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STATIC AND DYNAMIC ANALYSIS OF THE CABLEWAY

The paper is concerned with an analysis of behaviour of the cableway. On the basis of design data and results of adequate experiments, a physical model of cableway was formulated. The static of cableway was developed assuming a full nonlinear model based on elastic catenary curve. The tension of the rope and the reactive forces between the rope and the supports were calculated. Assuming various loadings of the rope, the relation between the tension in bottom and upper stations and the length of the rope was determined. The model describing the motion of the system is linear. Finite elements were used to formulate the model. Two methods of accelerating the system were investigated.

1. Introduction

Due to complex dynamics of cableway and random character of loads acting in the system, the description of cableway motion is difficult. Thus, the proper methods of experimental and numerical investigation of the system should be worked out. To improve the system's dynamics, the mathematical model emulating the static and dynamic properties of cableway installations is very helpful [1, 2].

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The results of experiments and numerical calculation presented in this paper concerns the cableway "Strugi Szczawiny", running from Korbielów to Hala Miziowa (the Pilsko Mountain, Beskid Żywiecki). There are two stations in this cableway installation: the upper station incorporating the driving system and the bottom station with the rope tensioning system. There are 22 supports between the stations. The rope connecting the two stations acts as a truck and a haul rope. 219 double chairs are mounted to the rope by conical grips. The maximal loading is 16000 kg.

The main purpose of the present study is to develop a mathematical model to be used for synthesis of a vibration control system in real life conditions.

2. Experiments

Acceleration and braking of the cableway causes local changes of rope tension. They are the reason of vibrations. Chairs vibrations are sensed by passengers, making them feel uncomfortable. Vibrations are more intensive in longer installations, where rope elasticity is of key importance.

The aim of the work was identification of cableway and description of static and dynamic behaviour of the cableway installation. An accurate model would allow for designing such a construction, in which the negative phenomena are reduced. Extensive tests were performed, involving the measurements of a chair's acceleration in three directions:

x – longitudinal direction (coinciding with the rope axis and the direction of travel),

y – transverse direction (horizontal and perpendicular to the direction of the travel),

z – direction normal to the rope axis.

Measurements were taken with an adhesive-mounted three-axial piezoelectric accelerometer. DAQ is a signal acquisition module for conducting high-accuracy measurements from integrated electronics piezoelectric sensors (IEPE). 24-bit resolution ADCs with 102 dB of dynamic range deliver and incorporate IEPE signal conditioning for the accelerometers. The input channels simultaneously digitize input signals up to 50 kS/s (kilo sample per second). A computer was used for an introductory analysis and acquisition of measurement signals. The measurements of acceleration were taken every few minutes, at the same time the cableway velocity signals were recorded. The measurement data include three ranges:

I – accelerating up to the nominal speed,

II – travel at the steady speed (the chair passes the support),

III – emergency braking between two subsequent supports.

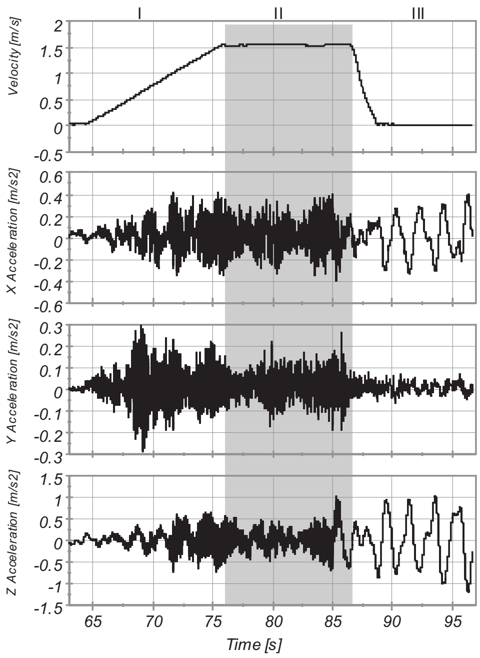

Fig. 1. Rope velocity and chair acceleration components in three directions

Selected results of measurement are shown in Fig. 1. The first graph presents velocity of the rope and the three others present components of acceleration associated with chair vibrations.

During emergency braking, all braking systems were applied. After braking, the rope and chairs oscillate with frequency of about 0.5 Hz. The largest accelerations are registered in the x-and z-direction. The accelerations in the transverse direction are negligibly small.

3. Static and dynamic analysis of cableway

The rope in the cableway is subjected to forces distributed along its length and forces applied at the end points of the overall structure. The load distributed along the rope results from the rope's mass and the mass of passenger chairs attached to the rope. The rope balance system is placed at the end point. In the installation considered in this study, the tensioning force is produced by the weight of the mass placed at the bottom station. At the upper end, the driving moment (or braking force) is applied to the driving wheel.

Due to a strongly nonlinear behaviour of the rope, the analysis of static and dynamic becomes a complex task, and the motion of a cableway during braking and accelerating is particularly difficult to capture.

3.1. Static analysis

The equilibrium state of cableway can be described using various methods [4, 3]. In this calculation, a nonlinear model based on a catenary was chosen. The equilibrium equations of a catenary passing through the points located on supports should be solved in order to obtain the length of the rope in each of two tracks and the vertical component of tension.

An assumption is made that the mass of each chair is uniformly distributed along the rope. Thus, the linear density of the rope is adequately increased.

In order to describe the static equilibrium of rope in each span, the following parametric equations of catenary were used:

$$\begin{aligned} x &= \frac{N_0}{g\mu}\left(\operatorname{asinh}\left(\frac{(s-s_m)g\mu}{N_0}\right)+\operatorname{asinh}\left(\frac{s_m g\mu}{N_0}\right)\right) \\ y &= \sqrt{\left(\frac{N_0}{g\mu}\right)^2+s_m^2}-\sqrt{\left(\frac{N_0}{g\mu}\right)^2+(s-s_m)^2} \end{aligned} \tag{1}$$

where: (x, y) – coordinates in local Cartesian system, N_0 – horizontal component of rope tension, μ – linear density, g – gravitational acceleration, s – arc-coordinate, s_m – catenary parameter. Equations (1) are supplemented by the equation describing the rope tension:

$$N = \sqrt{N_0^2 + ((s-s_m)\, g\mu)^2} \tag{2}$$

In the above equations, the arc coordinate is equal to zero at the point where the rope is supported ($x = 0$, $y = 0$). The coordinates of the second support point and the tension at the bottom end of the span are known. The length of the rope and the rope tension at the upper end of the span can be determined. The calculation starts from the bottom span, where the tension at the end point is equal to a half of the weight of the tensioning mass. Then, the adjacent span is taken into account in calculations. The rope tension determined for the previous span is used in calculations concerning the next span.

The tension at the bottom end of the rope is equal to *60* kN. Taking into account the mass of passenger chairs, the unit mass was assumed to be *17.4* kg/m (chairs are empty) or *28.1* kg/m (full load – chairs with passengers). The local coordinate system is introduced in each span. Selected results of calculations are presented in Fig. 2. The graphs illustrate the forces at the upper and bottom points of the rope as functions of the length of rope in two cases: with full load (Fig. 2a) and without the load of passengers (Fig. 2b).

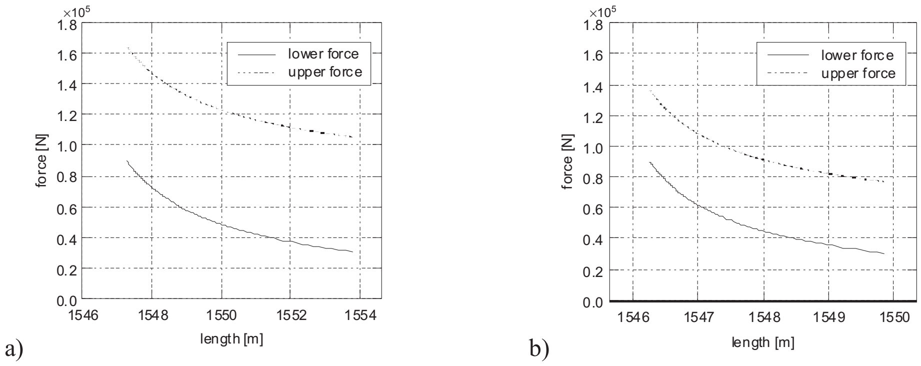

Fig. 2. Rope tension in bottom and upper stations versus the length of rope: a) full load, b) chairs are empty

It is apparent that the driving friction wheel has to be blocked when one rope is loaded by passengers, and the second one is loaded by empty chairs only. Using the characteristics presented in Fig. 2, we can easily calculate the static displacement of the mass in the tensioning system due to non-symmetric loading of ropes.

3.2. Study of dynamic behaviour

The dynamic behaviour of the cableway can be described using the system of ordinary and partial differential equations of motion [5]. They are difficult to solve analytically. For this reason, we introduce a simplified, finite-element model of the cableway. Finite-element method is frequently used in solving the static and dynamic problems of rope structures [6].

Similarly as in the static calculations, we assume that the mass of each chair is uniformly distributed along the rope. The bottom and upper wheels, as well as the mass in tensioning system, are assumed to be rigid bodies. The coordinate x_1 describing the displacement of the tensioning mass and the angles φ_1, φ_2 describing the revolutions of wheels are joined the nodal displacements $q_1,....,q_{40}$ of finite elements that describe the motion of the rope. Finally, the following vector of generalized coordinate is defined $[x_1, r\varphi_1, q_1, ..., q_{20}, r\varphi_2, q_{21}, ..., q_{40}]^T$. We assume that at $t = 0$ the nodes are located at the point placed on supports (cableway towers). Thus, each span is modelled by only one finite element. The coordinates are shown schematically in Fig. 3.

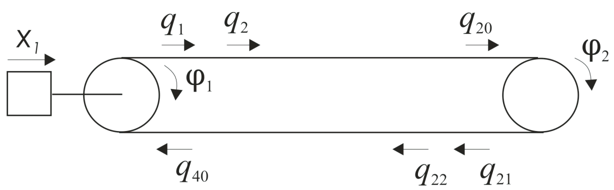

Fig. 3. Scheme of coordinates used in calculations

To create the FEM model of a rope, linear interpolation functions are applied to each line element. The tangent displacement u_e inside the element is interpolated using the corresponding nodal displacements by:

$$u_e = \bar{\psi}^T \bar{q}_e \tag{3}$$

where: $\bar{q}_e = [q_{e1}, q_{e2}]^T$ is the nodal displacements vector for element and $\bar{\psi} = [\psi_1, \psi_2]^T$ is the vector of interpolation functions.

The normal displacement w_e inside the element can be interpolated using nodal displacement and quasi-static equations. Thus, the analysis of the cableway motion can be reduced to solving a system of ordinary differential equations.

The calculations were performed for the following parameters: length of each rope $l = 1548$ m, stretching stiffness $EA = 1.9 \times 10^8$ N, unit mass of the rope with full chairs 28.1 kg/m, unit mass of the rope with empty chairs 17.4 kg/m.

It is apparent that the motion is a superposition of translation and vibration motion. The natural frequencies and modes are the basis for dynamic analysis of the systems. The spectrum analysis shows that the amplitudes of vibrations significantly increase for the frequencies close to the system's natural frequencies. Using the FE model of the cableway, one can calculate the natural frequencies and mode shapes. The results are shown in Tables 1 and Fig 4.

Table 1.

Non-zero natural frequencies

Mode Number	Natural frequency [Hz]
1	0.40
2	0.93
3	1.24
4	1.87
5	2.12
6	2.83

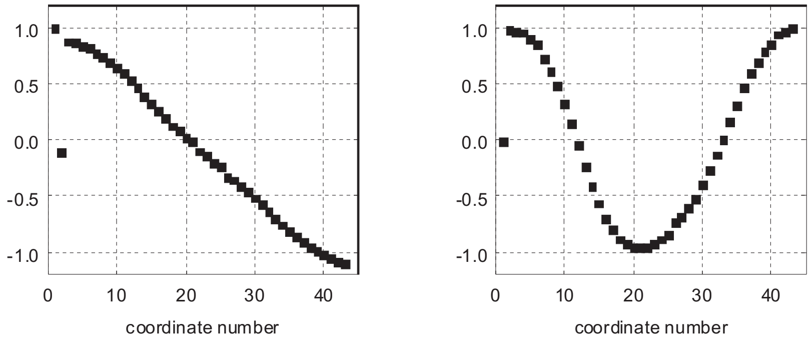

Fig. 4. The first and the second mode shapes of vibration

Taking into account the results of the modal analysis, the following conclusions can be formulated:

– Because of the large rope length, the first non-zero frequency is very low. Neighbouring natural frequencies are separated, in general, only by about 0.4 Hz. Thus, the cableway is very sensitive to any loads and disturbances.
– The displacement of tensioning mass takes the maximum value for the modes in which the displacements in each rope have the same directions.

When the full loaded cableway is started, displacements and accelerations strongly depend on the driving moment control. Two cases of the control were considered. In the first case, the dynamic component of driving moment is constant up to the moment when the velocity reaches the desired value. In the second case of the control, the angular acceleration of the driving wheel is constant until the velocity achieves the desired value.

Selected results of calculations of the cableway behaviour under a constant driving moment are presented in Figs. 5-7. The constant driving moment is applied to the driving wheel for 11.5 s. Fig.5 shows the dynamic component of driving force (the dynamic component of driving moment divided by the radius of driving wheel) as a function of time.

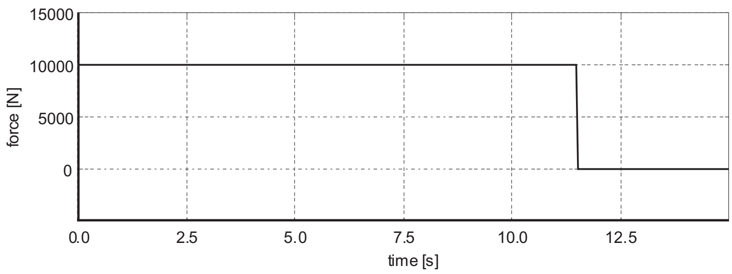

Fig. 5. Dynamic component of driving force

Arc displacement of the point placed on the driving wheel and its tangent velocity are presented in Fig. 6. As shown, the increase of velocity is not a smooth function. The motion has an oscillating component.

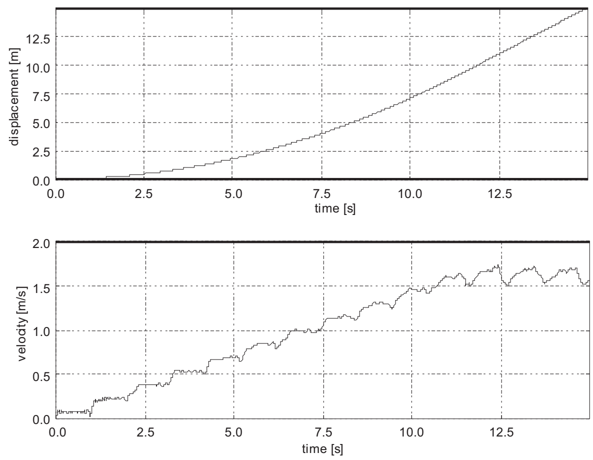

Fig. 6. Arc displacement and tangent velocity of the point placed on driving friction wheel

Fig.7 presents the displacement and the velocity of the tensioning mass. As shown, the main frequency of vibrations is equal to the first natural frequency (Tab. 1). The motion of tensioning mass is very important for operation of the cableway because it is directly associated with proper tension of the rope.

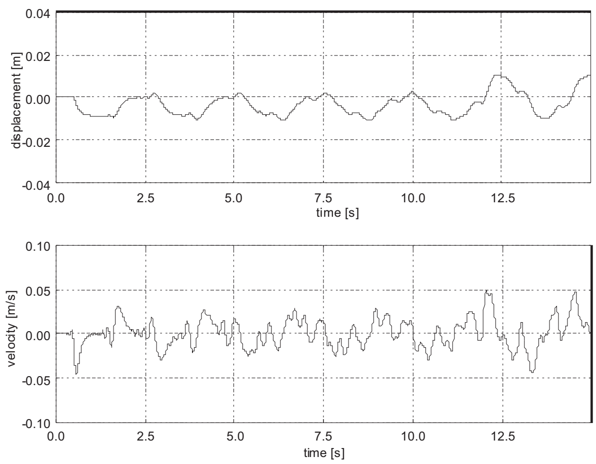

Fig. 7. Displacement and velocity of tensioning mass

In the following calculations it was assumed that the cableway accelerates from 0 to 1.5 m/s in 11.5 s and the acceleration of driving wheel is constant.

In this case, the motion of cableway is different than that in the case with constant dynamic moment. Some selected results, arranged in the same way as in the previous case, are presented in Figs. 8-10.

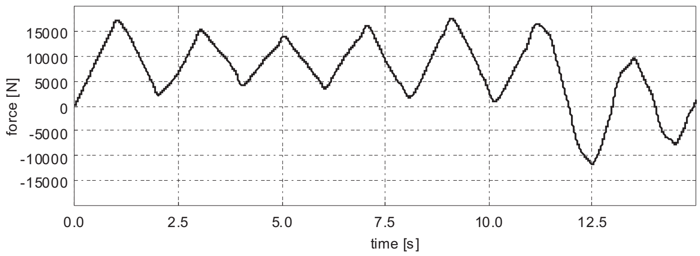

Fig. 8. Dynamic component of driving force

As shown in Fig. 8, the driving force has an oscillating component in the period when acceleration of the driving wheel is constant, and later when the driving wheel rotates continuously.

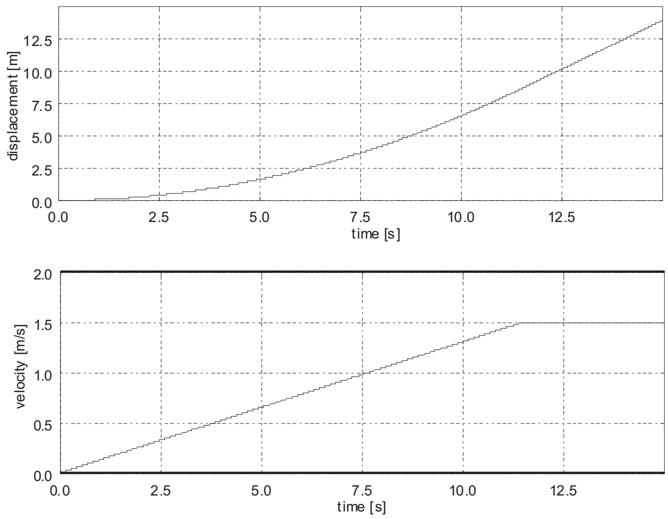

Fig. 9. Arc displacement and tangent velocity of driving wheel

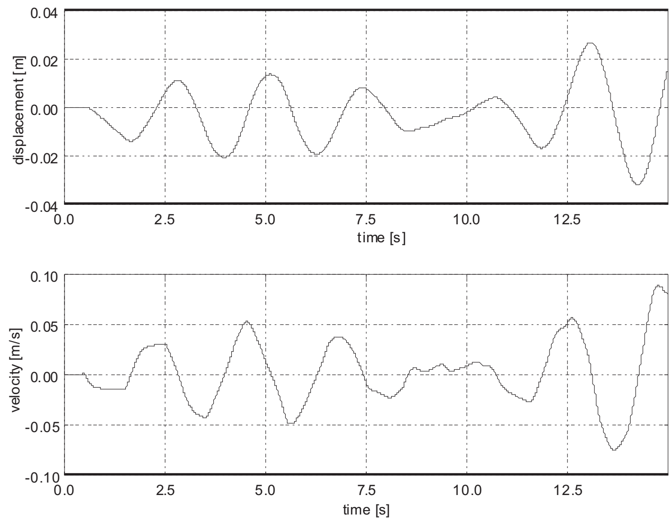

Fig. 10. Displacement and velocity of tensioning mass

The values of displacement and velocity of the tensioning mass are greater than those in the case with constant moment. During acceleration of the cableway, the displacement of tensioning mass takes positive values. The positive value of displacement is associated with the decrease of rope tension (see Fig. 3) and a simultaneous decrease of forces between the rope and the support rollers placed on the supports.

In order to prevent this effect and improve the system's dynamics, a proper damper can be mounted in the tensioning system. The damper does not act on the mass when it stretches the rope. The damper prevents the motion of the mass when it releases the rope. The results of numerical simulation are presented in Fig. 11. As in previous calculations, it was assumed that the angular acceleration of motion of the driving wheel is constant. The damping coefficient equals 1.5×10^5 Ns/m. It was found out by a process of trial and error.

The displacements presented in Fig. 11 are more advantageous than those presented in Fig. 10. During acceleration of the cableway with additional damper, the displacement of the tensioning mass is negative. This means that the rope is stretched better than in the case without the damper.

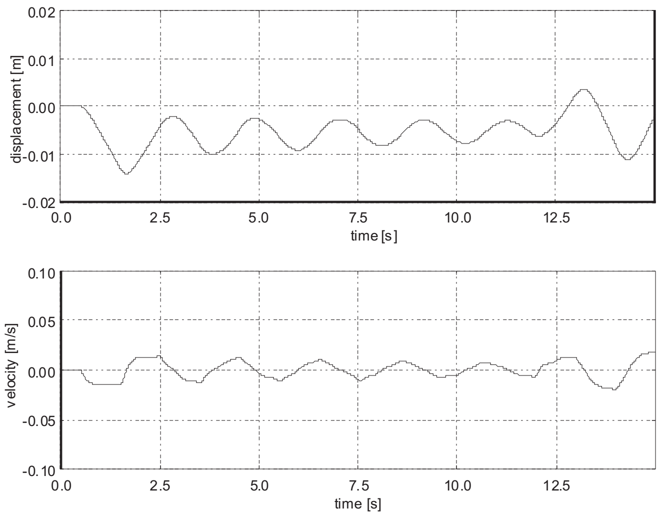

Fig. 11. Displacement and velocity of tensioning mass with additional damper

4. Conclusion

A cableway is a very complex system. It is important to have an accurate, general model which makes it possible to predict effects of possible dynamic behaviours of the cableway in various situations. In this paper, a model using finite-element method is presented. In spite of simplifying assumptions, the model is useful for calculations of characteristics of the overall system e.g. accelerating and braking of the cableway, and the behaviour of the tensioning system. The model should be extended, if necessary, to describe the motion of some subsystems. Based on the results of the performed calculations, we can draw the following conclusion:

- As expected, the cableway is very sensitive to any loads and disturbances. The first non-zero natural frequency is very low. The neighbouring natural frequencies are separated, in general, only by about 0.4 Hz.
- During accelerating, the motion of the cableway depends on the driving moment control. Two cases of the control were considered. In the case when the angular acceleration of the motion wheel is constant, disadvantageous phenomena occur in tensioning system, which is associated with large displacement of the tensioning mass.
- In order to prevent the disadvantageous phenomena, a proper damper can be mounted in the tensioning system. The damper does not act on the mass when it stretches the rope. The proposed solution can be realized by applying a viscous damper with directional flow valve.

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Analiza statyczna i dynamiczna kolejki linowej

S t r e s z c z e n i e

W pracy przedstawiono ogólną analizę statyczną i dynamiczną kolejki linowej. Na podstawie danych konstrukcyjnych oraz wyników odpowiednio dobranych pomiarów zbudowano model obliczeniowy konstrukcji. W obliczeniach statycznych wykorzystano model nieliniowy, linię statycznej równowagi liny opisano krzywą łańcuchową. Wyznaczono siłę naciągu oraz siły oddziaływania liny na podpory. Do opisu dynamiki zbudowano model liniowy wykorzystujący metodę elementów skończonych. W obliczeniach dynamiki układu rozpatrzono dwa sposoby rozruchu kolejki. Wyniki obliczeń zostały zilustrowane na wykresach.