

# MINIMIZING LENGTH IN A MIXED MODEL TWO SIDED ASSEMBLY LINE USING EXACT SEARCH METHOD

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**ABSTRACT**

In the two-sided mixed-model assembly line, there is a process of installing two single stations in each position left and right of the assembly line with the combining of the product model. The main aim of this paper is to develop a new mathematical model for the mixed model two-sided assembly line balancing (MTALB) generally occurs in plants producing large-sized high-volume products such as buses or trucks.

According to the literature review, authors focus on research gap that indicate in MTALB problem, minimize the length of the line play crucial role in industry space optimization. In this paper, the proposed mathematical model is applied to solve benchmark problems of two-sided mixed-model assembly line balancing problem to maximize the workload on each workstation which tends to increase the compactness in the beginning workstations which also helps to minimize the length of the line.

Since the problem is well known as np-hard problem benchmark problem is solved using a branch and bound algorithm on lingo 17.0 solver and based on the computational results, station line effectiveness and efficiency that is obtained by reducing the length of the line in mated stations of the assembly line is increased.

**KEYWORDS**

Two-sided assembly line balancing, mixed model, mathematical model, Lingo-17 solver.

## Introduction

An assembly line is a production process where different operations perform on raw material during the transfer process through conveyer, different machine and workers perform work on the product after that finally, unfinished material converted into finished produced. The assembly line balancing (ALB) was presented by Henry Ford in automobile plants [1].

In generalized assembly line balancing (GALB) researcher introduced many assumptions; these assumptions play a significant role in solving real-world ALB problems. In two-sided assembly line balancing (TALB) product can be manufactured or assembled in the parallel direction both at the left and right sides of the lines. A task in TALB will have direction

restriction to perform because of the use of both sides of the line. The directions of the task can be left, right, either used in TALB. Here right task and left task allocation are fixed but either task can be assigned on right or left according to space and worker comfort. The combination of the station where the task is performed left side (LS) and right side (RS) is called mated station (MS) [2, 3].

Assembly lines can be arranged dependent on the models assembled on the line are single product model based assembly line and mixed product model based assembly line. In a mixed model assembly line more than a single item, the model is gathered on the line with no arrangement time. A mixed model-based assembly line avoids constructing several lines, satisfy customer demands, and minimize workers. Mixed-model based assembly lines give greater adaptability

ty of reacting to customer requests on schedule and capture the worldwide market in the high competition phase. When the arrangement of assembling more than one product model on each neighboring line of TALB, The authors get a new line configuration called MTALB. Generally, such type of assembly line is utilized for large volume production in huge enterprises and large company [4]. Figure 1 shows the configuration of MTALB.

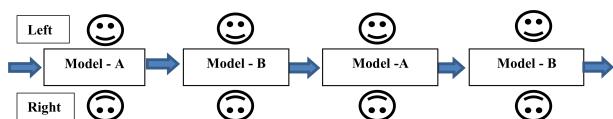


Fig. 1. Configuration of MTALB.

## Literature review

In [5] is presented a mathematical model with an ant colony based optimization algorithm for tackling a MTALB problem with an objective of limits the quantity of stations of the line. The authors considered zoning constraint that provides better results. Ozcan [6] addressed a mathematical model and established a simulated annealing algorithm to minimize the weighted smoothness index and maximize the weighted line efficiency. In this paper, the author considered first objective minimizes the number of MS and the second objective limits the quantity of stations with the consideration of cycle time.

Chutima [7] presented a negative knowledge-based particle swarm optimization to solve the multi-objective problem of MTALB problem with the objective of limits the quantity of MS when the cycle time is given. Negative knowledge-based particle swarm optimization used the knowledge of the particle relative positions to generate new solutions. Aghajani [8] addressed MTALB problem to minimize the cycle time if the quantity of MS is known as an objective to solve the problem. The authors presented a mathematical model for the robotic MTALB problem and proposed a meta heuristic method based on a simulated annealing (SA) method for the optimization of problem. Rabbani [9] presented a new U shaped layout for the MTALB problem with the first objective to decrease the cycle time and second objective to reduce the number of workstations. The authors are considered new constraints such as under zoning and synchronism constraints and developed genetic algorithms to solve it optimally.

Kucukkoc [10] presented a modified assembly line balancing for companies to fulfill customer demands on time with existing resources. The authors performed a mathematical formulation for simultane-

ous and sequencing concept and developed an agent based ant colony optimization algorithm to solve it optimally. Yuan [11] addressed TALB problem to minimize the number of MS and a total number of stations, when the cycle time is known . A honey bee mating optimization method is proposed to solve this problem. Zhang [12] In this paper, the author introduced a MTALB for Type-II problem, where authors collect the real data to solve industrial case study. This paper considered combining mismatched task groups, dissimilar from negative zoning constraints. Kucukkoc [13] addressed MTALB with the objective of minimize the cycle time and the number of workstations. The authors solved a real world problem by collecting company data and applied the ant colony optimization (ACO) algorithm for resolving such complex problem and get optimal result.

Delice [14] presented an improved PSO algorithm to solve the MTALB problem by considering minimize the number of MS as the first objective and limits the quantity of stations as the second objective when the cycle time is given. Li [15] addressed TALB problem with two objectives, the first one is to minimize the weighted line efficiency, and the second one is the weighted smoothness index. A novel multi objective hybrid imperialist competitive algorithm is developed to solve it optimally.

There is a lot of research done on TALB, but the literature review indicates that few numbers of researchers and authors emphasis on MTALB. According to the literature review, the authors considered the research gap and focus on MTALB problem to minimize the length of the assembly line. This objective plays a very crucial role in several industries that's why authors considered the tricycle case study problem. This paper generally presents the following input that can help the researcher to solve assembly line problems:

- 1) The main aim of this work is to propose a mathematical model with station oriented objective that is minimize the length on mated stations for the feasible task allotment in the MTALB problem.
- 2) The new mathematical model is verified on T(9), T(12), T(16) benchmark problem after that solve the tricycle assembly line case study problem to obtain the optimal solutions. Here mathematical model solved problems by Lingo solver to optimize problems and get the feasible solutions.
- 3) In solutions, the results of the exact solution approach and theoretical least numbers of workstations are compared. The comparison indicates that the exact solution approach provides improved solutions to reduce idle time, minimize station length, and minimize the number of workstations.

In further section, the paper is structured as follows. In Sec. 3, the authors described the overview of problem structure, and mathematical model of the MTALB problem with its assumptions, objective function, notations, decision variable, and constraints. In this paper solution approach and benchmark problem solutions considered in Sec. 4. Section 5 indicates case study problem data and its computational results and analysis. Section 6 of this paper are presented conclusions and future work that is very helpful for the researcher.

## Problem statement

### Motivation

According to the literature review industries facing assembly line balancing problems such as space optimization, idle time minimization, etc. that motivates the author to think about the improvement in this direction. The author visited a tricycle assembly plant ('XYZ' plant) of Madhya Pradesh (India). The author found that the assembly plant designed according to the single-sided mixed model assembly line; that's why the length of stations and idle time is high. Based on the recent research work in the area of MTALB, the authors are motivated to improve the existing assembly line of 'XYZ' plant by converting the same into the MTALB. Hence, the line efficiency, workers efficiency, and the total productivity of the plant increase when the plant is designed using the MTALB concept.

### Assumptions

The MTALBP mentioned assumptions that are following [2]:

- similar production characteristics models are produced,
- a straight line type layout is considered,
- deterministic task times are considered,
- demand-based on the planning horizon helps in cycle time calculation,
- Workers can implement tasks on both left and right sides of the line,
- some tasks can be implemented at a single side of the line; on the other hand, some tasks as per the requirement perform either side of the line,
- parallel assembly stations are not considered,
- precedence relationships of all models are known,
- the operator's movement time is not added in process time,
- breakdown time and non-productive time are not considered,
- inventory is not allowed in the production process,

- some tasks are always common for all models,
- task time of common tasks may or may not be the same for different models.

### Mathematical modelling

#### Decision

#### Variable

#### Symbol

|            | Description  |
|------------|--|
| $X_{mabc}$ | 1 if $i$ -th task allocated to the model $m$ for station $b$ on side $c$<br>0 otherwise  |
| $st_{ma}$  | starting time of $a$ -th task for model $m$  |
| $t_a$      | task process time of $a$ -th task  |
| $ss_{bc}$  | 1 if $a$ -th task utilized station $b$ on side $c$<br>0 otherwise  |
| $Z_{ad}$   | 1 if $a$ -th task allocated before $d$ -th task in the same mated station<br>2 if $d$ -th task allocated before $a$ -th task in the same mated station |
| $ms_b$     | 1 if $a$ -th task utilized mated station $b$<br>0 otherwise  |

#### Notation

#### Symbol

|           | Description  |
|-----------|--|
| $a$       | index of assembly task; $a = 1, 2, \dots, n$ ; where $n$ shows the total number of tasks in model $m$      |
| $b$       | index of station; $b = 1, 2, \dots, B$ ; where $B$ represent set of all stations                           |
| $m$       | index of model; $m = 1, \dots, M$ ; where $M$ shows he total number of models                              |
| $c$       | index of mated-station direction; where $C$ represent set of all directions<br>1 Left side<br>2 Right side |
| $T_a$     | $a$ -th task completion time   |
| $\mu$     | a big positive number  |
| $CT$      | cycle time   |
| $d1_a$    | 0 if $a$ -th task assigned on left side (LS)<br>1 otherwise  |
| $d2_a$    | 0 if $a$ -th task assigned on right side (RS)<br>1 otherwise   |
| $R^+$     | a positive real integer number   |
| $st_{ma}$ | starting time of model $m$ for task $a$  |
| $t_{ma}$  | processing time of model $m$ for task $a$  |
| $S(a)$    | immediate successors of $a$ -th task   |
| $P(a)$    | immediate predecessors of $a$ -th task   |

### Objective function

Objective function Eq. (1) represent, increase the compactness (workload) in the new opening workstations that help to reduce the length of the line and minimize idle time.

Here  $(B - b + 1)$  is a significant part that value is higher for opening stations and minimum value de-

signed for ending stations. This objective function signifies the square sum of each station workload for maximizing the task assignment capacity (workload) on each mated station. These objective functions directly reduce idle time, reduce the number of mated stations, and minimize length on the MTALB.

$$\text{Max } Z = \sum_{b=1}^B (t_{ma} \times x_{mabc}(B - b + 1))^2, \quad (1)$$

## Constraints

$$\sum_{b=1}^B \sum_{c=1}^2 x_{mabc} = 1, \quad (2)$$

$$\forall m \in M; \quad \forall a \in A,$$

$$\sum_{b=1}^B (d_1 \times x_{mab1} + d_2 \times x_{mab2}) = 1, \quad (3)$$

$$\forall m \in M; \quad \forall a \in A,$$

$$\sum_{c=1}^2 x_{mabc} \times (st_{ma} + t_{ma}) \leq b \times ct, \quad (4)$$

$$\forall m \in M; \quad \forall a \in A; \quad b \in B,$$

$$\sum_{c=1}^2 (x_{mabc} \times (b - 1) \times ct) \leq st_{ma}, \quad (5)$$

$$\forall m \in M; \quad \forall a \in A; \quad b \in B,$$

$$\sum_{b=1}^B \sum_{c=1}^2 b \times x_{mdbc} - \sum_{b=1}^B \sum_{c=1}^2 b \times x_{mabc} \leq 0, \quad (6)$$

$$\forall m \in M; \quad \forall a, d \in I; \quad d \in p(i),$$

$$st_{md} - st_{ma} + \mu \times \left( 1 - \sum_{c=1}^2 x_{mabc} \right) \\ + \mu \times \left( 1 - \sum_{c=1}^2 x_{mdbc} \right) \geq T_{ma}, \quad (7)$$

$$\forall m \in M; \quad \forall a, d \in A; \quad a \in P(d);$$

$$\forall b \in B; \quad c \in C,$$

$$st_{md} - st_{ma} + \mu \times (1 - x_{mabc}) + \mu \times (1 - x_{mabc}) \\ + \mu \times (1 - \mu \times (1 - z_{ad})) \geq T_{ma} \quad (8)$$

$$\forall m \in M; \quad \forall a, d \in A; \quad d \notin p(a);$$

$$a \notin p(d); \quad \forall c \in C; \quad b \in B,$$

$$st_{ma} - st_{md} + \mu \times (1 - x_{mabc}) + \mu \times (1 - x_{mdbc}) \\ + \mu \times (z_{ad}) \geq T_{md}, \quad (9)$$

$$\forall m \in M; \quad \forall a, d \in A; \quad d \notin p(a);$$

$$a \notin p(d); \quad \forall c \in C; \quad b \in B,$$

$$x_{mabc} \in \{0, 1\}, \quad (10)$$

$$\forall a \in A; \quad \forall m \in M; \quad b \in B; \quad \forall c \in C, \\ ss_{bc} \in \{0, 1\}, \quad (11)$$

$$ms_b \in \{0, 1\}, \quad (12)$$

$$z_{ad} \in \{0, 1\}, \quad (13)$$

$$\forall a, d \in A; \quad d \notin p(a); \quad a \notin p(d), \\ st_{ma} \in R^+, \quad (14)$$

$$\forall a \in A; \quad \forall m \in M.$$

Constraints-2 indicate that all the tasks are allocated to the workstation that is called assignment constraint. Constraints-3 shows that each task is allocated only once in any direction. Constraint-4 indicates that task should be allocated based on the capacity of workstation also called capacity constraint. Constraint-5 is ensures that the accomplishment time of predecessor those are immediate of that particular task must be equal to or lesser than the starting time of any task in the precedence relation. Constraint-6 considered the precedence relationship of the task, are called precedence constraints. Constraint-7 to Constraint-9 is representing the TALB sequence constraint. Constraint-7 will be active if both task  $d$  and task  $a$  are allocated in same MS on opposite side otherwise this constraint will not work. Constraint-8 and 9 will work when tasks  $d$  and  $a$  do not follow any precedence relationship and are allocated on the same mated station. Constraint-8 becomes  $st_{md} - st_{ma} \geq t_{ma}$ . If  $a$  is allocated earlier than  $d$ , if not on that case constraint-9 becomes  $st_{ma} - st_{md} \geq t_{md}$ . Constraints-10 to constraints-13 are binary variable constraints. Constraint-14 indicates that the opening time of every task must possess a positive value of integer.

## Problem data and method

### Solution approach

The flowchart of the solution approach for the MTALBP mentioned in Fig. 2. Firstly the new mathematical programming model is proposed according to the research gap and companies' requirement to solve MTALBP. In the next step verified mathematical model based on the benchmark problem data. In further action, after getting optimal results of the benchmark problem, the authors applied this concept to solve the industrial case study problem. In the last step, check whether the objective is achieved or not if achieved than process end if not on that case

again work on a mathematical model to get the ideal results.

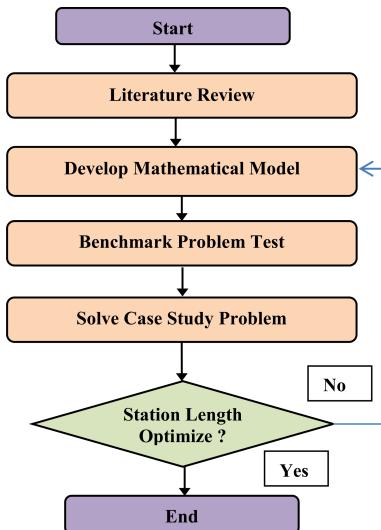


Fig. 2. Flow chart of solution approach.

### Benchmark problem data and results

This section represents benchmark problem data as well as solutions of benchmark problem those are mentioned in the literature to solve MTALB problem. Appendix shows the data of problem T(9), T(12), T(16), where 9, 12, 16 represent tasks respectively according to the left side, either side, right side. Additional, it indicates processing time of task for both the models A and B and its immediate predecessor. MTALB problem solved by exact solution approach provide exact solution those are feasible for the benchmark T(9), T(12), T(16) problems. Lingo solver is utilized to solve the mathematical models to get feasible and ideal results. Theoretical least number of stations can be calculated by the equation mentioned below:

$$\begin{aligned} \text{Theoretical least number of stations} \\ = (\text{Overall task time/Cycle time})^+. \end{aligned} \quad (15)$$

In equation fifteen (Z)+ indicates the lowest integer greater than or equals to Z. Benchmark problem

is solved by taking the maximum value of task time for model A and model B, that maximum value of task time considered as a mixed model task time. Mixed model task time can be either model A task time or model B task time for each task. Calculation of cycle time done based on the demand and available time. After considering the mixed model task time, cycle time is calculated three for problem T(9) and T(12). Problem T(16) is solved based on cycle time ten which the authors get by the calculation of mixed model task time. Based on the Eq. (15) benchmark problem T(9), T(12), T(16) theoretical least number of workstations are calculated those are mentioned below:

Problem T(9) theoretical [19/3]+ 7  
least number of stations

Problem T(12) theoretical [28/3]+ 10  
least number of stations

Problem T(16) theoretical [98/10]+ 10  
least number of stations

Figure 3 indicates that in mated station-1, task 1 is allocated for model A on the LS of mated-station; similarly, for model B, tasks 3 and task 6 are allocated to the LS in the feasible task allotment of T(9) problem. Tasks 2 are allocated on the RS of mated-station for model A; similarly, for model B tasks 2 and task 9 are distributed to the RS of the MS. A total of 3 numbers of mated stations are utilized to assigned 9 tasks. In the optimal task, assignment results light blue color indicates task processing time, dark orange color shows no task processing time that is also called idle time. The task number placed inside the box in the task assignment solution.

Figure 4 shows that there are four mated stations for the ideal task allotment in T(9) problem. Results indicate that the idle time of mated-stations for both models A, B is very less and tasks are arranged in a systematic manner without violating precedence relationship in the last mated station.

|                |         |      |   |   |      |   |      |
|----------------|---------|------|---|---|------|---|------|
| LEFT           | Model A |      | 1 |   | 4    |   | 7    |
|                | Model B | 3    | 6 |   |      |   | 8    |
| RIGHT          | Model A |      | 2 |   | 5    | 6 | 9    |
|                | Model B | 2    |   | 9 |      | 5 |      |
| Mated Stations |         | MS-1 |   |   | MS-2 |   | MS-3 |

Fig. 3. Ideal task allotment in T(9) problem.

|                       |                |             |  |   |             |   |             |    |             |
|-----------------------|----------------|-------------|--|---|-------------|---|-------------|----|-------------|
| <b>LEFT</b>           | <b>Model A</b> | 1           |  | 4 |             | 7 |             | 6  | 9           |
|                       | <b>Model B</b> | 1           |  |   | 4           | 9 | 11          | 12 | 10          |
| <b>RIGHT</b>          | <b>Model A</b> | 2           |  |   | 5           | 3 | 8           |    | 10          |
|                       | <b>Model B</b> | 2           |  |   | 5           | 8 | 7           | 10 |             |
| <b>Mated Stations</b> |                | <b>MS-1</b> |  |   | <b>MS-2</b> |   | <b>MS-3</b> |    | <b>MS-4</b> |

Fig. 4. Ideal task allotment in T(16) problem.

|                       |                |             |   |   |             |   |             |    |             |    |             |   |
|-----------------------|----------------|-------------|---|---|-------------|---|-------------|----|-------------|----|-------------|---|
| <b>LEFT</b>           | <b>Model A</b> | 1           | 3 | 6 |             |   | 7           | 8  | 13          | 16 |             |   |
|                       | <b>Model B</b> | 2           | 1 | 6 | 1           | 8 | 1           | 12 | 11          | 14 | 16          | 1 |
| <b>RIGHT</b>          | <b>Model A</b> | 2           | 1 | 3 | 5           |   | 10          |    | 11          | 14 | 15          | 1 |
|                       | <b>Model B</b> | 1           | 4 | 1 | 7           | 1 | 9           | 1  | 13          | 15 | 1           | 1 |
| <b>Mated Stations</b> |                | <b>MS-1</b> |   |   | <b>MS-2</b> |   | <b>MS-3</b> |    | <b>MS-4</b> |    | <b>MS-5</b> |   |

Fig. 5. Ideal task allotment in T(16) problem.

Figure 5 indicates that in mated station-1, task 1,3,6 for model A are allocated on the LS of the mated station; similarly, tasks 2 and task 6 for model B are allocated to the LS of the MS in the feasible task allotment of T(16) problem. A total of five numbers of mated stations is utilized to assigned 16 tasks, and task 15 is the only task that is allocated on the RS of the mated station.

The efficiency of benchmark problem T(9), T(12), T(16) for MTALB are mentioned in Table 1 that shows that the efficiency of problem T(9) is 73.21% for model A and 61.13% for model B. Results indicates that the total number of single station are fewer as compared to the theoretical least quantity of stations for all the benchmark problems.

Table 1  
Efficiency for MTALB problems.

| Efficiency [%] | Model 1 | Model 2 |
|----------------|---------|---------|
| T(9)           | 73.21   | 61.13   |
| T(12)          | 90.65   | 82.23   |
| T(16)          | 61.14   | 61.14   |

### Case study problem data and results

MTALB problem solved by the exact solution approach provides the exact solution that is feasible for the case study problem. The case study is done using Lingo 16.0 solver with Intel Core i5, 4GB of RAM, and 3.20 GHz processor. Lingo 16 solver utilized branch and bound optimization method based

on an exact solution approach to solve the case study problem and give ideal solutions.

Data collected from plant 'XYZ' building a tricycle. A total of 24 tasks are identifying for the assembling of the tricycle. Here stopwatch is used to record the task time where the task time recorded from the point when workers start to perform and from the point when workers end to perform. Idle time has not considered the brake down and maintenance time. Table 2 shows the detail of tasks with their task description, task side, model A task time, and model B task and task immediate predecessor.

This section indicates the result of case study problem where Fig. 6 shows that in mated station-1, task 2 and task 5 are allocated on the LS of mated-station for model A and task 2 and task 3 are allocated for Model B. Tasks 3 and task 4 for model A is allocated to the RS and task 4 and task 5 for model B are allocated to the RS in the feasible task allotment of case study problem. A total of 7 numbers of mated stations is utilized to assigned 24 tasks. Results indicate that for the case study problem, the total numbers of mated stations are less as compared to the theoretical least number of stations.

Table 3 indicates the efficiency of the case study problem. Now the new efficiency of assembly lines is approx. 86.50% for model A and 78.24% for model B. According to the managerial aspect, this will help in utilizing the workforce efficiently and to eliminate the extra use of resources.

Table 2  
Data of case study problem.

| Task no | Task description                      | Side | Model A process time | Model B process time | Immediate predecessors |
|---------|---------------------------------------|------|----------------------|----------------------|------------------------|
| 1       | Labels assembly                       | L    | 3                    | 0                    | —                      |
| 2       | Middle bearing assembly               | L    | 7                    | 9                    | —                      |
| 3       | Main gear and pedals arms assembly    | R    | 7                    | 9                    | —                      |
| 4       | Pedals assembly                       | R    | 5                    | 7                    | —                      |
| 5       | Front part assembly                   | L    | 4                    | 6                    | 2                      |
| 6       | Handle bar assembly                   | E    | 3                    | 4                    | 2,3                    |
| 7       | Front tyre mudguard assembly          | R    | 0                    | 4                    | 3                      |
| 8       | Front tyre wheel assembly             | E    | 3                    | 0                    | 5                      |
| 9       | Front tyre brake assembly             | E    | 6                    | 9                    | 6                      |
| 10      | Lock assembly                         | E    | 4                    | 0                    | 7                      |
| 11      | Horn fitment                          | L    | 0                    | 4                    | 1                      |
| 12      | Head lamp fitment                     | L    | 3                    | 8                    | 8,9                    |
| 13      | Rear part assembly                    | E    | 3                    | 8                    | 9                      |
| 14      | Gear assembly                         | R    | 9                    | 5                    | 9,10                   |
| 15      | Rear tyre wheel assembly              | R    | 5                    | 0                    | 4                      |
| 16      | Rear tyre mudguards assembly          | L    | 9                    | 7                    | 11                     |
| 17      | Saddle assembly                       | E    | 2                    | 5                    | 12                     |
| 18      | Suspension assembly                   | E    | 7                    | 4                    | 13                     |
| 19      | Chain assembly                        | E    | 9                    | 8                    | 13,14                  |
| 20      | Cables fitting                        | R    | 9                    | 4                    | 15                     |
| 21      | Rear tyre axle guard assembly         | L    | 8                    | 3                    | 16,17                  |
| 22      | Chain guard fitting assembly          | E    | 0                    | 8                    | 18                     |
| 23      | Plastic parts and fitting accessories | R    | 9                    | 7                    | 19,20                  |
| 24      | Alignment check process & Inspection  | E    | 9                    | 7                    | 20                     |

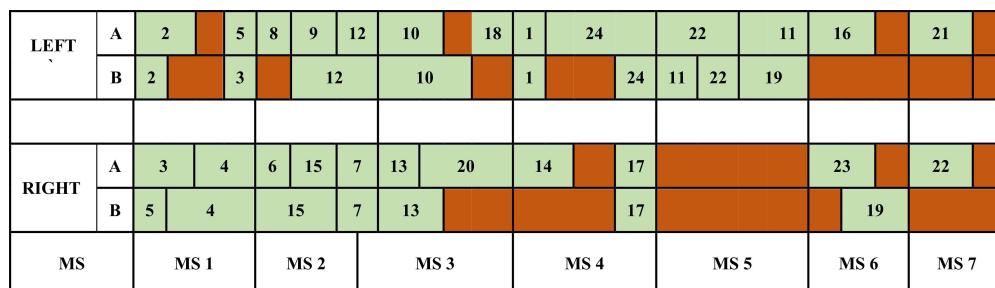


Fig. 6. Ideal task allotment for case study problem.

Table 3  
Efficiency for case study problem.

| Efficiency [%]     | Model A | Model B |
|--------------------|---------|---------|
| Case study problem | 86.50   | 78.24   |

## Conclusion and future research

A real-life case study with the MTALB concept is presented in this paper, which considers two different assembly line models with a straight layout. The research gaps and the proposed mathematical model mentioned in literature leads to carry out a study

of the tricycle ALB problem. The authors addressed a new mathematical model for solving the MTALB by considering the minimization the length that reduce the number of mated stations and reduce idle. Numerical example problem such as T(9), T(12), and T(16) is solved utilizing the proposed way to validate the efficiency of the proposed model. The proposed mathematical model can explain the numerical problem, in a reasonable time; therefore, the authors applied this methodology to resolve the case study problem.

The result of the case study shows that there is a significant improvement in the efficiency of the

plant. The new efficiency of assembly lines is approx. 86.50% for model A and 78.24% for model B. The experimental result shows that reduced workstations also reduces the length of the assembly line and increase the space in the plant. This will help the industry to utilize the workforce efficiently and to eliminate the extra use of resources.

In future work, a multi-objective model can be developed for MTALB, and the proposed mathematical model can also be applied with the stochastic approach. New meta-heuristic methods such as whale optimization, grey wolf optimization can be used for this mathematical model to get the feasible solution for the MTALB problem. According to the authors, MTALB problem can also be solved for different assembly's line layout such as parallel line and U line in future work. According to the industrial aspect, more realistic constraints, for example, position constraint, and distance limitations, can be very helpful to extend this work.

## Appendix

Table A  
Data of T(9) problem.

| Task no | Side | Model A time | Model B time | Immediate predecessors |
|---------|------|--------------|--------------|------------------------|
| 1       | L    | 2            | 0            | —                      |
| 2       | R    | 3            | 1            | —                      |
| 3       | E    | 0            | 1            | —                      |
| 4       | L    | 3            | 0            | 1                      |
| 5       | R    | 1            | 3            | 2                      |
| 6       | E    | 1            | 1            | 2,3                    |
| 7       | E    | 2            | 2            | 4,5                    |
| 8       | L    | 0            | 3            | 5                      |
| 9       | E    | 1            | 1            | 6                      |

Table B  
Data of T(12) problem.

| Task no | Side | Model A time | Model B time | Immediate predecessors |
|---------|------|--------------|--------------|------------------------|
| 1       | L    | 2            | 3            | —                      |
| 2       | R    | 3            | 3            | —                      |
| 3       | E    | 2            | 0            | —                      |
| 4       | L    | 3            | 2            | 1                      |
| 5       | E    | 1            | 2            | 2                      |
| 6       | L    | 1            | 0            | 3                      |
| 7       | E    | 3            | 2            | 4,5                    |
| 8       | E    | 3            | 1            | 5                      |
| 9       | E    | 2            | 1            | 5,6                    |
| 10      | E    | 2            | 3            | 7,8                    |
| 11      | E    | 0            | 2            | 9                      |
| 12      | R    | 0            | 1            | 11                     |

Table C  
Data of T(16) problem.

| Task no | Side | Model A time | Model B time | Immediate predecessors |
|---------|------|--------------|--------------|------------------------|
| 1       | E    | 6            | 0            | —                      |
| 2       | E    | 5            | 2            | —                      |
| 3       | L    | 2            | 0            | 1                      |
| 4       | E    | 0            | 9            | 1,2                    |
| 5       | R    | 8            | 0            | 2                      |
| 6       | L    | 4            | 8            | 3                      |
| 7       | E    | 7            | 7            | 4,5                    |
| 8       | E    | 4            | 3            | 6,7                    |
| 9       | R    | 0            | 5            | 7                      |
| 10      | R    | 4            | 1            | 7                      |
| 11      | E    | 6            | 3            | 8                      |
| 12      | L    | 0            | 5            | 9                      |
| 13      | E    | 6            | 9            | 9,10                   |
| 14      | E    | 4            | 5            | 11                     |
| 15      | E    | 3            | 8            | 11,12                  |
| 16      | E    | 4            | 7            | 13                     |

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