: a set of external events, associated with the network, allow the crossing of certain transitions;
- The petri net is strongly connected: if all vertices have at least one input relation and at least one output relation;
- Timed petri nets consist of introducing the notion of time into the network path to describe systems whose operation depends on time. Two types of this extension are: timers linked to transitions (T-timed) or places (P-timed). Their application is to find minimum cycle time;
- High-level petri networks are combined with other diagrams and languages, like XML. Their application in logic programs;
- Colored petri nets: In this type of the most used networks, the token, distinguished and attached by any information, is marked by colors. The extensions developed by the authors are: colored stochastic networks and colored generalized stochastic networks. The exploitation of this type of network mainly as a simulation tool;
- Continuous and hybrid petri networks are a combination of discrete petri nets and continuous petri nets composed of continuous places and transitions for the modeling of hybrid dynamic systems such as storage systems with independent demand;
- Stochastic petri networks were introduced the first time to evaluate the operating safety of computer systems. The transitions have random firing times and follow the exponential law that allows to develop the mathematical properties of the Markov chain process. This concept is developed in the modeling with complex requirements for example the modeling of the production systems. Several classes of stochastic petri networks, with different characteristics mainly in the nature of the transitions used, are proposed for modeling and performance analysis. The transitions used in this type of network are timed with a random time that follows the exponential law. Then the classes of the stochastic petri networks developed are:
  - Generalized stochastic networks characterized by immediate transitions (null timeouts) and exponentially distributed stochastic transitions;
  - Stochastic and deterministic petri networks are an extension of generalized stochastic petri-networks. They include three types of transitions: immediate transitions, stochastic and deterministic temporizations. Thus, this type of network is developed by authors and detailed in Subsec. 2.1;

– Stochastic extended petri networks are formed of a single type of transitions which is the random timed transitions and distributed with any law. This type of network can be solved by discrete event simulation if the conditions are not satisfied.

The techniques of analysis of petri networks by Dicesare and al in [13] are: enumerated analysis, transformation analysis, structural analysis, simulation analysis. The general methods of analysis of Petri nets use the matrix representation to overcome the size of the network studied and therefore the "physical" limitation imposed by the graph. Petri net's analysis methods can be classified into the following three groups [14]: the recoverability tree method (accessibility); the approach of the matrix equation; reduction or decomposition techniques.

According to the literature review and extensions presented by the authors, stochastic petri networks are the most dominant in the work done most used in the modeling and evaluation of system performance. Since we are interested in modeling the supply management performance of the spare parts stock, we will use a new extension of stochastic petri networks proposed by Labadi in [4]. This extension is based on the introduction of the batch's notion in places and transitions. We will detail this type of petri nets in the next section.

Batch, deterministic and stochastic petri nets

The application of stochastic petri networks is for the modeling and performance analysis of discrete systems. Logistics systems (especially supply of stocks) are assimilated to stochastic systems with discrete events with batch behavior (lots of different sizes and variables). To model this type of systems, batch deterministic and stochastic petri networks are developed by Labadi in [4]. A literature review done by Labadi and Chen in [16], represents the advantages of Petri nets as a modeling and analysis tool for logistic systems considered as systems with discrete events. stochastic and deterministic batch petri networks is a class of petri nets, an extension of stochastic and deterministic petri networks. The batch control aspect is introduced into the stochastic and deterministic petri networks [4, 17, 18]. In other ways, there are two types of places: discrete and lots [18]. This type of network is linked with the discrete petri net by M-markings, μ-markings and impact matrices. Labadi and al represent in [18] the operating rules and the specific analysis methods, for this type of developed network, includes the cover tree and the reduction and transformation techniques. The new components introduced in this template are batch place, batch tokens, and batch transitions. Labadi has defined in [4] two types of marking for both types of places for a graph. The μ-marking represents the number of tokens for a discrete place or a multi-set of whole or empty numbers for the batch tokens. The M-marking for a batch place is the sum of sizes of all batchs. For a discrete place, the M-marking is equal to its μ-marking. Thus, Labadi has determined in [4] the dynamics of the batch deterministic and stochastic petri network through the mathematical formulation of system evolution, discrete operating rules (validation of a discrete transition, crossing of a discrete transition, interpretation of the discrete operation), batch operation rules (validation of a batch transition, crossing of a batch transition, interpretation of batch operation) and finally the relations between the two types of operation.

Labadi has analyzed in [4] the behavior of the deterministic and stochastic petri network with respect to classical petri nets. Thus, he developed the incidence matrix and the state equation that manages the dynamic behavior of batch and discrete tokens at the same time. Then, the crossing policies associated with the lots (arrival order policy, random choice policy and sorting policy) and transitions (service and execution policies) are well handled by the author. The situations of the conflicts occur firstly in the case where a set of discrete and immediate transitions validated by the same μ-marking, secondly in the case of the squares places, when it is a set of tokens lots that can validate at the same time the same transition. So, he dealt with discreet crossing conflicts and lots and their resolution policies. Labadi has defined a third type of conflict. This conflict is the discrete/batch crossing conflict that represents the conflict between a set of immediate transitions of batch and discrete types. This conflict is solved in the same way that other conflicts presented with the application of the strategies set system modeling lord [4]. The essential properties of the deterministic and stochastic petri network are the boundlessness (a number of fine states), the liveliness and the blocking (for example: the blocking batch) specifying the activity of the transitions separately and the operation of the network, the host state and reversibility (network reset) [4]. The analysis techniques of batch deterministic and stochastic petri networks, discussed by Labadi in [4], are divided into two types:

- Qualitative analysis techniques, giving the possibility of searching for the essential properties of the network, operate on non-timed models. The five techniques developed are the enumeration analysis techniques (the construction of the graph of μ-markings and coverage associated with the network), the invariant techniques (P-invariant and T-invariant analysis), the conventional reduction techniques. and specific, the techniques by the associated discrete petri net network and the transformation techniques (passage from a batch deterministic and stochastic petri network to a classical petri network).

- Performance evaluation techniques, which we are interested in, fall into two broad approaches and Labadi presented in [4] the general procedure of each approach:
- An analytical approach based on the μ-markings graph manipulated simultaneously with the associated stochastic process characterized by a set of performance indicators. The batch deterministic and stochastic petri network is bounded and produces finite states. Thus, the techniques of analysis of the process of μ-markings of this type of network maintain the same general principle as those of stochastic petri networks.
- An approach based on discrete event simulation is remarkable for complex systems to make an analytical study. The mathematical syntax of the model makes it possible to build the simulation tool.

Labadi applied in [4] the type of network developed and the techniques presented on the management policy (s, S). We will adopt what is done by changing the policy and introducing the notion of obsolescence.

Description of the model

The inventory management policy, considered in the procurement system, is (T, s, S). Demand is stochastic and follows the fish process. The lead time is stochastic and follows the exponential law. The maintenance service request is lost or postponed when there is an stock's shortage due to a dead stock or supply delay too long and exceeds what is expected. This reported request will influence the maintenance action and later on the production system. The nodes of the Petri net are divided into two categories: transitions and places. In Table 1, we present the nodes of the proposed model which is presented in Fig. 1.

The petri net, which models the studied system, is represented by Fig. 1. The stock is modeled by the place $P1$ and its stock level by $M(P1)$. Place $P2$ photographs the stock level and the current supply orders. The stock's output, modeled by transition $t1$, represents the request of the maintenance service to carry out corrective or preventive maintenance interventions. The transition $t1$ considered exponential in the literature review since the dates are distributed exponentially. The crossing of the transition $t1$ takes place when the available stock $M(P1) > 0$ at time intervals distributed exponentially, which leads to decrease the M-markings of the places $P1$ and $P2$.

Table 1
Designation of the nodes of the proposed model.

Node	Designation
	Places
$P1$	Model the stock, its stock level M $(P1)$
$P2$	Model the stock level and the current orders $M(P1) + M(P3)$
$P3$	Model supply orders in progress
$P4$	Token to generate a new revision period and inspection of the condition and service life of the spare parts stock
$P5$	Place for inspection of the state of the spare parts stock level
$P6$	Place for obsolescence inspection of stock (life time)
	Transitions
$t1$	Model the output of the stock and delivery of the maintenance service request
$t2$	Model the order supply operation
$t3$	Model the ordering process
$t4$	Model the state of the sufficient stock level
$t5$	Model the deterministic delay

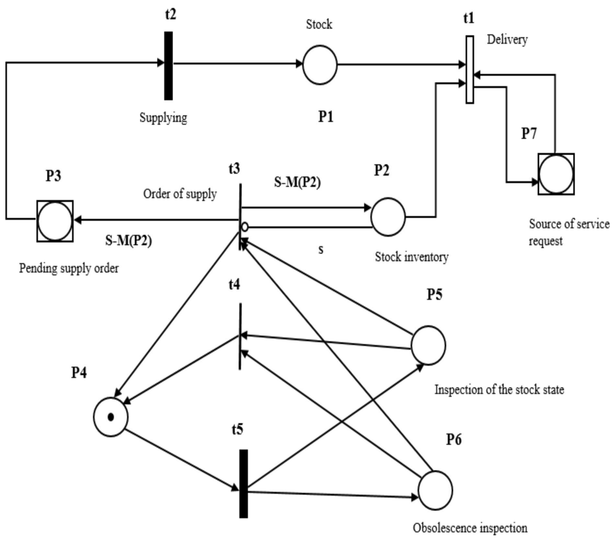

Fig. 1. Model of the analytical study system by batch deterministic and stochastic petri networks.

The stock level $M(P2)$ of the stock $P1$ is monitored by the inhibitory arc connecting the transition $t3$ and the place $P2$, whose the weight corresponds to the release's threshold of a supply order (firing the immediate transition $t3$). If $M(P2)$ falls below the threshold s, an order of quantity supply $S - M(P2)$ immediately passed by crossing the transition $t3$. In other ways, the transition $t3$ and its corresponding arcs model the command's release, the weight associated with these arcs corresponds to the quantity of the command which depends on the modeled policy. The firing time of the transition $t3$ is considered zero, that is to say the transition $t3$ is an immediate transition. The supply operation is made by firing the transition $t2$ whose the firing's times are distributed exponentially. Since the policy we have considered in our system is a policy with the periodic review. The revision period is determined by the deterministic delay associated with the transition $t5$. At each period, the transition $t5$ is fired and two tokens are placed: one in the place $P5$ and the other in the place $P6$, which means that an inspection on the state and the lifetime of the stock is in progress.

During the inspection of the stock status, there are two cases either to fire the $t3$ transition to place a batch order to complete the stock, or the $t4$ transition when the stock level is considered sufficient. The transition $t3$ has priority over the transition $t4$ ($\Pi(t3) > \Pi(t4)$). The system also goes through an inspection over the life of the parts in the stock, otherwise check the possibility of having a dead stock (obsolescence). In this inspection, the transition $t3$ is also a priority in firing over the transition $t4$. So we generalize in the both cases of the transition $t3$ is priority over the transition $t4$. After the firing in all cases, a token deposited in place $P4$ to generate a new inventory revision period.

Analytical study of the system

This technique consists in studying the system based on the states of the graph of the μ-markings which are divided into two types: the unstable states deduced from the firing of the immediate transitions and the tangible states retained from the firing of the deterministic timed transitions or stochastic. The parameters of the stock management policy considered (T, s, S), in the model presented in Fig. 1, are: (5, 2, 6). The generation of the maintenance service request is modeled by the loop composed of the place $P7$ and the transition $t1$. The transition $t1$ can be validated by different lots available in place $P7$. We consider that there are two possible sizes of the maintenance service request, hence the μ-marking of the $P7$ place is $\mu(P7) = \{1, 2\}$. The initial μ-marking is $\mu 0 = (6, 6, \emptyset, 0, 0, \{1, 2\})$. The initial μ-marking of places $P1$, $P2$ and $P3$ does not affect this study.

Evolution of the system

The μ-markings graph, one of the essential tools in the analytic approach, represents the evolution of the system modeled by the batch deterministic and stochastic petri networks. Each state of the system is described by the μ-markings and each firing represents the execution of an operation (the request delivery, the order placing and the order supply).

Starting from the initial μ-marking and considering all the possible firings, the μ-markings graph, presented in Fig. 2, is built by the repetition of its principle which consists in firing validated transitions of each new μ-marking reached. Finally, do not forget to add the batch crossing indices associated with their batch transitions.

To lighten the graph and the resolution method, as recommended in the analytical study procedure, the unstable states are eliminated by merging the tangible states. Then, this graph is isomorphic to a continuous time Markov chain which will make it possible to determine the associated stochastic process and its characterization. The graph of the associated Markov chain is represented in Fig. 3.

Resolution of the associated stochastic process

After the precision of the Markovian stochastic process associated to the studied system, we move on to specify the characteristics of the process (the transition matrix and the probabilities' distribution of the network's states). First, we start with the first characteristic of the process. The Markov process generator (or the transition matrix) Q, associated with the stochastic process of μ-markings, is constructed from the associated Markov chain graph as a function of the transition rates between the different possible markings. To simplify the complexity of the system's study and the associated system's resolution, we consider the two following cases:

- $\lambda 1_{[1]} = \lambda 1_{[2]} = \lambda 1$ means that the two types of demand under consideration follow the same exponential law.
- $\lambda 2_{[5]} = \lambda 2_{[6]} = \lambda 2$ means that the size of the lot does not influence the lead times.

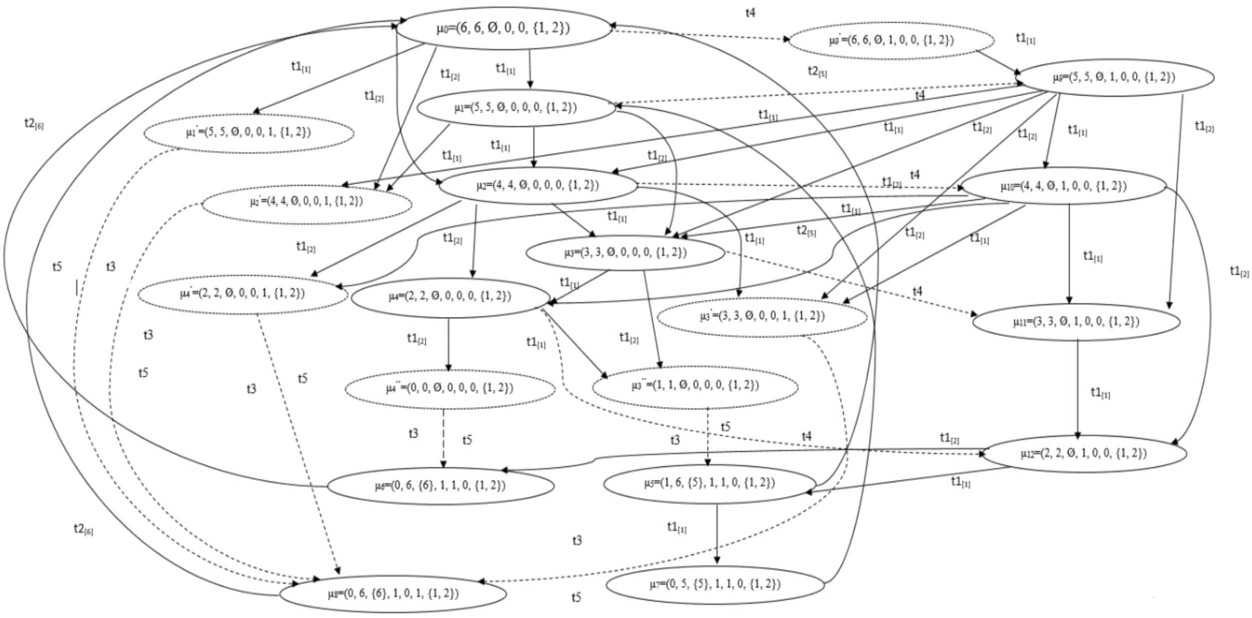

Fig. 2. The system's μ-marking graph.

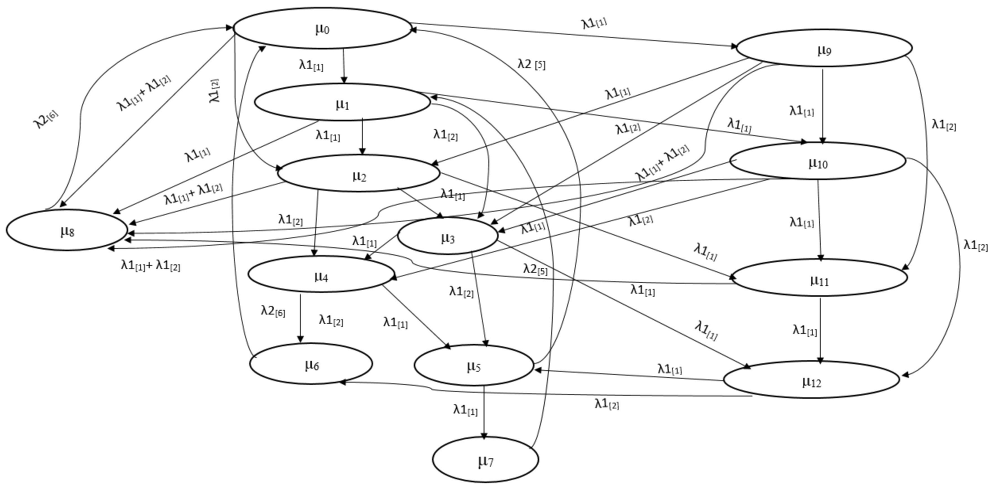

Fig. 3. The associated Markov chain graph.

Table 2
Simplified transition matrix.

		$\mu0$	$\mu1$	$\mu2$	$\mu3$	$\mu4$	$\mu5$	$\mu6$	$\mu7$	$\mu8$	$\mu9$	$\mu10$	$\mu11$	$\mu12$
Q	$\mu0$	$-5\lambda1$	$\lambda1$	$\lambda1$	0	0	0	0	0	$2\lambda1$	$\lambda1$	0	0	0
	$\mu1$	0	$-4\lambda1$	$\lambda1$	$\lambda1$	0	0	0	0	$\lambda1$	0	$\lambda1$	0	0
	$\mu2$	0	0	$-5\lambda1$	$\lambda1$	$\lambda1$	0	0	0	$2\ \lambda1$	0	0	$\lambda1$	0
	$\mu3$	0	0	0	$-3\lambda1$	$\lambda1$	$\lambda1$	0	0	0	0	0	0	$\lambda1$
	$\mu4$	0	0	0	0	$-2\lambda1$	$\lambda1$	$\lambda1$	0	0	0	0	0	0
	$\mu5$	$\lambda2$	0	0	0	0	$-\lambda1-\lambda2$	0	$\lambda1$	0	0	0	0	0
	$\mu6$	$\lambda2$	0	0	0	0	0	$-\lambda2$	0	0	0	0	0	0
	$\mu7$	0	$\lambda2$	0	0	0	0	0	$-\lambda2$	0	0	0	0	0
	$\mu8$	$\lambda2$	0	0	0	0	0	0	0	$-\lambda2$	0	0	0	0
	$\mu9$	0	0	$\lambda1$	$\lambda1$	0	0	0	0	$2\lambda1$	$-6\lambda1$	$\lambda1$	$\lambda1$	0
	$\mu10$	0	0	0	$\lambda1$	$\lambda1$	0	0	0	$2\lambda1$	0	$-6\lambda1$	$\lambda1$	$\lambda1$
	$\mu11$	0	0	0	0	0	0	0	0	$\lambda1$	0	0	$-2\lambda1$	$\lambda1$
	$\mu12$	0	0	0	0	0	$\lambda1$	$\lambda1$	0	0	0	0	0	$-2\lambda1$

In this case, the transition matrix associated with the system is simplified as shown in Table 2. The states' probability distribution is a function of the demand rate and the supply rate of the steady-state system. It is represented by a line vector Π and obtained by solving the system (1). The vector of the distribution, found after the resolution of the system (1), is presented by the Table 3

$$\begin{cases} \Pi.Q = 0 \\ \sum_i \pi_i = 1 \end{cases} \tag{1}$$

Table 3
Vector of the probability distribution of states.

π_0	$8640\lambda_2^2{+}5898\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_1	$2160\lambda_2^2{+}3319\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_2	$2448\lambda_2^2{+}2040\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_3	$2216\lambda_2^2{+}2353\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_4	$2632\lambda_2^2{+}2555\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_5	$7378\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_6	$5025\lambda_1^2{+}5162\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_7	$7378\lambda_1^2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_8	$24465\lambda_1^2{+}30660\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_9	$1440\lambda_2^2{+}983\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_{10}	$600\lambda_2^2{+}717\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_{11}	$2244\lambda_2^2{+}1870\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$
π_{12}	$2530\lambda_2^2{+}2470\lambda_1\lambda_2/36868\lambda_1^2{+}65405\lambda_1\lambda_2{+}24910\lambda_2^2$

System performance evaluation

Using the performance indicators formalized by the paper [4] and the states' distribution, the performances evaluation of the modeled system is done by the analysis of the parameters' effect related to the demand and the supply on the performances' variation which depend on these two parameters. To simplify the indicators' calculations, we consider: $\lambda_{1_{[1]}} = \lambda_{1_{[2]}} = \lambda_1$ and $\lambda_{2_{[5]}} = \lambda_{2_{[6]}} = \lambda_2$

- Average stock:

The function of the average stock is given by the average M-marking of the place $P1$. Its formula, according to the two parameters λ_1 and λ_2, is:

$$S_{moy}(\lambda_1,\lambda_2) = M(p1)_{moy} = \sum_{i=0}^{12} \pi_i * \mu(p1), \tag{2}$$

$$S_{moy}(\lambda_1,\lambda_2) = \frac{105736\lambda_2^2 + 98023\lambda_1\lambda_2}{36868\lambda_1^2 + 65405\lambda_1\lambda_2 + 24910\lambda_2^2}.$$

- Average storage cost:

The function of the average storage cost, according to the two parameters λ_1 and λ_2, is given by the multiplication of the average stock function by the unit storage cost Cs:

$$CS_{moy} = C_s * S_{moy} = C_s * M(p1)_{moy} = C_s * \sum_{i=0}^{12} \pi_i * \mu(p1), \tag{3}$$

$$CS_{moy} = C_s * \frac{105736\lambda_2^2 + 98023\lambda_1\lambda_2}{36868\lambda_1^2 + 65405\lambda_1\lambda_2 + 24910\lambda_2^2}.$$

- Probability of the empty stock:

The probability of the stock break, according to the μ-markings graph, is based on the μ-marking of the place $p1$ when it is equal to zero ($\mu(p1) = 0$) in three cases $\mu6$, $\mu7$ and $\mu8$. The function giving this probability is expressed as a function of λ_1 and λ_2 by:

$$Prob_{S=0}(\lambda_1,\lambda_2) = Prob(\mu(p1) = 0) = \pi_6 + \pi_7 + \pi_8, \tag{4}$$

$$Prob_{S=0}(\lambda_1,\lambda_2) = \frac{36868\lambda_1^2 + 35822\lambda_1\lambda_2}{36868\lambda_1^2 + 65405\lambda_1\lambda_2 + 24910\lambda_2^2}.$$

- Supply frequency:

Based on the μ-markings graph, the average supply frequency corresponds to the average crossing frequency of the t2 transition. According to the μ-marking graph, the crossing of this transition is done in the following cases:

- μ_5 and μ_7 represent the supply with a size of the supply order which is equal to 5 when firing $t2_{[5]}$.
- μ_6 and μ_8 represent the supply with a size of the supply order which is equal to 6 when firing $t2_{[6]}$.

Then, $S(t2_{[5]}) = \{\mu_5,\mu_7\}$ and $S(t2_{[6]}) = \{\mu_6,\mu_8\}$ represent the set of states where the transition $t2$ is fired with the lot corresponding crossing.

Then, the calculation of the average supply frequency is done by the sum of the average frequencies of the possible firings of the transition $t2$. The expression of the frequency, as a function of $\lambda1$ and $\lambda2$, is presented as follows:

$$FA_{moy}(\lambda_1,\lambda_2) = \sum_i F(t2[\mathrm{i}]) = \sum_x \sum_{i|\mu_i \epsilon S(t2_{[x]})} \lambda 2_{[x]} * \pi_i, \tag{5}$$

$$FA_{moy}(\lambda_1,\lambda_2) = \frac{36868\lambda_1^2\lambda_2 + 43200\lambda_1\lambda_2^2}{36868\lambda_1^2 + 65405\lambda_1\lambda_2 + 24910\lambda_2^2}.$$

- Average cost of placing an order:

The order supply orders have passed through the transition $t3$. For each supply order, there is a corresponding crossing of the transition $t2$. Hence, we can deduce that the average crossing frequency of the transition $t3$ (order of supply) is equal to the crossing frequency of the transition $t2$ (supply of the stock).

Then, the expression of the ordering cost function is in the form of a product of the average supply order frequency and the unit price of placing a C_c order.

$$CC_{moy}(\lambda_1,\lambda_2) = C_c * F(t3) = C_c * F(t2),$$
$$CC_{moy}(\lambda_1,\lambda_2) = C_c * \frac{36868\lambda_1^2\lambda_2+43200\lambda_1\lambda_2^2}{36868\lambda_1^2+65405\lambda_1\lambda_2+24910\lambda_2^2}. \quad (6)$$

- Average purchase cost:

The supply of the stock is done by the purchase of the order via an external supplier by firing the transition $t2$. The average purchase frequency corresponds to the average supply frequency since the transition $t2$ is fired by the different sizes of batch and also represents the action of the purchase of parts. The average purchase cost is expressed by the following function with C_a unit purchase cost of a product unit.:

$$CA_{moy}(\lambda_1,\lambda_2) = C_a * \sum_x x * F(t2[\mathrm{x}])$$
$$= C_a * \sum_x x * \sum_{i|\mu_i \epsilon S(t2_{[\mathrm{x}]})} \lambda 2_{[\mathrm{x}]} * \pi_i, \quad (7)$$
$$CA_{moy}(\lambda_1,\lambda_2) = C_a * \frac{213830\lambda_1^2\lambda_2+251822\lambda_1\lambda_2^2}{36868\lambda_1^2+65405\lambda_1\lambda_2+24910\lambda_2^2}.$$

- Coverage rate:

The coverage rate represents the ratio of average stock to average demand. The maintenance service request is modeled by the transition $t1$. It is executed when the stock is available by crossing the transition $t1$ in all cases of the demand at the same time, this will lead to the decrease of the stock level. Then, the average demand is equal to $\sum_x x/\lambda 1_{[\mathrm{x}]}$. Regarding the average stock, it already calculated based on the marking of the place $p1$. Finally, the expression of the coverage rate is given by the following function:

$$TC(\lambda_1,\lambda_2) = \frac{S_{moy}(\lambda_1,\lambda_2)}{D_{moy}(\lambda_1,\lambda_2)} = \frac{M(p1)_{moy}}{\sum_x x/\lambda 1_{[\mathrm{x}]}},$$
$$TC(\lambda_1,\lambda_2) = \frac{105736\lambda_1\lambda_2^2+98023\lambda_1^2\lambda_2}{110604\lambda_1^2+196212\lambda_1\lambda_2+74730\lambda_2^2}. \quad (8)$$

- Average shortage cost:

Out of stock is having a zero stock and can be caused by a delay in the orders supply. Then, according to the proposed system, the delivery of the orders or the replenishment of the stock is done by the crossing of the transition $t2$. The cost function of the stock break is written by the following expression:

$$CR_{moy}(\lambda_1,\lambda_2) = C_{ru} * \lambda_2 * Prob_{S=0}(\lambda_1,\lambda_2),$$
$$CR_{moy}(\lambda_1,\lambda_2) = C_{ru} * \frac{36868\lambda_1^2\lambda_2+35822\lambda_1\lambda_2^2}{36868\lambda_1^2+65405\lambda_1\lambda_2+24910\lambda_2^2}. \quad (9)$$

With C_{ru} cost of breaking the unit stock

- Probability of having a dead stock

In our system, there is a life-cycle inspection of the stored spare parts. We consider that the place $p6$ is modeled in a binary way, that is to say when we have a dead stock, the μ-marking of $p6$ equal to 1 and 0 in the opposite case. Then, the function giving the probability of the dead stock, based on the μ-marking of the place $p6$, is expressed as a function of λ_1 and λ_2 by: (DV = life-time)

$$Prob_{DV=0}(\lambda_1,\lambda_2) = Prob(\mu(p6) = 1) = \pi_8,$$
$$Prob_{DV=0}(\lambda_1,\lambda_2) = \frac{24465\lambda_1^2+30660\lambda_1\lambda_2}{36868\lambda_1^2+65405\lambda_1\lambda_2+24910\lambda_2^2}. \quad (10)$$

- Obsolescence cost

The dead stock generates an additional cost to the costs already considered. In the case of obsolescence, the level of the stock decreases. An order passed through the transition $t3$. The order is delivered by firing the transition $t2$. The cost of obsolescence is expressed by the following function with C_{ob} cost of unit obsolescence

$$CO_{moy}(\lambda_1,\lambda_2) = C_{ob} * \lambda_2 * Prob_{DV=0}(\lambda_1,\lambda_2),$$
$$CO_{moy}(\lambda_1,\lambda_2) = C_{ob} * \frac{24465\lambda_1^2\lambda_2+30660\lambda_1\lambda_2^2}{36868\lambda_1^2+65405\lambda_1\lambda_2+24910\lambda_2^2}. \quad (11)$$

System performance analysis

In concluding the analytical study, we present graphically some indicators presented above in functions the two parameters λ_1 and λ_2 in order to visualize the performances variation of the system compared to the rate of the maintenance service's request and the supply delay. From Fig. 4, we observe that the average stock increases when the rate of delay λ_2 increases i.e. the supply time decreases, on the other hand, it decreases when the rate of demand increases. Regarding the average storage cost, its pace is similar to the average stock since it is sufficient to multiply the latter by a coefficient (the unit cost of storage) to obtain the average cost. Conversely, in the first case, we notice from Fig. 5 that the two parameters influence the frequency of supply in the same way. It increases when demand and delay rates increase (delivery time decreases). According to Figs 6 and 7, the probabilities of the empty stock and the dead stock are the same as the difference in values. We have noticed that the probability increases when λ_2 is small, otherwise when the delay is large which is logical. Thus, the probability increases if the demand increases.

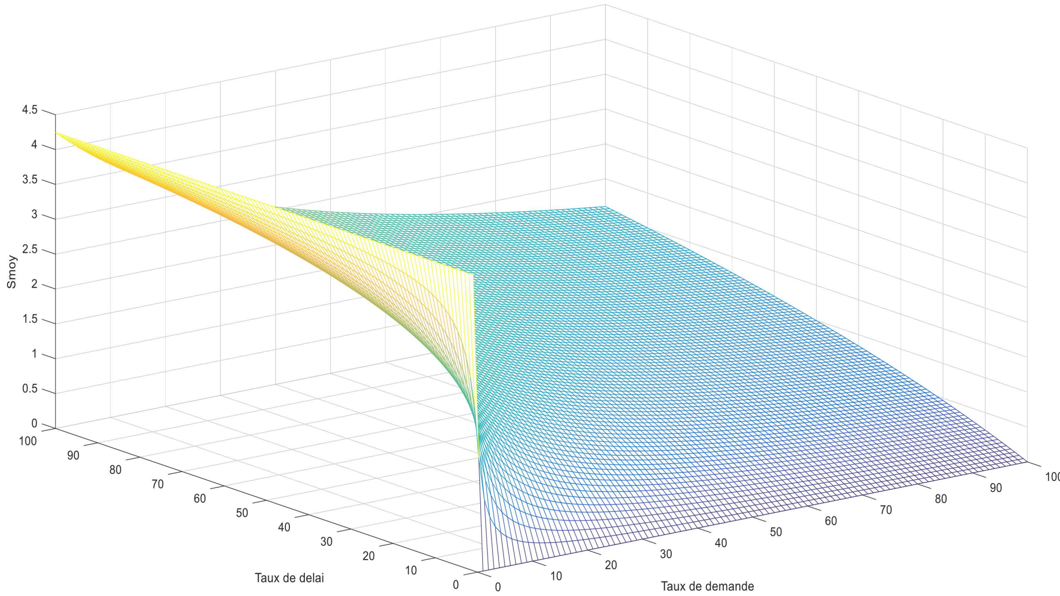

Fig. 4. The change in the average stock as a function of λ_1 and λ_2.

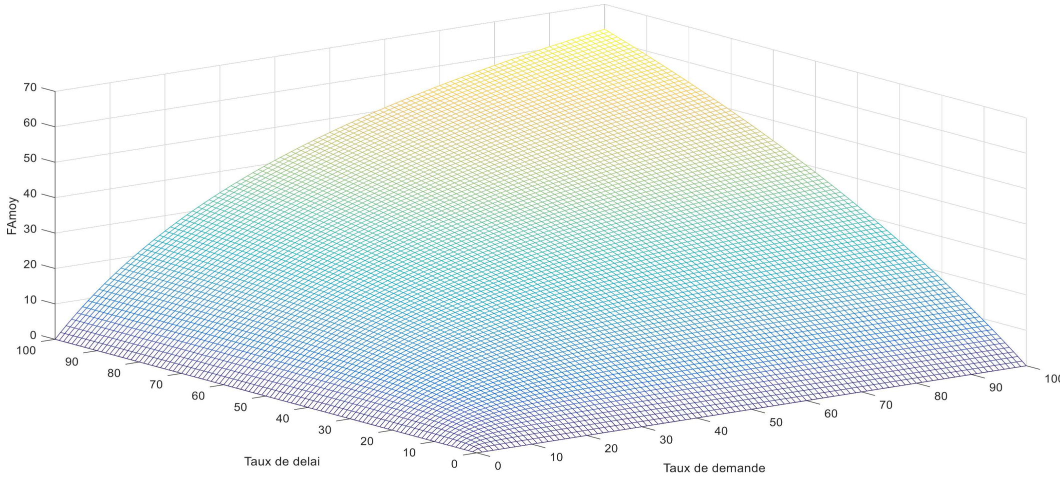

Fig. 5. The variation of the supply frequency as a function of λ_1 and λ_2.

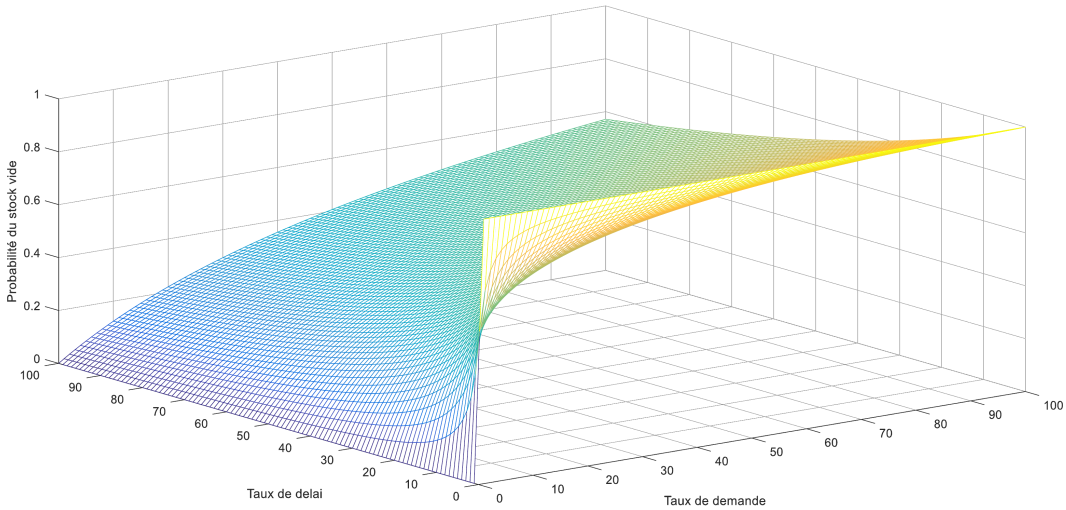

Fig. 6. The probability variation of the empty stock as a function of λ_1 and λ_2.

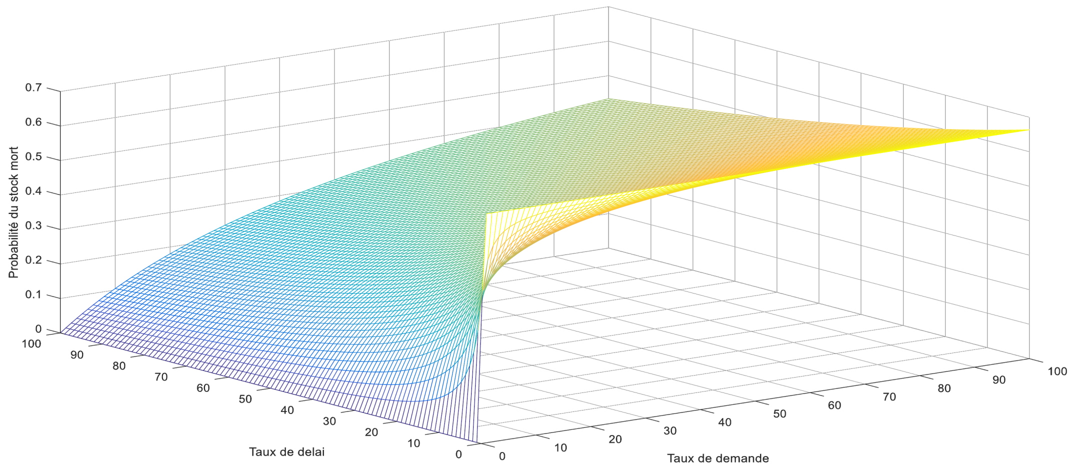

Fig. 7. The probability variation of the dead stock as a function of λ_1 and λ_2.

Analysis and synthesis

Through the analytical study and the simulation study done previously, we presented the effect of the parameters considered on the system: obsolescence (lifetime) and politics. We compare the integration of these parameters in the system with the Bayesian model proposed by Ghorbel [2], the reference model in [4] and the model of Fattah and al in [6] which combine petri nets with the SCOR tool. For the analytical study of these four models, we present a summary of these models in the form of Table 4.

Table 4
Summary of the three models.

		Models by Petri Networks			Bayesian Model proposed in [2]
		Labadi in [4]	Fattah and al in [6]	Our model	
System parameters	Tools Used	Batch Deterministic and stochastic petri networks.	– Batch Deterministic and stochastic petri networks. – SCOR model.	Batch Deterministic and Stochastic Petri Nets	Bayesian networks
	Policy considered	Policy with continuous control $(s, S) = (4, 6)$	Policy with continuous control $(s, S) = (2, 4)$	Policy with periodic control $(T, s, S) = (1, 2, 6)$	Policy with periodic control (T, s, S) 3 cases replenishment
	Markov chain graph	9 states	14 states	12 states	Do not use a Markov chain 12 graph classes
	Criteria considered	General supply case	Production workshop considered	Obsolescence risk Inventory inspection	Out of stock
Analytical study	Indicators	Average stock Probability of empty stock Supply frequency Coverage rate	Average stock Probability of empty stock Supply frequency	Average stock Probability of empty stock Supply frequency Coverage rate Probability of having a dead stock	Expected rupture Unavoidable rupture in forced replenishment and out of period/delayed replenishment Expected replenishment
	Costs	Average storage cost Average purchase cost Average cost of placing an order Average cost of the stock break	Average storage cost	Average storage cost Average purchase cost Average cost of placing an order Average cost of the stock break Obsolescence cost	Stock level Storage cost Purchase cost Cost and cumulative duration of shortage

According to the assumptions considered in each model, the expressions of the indicators considered are different, which is logical. On the other hand, according to the plots of some performance measures presented by Labadi in [4] and by our work presented in this paper, we notice that the average stock rate in functions the two parameters (rate of demand and delay) and the same in both models, but the difference is in terms of values: those obtained by our model is less than those presented in [4]. On the other hand, the pace of the probability of empty stock presented in [4] is different from that of our module. This differentiation is due to the parameter of obsolescence considered. This parameter is presented by the probability of having a zero life since it represents the dead stock and leads to having a zero stock (the stock break). So, the conclusion obtained, through the analytical study, is the consideration of obsolescence helps to minimize the average stock and the risk of scarcity. On the other hand, the influence of obsolescence is evaluated by studying the simulation system. We note that this parameter affects the average stock and transitions related to the ordering and supply of orders as well as the demand of the maintenance department.

Comparing our model with the particle model proposed by Ghorbel in [2] for the performance evaluation, we first notice that the graphical classes obtained by the Bayesian particle model are based on the binary states of the replenishment indicators and allow the prevention of risks of stock outage and cost evaluation depending on the level and condition of the stock. So, the difference between two models is at the level of the constitution and the use of the indicators in the performance evaluation. The evaluation of the supply by stochastic networks is based on the states of the system considered, on the other hand on the evaluation based on Bayesian networks, it is made taking into account the states of the performance indicators considered.

Conclusion

In this paper, we discussed performance evaluation in supply management using batch stochastic and deterministic petri networks, incorporating a new parameter. For this, we first presented a literature review on petri networks the first section. Afterwards, we presented the model adopted for the spare parts procurement management by considering: on the one hand the inventory level inspection which minimizes the number and the cost of placing orders, on the other hand the risk obsolescence presented by the inspection of the stored parts' life time. This last parameter helps to minimize the risk of out of stock, avoid production stoppage and take into account the breakage of spare parts in the market.

We applied one of the performance evaluation techniques: the analytic study. First, we noted that policy and lifetime inspection (risk of obsolescence) influenced stock behavior, and secondly that initial network marking also resulted in performance analysis in the study analytic. From the comparison of our model and existing models based on the same type of petri net, we noticed the added parameters positively influenced the behavior of the stock. Among, the stock characteristics, influenced by inventory level inspection and inspection of spare parts lifetimes, are the level of stock that is improved over the existing stock. We aim to begin in the future work to start the performance evaluation of joint management of spare parts management and maintenance.

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