T H E A R C H I V E O F M E C H A N I C A L E N G I N E E R I N G

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CALCULATION OF SQUEEZING FORCES DURING LONGITUDINAL ROLLING FOR THE FINAL PASSAGE

In this paper, the author derives theoretical formulae for calculating of squeezing forces. This report is the first one concerning the method of forming stepped shafts by longitudinal cold rolling. The formulae of the radial squeezing forces for the final passage of longitudinal rolling were calculated under the Huber hypothesis of plastic deformation and maximum shear stress.

1. Description of the method of longitudinal rolling

The method of longitudinal rolling is a new technique of accurate cold working of stepped shafts [1]. The method allows for shaping the shafts of any symmetrical cross-section, e.g. circular, square, hexagonal etc. The shafts can be produced from stocking material in the form of an alloy steel rod quenched and tempered to 36 ÷ 38 HRC.

The general scheme of forming a stepped shaft of circular cross-section by longitudinal cold rolling is presented in Fig. 1. The shaft (1), fixed in the way allowing for its elongation in the forming process, is placed between two shaping rollers (2), each of whom is supported by two support sleeves (4) on the bearing axle (3).

The forming sequence consists of four steps marked by arrows a – b – c –d in Fig. 1 and turn of the shaft through an angle depending on the transverse profile of the formed shaft step.

The cycle of forming sequence consists of the following stages:

- moving of the forming rollers to position a – b,
- the squeezing stage b – c,
- the longitudinal rolling stage c – d,

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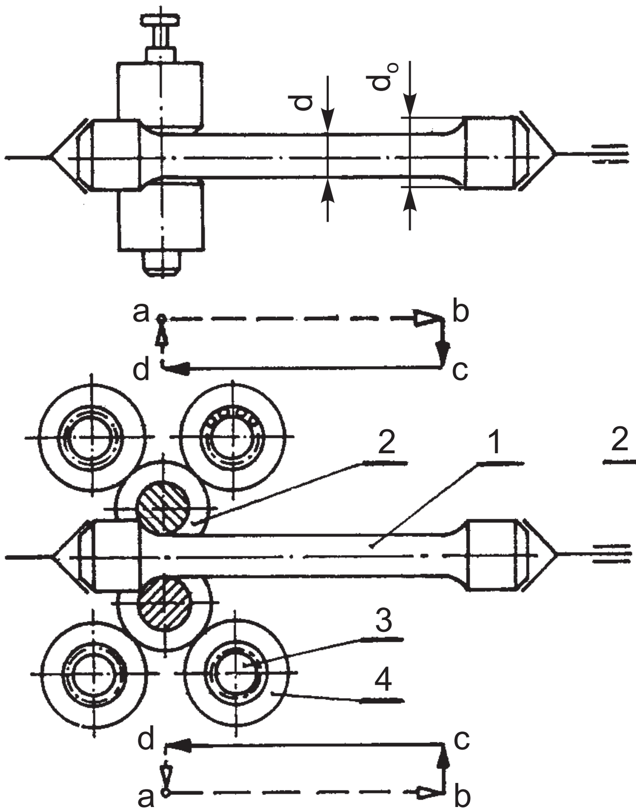

Fig. 1. The general scheme of longitudinal cold rolling of stepped shaft; d_o – initial diameter, d – diameter of stepped shaft

– the retracting stage d – a, [–] the returning stage a – b, in which the pressing mechanism travels from the bumper to the initial position and the formed shaft is rotated.

The full formation of the shaft step is completed after several passages. At that time, the required reduction of the shaft cross-section dimensions and elongation is obtained.

2. Calculation of squeezing forces in the process of longitudinal cold rolling during final passage

The general aim of the original method of multiple forming of the shaft cross-section is the reduction of step diameter by small deformation. After the final passage, we can obtain step diameter (less than IT-8), surface roughness (less than 0,63 μm) and the surface layer without cracks, scratches etc.

During the final passage, the contact between the shaft and the forming roller is only on the torus profile. Assuming the static model of squeezing of the roller with torus profile (see Fig. 2), we can obtain the formulae (1, 2, 2', 3, 3', 3") for the squeezing forces during the final passage in the longitudinal rolling.

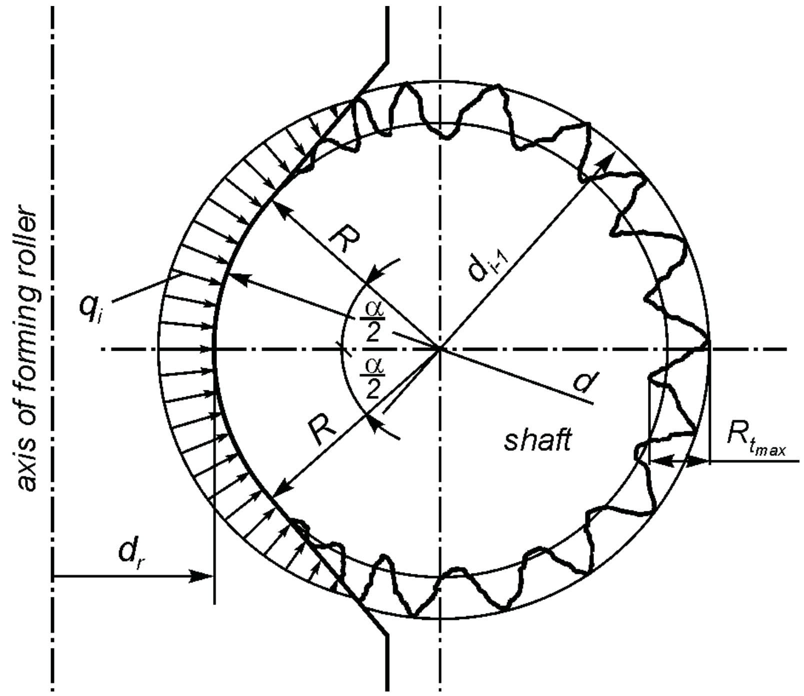

Fig. 2. The scheme of deformation surface roughness of step shaft during final passage

2.1. Simplification assumptions

Simplification for calculations of radial squeezing force are the following:
- the depth of plastic deformation $\delta = 2R_{t_{max}}$ (see Fig. 3), but it is always greater than the roughness of the original work surface;
- the forming roller is deformed only elastically, but the profiling shaft is subject to elastic and plastic deformation (upper zone is elastic, lower is plastic);
- in consequence of interaction of the roller and the shaft, there are plastic deformations on the depth δ, but in the shaft there are only elastic deformation (see Fig. 3);
- the layer of plastic deformation does not change the characteristics of the squeezing force [2];

– both bodies (the roller and the shaft) interact statically, therefore the contact pressure is the same as the pressure on the surface.

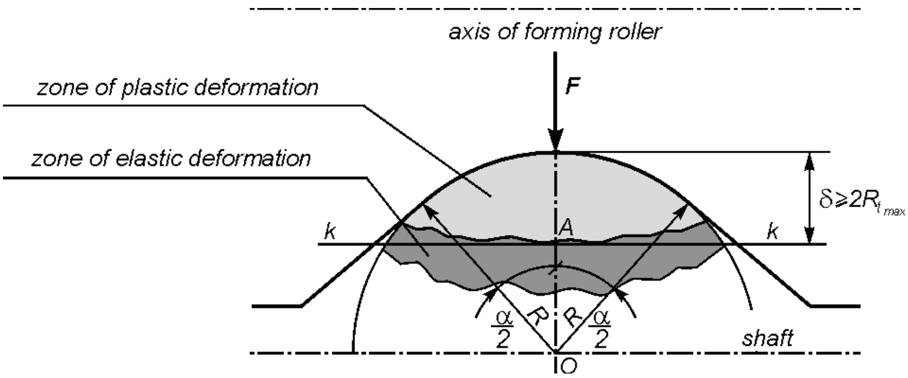

Fig. 3. Fragment of plastic deformation zone during concentrated force on the forming roller of torus profile

Considering the above assumptions, we will consider the following three cases:

1. $F_1 = f_1\ (R_{t_{\max}}, R_e)$ – Boussinesqu solution, which does not allows for curvatures of the surfaces;
2. $F_2 = f_2\ (R_{t_{\max}}, R_{\mathrm{e}}, k_N, k_P)$ – Hertz solution, which partially allows for the curvature of the surfaces (e.g. cylinder or sphere);
3. $F_3 = f_3\ (R_{t_{\max}}, R_{\mathrm{e}}, k_N', k_P')$ – Bielajev solution, which is the most general one, allowing for any curvature of the surfaces given by Landau – Lifszyc equations.

Where:

F_1 – static squeezing force between roller and shaft at rest,
$R_{t_{max}}$ – maximal roughness of surface before longitudinal rolling,
R_{e} – creep limit for the shaft material,
k_N, k_P – curvatures of the roller and shaft respectively.

Accepting the assumptions (2.1), we search for the function $F = f\ (R_{t_{max}}, R_{\mathrm{e}}, k_N, k_P)$ in the above three cases.

2.2. Boussinesqu approach – solution F_1

We assume that the interaction between the roller and the shaft is replaced by the concentrated force F which does not allow for curvature of the contact surface. We choose a coordinate system such that the force F has only the z component and acts along the z – axis ($x = y = 0$).

The stresses are the following (according Boussinesqu theory) [3]:

Due to axial symmetry of the contact surface, the difference of stresses $\left(\sigma_y - \sigma_z\right)$ between elasticity and plasticity domains equals τ_{max}, hence

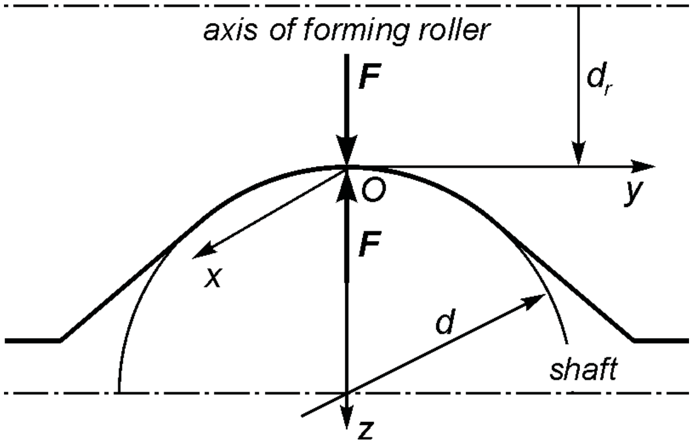

Fig. 4. Contact of the forming roller with the shaft in Cartesian coordinates x, y, z

$$\sigma_z = -\frac{3F}{2\pi} z^3 \left(x^2 + z^2\right)^{-\frac{5}{2}}; \qquad \sigma_y = -\frac{3F}{2\pi} z^2 y \left(x^2 + z^2\right)^{-\frac{5}{2}}.$$

$$\tau_{max} = \frac{\sigma_y - \sigma_z}{2}.$$

Substituting the above formulae of σ_y and σ_z into τ_{max}, we obtain

$$R_e = \frac{3F}{2\pi}\left[yz^2 \left(x^2 + z^2\right)^{-\frac{5}{2}} - z^3 \left(x^2 + z^2\right)^{\frac{5}{2}} \right].$$

On the line $k - k$ there is $\tau_{max} = 0,5\ \mathrm{R_e}$ and $z = 2R_{t_{\max}}$, hence

$$F = \frac{8}{3}\pi \cdot R_e \cdot R_{t^2_{\max}}, \tag{1}$$

It can be shown that application of M. T. Huber hypothesis leads to the same result.

2.3. Hertz approach – solution F_2

In solution F_2, according to Hertz [3], the components of the stress along the z – axis at $x = y = 0$ for two spherical bodies can be described by the following formulae:

$$\sigma_y = \sigma_1 = \frac{1}{2} q \left[\frac{2(1+\nu)\cdot z}{\sqrt{a^2 + z^2}} - (1 + 2\nu) - \left(\frac{z}{\sqrt{a^2 + z^2}} \right)^3 \right];$$

$$\sigma_z = \sigma_2 = q\left[\left(\frac{z}{\sqrt{a^2+z^2}}\right)^3 - 1\right].$$

Appling the τ_{max} hypothesis (as in 2.2), we have:

$$\tau_{max} = \frac{\sigma_1-\sigma_2}{2} = \frac{1}{2}q\left[\frac{2(1+\nu)\cdot z}{\sqrt{a^2+z^2}} - (1+2\nu) - \left(\frac{z}{\sqrt{a^2+z^2}}\right)^3 - 2\left(\frac{z}{\sqrt{a^2+z^2}}\right)^3 + 2\right].$$

Using $z = 2R_{t_{\max}}$ and $q = q_0 = \dfrac{3F}{2\pi a^2}$ we get:

$$F = \frac{\pi a^2 R_e}{3}\left[-\frac{1+2\nu}{2} + (1+\nu)\frac{2R_{t_{\max}}}{\sqrt{a^2+4R_{t_{\max}}}} - \frac{3}{2}\left(\frac{2R^2_{t_{\max}}}{\sqrt{a^2+4R^2_{t_{\max}}}}\right)^3 + 1\right]^{-1}.$$

In the above equation we have introduced a new parameter a ($a = \sqrt{a' \cdot b'}$, half – axis of the ellipse), which can be obtained from the formula $a = \sqrt[3]{\frac{3}{2}\frac{(1-\nu)^2 F}{2E(k_N+k_P)}}$.

In the case of steel shafts ($\nu = 0,3$) we have:

$$F = \frac{\pi a^2 R_e}{15}\left(1 - \frac{2R_{t_{\max}}}{\sqrt{4E(2R\cdot d + 2d_r\cdot R - d\cdot d_r)}}\right). \tag{2}$$

On the other hand, during longitudinal rolling of the round cross-section of the shaft $k_N = \dfrac{2}{d_r} - \dfrac{1}{R}$; $k_P = \dfrac{2}{d}$ (see Fig. 2) and a is given by:

$$a = \sqrt[3]{\frac{3}{4}\frac{(1-\nu)^2 F\cdot d\cdot R\cdot d_r}{E(2R\cdot d + 2d_r\cdot R - d\cdot d_r)}}. \tag{2'}$$

We have obtained two equations which enable one to compute the pressing force. The geometric parameters: d, R i d_r are shown in Fig. 2.

2.4. N. M. Bielajev approach – solution F_3

Solution F_3, according to Bielajev [4], is applied for the model shown in Fig. 4. Formulae for the components of stress tensor along the z – axis (at $x = y = 0$) are the following:

$$\sigma_x = \frac{3F}{4\pi} z(1-\nu)\int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)^3(b^2+s)s}} - \frac{3F}{4\pi} z 2\nu \int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)(b^2+s)s^3}} + \\ - \frac{3F}{4\pi}(1-2\nu)\int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)^3(b^2+s)}};$$

$$\sigma_y = \frac{3F}{4\pi} z(1-\nu)\int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)(b^2+s)^3 s}} - \frac{3F}{4\pi} z 2\nu \int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)(b^2+s)s^3}} + \\ - \frac{3F}{4\pi}(1-2\nu)\int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)(b^2+s)^3}};$$

$$\sigma_z = -\frac{3F}{2\pi} \frac{1}{\sqrt{(a^2+z^2)(b^2+z^2)}}; \qquad \tau_{xy} = \tau_{yz} = \tau_{zx} = 0,$$

where a i b are half-axes of the ellipsis of the contacting bodies.

From tables of integrals [5, 6], for $b > a$, we get:

$$I_l = \int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)^3(b^2+s)s}} = \frac{2b}{(b^2-a^2)a^2}\mathcal{E}(\varphi,k) - \frac{2}{(b^2-a^2)b}\mathcal{E}(\varphi,k) + \\ -\frac{2z}{a^2}\frac{1}{\sqrt{(b^2+z^2)(a^2+z^2)}};$$

$$I_2 = \int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)(b^2+s)s^3}} = \frac{2}{a^2 z}\sqrt{\frac{a^2+z^2}{b^2+z^2}} - \frac{2}{a^2 b}\mathcal{E}(\varphi,k);$$

$$I_3 = \int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)^3(b^2+s)}} = \frac{2}{a^2-b^2}\left(1-\sqrt{\frac{a^2+z^2}{b^2+z^2}}\right);$$

$$I_4 = \int_{z^2}^{\infty} \frac{ds}{\sqrt{(a^2+s)^3(a^2+s)s}} = \frac{2}{b^2-a^2 b}[\mathcal{F}(\varphi,k)-\mathcal{E}(\varphi,k)];$$

$$I_5 = \int_{z^2}^{\infty} \frac{ds}{\sqrt{(b^2+s)^3(a^2+s)}} = \frac{2}{b^2-a^2}\left(1-\sqrt{\frac{a^2+z^2}{b^2+z^2}}\right),$$

where $\mathcal{E}(\varphi, k)$ and $\mathcal{F}(\varphi, k)$ are elliptic functions.

Introducing an additional variable $w = \left(\frac{a}{b}\right)^2$ and using the above integrals for computing the stress tensor we get [7]:

$$\sigma_x = \frac{3F}{\pi}\frac{1}{b^2(1-w)}\left\{\frac{z}{b}\left[\frac{1-nv}{w}\mathcal{E}(\varphi,k) - (1-v)\,\mathcal{F}(\varphi,k)\right] + \frac{1-2v}{2}\left(1-\sqrt{\frac{b^2+z^2}{b^2w+z^2}}\right) + \right.$$
$$\left. - \frac{1-w}{w}\frac{b^2wv + x^2}{\sqrt{(b^2w+z^2)(b^2+z^2)}}\right\};$$

$$\sigma_y \frac{3F}{\pi}\frac{1}{b^2(1-w)}\left\{\frac{z}{b}\left[\frac{v-w}{w}\,\mathcal{E}(\varphi,k) + (1-v)\,\mathcal{F}(\varphi,k)\right] - \frac{2v-w}{2w}\sqrt{\frac{b^2w+z^2}{b^2+z^2}} - \frac{1-2v}{2}\right\};$$

$$\sigma_z = \frac{3F}{\pi}\frac{1}{b^2(1-w)}\left[\frac{b^2(1-w)}{2}\frac{1}{\sqrt{(b^2w+z^2)(b^2+z^2)}}\right],$$

where elliptic functions $\mathcal{E}(\varphi, k)$ i $\mathcal{F}(\varphi, k)$ are approximated as follows:

$$\mathcal{E}(\varphi,k) = \mathrm{arcctg}\frac{z}{b} \circ \left[1 - \frac{1}{4}(1-w) - \frac{3}{64}(1-w)^2\right] +$$
$$+\frac{z}{b}\frac{1}{1+\left(\frac{z}{b}\right)^2}\left[\frac{1}{4}(1-w) + \frac{3}{64}(1-w)^2 + \frac{1}{32}(1-w)^2\frac{1}{1+\left(\frac{z}{b}\right)^2}\right];$$

$$\mathcal{F}(\varphi,k) = \mathrm{arcctg}\frac{z}{b} \circ \left[1 + \frac{1}{4}(1-w) - \frac{9}{64}(1-w)^2\right] +$$
$$-\frac{z}{b}\frac{1}{1+\left(\frac{z}{b}\right)^2}\left[\frac{1}{4}(1-w) + \frac{9}{64}(1-w)^2 + \frac{3}{32}(1-w)^2\frac{1}{1+\left(\frac{z}{b}\right)^2}\right].$$

From the Huber hypothesis we get:

$$\sigma_{red} = \sqrt{\frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right]}; \quad \tau_{xy} = \tau_{yz} = \tau_{zg} = 0.$$

On the line $k - k$ (Fig. 3) the reduced stress is $\sigma_{red} = R_e$, and the coordinate $z = \delta = 2R_t$.

Using the computed stress tensor and collecting the terms under the square root, we obtain the pressing force F for final passage during longitudinal rolling:

$$F=\frac{\sqrt{2}\pi}{3}R_e b^2(1-2)\left\{\left[\frac{\delta}{b}\left[\frac{v-w}{w}\,\mathcal{E}\,(\varphi,k)+(1-v)\,\mathcal{F}\,(\varphi,k)\right]-\frac{1-2v}{2}+\right.\right.$$
$$\left.+\frac{w(b^2+\delta^2)-2v(b^2w+\delta^2)}{2w\sqrt{(b^2w+\delta^2)(b^2+\delta^2)}}\right]^2+$$
$$+\left[\frac{\delta}{b}(1-v)\left[\frac{1+w}{w}\,\mathcal{E}\,(\varphi,k)-2\,\mathcal{F}\,(\varphi,k)\right]+1-2v-\frac{b^2w(1+w)(1-2v)-2\delta^2[v(1+w)-1]}{2w\sqrt{(b^2w+\delta^2)(b^2+\delta^2)}}\right]^2+$$
$$\left.+\left[\frac{\delta}{b}\left[\frac{1-nv}{w}\,\mathcal{E}\,(\varphi,k)-(1-v)\,\mathcal{F}\,(\varphi,k)\right]+\frac{1-2v}{2}+\frac{b^2w^2(2v-1)=\delta^2[2(nv-1)+w]}{2w\sqrt{(b^2w+\delta^2)(b^2+\delta^2)}}\right]^2\right\}^{-0,5} \tag{3}$$

where $w=\dfrac{a^2}{b^2}<1$ and $b>a$, $k=\sqrt{1-w}$, $\varphi=\arcsin\sqrt{\dfrac{B^2}{b^2}+\delta^2}=\mathrm{arctg}\,\dfrac{\delta}{b}$.

In formula (3), the pressing force F depends on three variables, i.e. $F=f\,(\delta,b,w)$, because the other parameters R_e and v are given. They can be obtained from the curvatures taking the following quantities form the work by Landau – Lifszyc [8]:

$$A=\frac{3F\left(1-\nu^2\right)}{2\pi E}\int_0^\infty\frac{ds}{\left(a^2+s\right)\sqrt{\left(a^2+s\right)\left(b^2+s\right)s}};$$

$$B=\frac{3F\left(1-\nu^2\right)}{2\pi E}\int_0^\infty\frac{ds}{\left(b^2+s\right)\sqrt{\left(a^2+s\right)\left(b^2+s\right)s}}.$$

where A and B are given by following equations:

$$2(A+B)=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_1^{`}}+\frac{1}{R_2^{`}};$$

$$4(A-B)=\left[\left(\frac{1}{R_1}-\frac{1}{R_2}\right)^2+\left(\frac{1}{r_1^{`}}-\frac{1}{R_2^{`}}\right)^2\right]+2\cos 2\varphi\circ\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\left(\frac{1}{R_1^{`}}-\frac{1}{R_2^{1}}\right).$$

where: $R_1=0,5d_r$, $R_2=R$, $R_1^{`}=0,5d$, $R_2^{`}=\infty$, $\varphi=180^\circ$.

The above integrals and specific values of the geometric parameters enable us to obtain the two final equations:

$$F = \frac{2}{3}\frac{E}{1-\nu^2}b^2(1-w)\frac{w(d_r+d)}{d_r \cdot d\left\{\left[1-\frac{1}{4}(1-w)-\frac{3}{64}(1-w)^2\right]-w\left[1+\frac{1}{4}(1-w)+\frac{9}{64}(1-w)^2\right]\right\}}, \tag{3}$$

$$F = \frac{2}{3}\frac{E}{1-\nu^2}b^2(1-w)\frac{1}{2R\left[\frac{1}{2}(1-w)+\frac{3}{16}(1-w)^2\right]}. \tag{3''}$$

In this way, the three equations $\begin{cases}(3)\\(3')\\(3'')\end{cases}$ define uniquely the pressing force F as a function of three variables δ, w, b in the implicit form (implicit function theorem). It must be emphasised that we also need the assumption $w < 1$, as it assumed in Bielajev theory. The complicated nonlinear form of equations (3, 3', 3") does not allow for expressing the function $F(\delta, w, b)$ in an explicit form, except of very special cases. However, the solution can be calculated by numerical methods.

Figure 5 shows an example of such a numerical solution (in the form of graphs $F(\delta)$ for different values of R_e).

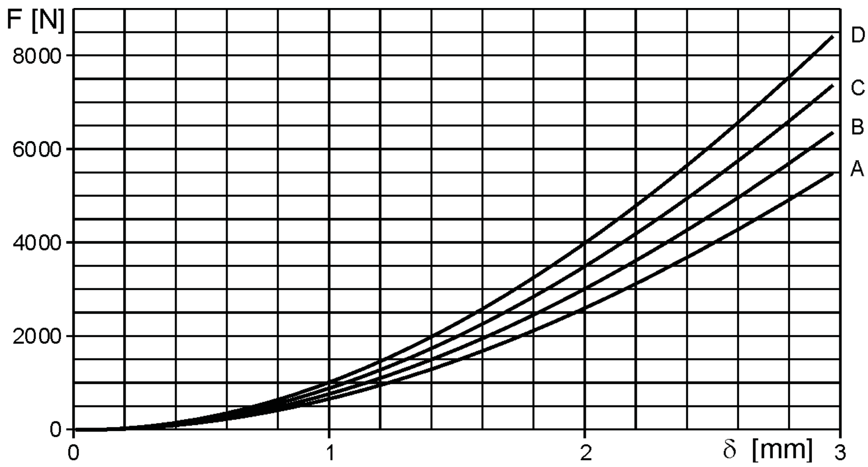

Fig. 5. Graphs of squeezing forces $F = f(\delta, R_e)$ for values r = 12 mm, D = 40 mm, d = 24 mm; R_e = (A = 300 MPa, B = 350 MPa, C = 400 MPa, D = 450 MPa), $E = 2,1 \cdot 10^4$ MPa, ν = 0,3.

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Obliczanie siły nacisku podczas walcowania wzdłużnego dla przejć wykańczających

Streszczenie

W pracy opisano metodę kształtowania wałków stopniowych w procesie walcowania wzdłużnego na zimno. Przedstawiono teoretyczne zależności pozwalające obliczyć wartość promieniowej siły nacisku w przejściach wykańczających. Zależności te opracowano w oparciu o hipotezy odkształcania plastycznego M. T. Huberta.