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## THE SURFACE OF A TOOTH FLANK OF A REAL FELLOWS CHISELLED GEAR AND CAD MODEL

The paper compares the geometrical surface structure of modelled tooth flanks of cylindrical gear obtained by a three dimensional simulation of gear generation with the geometrical surface structure of real gear obtained through chiselling by Fellows method. The paper presents the methodology of modelling tooth flanks of cylindrical gears in the CAD environment. The modelling consists in computer simulation of gear generation. The computer simulation of the gear generation was performed in the Mechanical Desktop environment. Metrological measurements of the real gear were carried out using a coordinated measuring machine and a profilometer.

### 1. Introduction

The surface stereometry and topography of gear tooth flanks and research on these properties have always been a major technological problem and are decisive for the operational value of gears. The research includes the semi-finished product phase, the stereometry perfection [7], [10], [11], [14] and its final formation taking into account the effectiveness and cost of production. Matrix, vector and differential equations are used for a tooth flank description [1], [2], [3]. They also make it possible to describe complicated geometric shapes of gear flanks. While making use of the envelope curve condition and the operational continuity of the surface contact of the tool and the generated tooth, a system of equations in their open parametric form is aimed.

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A model of the stereometric shape of the gear tooth flank and its surface topography can be defined also in the CAD environment. The method is based on the procedure of logical subtraction the solids representing the tool and the generated object. In the CAD environment, it is possible not only to obtain a complete model of gear teeth but also simulate the generation process [4], [8], [9], [12]. The cutting edge of a tool that is computer modelled can have an arbitrarily defined profile. Taking the simulation parameters corresponding to the real generation parameters, it is possible to obtain a solid gear wheel model without the errors resulting from the real generation process [5], [6], [13], [14]. Supposing that the machining simulation runs under ideal conditions, it is possible to determine the effect of tool setting and the machine-tool-chuck-object-tool system on gear tooth making precision. Model surface of tooth flanks allows analyzing monomial flank deviations, as well as radial composite and runout ones.

## 2. Characteristic of the real gear and CAD model subjected to tests

The parameters of the gear were introduced in Table 1. The semi-finished toothed wheels were made by die forging. The material was alloy, low-carbon steel for carbonizing of the AMS 6265 sort. The blank was of 30-35 HRC. The envelopes of the wheels were thermally toughened up to 35÷41 HRC. Gear generation chiselling was carried out in three passes by Fellows method, using an LS186CNC slotter (Lorenz Ettlingen, Buchholz-Mendt, Germany). The pot-type cutter, also a Lorenz product, with 56 teeth, was made of ASP 2030 steel and covered with a layer of titanium nitride (TiN). The in-feed method, with revolution feed, of 3.4, 0.5 and 0.1 mm incision depth was applied. The in-feed was 0.003 mm/double cutter pitch and revolution feed for three passes was respectively 0.285, 0.228 and 0.104 mm/double cutter pitch. Consequently, the number of the double cutter pitches on the scale between adjacent teeth was 20, 25 and 55. The speed of the to-and-fro motion of the cutter was 20, 25 and 30 m/min, respectively. The cutting fluid was Ferrocol EB oil (absolute viscosity at 40°C, 7 mm<sup>2</sup>/s). The radial runout of the control cylindrical surface of the cutter and the cylindrical surface of the generated gear rim did not exceed 0.01 mm.

Modelling tooth flanks in the CAD environment makes use of commands of turning, copying, and taking away the drawn solids of the wheel and the tools. The reciprocal turning of the toothed wheel and the tool is not smooth,

but rather stroke-like in character. The simulation procedure in the CAD environment is follows:

**rotate** {revolution of generated gear},

**rotate** {revolution of tool},

**copy** {tool copy and its placement on the turning diameter of the wheel},

**subtract** {subtraction of the dipped tool solid volume from the gear wheel model},

**rscrip**t {restarting the procedure}.

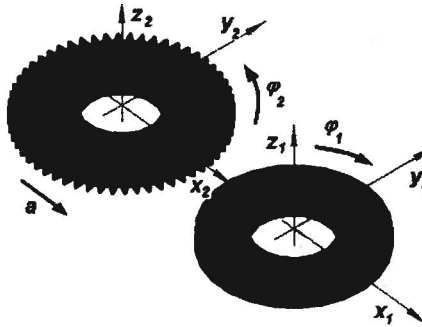


Fig 1. Simulation of straight tooth cylindrical gear generation with Fellows chiselling

The kinematics of the chiselling simulation was presented in Fig. 1. The tool is a solid model of a pinion cutter. The turn of the tool by angle  $\phi_1$  is related to the wheel turn  $\phi_2$ , in accordance with the transmission ratio of the technological gear. Example of the flanks of the teeth obtained through chiselling simulation is presented in Fig. 2.

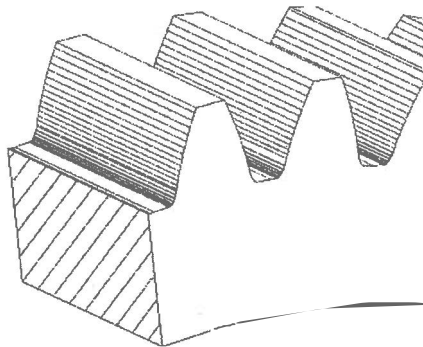


Fig. 2. Tooth flanks of a toothed wheel obtained with Fellows chiselling simulation

Table 1.

Gear data

| Specification   | Toothed wheel rim                                  |
|---|--|
| 1. Normal module (mm)   | 1,814  |
| 2. Normal pressure angle (°)  | 20   |
| 3. Width toothed wheel rim (mm)   | 6,223-6,477  |
| 4. Pitch diameter circle (mm)   | 88,9000  |
| 5. Root diameter circle (mm)  | 84,320-84,701                                      |
| 6. Number of teeth (-)  | 49   |
| 7. Form diameter circle (mm)  | 86,528   |
| 8. Base diameter circle (mm)  | 41,7690  |
| 9. Tooth fillet R (mm)  | min 0,711  |
| 10. Form diameter roll angle (°, ', '')   | 15°27'54''   |
| 11. Deviation involute. Tangential to involute profile.<br>On tooth point,<br>On form diameter (mm) | 0,0075 <sup>-0,02</sup><br>0,0150 <sup>-0,01</sup> |
| 12. Total distortion error (mm)   | 0,005  |
| 13. Transverse pitch error fpt (mm)   | 0,02   |
| 14. Total cumulative pitch error Fp (mm)  | 0,088  |
| 15. Arc tooth thickness (mm)  | 2,845-2,939  |
| 16. Reference data measurement over (mm),<br>diameter wires 3.135±0.001 (mm)                        | 93,246-93,480                                      |
| 17. Both sides class except lead (-)  | 6  |
| 18. Break edges of end profile (mm)   | 0,076-0,381  |
| 19. Break tooth tip edges (mm)  | max 0,127  |

To obtain the model of the surfaces of the analysed gear teeth (Table 1), the following method of calculating the parameters of a computer simulation of gear generation was adopted:

The tool, a pinion modular cutter, makes 55 double strokes on the circular pitch. The angle of the pitch of the toothed wheel is 7.347 (°). Because of this, one stroke of the tool corresponds to 0.134 (°) of the wheel turn. Assuming that the tool has 56 teeth, its angular displacement per one stroke is 0.117 (°).

Applying the calculated angles as arguments in the presented commands of the generated programme simulation, one obtained three – dimensional models of the tooth flank surfaces of cylindrical gear wheels.

### 3. The course and result of the research

The stereometric shape of the tooth flanks of the gear wheels (Fig. 3) was measured with a CNC coordinate measuring machine, PNC model,

Klingelnberg Sohne (Remscheid, Germany). The stylus ended with a spherical surface of a 1mm radius.

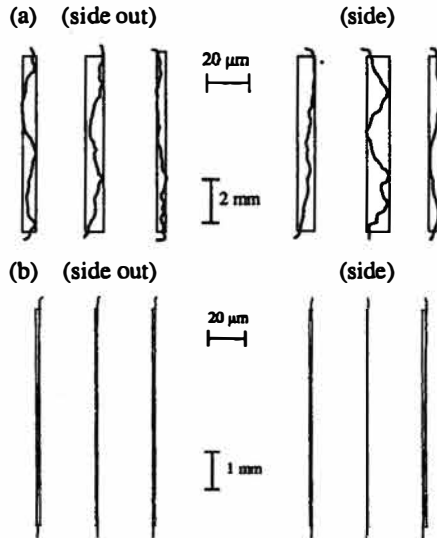


Fig. 3. Tooth profile deviation along the tooth height (a) and helix deviation (b) from side out and side for one tooth space of a Fellows chiselled gear wheel

The measured coordinates and the software offered by the producer of the measuring machine made it possible to compute single pitch deviation  $f_{tp}$ , total cumulative pitch deviation  $F_p$ , total profile deviation  $F_a$ , total helix deviation  $F_\beta$ , tooth thickness variation  $R_s$  and runout  $F_r$  (Table 2). The Fellows method of gear tooth generation ensures that precision of the tooth flanks getting out of machining is better than those getting into machining (Table 2). It was found after having examined all the tangential composite deviations  $F_a, F_\beta, f_{pt}$  and  $F_p$ .

Table 2.

Selected surface stereometry precision parameters of gear tooth flanks (standard deviations in brackets)

| Profile parameters       | Technological surface |                   |                   |                   | Modelling surface |                   |
|--------------------------|-----------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                          | Helix                 |                   | Profile           |                   | Profile           |                   |
|                          | Primitive profile     | Roughness profile | Primitive profile | Roughness profile | Primitive profile | Roughness profile |
| Pa, Ra ( $\mu\text{m}$ ) | 0.34                  | 0.14              | 1.12              | 0.46              | 0.46              | 0.04              |
| Pq, Rq ( $\mu\text{m}$ ) | 0.39                  | 0.20              | 1.34              | 0.56              | 0.80              | 0.05              |
| Pt, Rt ( $\mu\text{m}$ ) | 1.70                  | 1.48              | 5.29              | 2.83              | 3.51              | 0.32              |
| Psk, Rsk (-)             | -0.12                 | -0.86             | -0.48             | -0.02             | -0.31             | -0.40             |
| Pku, Rku (-)             | 1.94                  | 6.34              | 2.29              | 2.84              | 2.33              | 4.69              |

Then, a measurement of the topography of the tooth surface of the gear wheels in the middle of their width was carried out. A Talyscan 150 Taylor Precision (Leicester, GB) three dimensional profilometer was used. For the tooth space flanks getting into and out of machining the square measuring area was 3 mm. 360000 results were obtained (600 lines, 600 measuring points each). The sampling step and the spacing of the measuring tracks were 5  $\mu\text{m}$ . The analysed length of the profile and the helix of the chiselled one was 2550  $\mu\text{m}$  (85 points with a 30  $\mu\text{m}$  step). Example results are shown in Figure 4.

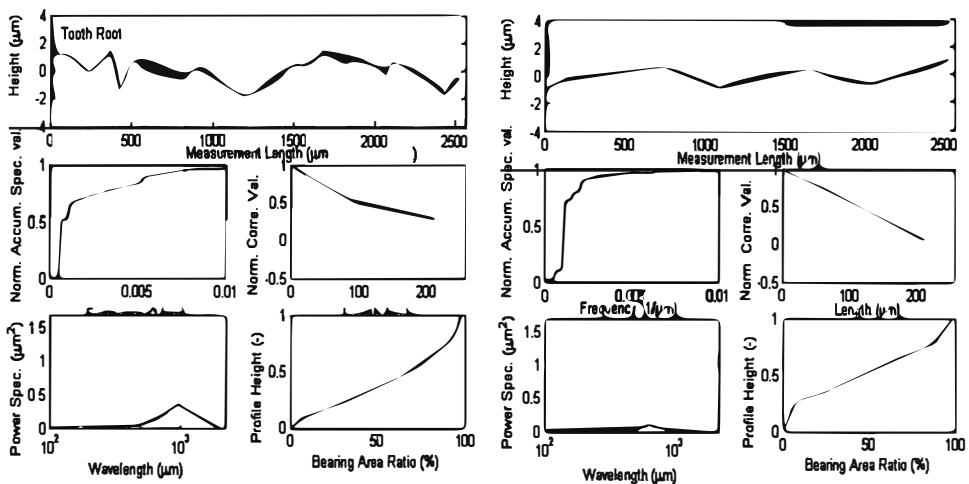


Fig. 4. Roughness profile of a Fellows chiselling real gear tooth flank for tooth profile (a) and helix (b). Characteristics of surface roughness profile: cumulated power spectral density, autocorrelation function, Spectrum of power spectral density, autocorrelation function, spectrum of power spectral density, and material ratio curve: Gauss's filters  $\lambda c=2.5\text{mm}$  were used

Model flank profiles of chiselled teeth were analysed at a normal cross – section and separated with an interpolating circle (associated profile) of a 15.0844 mm radius. The circle was determined by two points of the tooth flanks situated on the wheel rollers with the radii of 43.1868 mm and 45.6875 mm. The flanks of the model obtained through chiselling simulation were analysed along a 2691.2  $\mu\text{m}$  teeth profile height (80 points with a 33.64  $\mu\text{m}$  step). Example results are shown in Fig 5.

In the model of the tooth flank, the pitch helix is a straight line.

The characteristic of the topography of the tooth flanks on the basis of the profile and helix is given in Table 3. These values refer to both a technologically worked surface coming out of the machining and a model surface. The flanks of the teeth chiselled by Fellows method have a smaller

Table 3.

Expected parameter values of profile and line of technological surface (separated from measurement with a Talyscan 150 profilometer) and modelling flank surface of teeth getting out of Fellows chiselling

| Profile parameters       | Technological surface |                   |                   |                   | Modelling surface |                   |
|--------------------------|-----------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                          | Helix                 |                   | Profile           |                   | Profile           |                   |
|                          | Primitive profile     | Roughness profile | Primitive profile | Roughness profile | Primitive profile | Roughness profile |
| Pa, Ra ( $\mu\text{m}$ ) | 0.34                  | 0.14              | 1.12              | 0.46              | 0.46              | 0.04              |
| Pq, Rq ( $\mu\text{m}$ ) | 0.39                  | 0.20              | 1.34              | 0.56              | 0.80              | 0.05              |
| Pt, Rt ( $\mu\text{m}$ ) | 1.70                  | 1.48              | 5.29              | 2.83              | 3.51              | 0.32              |
| Psk, Rsk (-)             | -0.12                 | -0.86             | -0.48             | -0.02             | -0.31             | -0.40             |
| Pku, Rku (-)             | 1.94                  | 6.34              | 2.29              | 2.84              | 2.33              | 4.69              |

irregularity height along the helix than in the profile. It is confirmed by the power spectral density characterized by a long range of wave length. The strongest are the waves of primitive profiles of  $650 \mu\text{m}$  and those of roughness profiles of about  $350 \mu\text{m}$ . The values of these characteristic wave lengths of profiles for model tooth flank surfaces are about  $1200 \mu\text{m}$  and  $150 \mu\text{m}$ , respectively. The application of a Gaussian filter made it possible to get rid of the primitive surface and profile error.

Slight differences were found while comparing the height of the helix profile of real wheels with that of modelling wheels. The pitch helix of a chiselled real wheel shows irregularities of total height  $Pt=1.70 \mu\text{m}$ ,  $Rt=1.48 \mu\text{m}$ . In the model of the tooth flank, the pitch helix is a straight line. Slightly

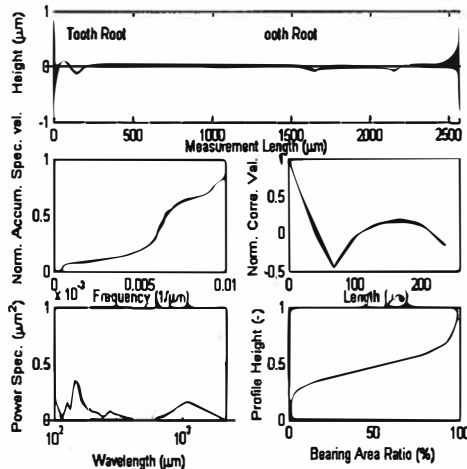


Fig. 5. Roughness profile (b) of model surface of Fellows chiselled gear tooth flank for tooth profile from Mechanical Dektop data. Characteristics of surface roughness profile: cumulated power spectral density, autocorrelation function, spectrum of power spectral density, and material ratio curve: A  $\lambda c=0.8$  mm Gauss's filter was used

greater differences follow from the comparison of real and model wheels along the profile height. The flank surface of a chiselled gear wheel has a tooth profile of heights  $P_t=5.29 \mu\text{m}$ ,  $R_t=2.83 \mu\text{m}$ . The corresponding values for the model surface are  $P_t=3.51 \mu\text{m}$ ,  $R_t=0.32 \mu\text{m}$ . Differences in respect to the height of profile irregularities, their spacing and slope as well as the peak heights, their density and curvature were found. It confirms a great effect of the physical phenomena of machining on the parameters of tooth flank surface topography.

Making use of a conregular model of surface topography, similar profile lengths of rising and falling irregularities profiles were found. It refers to tooth profiles of both real and model wheels (Fig. 6). The profile of the helix of machined wheels is asymmetric. Simultaneous slope of the flank profiles for rising irregularities is different from falling ones. The technological, worked flanks show considerable differences between these slopes on the helix. However, model surfaces have the same slope of rising and falling surface irregularities of the helix. Contour profiles along the tooth height of technologically worked

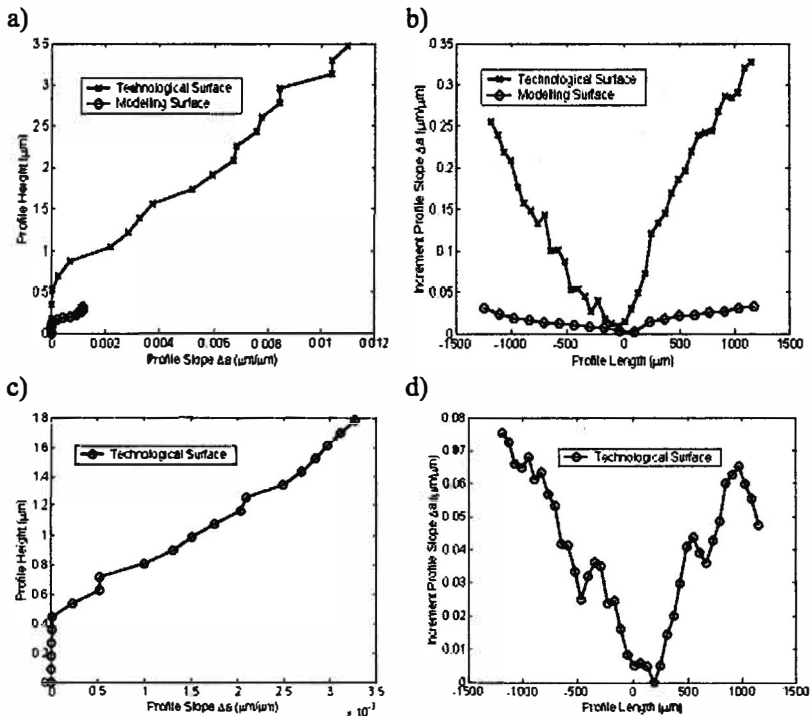


Fig. 6. Characteristics of surface roughness profile slope of side out Fellows chiselled gear tooth flank along tooth height (a), (b) and along helix (c), (d). Characteristic of profile height with regard to slope (a), (c). Slope increase along profile length (b), (d).



and a model surface have a similar slope diversification. The occurring slope differences of helix profile for rising and falling segments were thought to be due to the effect of physical phenomena of the machining process.

#### 4. Conclusion

The presented method of modelling cylindrical gear flanks allows a precise definition of the stereometric shape and surface topography. The method is particularly useful as it enables obtaining tooth profiles of any degree of complication. The models obtained in this way may be analysed along the tooth contour as well as along the helix at any section. Such an analysis is difficult, and often impossible when the surface is characterized by mathematical equations.

Tooth flanks getting into chiselling have bigger roughness height, spacing, slope, peak curvature and have a greater density than those getting out of machining. In the case of model gears, there is no difference between flanks coming into and getting out of machining. The model toothed wheels also have smaller tooth profile and pitch helix deviation values than real gear wheels. It is due to the fact that during the simulation it is impossible to allow for extra physical phenomena that accompany real machining.

The experimental results confirmed the correctness of the developed model of the tooth flanks of a cylindrical gear wheel as to the character of the surface topography after Fellows chiselling.

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### **Powierzchnia boku zęba rzeczywistego koła zębatego dłutowanego Metodą Fellowsa i Modelu CAD**

#### **Streszczenie**

W pracy porównano strukturę geometryczną powierzchni bocznej modeli zębów koła walcowego, uzyskanych metodą symulacji obróbki w układzie trójwymiarowym, ze strukturą geometryczną powierzchni zębów koła fizycznie wykonanego w procesie technologicznym dłutowania metodą Fellowsa. Przystawiono również metodykę modelowania powierzchni bocznych zębów kół zębatych walcowych w środowisku CAD. Przebieg modelowania stanowi symulację obróbki obwodniowej kół zębatych. Symulację obróbki przeprowadzono w środowisku programu Mechanical Desktop. Pomiary metrologiczne rzeczywistego koła zębatego przeprowadzono współrzędnościową maszyną pomiarową oraz profilometrem.