10.24425/acs.2019.131232

Archives of Control Sciences
Volume 29(LXV), 2019
No. 4*, pages 687–697*

State estimation of networked control systems over limited capacity and dropout channels

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This paper investigates state estimation of linear time-invariant systems where the sensors and controllers are geographically separated and connected over limited capacity, additive white Gaussian noise (AWGN) communication channels. Such channels are viewed as dropout (erasure) channels. In particular, we consider the case with limited data rates, present a necessary and sufficient condition on the data rate for mean square observability over an AWGN channel. The system is mean square observable if the data rate of the channel is larger than the lower bound given. It is shown in our results that there exist the inherent tradeoffs among the limited data rate, dropout probability, and observability. An illustrative example is given to demonstrate the effectiveness of the proposed scheme.

Key words: linear time-invariant systems, limited capacity, observability, state estimation, networked control, data rate

1. Introduction

Networked Control Systems (NCSs) are control systems where the sensors, actuators and controllers are geographically separated and connected over communication channels, which operate subject to communication limitations, such as random delays, packet dropouts, and data rate constraints [1]. NCSs have attracted increasing attention in recent years. Many networked control and estimation strategies are designed in presence of communication limitations [2].

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This work is partially supported by China Postdoctoral Science Foundation funded project (No. 2013M530134), the Open Foundation of Automatic Weapons and Ammunition Engineering Key Disciplines of Shenyang Ligong University (No. 4771004kfx02). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

Received 13.02.2018. Revised 07.11.2019.

A high-water mark in the study of quantized feedback using data rate limited feedback channels is known as the data rate theorem. The intuitively appealing result was proved in [3–5], indicating that it quantifies a fundamental relationship between unstable physical systems and the rate at which information must be processed in order to stably control them. This result was generalized to different notions of stabilization and system models, and was also extended to multi-dimensional systems [6–8]. Control under communication constraints inevitably suffers signal transmission delay, data packet dropout and measurement quantization which might be potential sources of instability and poor performance of control systems [9–11].

In [12], a quantized-observer based encoding-decoding scheme was designed, which integrated the state observation with encoding-decoding. The paper [13] addressed some of the challenging issues on moving horizon state estimation for networked control systems in the presence of multiple packet dropouts. It was shown in [14] that maxmin information was used to derive tight conditions for uniformly estimating the state of a linear time-invariant system over a stationary memoryless uncertain digital channel without channel feedback. The case with both measurement quantization and control signal quantization was considered in [15] and the case of LQG systems subject to both input and output quantization was addressed in [16]. Networked control systems may be formulated as Markovian jump systems [17]. The problem of stability analysis and stabilization was investigated for discrete-time two-dimensional (2-D) switched systems in [18].

In this paper, we focus on the state estimation problem for linear time-invariant systems, where the sensors and controllers are connected over limited capacity, additive white Gaussian noise (AWGN) communication channels. We will examine the role that the data rate of such channels has on observability, and derive the necessary and sufficient condition on the data rate for mean square observability.

The remainder of this paper is organized as follows: Section 2 introduces problem formulation; Section 3 deals with the state estimation problem over AWGN communication channels; The results of numerical simulation are presented in Section 4; Conclusions are stated in Section 5.

2. Problem formulation

In this paper, we are concerned with the following linear time-invariant system:

$$X(k+1) = AX(k) + FW(k), \tag{1}$$

$$Y(k) = CX(k), \tag{2}$$

where $X(k) \in R^n$ denotes the state process, $Y(k) \in R^m$ denotes the measured output, and $W(k) \in R^q$ denotes the process disturbance. A, C, and F are known constant matrices with appropriate dimensions. Here, it is assumed that the pair (A, C) is observable. The sequence $\{W(k)\}_{k\in Z^+}$ is independent identically distributed (i.i.d.) with distribution $W(k) \sim N(0, \theta_w)$ and the initial state $X(0) \sim N(0, \theta_0)$ is independent of $W(k)$.

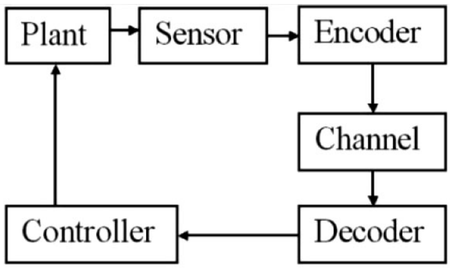

Figure 1: Networked control systems

The sensors and controllers are connected over a limited capacity, additive white Gaussian noise (AWGN) communication channel. Such a channel is described by the channel input $\alpha(k)$, channel output $\beta(k)$, and the channel noise $n(k)$. We define $\hat{X}(k)$ and $Z(k)$ as the state estimate and the estimation error, respectively. Then, we have

$$Z(k) = X(k) - \hat{X}(k). \tag{3}$$

Here, the plant state is encoded, and $R(k)$ bits of the information on the plant state are transmitted via the communication channel. Clearly, $R(k)$ is a time-varying variable. We define R as the mathematical expectation of $R(k)$. The existence of the channel noise often results in data packet dropout. Let p_e denote the dropout probability.

At each $k \geqslant 0$, the encoder is defined as

$$\alpha(k) = \delta\big(k,\ Y(0),\ Y(1), \cdots, Y(k)\big). \tag{4}$$

At each $k \geqslant 0$, the decoder is defined as

$$\hat{X}(k) = \varphi\big(k,\ \beta(0),\ \beta(1), \cdots, \beta(k)\big). \tag{5}$$

Notice that, the number of consecutive lost packets is unbounded in presence of the channel noise. This implies that the estimation error is unbounded too. Thus, we adopt a notion of mean square observability.

Definition 1 *Consider the system (1). For a finite $\mu > 0$, the system (1) is mean square observable if there exists an encoder and a decoder such that the estimation error of the plant state satisfies the following condition:*

$$\limsup_{k\to\infty} E\|Z(k)\| \leqslant \mu < \infty. \tag{6}$$

The objective of this paper is to examine the role that the limited data rate has on observability of the system (1), and to derive the necessary and sufficient condition on the data rate for mean square observability.

3. State estimation over AWGN channels

This section deals with the state estimation problem for linear time-invariant systems over limited capacity and AWGN channels, presents a lower bound on the data rate above which there exists an encoder and a decoder such that the system (1) is mean square observable.

Let γ_i denote the ith eigenvalue of system matrix A. Then, we assume that there exists a nonsingular real matrix H that diagonalizes $A = H'\Phi H$, where

$$\Phi = \mathrm{diag}\left[\gamma_1,\ \gamma_2, \cdots, \gamma_n\right]. \tag{7}$$

In order to find the minimum data rate for observability of the system (1), we employ the transformations of coordinates. Namely, we have

$$\bar{X}(k) := HX(k), \tag{8}$$

$$\widetilde{X}(k) := H\hat{X}(k), \tag{9}$$

$$\bar{Z}(k) := HZ(k). \tag{10}$$

Then, the system (1) can be rewritten as

$$\bar{X}(k+1) = \Phi\bar{X}(k) + \bar{W}(k), \tag{11}$$

$$Y(k) = CH'\bar{X}(k), \tag{12}$$

where we define

$$\bar{W}(k) := HFW(k). \tag{13}$$

Then, we have the following result.

Theorem 1 *Consider the system (1) over an AWGN communication channel. Let p_e denote the dropout probability and let γ_i denote the i-th eigenvalue of system matrix A. Then, a necessary and sufficient condition on the data rate for mean square observability of the system (1) is the following:*

$$R > \sum_{i\in\Omega}\left(\frac{p_e}{(1-p_e)^2} + 1\right)\log_2|\gamma_i| \ \ \textit{(bits/sample)}, \tag{14}$$

with the set $\Omega := i \in \{1, 2, \cdots, n\} : \ |\gamma_i| \geqslant 1\}$.

Proof. First, we define

$$\bar{X}(k) := [\bar{x}_1(k)\ \bar{x}_2(k)\ \cdots\ \bar{x}_n(k)]', \tag{15}$$

$$\bar{Z}(k) := [\bar{z}_1(k)\ \bar{z}_2(k)\ \cdots\ \bar{z}_n(k)]', \tag{16}$$

$$\bar{W}(k) := [\bar{w}_1(k)\ \bar{w}_2(k)\ \cdots\ \bar{w}_n(k)]', \tag{17}$$

$$\widetilde{X}(k) := [\widetilde{x}_1(k)\ \widetilde{x}_2(k)\ \cdots\ \widetilde{x}_n(k)]'. \tag{18}$$

Then, it follows that

$$\bar{x}_i(k+1) = \gamma_i \bar{x}_i(k) + \bar{w}_i(k). \tag{19}$$

Notice that we have

$$W(k) \sim N\left(0, \theta_w\right), \tag{20}$$

$$X(0) \sim N\left(0, \theta_0\right). \tag{21}$$

This implies

$$\bar{x}_i(0) \sim N\left(0, \sigma_0\right), \tag{22}$$

$$\bar{w}_i(k) \sim N\left(0, \sigma_w\right), \tag{23}$$

where σ_0 and σ_w are two known constants.
At any time k, we set

$$\bar{x}_i(k) \sim N\left(c_i(k), \sigma_i(k)\right), \tag{24}$$

where $c_i(k)$ and $\sigma_i(k)$ denote the mathematical expectation and variance of $\bar{x}_i(k)$ Then, we have

$$\widetilde{x}_i(k) = c_i(k), \tag{25}$$

$$\bar{z}_i(k) \sim N(0, \sigma_i(k)). \tag{26}$$

Let $v(k) = 1$ indicate that the data packet is successfully sent to the decoder and in contrast, let $v(k) = 0$ indicate dropout of the data packet. Then, we define $e(k)$ as the number of consecutive lost packets at time k. Here, we give

$$e(k) = \begin{cases} 0, & \text{when } \ v(k) = 1, \\ e(k-1) + 1, & \text{when } \ v(k) = 0, \end{cases} \tag{27}$$

At time $k{+}1$, we have

$$\bar{x}_i(k+1) \sim N\left(c_i(k+1),\ \sigma_i(k+1)\right). \tag{28}$$

Furthermore, we set

$$\widetilde{x}_i(k+1) = c_i(k+1), \tag{29}$$

$$\bar{z}_i(k+1) \sim N(0, \sigma_i(k+1)). \tag{30}$$

Then, we have

$$\sigma_i(k+1) = \begin{cases} \dfrac{\gamma_i^{2(e(k)+1)}}{n_i^2(k)}\sigma_i(k) + \sigma_w, & \text{when } |\gamma_i| \geqslant 1, \\ \gamma_i^2\sigma_i(k) + \sigma_w, & \text{when } |\gamma_i| < 1, \end{cases} \tag{31}$$

where $n_i(k)$ denotes the corresponding quantitative level. It follows from (20), (21), and (31) that

$$\sigma_i(k) = \begin{cases} \prod\limits_{j=0}^{k-1} \dfrac{\gamma_i^{2(e(j)+1)}}{n_i^2(j)}\sigma_0 + \sum\limits_{d=1}^{k-1}\prod\limits_{l=d}^{k-1}\dfrac{\gamma_i^{2(e(l)+1)}}{n_i^2(l)}\sigma_w + \sigma_w, & \text{when } |\gamma_i| \geqslant 1, \\ \prod\limits_{j=0}^{k-1} \gamma_i^{2(e(j)+1)}\sigma_0 + \sum\limits_{d=1}^{k-1}\prod\limits_{l=d}^{k-1}\gamma_i^{2(e(l)+1)}\sigma_w + \sigma_w, & \text{when } |\gamma_i| < 1. \end{cases} \tag{32}$$

Thus, we have

$$\lim_{k\to\infty} \sigma_i(k) < \infty, \tag{33}$$

if the data rate $r_i(k)$ corresponding to $\bar{x}_i(k)$ satisfies the following inequality:

$$r_i(k) > (e(k)+1)\log_2|\gamma_i| \quad \text{(bits/sample)}. \tag{34}$$

This implies that

$$\limsup_{k\to\infty} E\|Z(k)\| < \infty, \tag{35}$$

if the data rate $R(k)$ corresponding to $\bar{X}(k)$ satisfies the following inequality:

$$R(k) > \sum_{i\in\Omega}(e(k)+1)\log_2|\gamma_i| \quad \text{(bits/sample)}, \tag{36}$$

where we define the set

$$\Omega := i \in \{1, 2, \cdots, n\} : |\gamma_i| \geqslant 1\}. \tag{37}$$

This gives

$$R > \sum_{i\in\Omega}\left(\frac{p_e}{(1-p_e)^2}+1\right)\log_2|\gamma_i| \quad \text{(bits/sample)}. \tag{38}$$

The proof of sufficiency is complete. Next, we give the proof of necessity.

If we have

$$\limsup_{k\to\infty} E\|Z(k)\| < \infty, \tag{39}$$

$\sigma_i(k)$ must be bounded as $k \to \infty$. It follows from (32) that

$$r_i(k) > (e(k)+1)\log_2|\gamma_i| \ \text{(bits/sample)}, \quad i \in \Omega, \tag{40}$$

where

$$\Omega := i \in \{1, 2, \cdots, n\} : |\gamma_i| \geqslant 1\}. \tag{41}$$

Clearly, we have

$$R > \sum_{i\in\Omega}\left(\frac{p_e}{(1-p_e)^2}+1\right)\log_2|\gamma_i| \ \text{(bits/sample)}. \tag{42}$$

The proof of necessity is complete. □

Remark 1

1. *It is shown in Theorem 1 that there exist the inherent tradeoffs among the data rate, dropout probability, and observability.*
2. *If the dropout probability p_e is equal to zero (namely, no packet is lost), the condition in Theorem 1 can reduce to the existing result [1]:*

$$R > \sum_{i\in\Omega}\log_2|\gamma_i| \ \textit{(bits/sample)}. \tag{43}$$

3. *If the dropout probability p_e is equal to one (namely, all packets are lost), the data rate must satisfy the following condition:*

$$R > \infty \ \textit{(bits/sample)}. \tag{44}$$

It means that there exist no encoder and no decoder such that the system (1) is observable.

4. Simulation

In this section, we give a numerical example to demonstrate the effectiveness of the proposed result. Here, we consider an unmanned air vehicle, where three of all the states evolve in discrete-time according to

$$X(k+1) = \begin{bmatrix} 1.2422 & 1.5422 & -0.2543 \\ 0.2156 & -2.3247 & 0.6345 \\ 0.7834 & 0.2645 & 1.8336 \end{bmatrix} X(k) + 5.3211W(k), \tag{45}$$

$$Y(k) = X(k). \tag{46}$$

We set $X(0) = [3.24 - 3.512.56]'$, $p_e = 0.1$, $\sigma_w = 0.05$, and $\sigma_0 = 1$.

Here, we want to examine the role that the data rate has on observability of the system above. Let $R = 160$(bits/s). The corresponding simulation is given in Fig. 2. It is shown that the estimation error is bounded when the data rate is larger than the lower bound given in Theorem 1.

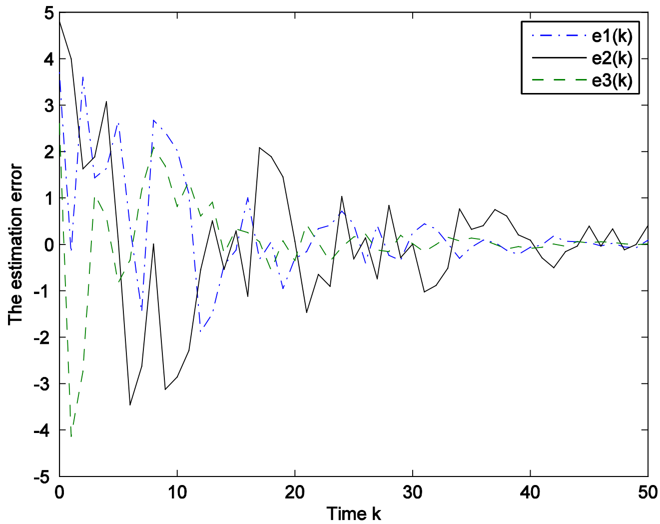

Figure 2: The estimation error responses when $R = 160$ (bits/s) and $p_e = 01$

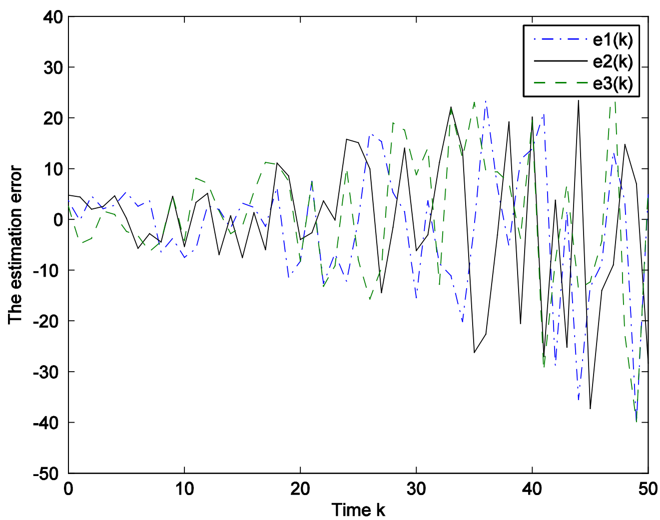

Figure 3: The estimation error responses when $R = 120$ (bits/s) and $p_e = 01$

If let $R = 120$ (bits/s) be less than the lower bound given by Theorem 1, the corresponding simulation is given in Fig. 3. It is shown that the estimation error is unbounded when the data rate is not large enough.

If let $R = 160$ (bits/s) be larger than the lower bound and let $p_e = 0.2$, the corresponding simulation is given in Fig. 4. It is shown that the estimation error is also unbounded when the dropout probability p_e continues to increase.

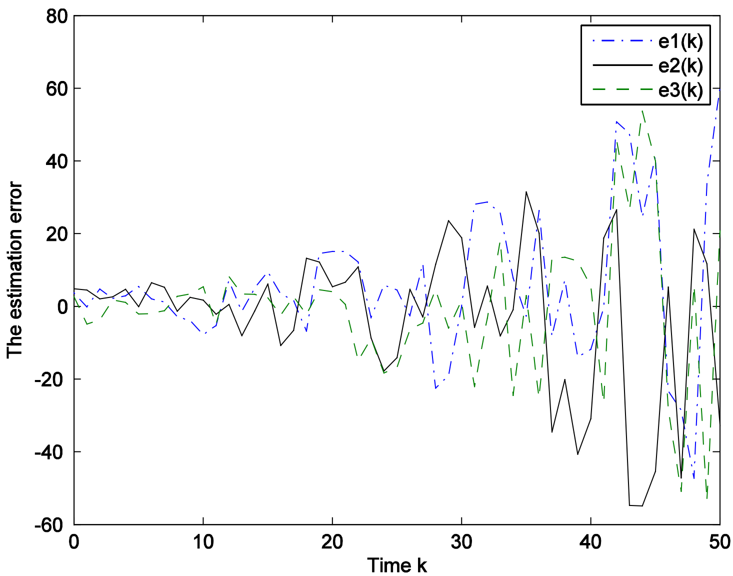

Figure 4: The estimation error responses when $R = 160$ (bits/s) and $p_e = 02$

Clearly, the data rate and dropout probability have important effects on observability of networked control systems.

5. Conclusions

In this paper, we considered linear time-invariant systems over limited capacity and AWGN communication channels, examined the role that the limited data rate has on state estimation. Here, we gave a notion of mean square observability and derived the necessary and sufficient condition on the data rate for mean square observability. It was shown that the data rate and dropout probability have important effects on observability. The system is mean square observable if the data rate of the channel is larger than the lower bound given in our results. The simulation results have illustrated the effectiveness of our results. The study of nonlinear system with limited data rate will be our future work.

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