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## GENERATION OF A SERVICE LOADING WITH THE DESIRED PROBABILITY DISTRIBUTION AND AUTOCORRELATION FOR FATIGUE TESTS USING MATLAB

The aim of the paper is to present a procedure for generating service loading for fatigue tests of materials and structures. The generated loading characterizes desired functions of probability distribution and autocorrelation. The proposed numerical procedure uses MATLAB toolboxes and consists of three steps: (a) generation of a sequence of real numbers with the desired autocorrelation function and with any probability distribution function; (b) generation of loading history with the desired probability distribution function; (c) rearrangement of loading history (mentioned in item b) based on a sequence of real numbers with the desired correlation (mentioned in item a).

### 1. Introduction

Fatigue tests of structural materials and machine elements should be performed under the same loading as service conditions. In order to determine parameters of real loading, it is necessary to register fragments of their histories during operation of the machine. Next, the registered histories are used for long-lasting generation of loading at the test stand. Another procedure used in laboratories is based on normalized loading histories, which are dependent on the tested structure. As an example, CARLOS (Car Loading Standard) [1], applied for tests of car suspension systems can be mentioned. When the loading histories are registered, it is possible to determine their probabilistic characteristics, and write them in a short form. For registration and reconstitution of service loading, extreme values of the Markov matrix are frequently used [1]. In this matrix, the elements represent probability of passage from one discrete loading level to another one. Fig. 1 shows a scheme

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of the Markov matrix with  $n$  discretization levels of extreme (maximum and minimum) loading values.

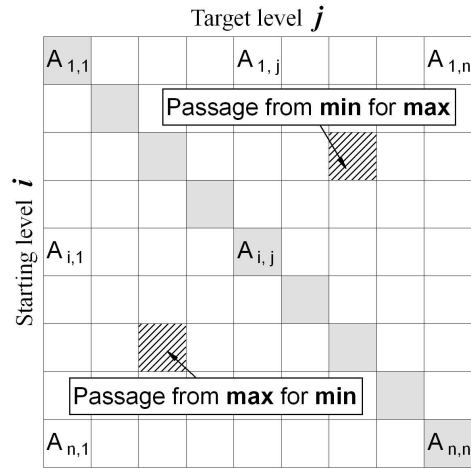


Fig. 1. The Markov matrix

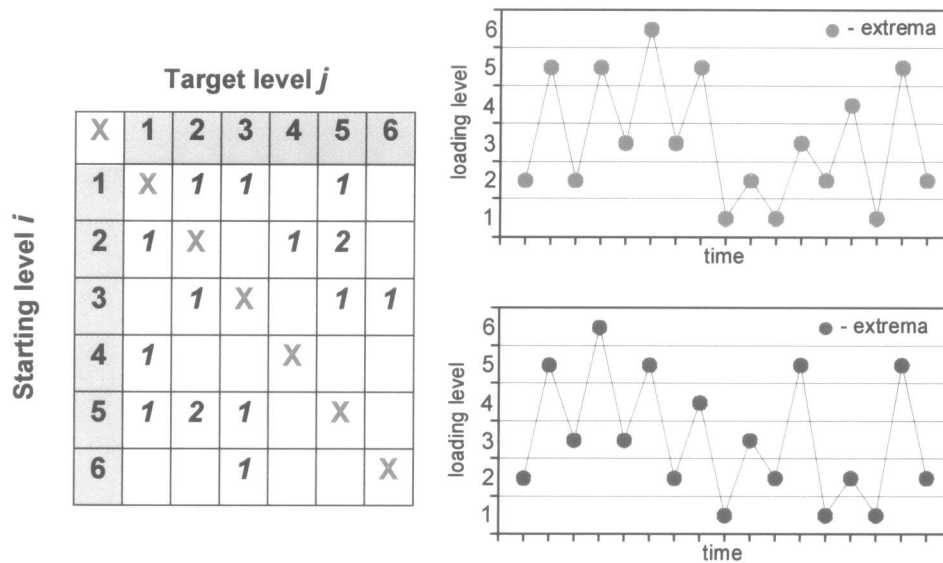


Fig. 2. The Markov matrix and two corresponding loading histories

The Markov matrix can be applied for reproduction of extreme values of the registered history at each laboratory. However, this matrix does not allow preserving a sequence of the extrema occurrence in the reproduced history in relation to the registered history, because only frequency of the

extrema occurrence has been written, not its position at time. It means that one matrix describes several histories of loading. As it can be observed in Fig. 2, from one Markov matrix two histories of loading with different sequences of extreme values are obtained.

## 2. The correlation transfer procedure

During fatigue tests of materials and structures in laboratories, it is necessary to keep the sequence of extrema of the service loading. In order to perform this condition, both measured in structure and reconstituted in laboratories histories should be described by the same functions of autocorrelation and probability distribution. Polge et. al in [2] present the correlation transfer method used for generation of signal based on given functions of probability distribution and autocorrelation. This method can be used for generation of loading history with required probability distribution and suitable sequence of occurrence of extrema measured during the machine service. The correlation transfer procedure is shown in Fig. 3.

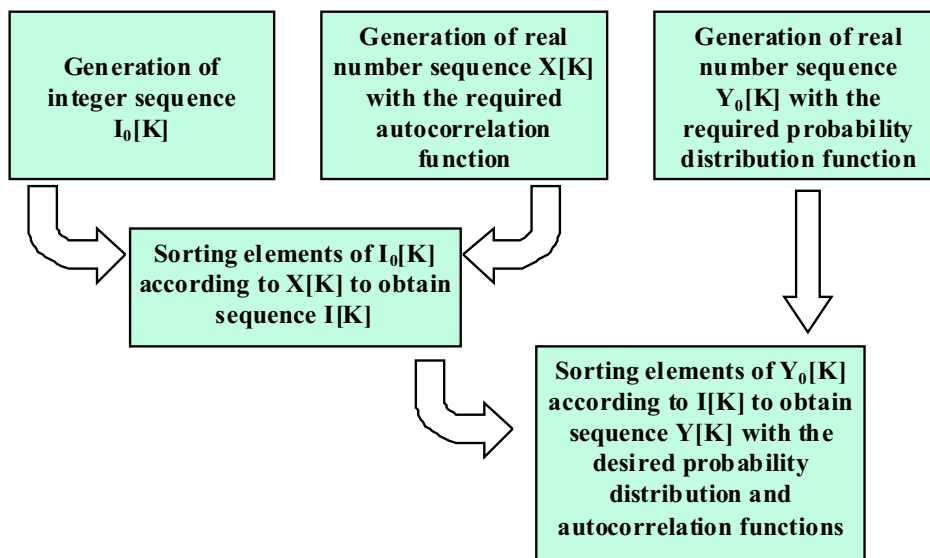


Fig. 3. A scheme of service loading generation with the correlation transfer method

In order to obtain the sequence  $Y[K]$ , the elements in the primary data set  $Y_0[K]$  are rearranged on the basis of an auxiliary sequence of real numbers  $I[K]$  with any probability distribution and the required autocorrelation function. An example of the correlation transfer procedure for  $K=10$  is presented below.

<b>1.</b>	<b>Generation of a sequence of real numbers <math>X[K]</math> with the required autocorrelation (with any probability distribution)</b> e.g. $X[K] = [3.8, 5.3, 2.1, 5.6, 6.2, 3.3, 9.3, 5.8, 2.0, 8.2]$
<b>2.</b>	<b>Generation of a sequence of integers <math>I_0[K]</math></b> e.g. $I_0[K] = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$
<b>3.</b>	<b>Rearrangement of elements of <math>I_0[K]</math> according to increasing values of <math>X[K]</math> to obtain sequence <math>I[K]</math></b> e.g. $X[K] = [3.8, 5.3, 2.1, 5.6, 6.2, 3.3, 9.3, 5.8, 2.0, 8.2]$ $I[K] = [4, 5, 2, 6, 8, 3, 10, 7, 1, 9]$ The element $X[9] = 2.0$ is the smallest in the sequence $X[K]$ so, it must be $I[9] = 1$ . The element $X[7] = 9.3$ is the biggest in the sequence $X[K]$ so, it must be $I[7] = 10$ .
<b>4.</b>	<b>Generation of a sequence of real numbers <math>Y_0[K]</math> with the desired probability distribution (with any autocorrelation)</b> e.g. $Y_0[K] = [5.6, 2.4, 8.6, 9.1, 3.7, 3.4, 6.8, 7.1, 9.1, 0.7]$
<b>5.</b>	<b>Rearrangement of elements of <math>Y_0[K]</math> according to order of <math>I[K]</math> to obtain sequence <math>Y[K]</math></b> e.g. $I[K] = [4, 5, 2, 6, 8, 3, 10, 7, 1, 9]$ $Y[K] = [3.7, 5.6, 2.4, 6.8, 8.6, 3.4, 11.1, 7.1, 0.7, 9.1]$

As a result, the rearranged sequence  $Y[K]$  is received. It represents the data set with desired functions of probability distribution and autocorrelation. The elements of the sequence  $Y[K]$  can represent instantaneous or only extreme values of loading, displacement, or strain histories used in fatigue tests.

### 3. Generation of data sets using MATLAB

The correlation transfer procedure requires the use of numerical methods of data set generation. It is necessary to create two sequences of numbers (with the desired probability function and with the required autocorrelation function). For this purpose, the MATLAB software can be used.

### 3.1. Generation of data sets with the desired probability distribution function

The service loadings for fatigue tests are characterised by particular probability distribution functions. The Gaussian, Rayleigh, Chi-square, gamma, and beta distributions are the most widely used. Generation of sequences with selected probability distribution in MATLAB requires declaration of their parameters and length of sequences. To generate the required sequences of numbers, the *random* command is used. The syntax of the *random* command is:

$$R = \text{random}('Type', Parameter1, \dots, ParameterN, \#rows, \#cols);$$

For the Gaussian distribution, expression 'norm' should be written instead of 'Type', *Parameter 1* is the mean value, and *Parameter 2* is the standard deviation. To generate, for example, time series of length  $K = 1000$ , containing the Gaussian distributed numbers with mean value 5 and variance 2, the following command should be used:

$$R = \text{random}('norm', 5, \text{sqrt}(2), 1, 1000);$$

In the case of the Rayleigh distribution (see Fig. 4), the following commands are used to generate a sequence of 1000 elements:

$$K = 1000;$$

$$B = 0.2;$$

$$Y = \text{random}('rayl', B, 1K);$$

where  $K$  – number of elements (length of the sequence),  $B$  – parameter of the Rayleigh distribution,  $Y$  – sequence with the Rayleigh distribution (in Fig. 3  $Y \equiv Y_0[K]$ )

### 3.2. Generation of data sets with the required autocorrelation function

In this case, it is necessary to generate a sequence of numbers  $X[K]$  with broad-band frequency (white noise). For this purpose, the sequence of uniform probability distribution can be used (Fig. 5), according to the following command:

$$X = \text{rand}(1, K);$$

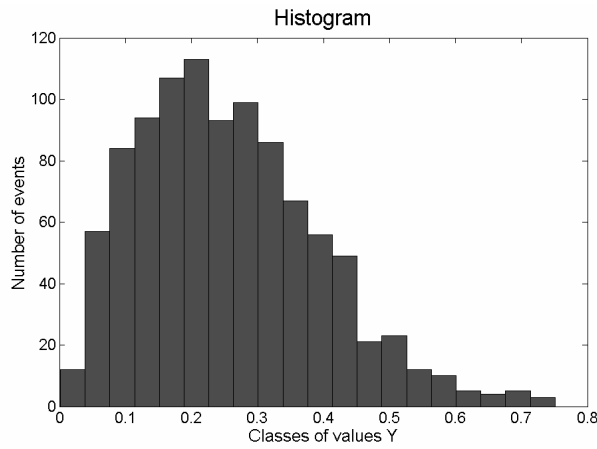


Fig. 4. Histogram of a generated sequence with the Rayleigh probability distribution

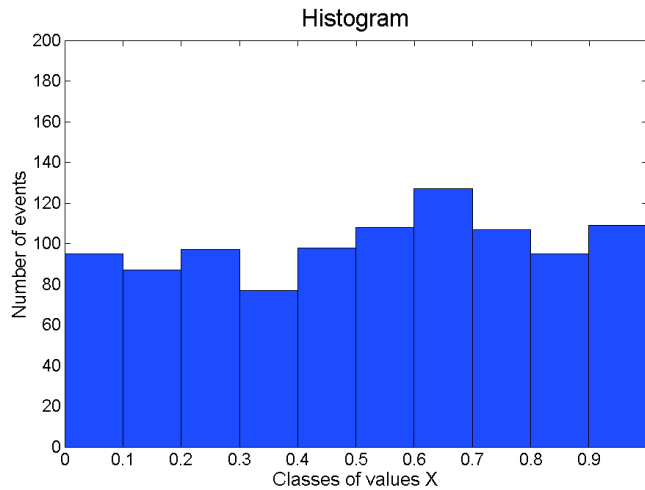


Fig. 5. Histogram of a generated sequence with uniform probability distribution

As it is known, the autocorrelation function  $R(\tau)$  is related to the power spectral density function  $S(\omega)$  by the Fourier transform

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)x(t + \tau) d\tau \stackrel{F}{\leftrightarrow} S(\omega) = \int_0^T R(\tau)e^{-j\omega\tau} d\tau. \quad (1)$$

where:  $\omega = 2\pi f$ ,  $f$  – frequency,  $T$  – time of observation. It means that through proper spectral density function in frequency domain it is possible to define the autocorrelation function in time domain. The frequency characteristics

of the signal can be formed with use of the filters according to the following relation

$$G_y(f) = |H(f)|^2 G_x(f), \quad (2)$$

where:  $G_x(f)$  – power spectral density of the input signal,  $G_y(f)$  – power spectral density of the output signal,  $|H(f)|$  – module of the transfer function of the filter.

Assuming that  $G_x(f)$  is known and  $G_y(f)$  is required, the transfer function for the filter is

$$H(f) = \sqrt{\frac{G_y(f)}{G_x(f)}}. \quad (3)$$

For a random sequence of number with constant power spectral density function  $G_x(f) = C$  (i.e. white noise, Fig. 5), the power spectral density of the output signal  $G_y(f)$  depends only on characteristic of the filter .

$$G_y(f) = C |H(f)|^2. \quad (4)$$

The Butterworth filter from MATLAB was chosen for modelling the shape of power spectral density. The following command should be used

$$[A, B] = butter(n, \omega_n)$$

to create the low-pass filter with  $n$  order and parameter  $\omega_n = \frac{\omega_{\min/\max}}{\omega_p}$ , where:

$\omega_{\min/\max}$  – angular frequency,  $\omega_p$  – sampling,  $A, B$  – vectors containing the coefficient of the numerator and denominator of the transfer function of the filter. For  $\omega_n$  defined as vector with two elements  $\omega_n = [\omega_1, \omega_2]$ , the band-pass filter is created with angular frequencies  $\omega_1 < \omega < \omega_2$ .

The following commands

$$Tp = 0.1;$$

$$wp = 2 * pi / Tp;$$

$$K = 1000;$$

$$WN = rand(1, n) * w = 50;$$

$$n = 3; [A, B] = butter(n, w/wp);$$

create the filter with sampling time  $T_p$ , sampling  $\omega_p = 2\pi/T_p$ , angular frequency  $\omega = 50$  and order  $n = 3$  and generates the random sequence  $WN$ . The Bode diagram of the described filter is presented in Fig. 6.

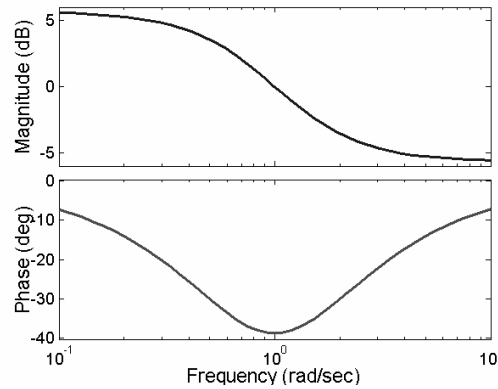


Fig. 6. The Bode diagram

To generate a sequence  $X[K]$  with the required autocorrelation function the sequence  $WN$  must be filtered with use of the command

$$X = filter(A, B, WN);$$

A sequence generated in this procedure is characterized by the required autocorrelation function. The next step is generation of an additional sequence of integers  $I_0[K]$ . To finalize the generation of the data set, the correlation transfer procedure must be used (see. Fig. 3). At first, the sequence  $I_0[K]$  is rearranged (on the basis of the sequence  $X[K]$ ) to get the pattern of the autocorrelation function of  $I[K]$  sequence. Then, the sequence  $Y_0[K]$  should be rearranged (on the basis of  $I[K]$  sequence) to obtain the sequence  $Y[K]$  with the desired functions of probability distribution and autocorrelation.

#### 4. Conclusions

1. The correlation transfer method is useful and efficient for reconstruction of service loading in laboratory fatigue test of materials and structures when sequence of the extreme values must be exactly preserved.
2. In order to reproduce the extreme values of loading with the desired sequence, the elements from the given Markov matrix should be sorted according to the sequence described by the autocorrelation function.



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#### REFERENCES

- [1] Shutz D., Klatschke H., Steinhilber H., Heuler P., Schutz W.: Standardized load sequences for car wheel suspension components, CARLOS, Final Report, LBF-Bericht No. FB-191, Darmstadt, 1990.
- [2] Polge R. J., Holliday E. M., Bhagawan B. K.: Generation of a pseudo-random set with desired correlation and probability distribution, Simulation, May 1973, pp. 153÷158.
- [3] MATLAB/HELP, The Language of Technical Computing, Version 6.0.0.88 Release 12, Copyright 1984-2000 The MathWorks, Inc.
- [4] <http://www.mathworks.com/products/statistics>, The MathWorks-Statistics Toolbox,

#### **Generacja obciążeń eksploatacyjnych o zadanych funkcjach gęstości prawdopodobieństwa i autokorelacji do badań zmęczeniowych z wykorzystaniem MATLABa**

##### Streszczenie

W pracy przedstawiono procedurę generowania obciążeń eksploatacyjnych wykorzystywanych do badań zmęczeniowych materiałów i konstrukcji. Generowane przebiegi czasowe obciążenia charakteryzują się zadanymi funkcjami gęstości prawdopodobieństwa i autokorelacji. Zaproponowana procedura numeryczna wykorzystuje moduły oprogramowania MATLAB (tzw. ToolBox) i składa się z trzech kroków:

- a) generacji ciągu liczb rzeczywistych o zadanej funkcji autokorelacji i dowolnym rozkładzie prawdopodobieństwa,
- b) generacji przebiegu obciążenia o określonej funkcji gęstości prawdopodobieństwa,
- c) zmiany położenia elementów przebiegu czasowego obciążenia z pkt b w oparciu o ciąg liczb rzeczywistych o zadanej funkcji autokorelacji z pkt a.