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Improved method of gear hobbing computer aided simulation

Simulation studies of the hobbing process kinematics can effectively improve the accuracy of the machined gears. The parameters of the cut-off layers constitute the basis for predicting the cutting forces and the workpiece stress-strain state. Usually applied methods for simulation of the hobbing process are based on simplified cutting schemes. Therefore, there are significant differences between the simulated parameters and the real ones. A new method of hobbing process modeling is described in the article. The proposed method is more appropriate, since the algorithm for the momentary transition surfaces formation and computer simulation of the 3D chip cutting sections are based on the results of hobbing cutting processes kinematics and on rheological analysis of the hob cutting process formation. The hobbing process is nonstationary due to the changes in the intensity of plastic strain of the material. The total cutting force is represented as a function of two time-variable parameters, such as the chip's 3D parameters and the chip thickness ratio depending on the parameters of the machined layer.

1. Introduction

Gear hobbing remains the widespread method of gear machining that allows for manufacturing gears of different modules, dimensions and weights. This process is a subject of permanent scientific research due to its high level of universality and reliability. Systemic analysis of the cut-off layers dynamic formation creates the basis for a complex research on the gear-cutting process. The reliability of geometric modeling and methods of quantitative estimation of machined surface layers influence the correctness of the decisions about hob designing, optimizing

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the cutting parameters, reducing the tool wear, improving machining productivity and quality of the finished gears, and reducing the cutting forces, dynamic loads, oscillations and vibrations.

Simulation and formalization of hobbing is one of the most complicated tasks, compared to other forming and cutting processes. It is due to the complexity of process kinematics and teeth gaps formation. The optimal method for qualitative and adequate estimation of volumetric-structural and quantitative chip cross sections analyses has not been developed yet, because the kinematics of this process is complicated and therefore it is difficult to simulate it. A critical literature review reveal that the majority of previous studies were dedicated to solving this problem by different methods [1–12]. The approximation of the cutting surfaces by means of spatial trajectories and of its cross-section with the surface of the wheel is a common feature for most of these methods. The analysis of cross section geometries allows for getting information about the cut-off layers and their parameters. The basic principle of these methods is schematically shown in Fig. 1.

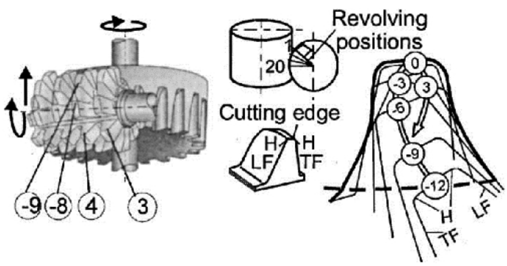


Fig. 1. An example of planar approach to geometric simulation of the hob cuter movement [3]

In some investigations, it is not taken into consideration that the gear rolling method must take into account the simultaneous axial displacement, and that each subsequent tool path meets a new generating revolving position of the hob cutter. Therefore, the calculation schemes for such an approach are planar [3, 8, 14]. On the other hand, in some studies, the displacement of the hob in the axial feed and the traces of the cutting hob teeth during previous cutting steps are not taken into account (Fig. 2) [1, 2, 18]. Also, many important features of the hob-cutting scheme are not taken into consideration in overwhelming majority simulation methods. Among them, one can mention subdividing the cutting hob teeth into groups, allowing for distinguishing which of them cut in current hob position; which are located at a certain distance relatively to the work gear flank (end gear surface), and which have already made the cutting action in the previous hob position.

The most important problems are associated with solving the mathematical modeling and research on various contact, power, thermal, stress-strain state, tribological conditions of cutting by the hob cutter. In the present research, a number of these problems have been solved, in particular:

- firstly, the task of developing a more precise and adequate generation method for 3D spline paths of the hob teeth and calculating the parameters of cross-section in hobbing generative;
- secondly, the application of the simulated results for prediction of the force load influence on the technological system. At the same time, the condition of unsteadiness of hobbing is taken into account.

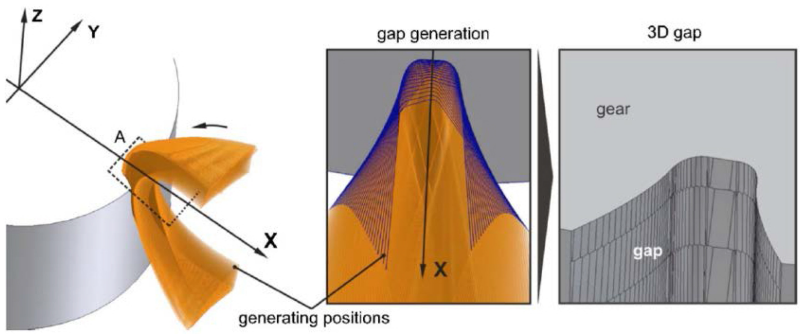


Fig. 2. An example of simulation in which feed is not taken into consideration as hob movement [1]

On this basis, a new approach to the modeling, computer simulation and prediction of elastic vibrations of the hobbing machine tool has been developed. This is based on a comprehensive cross-cutting investigation of interrelated factors and parameters of this process. The approach includes:

- simulation of the hobbing and the design of volume-structural model of unstrained chips;
- modeling the nonstationary harmonic cutting force as the external impact on the elastic system;
- modeling of transient process of the cutting force and its time delay in relation to the chip thickness change;
- experimental study on the dynamic characteristics of a gear machine tool;
- simulation of oscillations of a closed dynamic machine tool system with the nonstationary hobbing process, based on all the previous steps.

2. Simulation study results

2.1. Simulation of the chip cross section parameters

Spatial representation of kinematic movements of all cutting elements is the basis for developed of the method of the chip's graphic simulation in the working space during hobbing. The shape of the chips will be taken in the unstrained state.

Four different simultaneous kinematic movements are the main features of rolling generation kinematics such as hobbing. These are the following movements: hob rotation, tool movement in the axial feed direction, work gear rotation, as well as moving along the hob axis of the shape-generating toolpath of cutting elements located on the hob screw surface. The last one is the movement of the teeth of hob located on the helical surface, along the hob axis. The main problem for simulation is that the boundary of the surface, in which cutting in one gap occurs, constantly changes its profiles, location and shape according to the cutting hob teeth positions. This surface is formed on the workpiece, as some transition surface between the not-machined surface and partially treated gap. All the teeth that have made the cutting during the time preceding present machining moment are involved in the formation of the gap. Therefore, the shape and dimensions of the gap continuously change as a result of the removal of different allowances.

The representation of this transition surface is given in Fig. 3. There are shown the partially treated gap and traces of the machined gear teeth from previous cutting steps, which together with the contour of the gear contour spur determine the geometry of the machined surface layers.

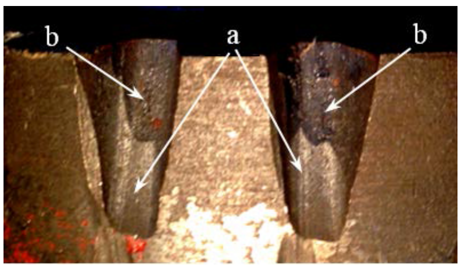


Fig. 3. The image of transition surface in the gap of partially machined gear: a – tooth mark from the present cutting step; b – tooth mark from previous cutting step

2.2. Basic principles and main research idea

The main principles of the developed methodology that distinguish it from similar ones are the following.

A. The chip, that is the layer of metal that is removed by each hob tooth on a helical surface, is bordered in space by the following surfaces:

- the cutting surface that is the same for every tooth, designed based on an initial generating rack. This surface is generated as a result of the tooth rotation around the hob axis;
- the external cylindrical surface of the gearwheel that depends on the hob cutter position at the active length;
- the composition of the cutting surfaces – splines of all hob teeth that have performed the cutting process in the present gap before this tooth.

The geometry characteristics of the cutting surface are the same for every hob with such parameters as: the module, the number of columns, the helix angle, the pressure angle of the generating rack, the outside diameter and feed magnitude. This surface is defined by the profile crowning of the generating rack tooth –

trapezoid with the angle of the sideline equal to the pressure angle. It is generated in hobbing by the rotation trajectory of the relative hob axis of this profile. The hob path is represented as a spline, which can be described both analytically and graphically using modern CAD systems by intersection of each cutter spline with a cylindrical surface of the workpiece. All the cutting hob teeth belonging to the same gear generating rack can also be represented as an array of splines (Fig. 4).

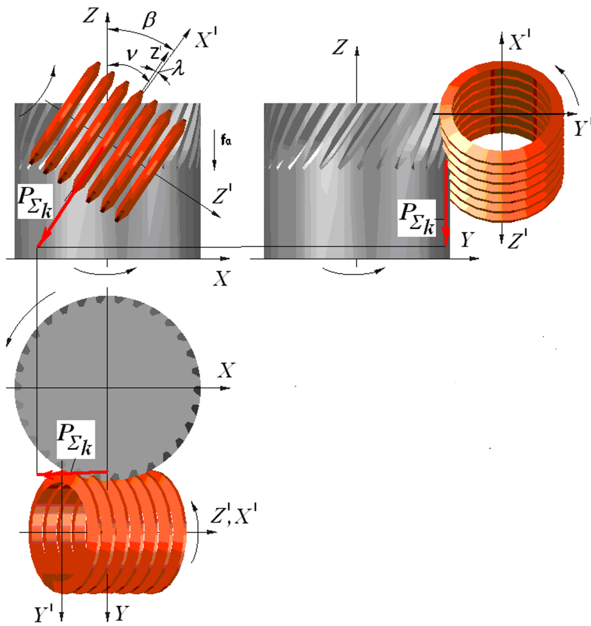


Fig. 4. Hob cutter image with the set of cutting teeth splines in hobbing process

The initial hob's revolving position is denoted by the angle ν that is determined by the coincidence of the hob helix direction with the direction of the machined gear tooth. In the known works [4, 9, 10] one assumes that the spline plane coincides with this angle, although the hob does not rotate along the hob helical direction, but around the hob axis. In effect it means that the plane in which the spline of a single tooth is perpendicular to the hob axis and inclined relative to the gap at an angle λ (Fig. 4). This is the **first difference** between the proposed simulation approach and the known methods.

B. The trajectories of the hob teeth revolution, which previously treated a certain gap, must be considered to determine the composition of the splines in the cutting space. This space is bounded by the cutting surface of the running hob cutter and the boundaries of the output cross sections. The current hob tooth performs cutting motion in the same direction relative to the cutter and the gearwheel with periodically revolving tool. The outer and inner boundaries of the cut-off layer remain the same during the steady cutting state. However, the tool inlet and outlet phases are the exceptions to the above simulation pattern. The boundaries of the cut-off layer change after every revolving position of the cutting tooth in that case. However, these processes are not considered in this paper. As stated above, the formation of a certain gap is carried out by all the cutters on its helical surface when the gearwheel is rotated together with the toll cutting machine table. The shape and dimensions of the treated surface, as well as the shape and size of the chips on all the teeth, are continuously changing. This machining cycle for a single gap is repeated periodically with workpiece revolution and is accompanied by the hob's displacement on axial feed magnitude.

The second significant difference of the advanced simulation method is associated with the hobbing kinematics unique features. The periodicity of the gear teeth machining means that the transition surface in the gap is formed partly in the present hob revolving position and partly in previous ones, according to the axial feed. In other words, the shaping cutting movements are displaced both in space and in time. Such hob cutters disintegration and the dynamic change of their revolving positions distinguishes the proposed method from the existing ones and allows for improving the 3D computer simulation accuracy [1–3, 7].

The scheme shown in Fig. 5 illustrates the above-mentioned difference between the present method and the other ones. This scheme depicts how the i-th tooth at a distance L_x from the central axis machines a particular gap at the suitable time. All teeth that are located behind the present tooth on the hob helical surface, that is at the distance $(L_a/2 - L_x)$ of working length of the hob cutter, generate this gap. This side of the partly machined profile is shown in blue in Fig. 5. However, another part of this gap profile has already been partially generated by the hob teeth located ahead of the present tooth, but in the previous revolving hob position in axial feed motion. This group of teeth and the appropriately generated profiles are marked in red.

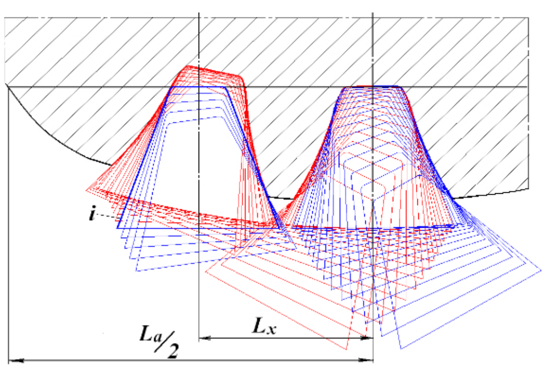


Fig. 5. The boundaries of the tool path and the machined gear trace for different cutting positions (blue) wheel teeth traces and the hob path in the length L_x in running cutter position relative to the direction of the axial feed; (red) wheel teeth traces and the hob path in the length $L_a/2 - L_x$ in previous cutter position relative to the direction of the axial feed

2.3. Calculation of elementary motion parameters

Initial data for simulation: m_n is the module; Z_h is the number of hob columns; D_{ah} is the outside diameter of cutter; Z_g is the number of gear teeth; D_a, D_f are tip and root diameters of the machined wheel, respectively; f_a is the axial feed; λ is the helix angle of the hob cutter; β is the helix angle of machined wheel; $\nu = \beta - \lambda$ is the setting angle of the hob position; $m_p = \dfrac{m_n}{\cos\nu}$ is the transverse wheel module.

A. Continuous cutting kinematic motions in the hobbing process can be decomposed into several elementary components.

The primary elementary movement is the hob rotation at one-corner step:

$$\tau = \frac{360^\circ}{Z_h}\,. \tag{1}$$

Then, the gear wheel rotation by the profiling angle ψ is described as [12]:

$$\psi = \frac{360^\circ}{Z_h Z_g}\,. \tag{2}$$

A certain point on the helical surface of the hob will relocate simultaneously along the generating line relative to the axis by the distance:

$$\Delta x = \frac{\pi m_n}{Z_h \cos\lambda}\,, \tag{3}$$

where $p = \pi m_n$ is the step of worm helical line.

At the same time, the hob displaces in the axial feed direction by the distance:

$$f_a' = \frac{f_a \cos\beta}{Z_h Z_g}\,. \tag{4}$$

For example, the hob's rotational speed, in the conditions when the cutting speed $V = 100$ m/min; module $m = 5$ mm; number of gear teeth $Z_g = 36$; the hob diameter $D_{ah} = 100$ mm and the number of columns $Z_h = 10$, is:

$$n = \frac{1000V}{\pi D_h} = \frac{1000 \cdot 100}{3.14 \cdot 100} = 318.5\ \text{min}^{-1}.$$

The period of hob revolution is 0.19 s, and the duration of the hob cutter rotation referred to a one-corner step is 0.019 s. During this time, the machined gear will rotate in the direction of the circular feed at the angle $\psi = 1^\circ$, and the cutter point located along the axis will move by a distance of $\Delta x = 1.57$ mm.

The initial hob active cutting length L_a is:

$$L_a = D_{fk} \tan\theta\,, \tag{5}$$

where the magnitude of the angle θ can be defined as:

$$\theta = \arccos \frac{D_f}{D_a} . \tag{6}$$

The total number of hob teeth that can be placed at the helical line L_a can be calculated as:

$$q = \frac{L_a}{\Delta x} = \frac{L_a}{\pi m_n} Z_h . \tag{7}$$

If the hob axis coincides with the gearwheel center axis, the hob's tooth current position will be denoted as "0". Thus, the theoretical number of teeth on the input and output part of the hob cutter can be calculated as $q_{\text{inp}} = q_{\text{out}} = q/2$. If one marks the teeth along the helical line from the extremity, then their sequence of numbers will be as follows: $(-q_{\text{inp}})$; $(-q_{\text{inp}}+1)$; $(-q_{\text{inp}}+2)$; . . . ; 0; 1; 2; . . . ; $(q_{\text{out}}-1)$; (q_{out}). The position of any i-th cutting tooth $(q_{\text{inp}} < i < q_{\text{out}})$ is characterized by the distance to the gearwheel center axis as $L_x = -L_a + i\Delta x$ and by the angle θ_i.

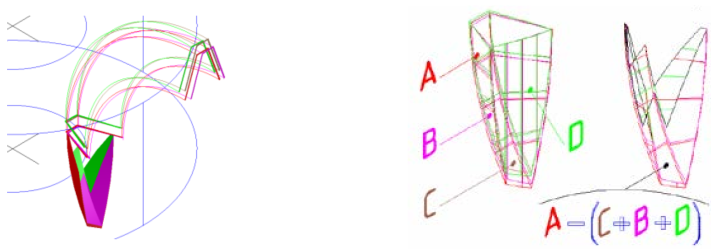
B. Taking into consideration the short duration of hob teeth elementary movements, one can assume that the gearwheel has a fixed position in the time interval between elementary cutting steps; the splines that describe the 4 cutting paths are immobile, and the continuous process is considered as a sequence of mutual positions of the hob and the workpiece in these discrete linear and angular positions. The period of change of these positions corresponds to the rotation time of the cutter for one corner step, in our example it is 19 msec. This is the **third difference** between the existing [1–3] hobbing simulation approaches and the present one.

2.4. Algorithm for the formation of momentary transition surface and 3D simulation of machined surface layers

The foregoing regularities and features of the simulation process are the basis of the algorithm for designing 3D tool splines and chip cutting sectional parameters. A schematic pattern of a graphical description of a geometric chip model is shown in Fig. 6. In accordance with the above procedures of the cutting process formalization, the forming cutter path is shaped by the:

- spline of the certain tooth (it will be marked with the index i – surface A (Fig. 6b));
- spline of the tooth located on the helical surface just before cutting tooth. For this, i-th spline is displaced to the $(-\Delta x)$ distance and rotated to the angle $(+\psi)$ in the XYZ coordinate system (it will be marked with the index $(i - 1)$ – surface B (Fig. 6b));
- spline of the tooth, which is in the previous position relative to the hob on the axial feed. For this, i-th spline is displaced by the f_a distance (it will be marked with the index j – surface C (Fig. 6b));

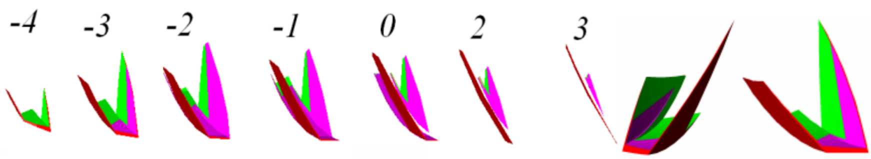
- spline of the tooth, which is located at a previous position relative to j-th tooth on the helical line of the hob cutter. For this, the j-th spline is displaced by the $(+\Delta x)$ distance and rotated by the angle $(-\psi)$ in the XYZ coordinate system (it will be marked with the index $(j-1)$ – surface D (Fig. 6b)).



(a) (b) (c)

Fig. 6. Graphical interpretation of the algorithm for the momentary transition surface formation as intersection of some splines: the spline of i-th tooth path and a set of additional splines forming 3D transition surface (a) and the cut-off layer (b, c)

The intersection of these four splines and the gear's cylindrical surface corresponds to the volumetric circuit of the chip, which is removed by i-th cutting tooth in such a position on the toothed wheel's chord, which is generated by the intersection of the hob's generating line with the flank face of the gear. The spline of the i-th teeth (a), and the set of splines, which form its 3D transition surface, and the 3D geometrical model of unstrained chip at the conventional (b) and the climb (c) feed, are shown in Fig. 6a. This simulation model is designed in the AutoCAD. The sequence of the cutting sections models on a running column of the hobbing cutter (a) and the shape of chip which is cut off by the second tooth ("−2") of the column are shown in Fig. 7b.



(a) (b)

Fig. 7. An example of geometrical simulation of the 3D geometrical model of unstrained chip (a) and a shape of chip which is cut off by the second tooth ("−2") of the current column (b)

The foregoing algorithm can be used to simultaneously design a set of all splines that fit the running hob's column. The spline of the current i-th rack position (marked in red), the spline of $(i-1)$-th rack position (marked in purple) and the spline of $(j-1)$-th rack position (marked in green) within the active length L_a are shown in Fig. 8.

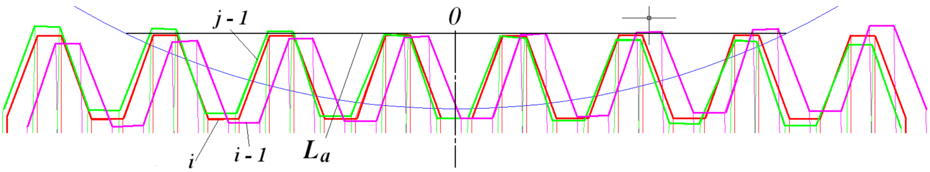


Fig. 8. Generating rack location at positions “i” (marked in red), “$i-1$” (marked in violet) and “$j-1$” (marked in green)

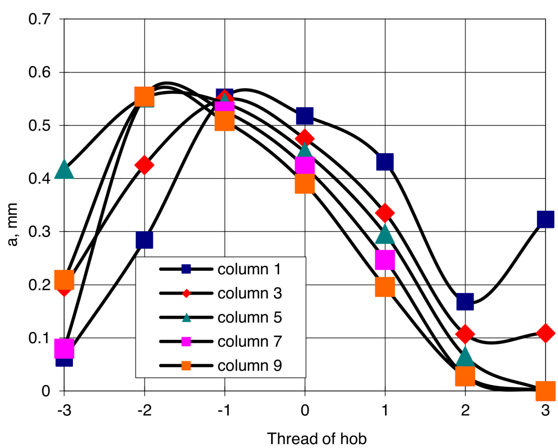
Table 1.
3D models of chips on columns and on cutting threads in positions “i” (marked in red), “$i-1$” (marked in violet) and “$j-1$” (marked in green)

Column No.	Thread No., tooth No.					
	Thread −3	Thread −2	Thread −1	Thread 0	Thread +1	Thread +2
Column 1	1	10	19	28	37	46
Column 3	3	12	21	30	39	48
Column 5	5	14	23	32	41	–
Column 7	7	16	25	34	43	–
Column 9	9	18	27	36	45	–
gear turning →						

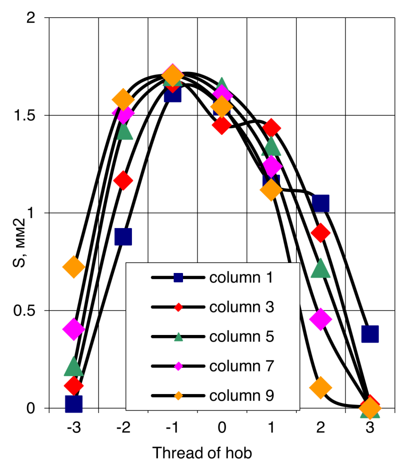
3D geometric models of the chips are given in Table 1 for such parameters as: coming feed $f_a = 2.5$ mm/rev; helical gear: $Z_g = 32$; $\beta = 32°$; $\lambda = 2°$; $\nu = 30°$; $m_n = 3.5$ mm, $m_p = \dfrac{m_n}{\cos\nu} = 4.06$ mm; $D_a = 152.25$ mm; hob: $Z_h = 9$; $D_{ah} = 90$ mm; $L_a = 84$ mm; the number of active cutting threads is $z = \dfrac{L_a}{\pi m_p} = 6.6$, we accept 7.

Let's enumerate the cutting threads, according to their location relative to the gear axial plane as: −3; −2; −1; 0; +1; +2; +3. The geometric models of chips are shown for the hob's columns with the double step: 1-3-5-7-9.

If we simulate the geometric models of chips that are removed by all hob teeth, then we can determine such important parameters of cutoff layer as: cross-sectional area, variable thickness and width of cut, the volume of chips on the front and side hob cutting edges, and the length of the contact between the cutting edge and the workpiece surface. The charts describing the dependence of the maximum cross-sectional area on the angle of hob rotation and thickness of chips on teeth and columns for the above initial data are shown in Fig. 9.



(a)



(b)

Fig. 9. Maximum thickness (a) and area (b) of the transverse cross-sections on the hob columns

For Fig. 9, it is indicated that "*a*" is the maximum chip thickness of the section and "*S*" is the maximum cross-sectional area of the chip. In this case, the hob cutter has 9 teeth and includes 7 columns within the outer diameter of the machined gear wheel. Such an aggregate makes a screw during hob cutter rotation. The graphs show how the maximum thickness of the chip section (Fig. 9a) and the maximum cross-sectional area (Fig. 9b) can be changed on each cutter's helical surface (vertical direction) and on each column (horizontal direction). The overall combination of these relationships is shown in Fig. 14 as a 3D graph.

3. Advanced simulation of the hobbing process

The solution to this problem allows us to calculate the cutting forces in hobbing process resulting from the volumetric loading of the hob cutter.

3.1. Cutting forces in gear hobbing

It is known [14, 15, 19] that the cutting force is a function of the area of the chips cross-section defined as:

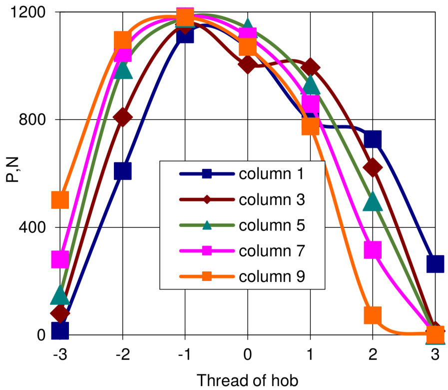
$$P = \tau \xi S, \tag{8}$$

where S is the area of the chip cross section (mm^2), ξ is the chip thickness ratio; τ is the ultimate shear strength of the workpiece material, MPa.

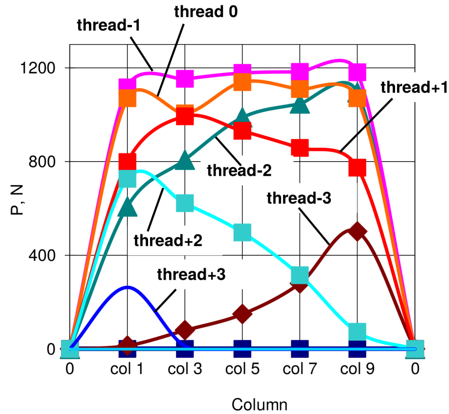
According to the maximum shear stress theory, the ultimate shear strength $[\tau]$ is calculated as a half of the ultimate tensile strength $[\sigma]$ [9]. For carbon steels ($\sigma = 600 \div 650$ MPa, $\tau = 300 \div 330$ MPa), the average value of the chip thickness ratio is within the limits $\xi = 1.75 \div 2.5$; for alloyed steels ($\sigma = 900 \div 1000$ MPa, $\tau = 450 \div 500$ MPa), this ratio is within the bounds $\xi = 2.3 \div 3.5$.

The elementary hobbing force appears on each active cutting edge of the hob tooth and changes along the arc of the cutting edge contact with the machined surface. The cutting process is a function of forces acting on all active teeth edges – on the tooth face, and on right and left tooth flanks. The total force acting on a certain column is defined as the geometric sum of forces on all its teeth that are simultaneously in contact with the workpiece. Like the elementary force on the hob teeth, the total force acting on the columns is a vector, the magnitude and direction of which varies according to a suitable regularity. The cutting process for each of the columns has specific parameters characterized by periodicity of workpiece revolution, and is accompanied by the hob's displacement in axial direction with the frequency corresponding to one turn of the hob cutter. However, as noted above, within one rotations cycle each cutting tooth cuts the chip with the same parameters and shape. Consequently, the chip's parameters (cross-sectional area and cut-off thickness) are the primary factors that lead to the cutting force variation.

The graphs shown in Fig. 10 characterize the change of the cutting force determined according to relation (8) and correspond to the sections parameters shown in Fig. 9. Output data: gear material is carbon steel AISI 1045 ($\tau = 330$ MPa), average value of the chip thickness ratio is $\xi = 2.1$. The dynamics of cutting force on the cutting teeth along the columns is shown in Fig. 10a. The dependence of the cutting force on the columns in the function of the angle of hob cutter rotation is shown in Fig. 10b.



(a)



(b)

Fig. 10. Changes of the cutting force versus angle of hob rotation on the hob threads (a) and on the columns (b)

3.2. Using the results of rheological simulation for the updated modeling of the hobbing process

The second factor that decides on the nonstationary character of the hobbing process is the instability of the plastic strain intensity of the treated material, which is due to the change of chip thickness ratio ξ. This value depends on the cutting depth and the width of chip, which continuously change in the hobbing process. Therefore, the graphs given above in Fig. 10 do not reflect the actual rules of the change in the cutting forces.

Contrary to the existing calculation methods of the ratio ξ, which usually are experimental, we have developed a method for determining this factor by the rheological simulation of the hobbing process [16, 17]. Using this model for analyzing non-stationary cutting allows us to set the change of chip thickness ratio in accordance with changes in the hob cutting depth. A fragment of the cutting simulation in the Deform 2D is shown in Fig. 11. It reflects the results of computer rheological simulation of the cutting process in the Deform-2D and characterizes a pattern of elastic and plastic strains during cutting with a certain cutting depth and axial feed. The graph depicted in Fig. 12, which is derived from this simulation, indicates that the intensity of the chip thickness ratio, which is included in relation (8), varies depending on the thickness of the cut layer. This is typical for the hobbing process. Therefore, the cutting force depends not only on the cross-sectional area of the slices, which also constantly changes, but also on the cutoff thickness. This is an important element of novelty, which until now was not possible to take into account.

Summary of the Deform 2D/3D Post Processor gives a possibility to analyze shear angle β as a standard procedure. Thus, chip thickness ratio ξ can be

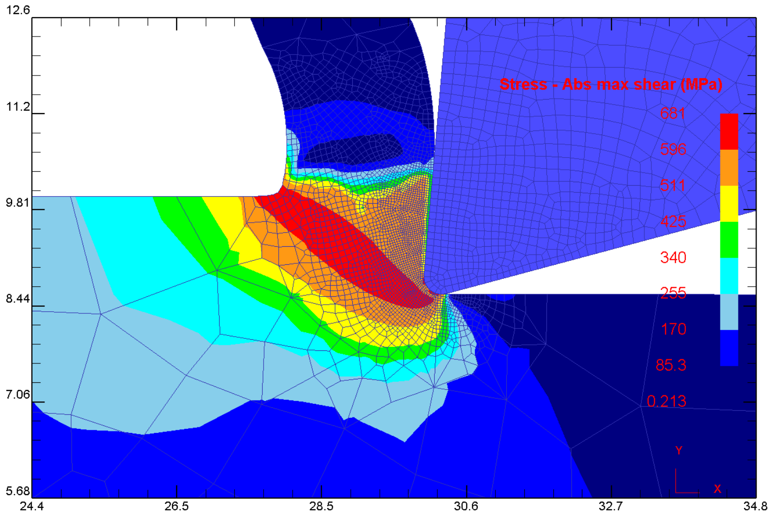


Fig. 11. A fragment of the cutting simulation in the Deform 2D

calculated as:

$$\xi = \frac{\cos(\beta - \gamma)}{\sin \beta}, \tag{9}$$

where γ is the rake angle of hob cutter.

The analysis of the simulation results with a variable depth of cutting allows us to determine the chip thickness ratio ξ. The graph of the ratio ξ dependence on hobbing thickness between 0.2÷1.7 mm is shown in Fig. 12 (the simulated curve is shown in blue and the approximated curve is marked in red).

If the temperature effects on the physical and mechanical properties of the machined material are ignored, taking the constant shear strength τ and using

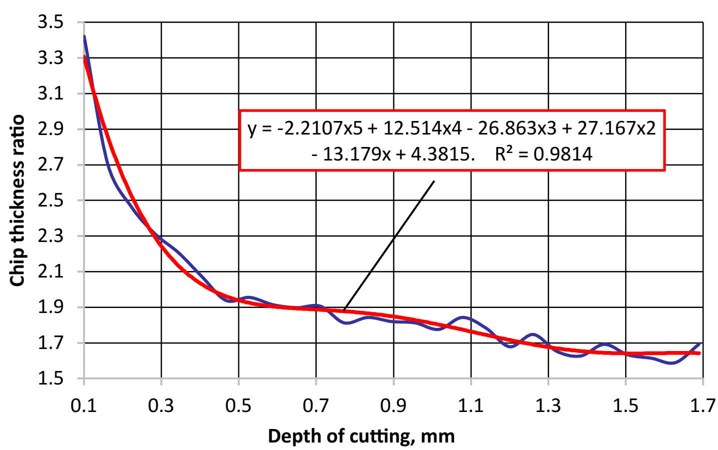
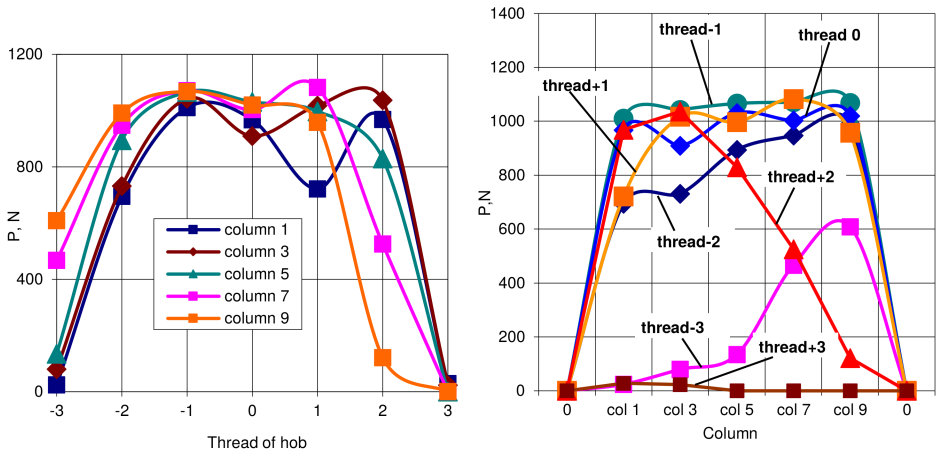


Fig. 12. The graph of the chip thickness ratio ξ depending on the cutting depth

the function $\xi = \xi(a)$ (Fig. 12), the corrected value of the cutting force can be simulated. Graphs of cutting forces, which are obtained taking into account the change in the chip thickness ratio, are shown in Fig. 13.



(a) (b)

Fig. 13. Updated dependency of the cutting force on the hob threads (a) and on the columns versus the angle of hob rotation (b), subject to the change of the chip thickness ratio

It is obvious that the corrected cutting force, obtained by taking into consideration the change in the cutting depth, differs both in form and in size from the cutting force calculated for the unchanged average values of the chip thickness ratio and constant parameters of the sections. The theoretical spatial power field of a hob cutter on the teeth (axis X) along the worm helical line and on the columns (axis Y) is shown in Fig. 14.

If we use the simulation results shown in Fig. 14, then the total force acting on each of the 9 hob columns can be calculated. This total force will be equal to the sum of the cutting forces on each of the teeth, assuming that the vectors of forces on each tooth of one column have the same direction. The total force will be conditionally applied to the central tooth and it will be oriented tangentially to the trajectory of rotation. The regularity of force changing on all columns per hob revolution characterizes the influence of the power load change on the technological system of the gear tool machine. The graph of change of the total cutting force during three hob rotations is shown in Fig. 15.

The maximum value of the total cutting force P_Σ on the columns is 5.1 kN, the average maximum force value is 4.8 kN, and the range of the maximum force is 0.7 kN, as it comes from the simulation results (Fig. 15). This force acts periodically with a frequency equivalent to the rotational speed of hob: $\nu = n_h$. The vector of force on k-th column $(\overrightarrow{P_{\Sigma k}})$ at a certain angle of rotation of the rake surface φ_m,

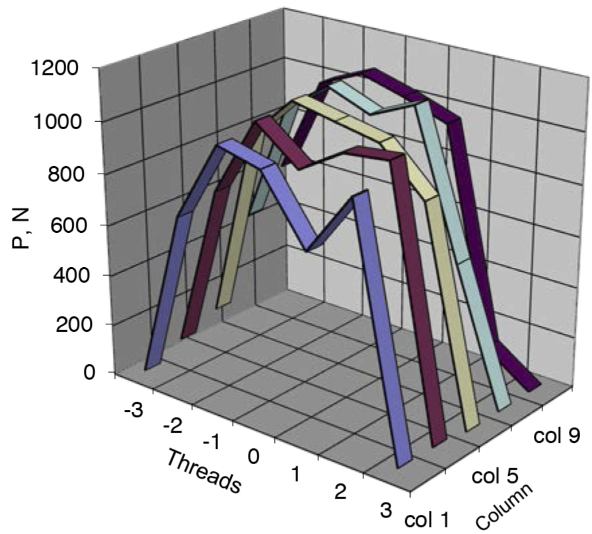


Fig. 14. 3D mode of cutting force in the dynamic hobbing process Gearwheel: $m_n = 3.5$ mm, $Z = 35$, steel AISI 45; hob: $Z_h = 9$; $D_{ah} = 90$ mm; axial feed rate is 2.5 mm/rev; cutting speed is 35 m/min

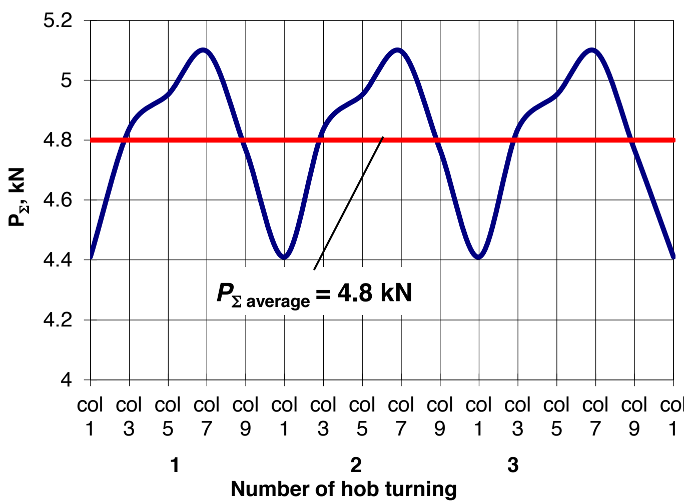


Fig. 15. Graph of changes in total cutting force during three hob cutter rotations

corresponding to the maximum thickness of section $a_{\max}$, is shown in Fig. 16. This is demonstrated to determine the effect this force has on the precision of the treated gearwheel. The action of this force projection on the plane of the wheel face creates a rotating torque T on the wheel axis (Fig. 4) with the value:

$$T_{\max} = 0.5 \cdot 10^{-3} P_{\Sigma} D_{\varphi} \cos \varphi_m \sin \nu \, . \tag{10}$$

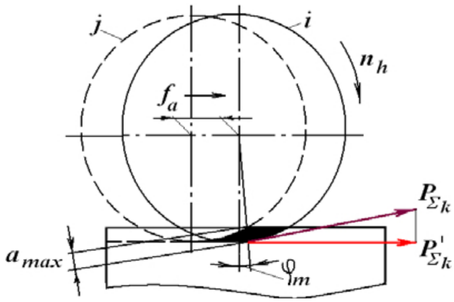


Fig. 16. Scheme of the hobbing forces, corresponding to the maximum cutting cross-section area

This torque is the cause of torsional elastic strains given as the maximum angular displacement $\Delta\varphi$ of the gear-hobbing machine table axis:

$$\Delta\varphi = \frac{T_{\max}}{J} \tag{11}$$

where J is the torsional stiffness of the gear-hobbing machine table, kN/rad.

According to relation (10), it is possible to calculate maximum value of the rotating torque as $T_{\max} = 111$ Nm for the foregoing data. If the torsional stiffness is $J = 12.7$ kN/rad, then the maximum angle of elastic torsion of the gear-hobbing machine table is 0.5°. This value corresponds to the maximum kinematic error of the treated gear.

The value of the cutting force changes on the teeth of the cutter helical surface during machining of one gear tooth gap (single-tooth cutting), which is carried out on the entire active length of the cutter, can be determined similarly as the cutting force on the columns. The graph of this force is shown in Fig. 17. The maximum value of the cutting force for the foregoing data is 1.07 kN, and the average force value is 909 N.

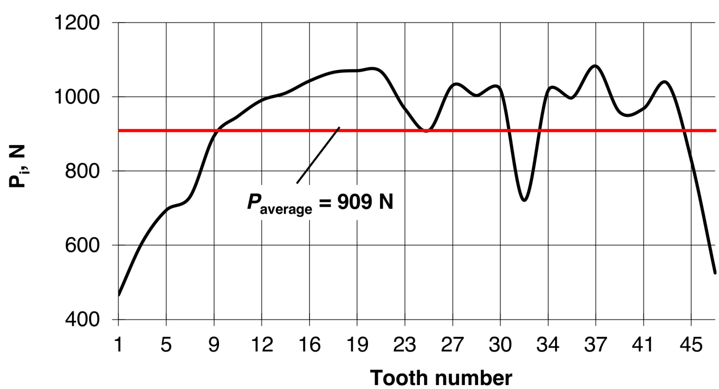


Fig. 17. Graph of changes of the cutting force on the teeth of the helical surface of the hob in single-tooth cutting mode (for one gap only)

4. Conclusions

1. A new methodology for simulation and analysis of real transitional surfaces in the gap between the gear teeth and spatial paths of hob teeth is proposed. The cross section of these surfaces, taking into account the change in their positions in the hobbing process, corresponds to the shape and dimensions of the unstrained chips, which are removed by all active teeth of the hob cutter. The method is based on discretization of hobbing process, which are reduced to the elemental kinematic motions. These elementary motions are decomposed in space and time into motions that precede the current cutting position along the helical cutter line; and to motions, which change the position for the workpiece and the hob cutter according to suitable rules in linear and angular directions. A modern CAD/CAE system (such as AutoCAD, Deform 2D/3D) is necessary for simulation of most important dynamic variable parameters characterizing the geometric shape, volume, cross-sectional area of the chips, the thickness of the cutting cross-sections, the contact length on the separate hob edges and the running cutting tooth, stress-strain state of the treated surface and machining cutters etc.

2. The total cutting force is calculated based on the simulation of chips cross-section parameters as a geometric sum of elementary forces that arise on separate teeth and columns of the hob. This allowed for simulating the power load as a variable in time cutting of different cycles and frequencies: on a single hob tooth, on a single cutting thread of hob, in one or several hob cutters rotations. The obtained 3D geometric models of cut-off layers and 3D unstrained chips made it possible to determine the force in two dimensions – on the active surface of the cutter length, that is, on its individual threads, and along the angle of column face rake rotation, that is, on individual columns. This gives an idea about load on the hob cutter in a spatial field of force and allows for calculating the magnitude of the cutting force at any point of hob active space.

3. The main parameter characterizing the intensity of plastic strain during the cutting process is the chip thickness ratio, defined by the Deform 2D, as a dynamical function of the thickness cut change. Due to this, the cutting force is represented as a function of two time-variable parameters – the cutting cross-section area and the ratio of chip thickness to the changeable cutting thickness. Thus, the hobbing process, which is a non-stationary process with parameters that change according to established regularities, can be simulated.

4. The proposed method of designing 3D geometric models of cut-off layers and unstrained chips basing on the decomposition of the complex generative gear cutting process into elementary interconnected and coordinated motions, can be used for other methods of cutting and shaping of the different types of gear wheels, such as helical, bevels, hypoid, worm, etc.

5. Corrected cutting forces, which are obtained based on this simulation method, can be used to predict the wear and rupture life of hobs; analyses of

the cutting process impact on the occurrence of oscillations in the elastic system of the gear tool machine; process control for ensuring the required accuracy of the treated gears.

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