www.czasopisma.pan.pl PAN www.journals.pan.pl
Metrol. Meas. Syst., Vol. 27 (2020) No. 1, pp. 33–50
DOI: 10.24425/mms.2020.131713

METROLOGY AND MEASUREMENT SYSTEMS
Index 330930, ISSN 0860-8229
www.metrology.pg.gda.pl

ANALYSIS OF THE INADEQUACY OF DETERMINING THE SPECTRAL CHARACTERISTICS OF OPTICAL FIBRE PERIODIC STRUCTURES BY WAY OF NUMERICAL MODELLING

Piotr Stępniak, Piotr Kisała

Lublin University of Technology, Institute of Electronics and Information Technology, Nadbystrzycka 38 A, 20-618 Lublin, Poland (✉ *stepniak.piotr@pollub.edu.pl*, +48 81 538 4309, *p.kisala@pollub.pl*)

Abstract
As part of the work, the error level of simulations of uniform optical-fibre Bragg gratings was determined using the transition matrix method. The errors were established by comparing the transmission characteristics of the structures obtained by simulation with the corresponding characteristics arrived at experimentally. To compile these objects, elementary properties of the characteristics were specified, also affecting the applications of Bragg gratings, and compared with each other. The level of error in determining each of these features was estimated. Relationships were also found between the size of the physical properties of Bragg gratings and the level of errors obtained. Based on the findings, the correctness of the simulation of structures with the said method was verified, giving satisfying results.

Keywords: numerical simulations, fibre Bragg grating, numerical modelling.

1. Introduction

Simulations of physical processes taking place in different centres are important for science, as long as these processes are relevant to it. Simulation in general enables to determine the result of the process, based on its input parameters and knowledge of its running. It also enables to check how the process will end when changes are made to its parameters or its run. Introducing such changes in a real process can be very difficult, unprofitable, or impossible to implement using available methods and means. That is why simulations are such an important tool in the study of processes in progress. By letting one see what is unnoticeable empirically, they are an invaluable help in the development of science.

Performing the simulation requires knowledge about the input data set of the process being studied. This means not only determining a set of variables and constant parameters affecting the process, but also the scope of significant values, their validity, and even their mutual correlation. This knowledge enables to build a model used in simulation. Thus, the model is an abstraction based on current knowledge, it is an idealised object with a specific level of detail. The method used is also influenced by the method used for the research. Computer simulations are used mainly where the process cannot be reproduced by analytical methods, or it is unprofitable in time due to its complexity. Such a situation takes place in the process of light flowing through an optical fibre studied in this work, with periodically modulated index of refraction in its core.

In order for the simulations to be used to analyse the phenomena occurring, it is necessary to know their level of accuracy. The result obtained as a result of the simulation will never be as precise as the result obtained in the laboratory, which depends on the model used, which is always a simplified object by adopting some assumptions, some of which may not be correctly defined due to incomplete knowledge about the subject process. An equally important reason for this state of affairs are the generalisations introduced by the simulation methods used. Depending on the type of calculations performed and the range of variables used in them, the source of inaccuracy can also be the arithmetic used in the computer program and – in the case of simulation of continuous signals – their sampling and quantisation. In this work the level of accuracy of fibre optic simulations of Bragg gratings was estimated without analysing the causes of inaccuracy.

2. Work methodology

Simulations were carried out on the basis of prepared models of uniform gratings. Structure models include their length and period. The value of the effective refractive index in the core of the optical fibre, on which the grating was created and the modulation value of this coefficient were also taken into account (the model also takes into account the possibility of a situation where the value changes over the length of the structure). However, it does not take into account the other parameters of the optical fibre, or data dependent on the way the grating is made.

Simulations were carried out based on the transition matrix method, which uses the theory of coupled modes, enabling to define equations whose solution leads to the spectral characterisation of the Bragg grating. This method approximates the waveform, which by definition introduces some inaccuracy. The transition matrix method treats the Bragg grating as a four-terminal network. This means that at both ends of the structure it assumes the existence of input and output waves, which are written in a vector form, as illustrated in Fig. 1.

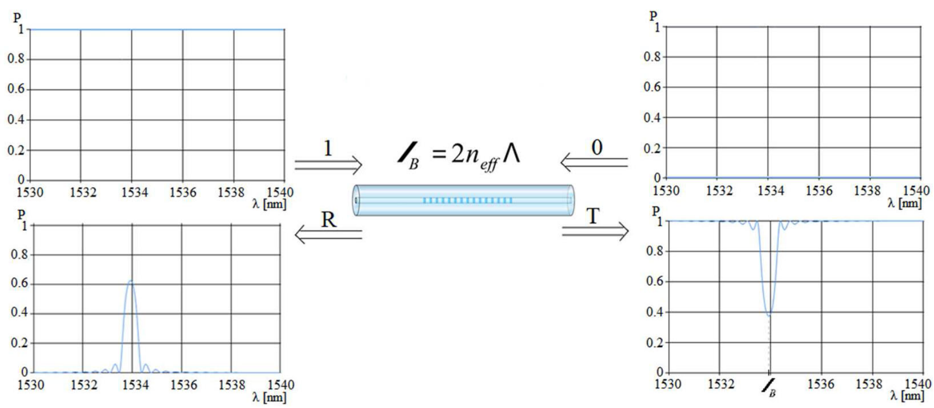

Fig. 1. A diagram showing the uniform Bragg grating as a four-terminal network. The model is introduced into the one-sided vector and the zero vector from the right, which in the method of the transition matrix are the boundary conditions that enable to write a system of equations necessary for the mathematical formulation of a solvable problem.

The transition matrix method introduces a logical division of the grating into equal sections, for which the calculations are performed separately, and the output vectors of one of the sections are at the same time the input vectors of subsequent sections [1]. As a result, calculations are carried out on full matrices, however for the purpose of this work, the mathematical details of these activities are omitted. However, it should be emphasised that the level of inaccuracy of this method depends on the number of sections to which the model structure has been divided.

This dependence is much greater in the simulation of apodised or chirped gratings (these are structures in which their length varies accordingly: the coefficient of modulation of the refractive index in the core, or its period). This is due to the fact that within the section the grating period and modulation depth of the effective refractive index in the structure must be constant, so increasing the number of sections leads to a better representation of the apodisation and chirp profiles. In the method described, these profiles are approximated by vectors constituting a set of values of profiled quantities. The length of the vector is determined based on the number of sections, and a single value is calculated by determining the arithmetic mean of the values at the section ends.

The simulations were carried out in a handwritten application dedicated to this purpose, written in the C# programming language, having a graphical interface based on the Windows Forms technology. The program uses decimal data types, thanks to which, in the calculations performed by the application, the accuracy of 28 decimal places was obtained [2]. It was verified that the values in variables stored during the algorithm operation had up to about 12 or even 16 digits significant for the results of calculations, due to the specifics of the simulation method chosen. These values lie at the limit of the accuracy range of the binary variables in C#, which is between 15 and 16 decimal places [2]. It was verified that the change of arithmetic from decimal to binary causes slight differences in the transmission spectrum of the Bragg grating obtained as a result of the simulation. Due to the above facts, it was concluded that the errors resulting from the roundings associated with the digital representation used in the calculation of variables are negligibly small and do not affect the results of comparing the spectra obtained by simulation with the spectra obtained in the laboratory.

The transmission spectra of the gratings obtained by the simulation were compared with the laboratory transmission spectra of the gratings obtained with the use of the phase mask method. Figs 2, 3 and 4 show the spectral characteristics measured in the transmission mode for various

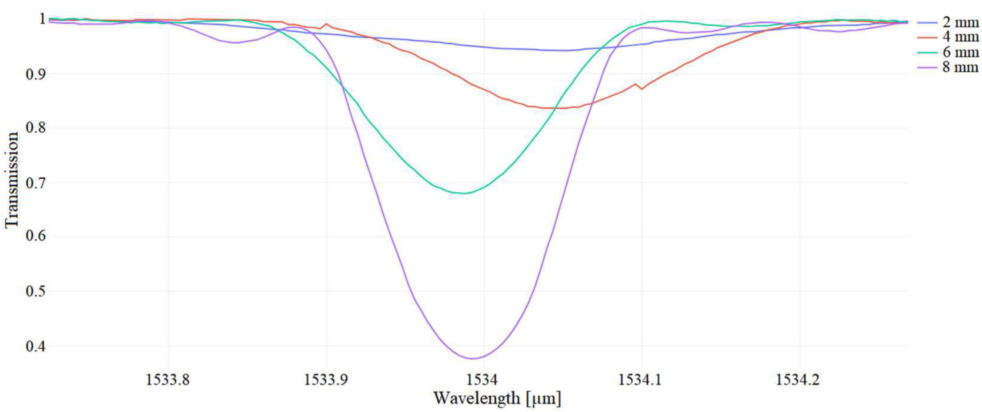

Fig. 2. A set of measured transmission characteristics of Bragg gratings of 530 nm and various lengths, respectively: blue – structure with a length of 2 mm; red – structure with a length of 4 mm; green – structure with a length of 6 mm; violet – an 8 mm long structure.

values of the length of the structures, their period and modulation depth of the refractive index. The structures tested in the laboratory were made using a Coherent excimer laser "Bragg Star M" (248 nm wavelength). The transmission characteristics were determined by passing a wide beam of light generated by SLED diodes through the gratings and registering the energy at the output of the structure by a spectrum analyser. The characteristics obtained in this way were recorded with a resolution of 4 nm in a window with a width of 5 μm, which means a record of transmitted energy for 1250 different wavelengths. Because the source used has a dependence of the value of the emitted energy on the wavelength, the transmission characteristic vectors obtained have been converted, neutralising the influence of this dependence. The conversion was made by approximating the correlation with a straight line determined on the basis of the extreme points that made up the transmission window, which could have been performed assuming that the wavelengths filtered by the tested gratings did not extend beyond that window. This condition was indeed always met, because the spectrum analyser was set in such a way that all the reflected radiation was in the middle of the window being analysed.

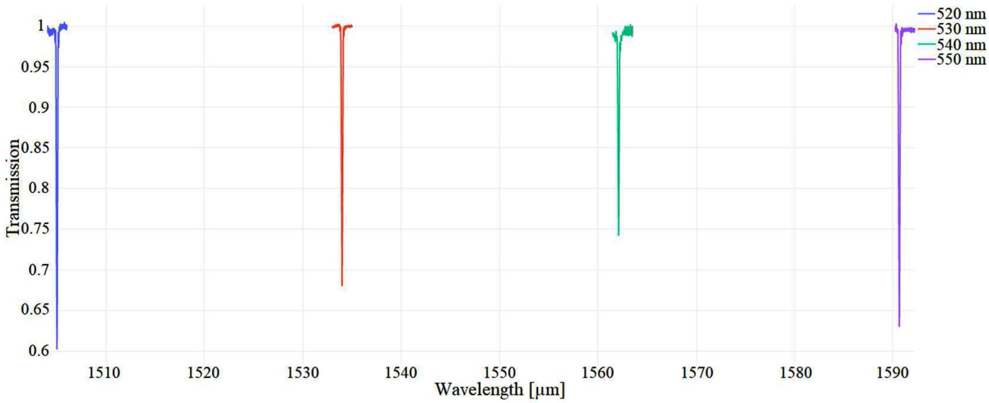

Fig. 3. A set of measured transmission characteristics of 6 mm long Bragg gratings and various periods, respectively: blue – structure with a period of 520 nm; red – structure with a period of 530 nm; green – structure with a period of 540 nm; violet – structure with a period of 550 nm.

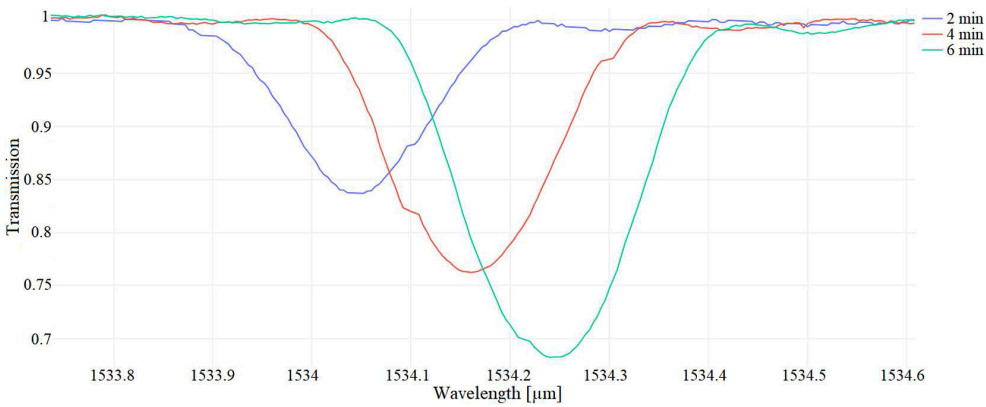

Fig. 4. A set of measured transmission characteristics of 4 mm long Bragg gratings, 530 nm period and various refractive index modulation depths reflected by the length of firing time of the structure: blue – 2 minutes; red – 4 minutes; green – 6 minutes.

Metrol. Meas. Syst., Vol. 27 (2020), No. 1, pp. 33–50
DOI: 10.24425/mms.2020.131713

The criteria for comparing the characteristics obtained by the simulation and those achieved in the laboratory are based on the convergence of their properties, which are crucial in the applications of gratings in fibre optic sensors. These properties are: Bragg wavelength, *i.e.* the wavelength which is transmitted to the smallest extent by the Bragg grating, determining the location of the characteristic peak in the transmission characteristic; the dynamics of the peak defining the level of reflection of the wavelength similar to the Bragg's length, expressed by the ratio of wave energy at the output of the optical fibre, to the energy at its input, at the wavelength at which this suppression is greatest (Bragg wave); *Full Width at Half Maximum* (FWHM) of the peak, describing the range of wavelengths that is the difference in wavelengths for which the ratio of their energy transmitted to their input energy is half of the maximum value, reduced by the amount of noise (in the sense of interrupting the monotonicity of the part of the characteristic containing the peak envelope) corresponding to the reflection of the waves by the structure being tested), measured separately for a wavelength shorter and separately for a wavelength longer than the Bragg wavelength (Fig. 5). The dynamics of the sideband determine the ratio of energy transmitted to the input (the greater the dynamics, the smaller the ratio), for a wave of length different from Bragg but close to it, for which there is a local maximum wave reflection, visible in the transmission characteristic as a side peak (much lower than the main peak, called the side band), where this coefficient relates to a side peak with higher dynamics. The scheme for determining this coefficient is shown in Fig. 6 [3].

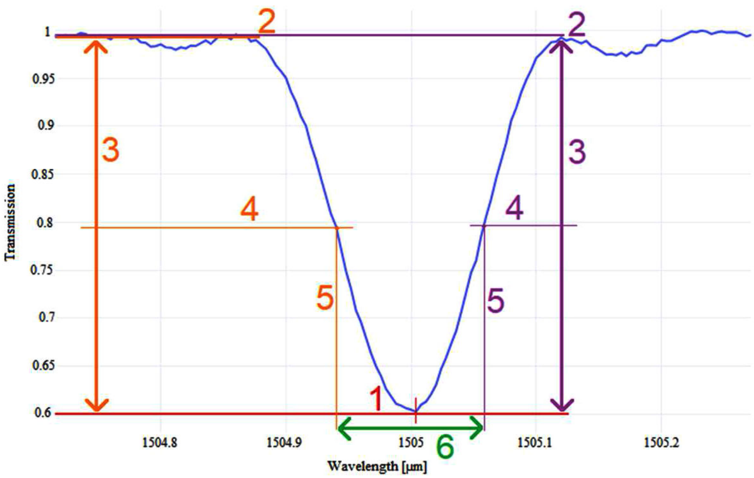

Fig. 5. A diagram showing the sequence of steps leading to the calculation of FWHM of the peak: Determining the minimum transmission value (1) and finding the Bragg wavelength (marked in red on the abscissa); Determining the transmission value for which the first change in monotonicity occurs in the vicinity of the Bragg wavelength (2). Calculating the difference between the obtained transmission coefficients (3) and determining the average value (4); Determining the wavelength for which the function assumes the calculated value (5). Steps 2–5 should be performed separately for two parts of the characteristic; Removing the determined wavelengths from each other will result in FWHM of the peak (6).

The comparison of characteristics was made by comparing each of the properties listed above. This action does not give a full picture of the differences between the objects compared, but has several indisputable advantages. First, it enables to compare parameters whose values create transient characteristics (defining the dependence of properties of transmission characteristics on the physical properties of structures) of Bragg gratings, and thus enables to determine the level of accuracy of such characteristics, created on the basis of numerical calculations. Secondly, as

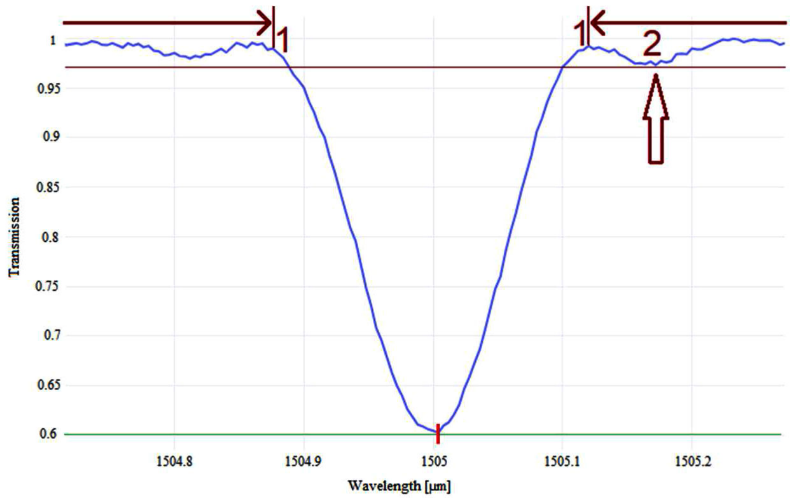

Fig. 6. A diagram showing the method of determining the first order side mode transmission coefficient ("sideband"). After determining the Bragg wavelength (marked in red), one should find points where the monotonicity of spectral characteristic ranges (1) changes. Next, one should determine the interval in the transmission window that does not contain given monotonous intervals constituting the main mode peak – this interval is marked in the upper part of the diagram. The minimum value of the transmission coefficient in this range (2) is the sought-after value.

already indicated, it compares the basic properties that affect the parameters of optical sensors (which can measure different physical quantities) built on the basis of Bragg gratings, because the mentioned properties of transmission characteristics have a direct impact on sensor parameters such as sensitivity, resolution and measurement range, as well as the requirements of the components used [4]. Another advantage of the adopted methodology of comparing characteristics is the ability to independently examine the relationship between the properties described, which would not be possible when using methods based on comparing the correlation of transmission characteristics as sets of points. In the methodology used, the compared data vectors were treated as objects with specific features, which enabled to isolate details at the expected level.

In this work, it was not only checked whether and to what extent the transmission characteristics of fibre-optic Bragg gratings obtained by simulation were consistent with those obtained in the laboratory, but also the dependence of this compliance on physical parameters of structures was examined. The measurements and simulations were carried out for the "reference grating" with set parameters, and then changed by fixed property values, such as length, period and modulation depth factor, performing three or four measurements and simulations for different values of one of the above properties, with constant values of other grating properties. Of course, changing the properties of the grating parameters in order to perform a laboratory measurement was associated with the need to create a new structure, therefore the number of examined cases is a compromise between the time and costs of the study, and with the possibilities to determine the effect of changes in individual properties on the divergence characteristics and to check whether these relationships are linear, which has been achieved.

The problem encountered during the research was the fact that it was impossible in the laboratory to determine the depth of modulation of the refractive index in the core of the optical fibre (in short: the modulation depth factor). This resulted in the inability to accurately determine the discrepancy between the calculated and actual values of this magnitude. The above problem makes it impossible to accurately determine the discrepancy for any other grating property studied,

Metrol. Meas. Syst., Vol. 27 (2020), No. 1, pp. 33–50
DOI: 10.24425/mms.2020.131713

unless the value of modulation depth factor is measured, calculated or at least estimated. This is because a precise check of the impact of changing one of the grating's properties on changes in its spectral characteristics requires introducing into the simulation model all other parameters whose values are consistent with their measured laboratory equivalents. The above problems were solved by performing two series of simulations. One series was made for the modulation depth determined on the basis of its calculation based on the laboratory gratings obtained in the model, assuming that the error of its determination is zero for these gratings. The second series was made for the depth of modulation obtained based on the assumption that it was directly proportional to the production time (interaction of ultraviolet radiation) of the grating, which is known for each case studied. The value of the parameter sought was calculated using the least-squares method, based on the measurements made for three gratings with different known production times. It is common knowledge that every simulation is burdened with error. What is also known is the occurrence of saturation of a long-exposed grating, the chemical bonds changed by UV radiation, which inhibits further increase in modulation depth during its further irradiation. Nevertheless, the use of results from two sets of simulations enabled, for each tested dependence of spectral characteristics and grating properties, to determine the range in which the simulation error was located in relation to the laboratory measurement, where one simulation set was made for boundary values and the other for expected values of modulation depth factor. Additionally, for each grating tested, the numerical calculations were made to verify whether the obtained value lay in the range determined by the methods described above and was close to the expected value, which in turn enabled to estimate the dependence of the modulation depth on the time of grating production. Based on the obtained transmission characteristics and using simulations, it was possible to determine the probable values of the mentioned factor, which enabled to verify which of the above calculation sets enables to determine the level of their errors more accurately.

Due to the problem presented above, the results of analyses presented in this publication are of comparative nature. Despite the fact that the exact value of the simulation error was not calculated, the estimation of this quantity by determining the range of the possible error values enabled to show the dependence of its change on the modification of individual physical properties of the examined gratings, and to show the values of these inaccuracies, as presented further in the work.

3. Obtained inaccuracies of numerical calculations and their dependencies

3.1. Inaccuracies of Bragg wavelength calculations

Bragg wavelength is strictly dependent on the parameters of the structure and, in accordance with the Bragg formula, is directly proportional to the period of the grating and the refractive index in the fibre core [5]. It would seem that the Bragg wavelength is therefore the easiest value to calculate. However, it results from the compilation (Table 1) that the calculated values differ

Table 1. A summary of Bragg wavelengths obtained on the basis of laboratory measurements and numerical calculations, together with calculation errors with respect to the measurement of a set of gratings with given parameters.

	Grating 1	Grating 2	Grating 3	Grating 4	Grating 5=2	Grating 6	Grating 7	Grating 8	Grating 9=3	Grating 10	Grating 11
Grating period [nm]	530	530	530	530	530	530	530	520	530	540	550
Grating length [cm]	2	4	6	8	4	4	4	6	6	6	6
Core refractive index	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471
Irradiation time [min]	2	2	2	2	2	4	6	2	2	2	2
Measured Bragg wavelength [nm]	1534.048	1534.052	1533.988	1533.992	1534.052	1534.160	1534.240	1505.004	1533.988	1562.100	1590.644
Calculated Bragg wavelength [nm]	1533.924	1533.924	1533.926	1533.926	1533.924	1533.924	1533.924	1504.982	1533.926	1562.869	1591.811
Absolute error [nm]	0.124	0.128	0.062	0.066	0.128	0.236	0.316	0.022	0.062	0.769	1.167

from the ones measured and the absolute error (which is the difference between the measured and calculated values) is different for individual gratings. It is worth noting that the obtained values of Bragg wavelength using numerical calculations were the same for two sets of simulations, differing in the assumed modulation coefficient of modulation of the refractive index in the core. This means that simulations of Bragg gratings using the transition matrix method do not include this factor in calculation of the Bragg wavelength. However, it cannot be inferred that the dependence of Bragg wavelength on the modulation depth factor does not exist. On the contrary – this fact, the causes of which should be seen in the technological process of producing the structures examined, may explain the various values of calculation error observed in relation to the measured value.

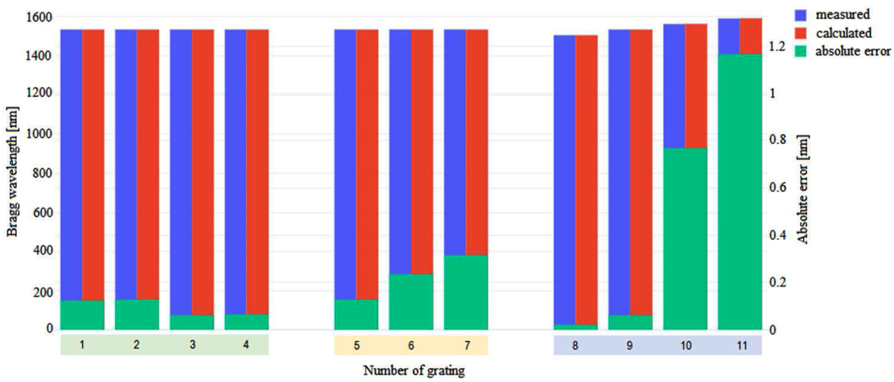

Fig. 7. The data from Table 1 in a graphical form. For each grating, the measured and calculated Bragg wavelength values were determined and the absolute error was obtained (the second axis of elevation was used, the scale of which is visible on the right side of the graph).

On the basis of the above list (Table 1), several interesting observations can be made. First of all, it should be noted that for the vast majority of cases the measured value is greater than the calculated one. Secondly, unless it is possible to determine the dependence of the error level on the length of the grating (gratings 1–4), the dependence of the error value on the time of firing the grating (structure 5–7) is visible. One can also note the dependence of the magnitude of error on the grating period, which increases significantly for structures with 540 nm and 550 nm (gratings 10 and 11), which can be seen in Fig. 7. Thirdly, it can be seen from the summary that for the two examined gratings with the highest period, the calculated value of Bragg wavelength is greater than the measured one, which is an exception to the rule observed for other cases. Before explaining these observations, it is necessary to consider whether the obtained errors are large or small (in the context of common applications of gratings and the possibility of using simulation by the adopted method). It should also be checked if they can only be the result of measurement uncertainty.

The best way to check the size of an error is to compare it to a reference value. Such a value may be a selected wavelength range, *e.g.* one in which the selected spectrum analyser works, or one in which the selected sensor operates. However, since the present work focuses on the comparison of (real and simulated) transmission characteristics of the same grating, a different (real) characteristic is its reference point, which is its *full width at half maximum* (FWHM). The FWHM defines for which wavelength the grating blocks the transmission of energy to an extent equal to at least half of the value for which this takes place for a wave of Bragg wavelength.

DOI: 10.24425/mms.2020.131713

The relative error can therefore be defined as the ratio of the absolute error to the FWHM of the spectrum, expressed as a percentage. Interpret it as the level of overlap between two characteristics (meaning the overlap of characteristics at half their peak). If the error is 0%, the two characteristics coincide. If the error is 100%, the two characteristics intersect at half their height. If the error is close to about 200%, the characteristics are in contact with the bases, and for higher errors they do not intersect. The following table (Table 2) contains values of absolute and relative errors for the numerical calculations of Bragg wavelength. It is worth noting that the proportions between the absolute errors differ from those between the relative ones, which results from differences in FWHM of spectra for individual gratings and can be observed by comparing the blue-marked fields symbolising the error size Table 2. These differences in a graphical form are presented in the graph below (Fig. 8).

Table 2. A list of Bragg wavelengths obtained on the basis of laboratory measurements and numerical calculations, together with calculation errors with respect to the measurement of a set of gratings with given parameters.

	Grating 1	Grating 2	Grating 3	Grating 4	Grating 5=2	Grating 6	Grating 7	Grating 8	Grating 9=3	Grating 10	Grating 11
Absolute error of Bragg wavelength [nm]	0.124	0.128	0.062	0.066	0.128	0.236	0.316	0.022	0.062	0.769	1.167
Measured full width at half maximum [nm]	0.23461	0.15763	0.12461	0.11874	0.15763	0.18149	0.18892	0.11842	0.12461	0.12652	0.14416
Relative error [%]	52.85	81.21	49.76	55.59	81.21	130.03	167.27	18.58	49.76	607.8	809.53

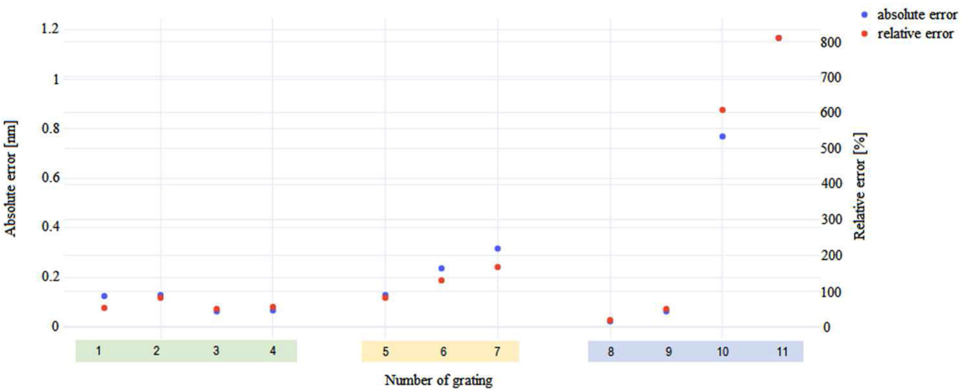

Fig. 8. A graph showing the relationship between the relative and absolute errors of the calculated Bragg wavelength and the size of the measured quantity. The full width at half maximum of the main mode peak, for each grating separately, was taken as the reference point when determining the relative terror.

Since the properties of transmission characteristic were determined on the basis of one spectrum, we cannot talk about type *a* measurement uncertainty. Therefore, the uncertainty of the whole measurement will only consist of type *b* uncertainty. The uncertainty of Bragg wave measurement for a real grating is negligible, as it is 0.001. However, it should be added to the uncertainty of Bragg measurement for a simulated grating, which depends on the uncertainty of the input parameters of simulations, which are the grating period and the index of refraction in the core. Taking into account only the uncertainty of the second factor (because the grating period is a property of precisely made phase masks) at 0.0001, we obtain a type *b* uncertainty of 1.04–1.1 (depending on a particular grating). In connection with the above, it should be concluded that relatively large values of errors obtained between the simulation and measurement results of the examined structures are not the result of measurement uncertainties.

The model used in the simulation, as already mentioned, does not take into account the method of producing the grating, nor the type of fibre doping on which it is stored. Therefore, there is no information in it about whether the process of irradiation occurs either periodically reducing or increasing the refractive index. Bragg wavelength is dependent on the refractive index, taking into account its modulation (probably from the average of its value over the length of the structure), and not from the value of the coefficient for the optical fibre core on which the grating is fired before it is made. This means that the value of Bragg wavelength determined by simulation is always burdened with error. Of course, this error can be reduced by adding to the model additional information about the optical fibre or the method of producing the grating. Nevertheless, accurate determining the profile of changes in the refractive index over the length of the grating is not feasible, which makes it impossible to eliminate this error. The above facts explain the dependence of the error on the numerical calculations made since the time of grating production. Increasing this time is associated with an increase in the modulation of the index of refraction in the core, and this in turn implies an increase in its average value over the length of the structure, which directly translates into an increase in the actual Bragg wavelength for the grating, which is not included in the simulation. However, the above cannot sufficiently explain the large error in calculating Bragg wavelength for gratings with periods of 540 nm and 550 nm. An error can either result from imperfectly created phase mask, on the basis of which the grating was made, or appear at another stage of the structure production (because in the work other gratings of such periods were not tested). Nevertheless, the cause of its occurrence is unknown to the authors.

An interesting observation is the occurrence of different error values for gratings of different lengths, but with the same period. These disproportions result from measurements (Fig. 2), because the calculated Bragg wavelength in this case is always the same. The reasons are probably related to the manufacturing process of the structure. In particular, they may be related to the inaccuracy of measuring its production time (which – as described above – significantly affects the change in the refractive index on which Bragg wavelength depends), as well as to imperfections in creation of the phase mask, which – due to the variable adjustment of the mask length to the grating length – may affect the structure period. Changes in temperature may also bring a slight contribution.

3.2. Inaccuracies in dynamics of reflection

The dynamics of reflection means the relative amount of light energy, for a Bragg wavelength, which does not pass through the structure (it is reflected). Graphically, in the transmission characteristic, it is interpreted as the height of the peak reflecting the operation of the structure as a selective mirror. The following list (Table 3) contains values calculated on the basis of two different grating models (explained in the chapter dedicated to the work methodology), which are the ratios of energy transmitted through the grating, to the energy entered into it, for specific wavelengths, consistent with Bragg wavelength. Due to the adopted procedure, it should be noted that a lower transmission value in a set means a greater dynamics of reflection. The table contains data for nine gratings with different parameters. It also includes the error values of numerical calculations, using each of the two models, differing in the depth of the refractive index modulation. By using colour in the table cells, discrepancies in the results obtained for different simulations were marked. Since the range of values that the transmission coefficient can take is in a range from 0 to 1, it should be noted that the absolute error of calculations of the property discussed can be simultaneously interpreted as a relative error.

As mentioned before, the adopted research methodology, due to the constraints described, does not allow conclusions to be drawn about the error value of simulations of individual gratings

Metrol. Meas. Syst., Vol. 27 (2020), No. 1, pp. 33–50
DOI: 10.24425/mms.2020.131713

Table 3. A list of Bragg wavelengths obtained on the basis of laboratory measurements and numerical calculations, together with calculation errors with respect to the measurement of a set of gratings with given parameters.

	Grating 1	Grating 2	Grating 3	Grating 4	Grating 5=2	Grating 6	Grating 7	Grating 8	Grating 9=3	Grating 10	Grating 11
Grating period [nm]	530	530	530	530	530	530	530	520	530	540	550
Grating length [cm]	2	4	6	8	4	4	4	6	6	6	6
Core refractive index	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471
Irradiation time [min]	2	2	2	2	2	4	6	2	2	2	2
Measured transmission for Bragg wavelength	0.94274	0.83687	0.68017	0.37622	0.83687	0.76231	0.68254	0.60234	0.68017	0.74190	0.63007
Calculated transmission for Bragg wavelength – edge factor	0.98175	0.93021	0.85363	0.76251	0.93021	0.76255	0.57320	0.84875	0.85363	0.85839	0.86289
Calculated transmission for Bragg wavelength – expected factor	0.95460	0.83701	0.68699	0.53910	0.83701	0.76255	0.68498	0.67828	0.68699	0.69556	0.70375
Absolute error – edge factor	0.03901	0.09334	0.17346	0.38629	0.09334	0.00024	0.10934	0.24641	0.17346	0.11649	0.23282
Absolute error – expected factor	0.01186	0.00014	0.00682	0.16288	0.00014	0.00024	0.00244	0.07594	0.00682	0.04634	0.07368

in isolation from the rest of the cases. For this reason, the most important task is to examine the error trend in data sets, made on the basis of gratings that differ only in one property. The structures identified in Table 3 as 5, 6 and 7 are the reference objects, based on the data for which the expected and boundary values of the modulation depth factor were estimated. Therefore, the related results have not been analysed. However, the dependence of the calculation error of the rebound dynamics on the simulation path was verified, depending on the grating length and its duration.

Analysing the results of measurements and simulations in the context of the influence of the grating length on the calculation error, it can be noticed that regardless of the modulation depth factor adopted, the calculated signal transmission value is greater than the measured value (Fig. 9), which – when determining the trend – enables to rely only on the absolute error values (ignoring the sign of the difference calculated in order to receive it). These values noticeably increase with an increase of the grating length, which is more pronounced for the set of data calculated for the modulation depth coefficient taking the boundary value. For the second case, this error is low for 2 cm, 4 cm and 6 cm long gratings, but increases very significantly for a structure of 8 cm

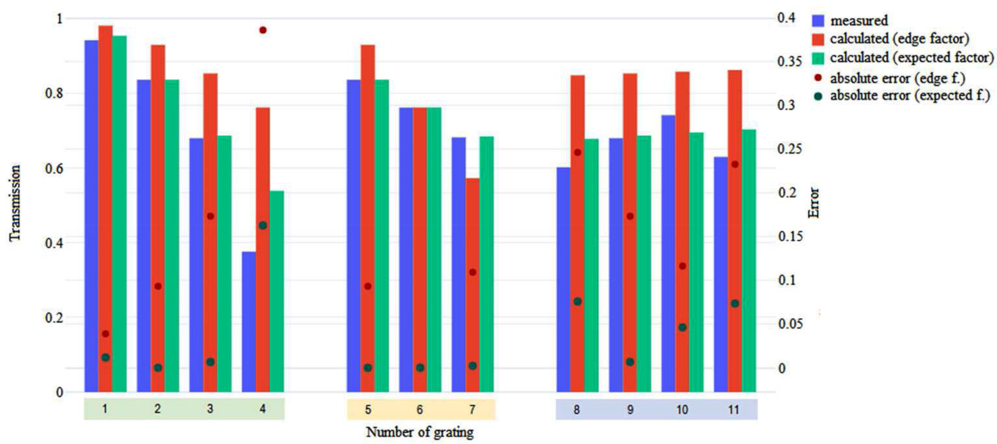

Fig. 9. A graph showing the relations between the measured and calculated values of the transmission coefficient for individual gratings included in Table 3, together with the information on absolute error values.

in length. In connection with the above, it should be concluded that as the length of simulated grating increases, the error of determining the dynamics of reflection increases, which, due to the possibility of using simulations of these structures, is an important conclusion.

While examining the influence of the grating period on the error of determining the dynamics of reflection based on simulations, no correlation of these values was observed. The values of errors received for variable time gratings are significant. In addition, the transmission values calculated for a wavelength equal to Bragg wavelength are in several cases smaller than the values measured, and for a few of them they are larger. What is more, there was no correlation between the value of the grating period and the level of these fluctuations for any of the groups of measurements included in the above list.

3.3. Inaccuracy in calculation of bandwidth of reflection band

The spread bandwidth is the wavelength range surrounding the Bragg wavelength for which the structure does not transmit the radiation in its entirety. Since the degree of its permeability changes at the edges of this range in a continuous way, to precisely define this range a commonly known measure of the *full width at half maximum* (FWHM) has already been used, also described in more detail in this paper. The adoption of such a measure enables to compare different characteristics with each other, and thus to determine the error of obtaining this value by simulation, as well as the possible occurrence (or lack) of its dependence on the physical parameters of the gratings. Table 4 gives values of the FWHM of the transmission characteristics of selected gratings with different parameters. The table contains values measured and calculated while taking into account two different assumptions, which is discussed in detail in the chapter dedicated to the research methodology. Included in it is the absolute error of calculation of the FWHM of the transmission characteristic, as well as (due to a large discrepancy of this property for different gratings tested) the relative error, which is a ratio of the absolute error to the calculated value.

Table 4. Comparison of the reflection widths expressed by the FWHM of the transmission characteristic, for a set of tested Bragg gratings, containing measured and calculated values (with the adoption of two different assumptions), the absolute errors of calculation being the difference moduli of the calculated and measured values and the relative errors being quotients of the absolute error and the calculated value.

	Grating 1	Grating 2	Grating 3	Grating 4	Grating 5=2	Grating 6	Grating 7	Grating 8	Grating 9=3	Grating 10	Grating 11
Grating period [nm]	530	530	530	530	530	530	530	520	530	540	550
Grating length [cm]	2	4	6	8	4	4	4	6	6	6	6
Core refractive index	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471
Irradiation time [min]	2	2	2	2	2	4	6	2	2	2	2
Measured reflection band breadth (FWHM) [nm]	0.23460	0.15763	0.12461	0.11873	0.15763	0.18149	0.18892	0.11842	0.12461	0.12652	0.14416
Calculated reflection band breadth (FWHM) – edge f. [nm]	0.36392	0.18759	0.13111	0.10447	0.18759	0.20892	0.24112	0.12667	0.13111	0.13572	0.14039
Calculated reflection band breadth (FWHM) – expected f. [nm]	0.36975	0.19882	0.14699	0.12409	0.19882	0.20892	0.22083	0.14246	0.14699	0.15166	0.15638
Absolute error – edge factor [nm]	0.12932	0.02996	0.00650	0.01427	0.02996	0.02743	0.05221	0.00825	0.00650	0.00920	0.00377
Absolute error – expected factor [nm]	0.13514	0.04119	0.02239	0.00536	0.04119	0.02743	0.03192	0.02404	0.02239	0.02514	0.01223
Relative error – edge factor	0.35535	0.15972	0.04961	0.13658	0.15972	0.13131	0.21652	0.06513	0.04961	0.06779	0.02685
Relative error – expected factor	0.36550	0.20719	0.15230	0.04317	0.20719	0.13131	0.14453	0.16876	0.15230	0.16576	0.07818

The research carried out shows the dependence of the error of determining the FWHM of the grating spectrum by way of simulation, on the grating spectrum's length. From the above table (Table 4, Fig. 10) it follows that for shorter gratings the error is larger and concerns both absolute

Metrol. Meas. Syst., Vol. 27 (2020), No. 1, pp. 33–50
DOI: 10.24425/mms.2020.131713

and relative views. For a grating with a length of 8 cm, a larger relative error was calculated assuming a linear increase in the modulation depth coefficient along with the grating fabrication time, than that for a 6 cm structure. However, it should be noted that in this case (longer grating) the calculated size is smaller than the measured value (unlike in the other cases), which in general could mean a reversal of the trend. However, considering the fact that this fluctuation is not present for calculations based on the assumption of a non-linear increase in the modulation depth coefficient along with the time of grating production, it can be concluded that such a scenario is very unlikely, and the fluctuation mentioned may be omitted in forming a general trend.

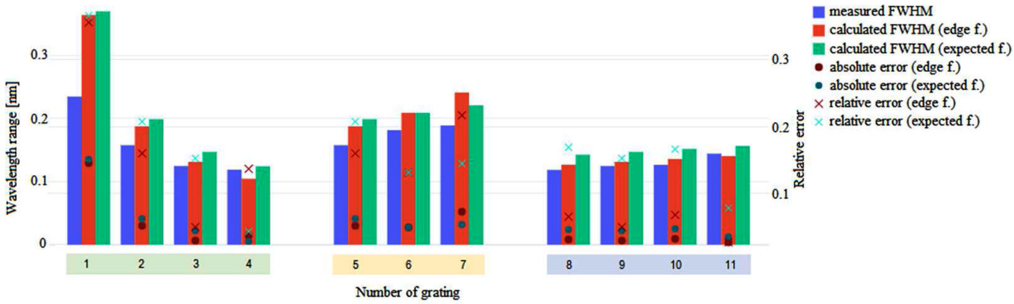

Fig. 10. A graph showing the relationships between the measured and calculated values of the FWHM of the main mode peak, as well as a comparison of relative and absolute calculation errors for individual gratings with the parameters given in Table 4.

Observations did not show the characteristic effect of the modulation depth of the refractive index in the core on the error in calculating the bandwidth of Bragg grating. The impact on this value was not observed for the variable grating period either. Attention was paid to a statistical decrease in the relative error of calculations of the FWHM of the grating spectrum (using the assumption of a non-linear increase in modulation of the refractive index along with an increase in grating production time) along with an increase in the structure period. However, this dependency is so weak that it cannot be ruled out that it is the result of inaccurate measurement and calculation.

3.4. Inaccuracy in calculations of sideband dynamics

Sidebands are a term appearing in the literature describing the phenomenon of a drop in transmission characteristic for uniform Bragg gratings with a length similar to Bragg wavelength, but different from it, for which there is a wavelength closer to Bragg wavelength, in which the structure has a higher transmission coefficient [6]. In the transmission characteristics of the sidebands, they are visible as additional peaks with low dynamics in relation to the main peak. This dynamics (or more precisely the dynamics of the peak with greater eminence) was measured and calculated, and the obtained values compared, which enabled to determine the error between the results obtained. Performing these activities for a set of gratings made it possible to check the existence of the dependence of these errors on the physical properties of the gratings, the results of which are included in the table below (Table 5).

Regardless of the criterion adopted in the calculations made, a relation was found between the magnitude of error of the value calculated in relation to the value measured of the dynamics of sidebands and the length of the grating, as well as the time of its production. Based on the research results, it was concluded that the longer the grating, the greater the inaccuracy of the dynamics of sidebands, determined by numerical calculations. Also, as the fabrication time increases, so does

Table 5. Comparison of the level of dynamics of reflection for sidebands, expressed by a ratio of the energy transmitted to the energy entered, for the wavelength corresponding to the centre of the web with higher dynamics in the transmission characteristic containing values calculated and measured, and the calculation error for a set of gratings with different properties.

	Grating 1	Grating 2	Grating 3	Grating 4	Grating 5=2	Grating 6	Grating 7	Grating 8	Grating 9=3	Grating 10	Grating 11
Grating period [nm]	530	530	530	530	530	530	530	520	530	540	550
Grating length [cm]	2	4	6	8	4	4	4	6	6	6	6
Core refractive index	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471
Irradiation time [min]	2	2	2	2	2	4	6	2	2	2	2
Measured side peak transmission	0.99777	0.99366	0.98737	0.95724	0.99366	0.99665	0.99415	0.98834	0.98737	0.97954	0.98588
Calculated side peak transmission – edge factor	0.99913	0.99652	0.99222	0.98629	0.99652	0.98630	0.96995	0.99193	0.99222	0.99250	0.99277
Calculated side peak transmission – expected factor	0.99778	0.99121	0.98056	0.96619	0.99121	0.98630	0.98037	0.97981	0.98056	0.98124	0.98190
Absolute error – edge factor	0.00136	0.00287	0.00485	0.02905	0.00287	0.01036	0.02421	0.00359	0.00485	0.01296	0.00689
Absolute error – expected factor	0.00001	0.00244	0.00681	0.00895	0.00244	0.01036	0.01379	0.00852	0.00681	0.00170	0.00398

this inaccuracy. The results of research on the influence of the grating period on the calculation error of the value mentioned are, however, ambiguous. The above relationships are presented in the graph below (Fig. 11).

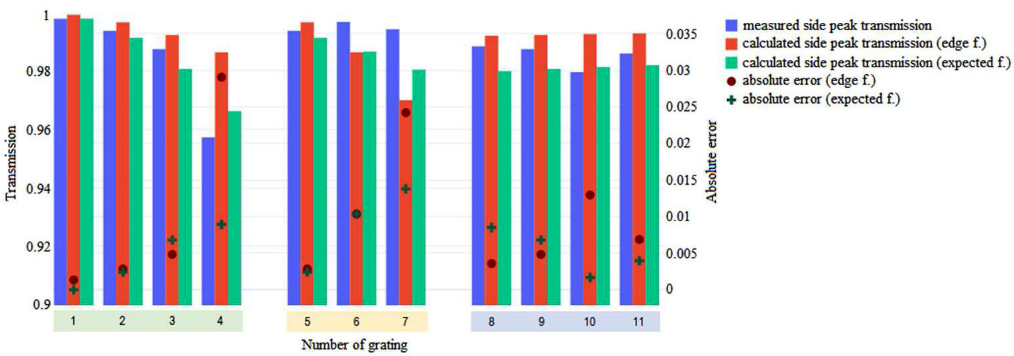

Fig. 11. A graph showing the relationships between the measured and calculated values of the dynamics of reflection for the first-order side modulus, and a list of absolute calculation errors for individual gratings with the parameters given in Table 5.

It is worth noting that both the calculated and measured values of sideband dynamics are small – the transmission coefficient for the corresponding wavelengths does not fall below a value of 0.95 (thus, the dynamics does not exceed a level of 0.05). For such small values, the values of measurement uncertainties are important in the process of deduction and inference. Due to one-time laboratory measurements and the complexity of the transition matrix method used in the calculations, the uncertainties of both type *a* and *b* are unknown. However, on the basis of the transmission characteristics obtained, one can determine the level of noise understood as inaccuracy in the interpretation of the transmission coefficient by the spectrum analyser. Based on the analysis of these characteristics for the wavelengths distant from Bragg wavelength, the noise value was determined, which decreases with an increase of the grating length and amounts to 0.0018 for a grating of 8 cm and 0.004 for a 2 cm grating, but does not depend on the time of grating manufacture. On this basis, it was found that the above-described correlations actually occur and cannot be the result of measurement inaccuracies.

DOI: 10.24425/mms.2020.131713

3.5. Values of measurement errors

The section dedicated to the research methodology describes the problem of the lack of the ability to measure the depth of modulation of the refractive index in the core of an optical fibre. It was also mentioned that in connection with the above, the inaccuracies obtained on the basis of measurements and calculations are approximate, because the model used in the simulations contains certain assumptions. It was decided to approximate the values of these errors, and to this end it was necessary to estimate the real value of the modulation depth factor. The above was accomplished by calculating – by simulation – the value of this coefficient for each of the structures used, for which calculated the dynamics of reflection will be equal to the dynamics of reflection of the generated grating. The use was made of the fact that the depth of modulation of the refractive index has, from all the studied properties of transmission characteristics, the greatest influence on the transmission coefficient for Bragg wavelength [7], by means of which the reflection dynamics is described. The results of these calculations are included in the table below (Table 6).

Table 6. A summary of estimated values of modulation depth factor containing boundary values, expected values, calculated values and the differences between the values expected and calculated for each of the gratings examined.

	Grating 1	Grating 2	Grating 3	Grating 4	Grating 5=2	Grating 6	Grating 7	Grating 8	Grating 9=3	Grating 10	Grating 11
Grating period [nm]	530	530	530	530	530	530	530	520	530	540	550
Grating length [cm]	2	4	6	8	4	4	4	6	6	6	6
Core refractive index	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471	1.4471
Irradiation time [min]	2	2	2	2	2	4	6	2	2	2	2
Edge factor of refractive index modulation depth. strongly dependent on irradiation time	0.000047	0.000047	0.000047	0.000047	0.000047	0.000094	0.000141	0.000047	0.000047	0.000047	0.000047
Expected factor of refractive index modulation depth. calculated for reference gratings	0.000075	0.000075	0.000075	0.000075	0.000075	0.000094	0.000113	0.000075	0.000075	0.000075	0.000075
Edge factor of refractive index modulation depth. weakly dependent on irradiation time	0.000103	0.000103	0.000103	0.000103	0.000103	0.000094	0.000085	0.000103	0.000103	0.000103	0.000103
Calculated (for each grating separately) factor of refractive index modulation depth	0.000084	0.000075	0.000076	0.000099	0.000075	0.000094	0.000113	0.000087	0.000076	0.000067	0.000088
Difference of calculated value from expected for reference gratings	0.000009	0.000000	0.000001	0.000024	0.000000	0.000000	0.000000	0.000012	0.000001	0.000008	0.000013

Based on the above tests, it can be concluded that the modulation depth factor is not directly proportional to the time of grating production. This may be related to the saturation phenomenon occurring during the production of Bragg gratings [8] and to the method used for the production – the phase mask method, whose corresponding technical process may result in a small change in the refractive index in the core over the entire structure, weakening modulation [9]. Regardless of the reason for this state of affairs, two conclusions can be drawn from the above studies. First, the calculated values of the modulation depth factor are close to the expected value and have a tendency to deviate towards a weak dependence on the time of firing the grating. Secondly, different values of the coefficient for gratings with the same production time were obtained. Such differences in general may be due to imperfections in the phase masks or the lack of precision in the measurement of time of grating production. In this case, however, they are most likely the result of inaccuracies in the way they are calculated. It should not be forgotten that the values were calculated on the basis of measured characteristics burdened with measurement errors and with the use of simulations, on the basis of which the accuracy of determining the modulation depth factor is unknown.

From the above conclusions, the most important is the convergence of the value of the expected modulation depth factor with the average of its calculated values, because it means that obtained error values for simulations carried out assuming the existence of the mentioned coefficient with expected values included in the above statements (Tables 3, 4 and 5), are close to reality. Using the above data, it is also possible to determine the relative error of the calculation of the values associated with the transmission characteristics of the Bragg gratings, dividing the absolute error value by the calculated value. The most important values of both types of calculation errors for these features are listed below.

The value of absolute error of calculations of the dynamics of reflection was on average 0.042 and its maximum value was 0.163. The average value of relative error of the calculation was 14.3% and the maximum value was 35.3%. The average value of absolute error in the calculation of the reflection band-span was 0.036 nm (relative error 16.2%), and its maximum value was 0.135 nm (relative error 36.5%). The mean value and absolute error of sideband dynamics calculations was 0.0062 (the maximum value was 0.0138). However, the mean value of relative error of these calculations was 34.3% (the maximum value was 75.6%).

The above data confirm the convergence of the transmission characteristics of the Bragg gratings, which result from the simulation of these structures, with the transmission characteristics of the generated gratings (the relative error between the key features of these characteristics is on average well below 50%). Nevertheless, one must take into account that the numerical calculations carried out with the use of a simplified model are burdened with an error whose approximate values have been given above. Of course, these values are determined for the adopted set of gratings. For a different set of structures, mean error values may differ from those set above.

4. Summary and conclusions

The most important conclusion drawn from the performed research is the fact that the transmission characteristics of uniform Bragg gratings, obtained by simulation, are similar to the transmission characteristics of gratings with the same parameters obtained as a result of measurements, and in most cases – at least partly – overlap with them. This means that the transition matrix method used in the numerical calculations enables a good representation of the characteristics of Bragg gratings. Nevertheless, this method is burdened with some errors, which verify the possibility of using this mapping for particular applications. The research described in this paper enabled to determine the approximate values of calculation errors, as well as their dependence on physical parameters of the grating. As a comparative criterion, four values were adopted describing transmission characteristics: Bragg wavelength, reflection dynamics, *full width at half maximum* (FWHM) and sideband dynamics.

It was found that for the examined cases the inaccuracy of the Bragg wavelength measurement varies between 0.022 nm and 1.167 nm. This inaccuracy, after reference to the FWHM of spectrum, assumes a relative value of 18% to 810%, which means that for some cases the error size is greater than the lattice FWHM. Positive correlations were also found between the time of production of the Bragg grating and the magnitude of error and between the period of structure and this size. The first of these correlations can be reduced by using a grating model in the calculation, taking into account the elements of technological process of its production.

There were made measurements of the level of inaccuracy in the calculation of the dynamics of reflection understood as the change of the transmission coefficient for the radiation of the Bragg wavelength, which for the tested examples is from 0.0068 to 0.1629. This error in relative terms amounted to a maximum of 35.3%. The dependence of the level of inaccuracy of calculations on the grating length has been observed, which increases with increasing the grating length.

It was found that the FWHM of the transmission characteristic can be calculated with an accuracy strongly dependent on the length of the structure itself. For a 2 cm grating, the calculations are less accurate and the relative error of the calculations is almost 37%, while for a 8 cm grating this error drops to 4.3% which is a very good result. The accuracy of FWHM calculations is also weakly dependent on the grating period, because the calculation error decreases with the increase of the period.

The dependence of the accuracy of sideband dynamics calculations on the physical properties of gratings was also observed, understood as the decrease of the transmission coefficient for the corresponding waves. This error is all the greater, the longer the structure and the greater the depth of modulation of the refractive index it has, the relationships being clearly visible for the absolute error, the maximum value of which is 0.014. They are less (but still) visible for relative errors, which reach 76%.

It was shown that the transmission characteristics of gratings can be simulated with a great accuracy, as evidenced by the average relative errors of the properties described above, measured for the gratings analysed in the studies: Bragg wavelength: 219%, reflection dynamics: 14%, reflectance bandwidth: 16%, dynamics of sidebands: 34%. It is also possible to improve these parameters by using a more complicated model, or by submitting the result of calculations using the transition matrix method, compensating the impact of changes in the modulation depth of the refractive index on its average value.

In addition, on the occasion of studying the inaccuracy of calculations, the modulation depth of the refractive index was approximated, showing that it is possible for a suitably prepared set of gratings. At the same time, it was shown that the increase of this coefficient is not directly proportional to the time of grating production, which may depend on the method of grating production.

In summary, the work proved that the transition matrix method is suitable for simulation of fibre optic Bragg gratings. It was shown that the transmission characteristics obtained using numerical calculations with this method are close to those of real gratings with parameters corresponding to the structures simulated. Relations between the size of Bragg grating parameters and the effectiveness of calculations were observed. On this basis, it can be concluded that simulations of these structures can be used not only in forecasting their transmission characteristics before they are created, but also as a tool enabling to determine unknown parameters of Bragg gratings based on their characteristics.

References

[1] Yamada, M., Sakuda, K. (1987). Analysis of almost-periodic distributed feedback slab waveguide via a fundamental matrix approach. *Applied Optics*, 26(16), 3474–3478.

[2] Saeed, J.M. (2019). A memory-efficient canonical data structure for decimal floating point arithmetic systems modeling and verification. *Turkish Journal of Electrical Engineering & Computer Sciences*, 27(1), 471–483.

[3] Zhang, W., Huang, W., Li, F. (2018). High-resolution fiber Bragg grating sensor and its applications of geophysical exploration, seismic observation and marine engineering. *Guangdian Gongcheng/Opto-Electronic Engineering*, 45(9).

[4] Jacques, A., Antonello, C., Andrea, C. (2011). Fiber Bragg Grating Sensors: Recent Advancements, Industrial Applications and Market Exploitation. *Bentham Science Publishers*, 10–14.

[5] Othonos, A., Kalli, K. (1999). *Fibre Bragg gratings: fundamentals and applications in telecommunications and sensing*.

[6] Chia-Ju, Y., Tzu-Hsiang, Y., You-Cheng, L., Bing-Hao, S., Yung-Jr, H. (2018). Apodization techniques for side-lobes suppression in silicon photonics waveguide gratings. *Lasers and Electro-Optics (CLEO)*.

[7] Stępniak, P., Kisała, P. (2017). Analysis of impact long period Bragg gratings parameters on their spectral transmission characteristics. *Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2017*, *Proc. of SPIE*, 10445.

[8] Kaim, S., Mokhov, S., Divliansky, I., Zeldovich, B.Y., Glebov, L.B., Smirnov, V., Lumeau, J. (2015). Saturation of multiplexed volume Bragg grating recording. *Journal of the Optical Society of America A: Optics and Image Science, and Vision*, 32(1) 22–27.

[9] Mahakud, R., Prakash, O., Kumar, J., Nakhe, S.V., Dixit, S.K. (2012). Analysis on the effect of UV beam intensity profile on the refractive index modulation in phase mask based fibre Bragg grating writing. *Optics Communications*, 285(24), 5351–5358.