KRZYSZTOF AUGUSTYNEK [*], IWONA ADAMIEC-WÓJCIK [**],
EDMUND WITTBRODT [***]

DYNAMIC ANALYSIS OF A SATELLITE WITH FLEXIBLE LINKS

The paper presents a spatial model of the satellite antenna with an arbitrary number of flexible arms. Such a system is an example of an open kinematic chain with a tree-like structure. The modification of the rigid finite element method is used to discretise flexible links. The equations of motion are derived from the Lagrange equations and the motion of the system is described using joint coordinates and homogenous transformations. Numerical simulations have been carried out to analyse how the method of extending the arms influences the dynamics of the system.

1. Introduction

The model of the satellite is an interesting example of a dynamically coupled system, in which occur. mutual interactions between the motion of the central rigid body and the attached arms. Additionally, the arms may be flexible and may undergo large base motion, which complicates modelling the dynamics of the system. A planar model of the satellite antenna with flexible links is presented in [7]. To model the flexibility, the authors applied a hybrid method which combines the finite segment method with the finite element method. The flexible link is replaced by a series of rigid segments which represent mass features of the continuum. The segments are connected by means of massless beam elements which reflect spring features of the system. The model takes into account clearance in joints. The presented experimental verification of the deployment process shows acceptable compatibility of

---

[*] *Faculty of Management and Computer Science, University of Bielsko-Biała, Willowa 2, 43-309 Bielsko-Biała, Poland; E-mail: kaugustynek@ath.bielsko.pl*

[**] *Faculty of Management and Computer Science, University of Bielsko-Biała, Willowa 2, 43-309 Bielsko-Biała, Poland; E-mail: i.adamiec@ath.bielsko.pl*

[***] *Faculty of Mechanical Engineering, Gdańsk University of Technology, Narutowicza 11/12, 80-952, Gdańsk, Poland; E-mail: ewittbro@sunrise.pg.gda.pl*

results. The modification of the rigid finite element method used for modelling flexible arms is presented in [2, 4]. Research devoted to modelling a hub-beam system is widely reported in the literature [5, 6, 8, 10]. Such systems can also be used as a model of the satellite. This paper presents a development of the model described in [4]. There one can find a spatial model of a satellite antenna with a rigid central body and four flexible arms. In this paper, we discuss the model of the satellite with an arbitrary number of flexible arms connected by means of rotary joints.

2. Mathematical model of satellite

It is assumed that the satellite consists of a central rigid body and flexible arms connected with this body (Fig. 1). In order to describe the motion of the system, joint coordinates and homogenous transformations are used. The local coordinate system {0} is connected with the central body. The origin of the system is placed in the center of the mass and its axes coincide with the principal axes of inertia of the body. The motion of the central body with respect to the inertial global system {} is defined by the vector of generalised coordinates:

$$\tilde{\mathbf{q}}^{(0)} = \left[\begin{array}{cccccc} x^{(0)} & y^{(0)} & z^{(0)} & \varphi_x^{(0)} & \varphi_y^{(0)} & \varphi_z^{(0)} \end{array} \right]^T \tag{1}$$

The transformation matrix from system {0} to {} can be written as:

$$\mathbf{B}^{(0)} = \left[\begin{array}{cccc} c_z^{(0)}c_y^{(0)} & c_z^{(0)}s_y^{(0)}s_x^{(0)} - s_z^{(0)}c_x^{(0)} & c_z^{(0)}s_y^{(0)}c_x^{(0)} + s_z^{(0)}s_x^{(0)} & x^{(0)} \\ c_z^{(0)}c_y^{(0)} & s_z^{(0)}s_y^{(0)}s_x^{(0)} + c_z^{(0)}c_x^{(0)} & s_z^{(0)}s_y^{(0)}c_x^{(0)} - c_z^{(0)}s_x^{(0)} & y^{(0)} \\ -s_y^{(0)} & c_y^{(0)}s_x^{(0)} & c_y^{(0)}c_x^{(0)} & z^{(0)} \\ 0 & 0 & 0 & 1 \end{array} \right] \tag{2}$$
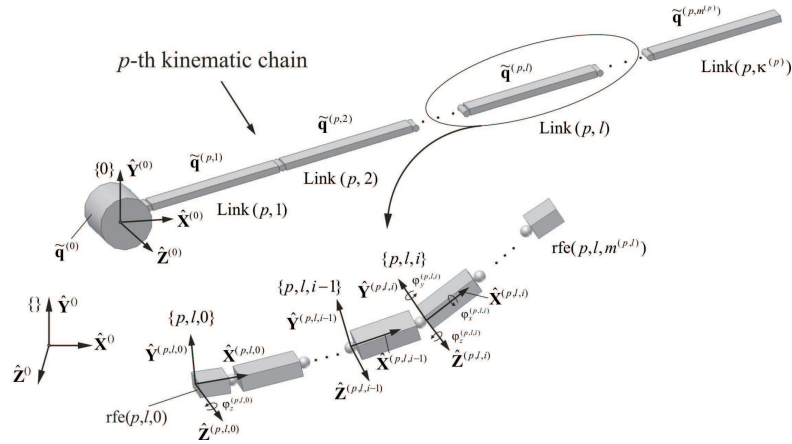
where: $s_\xi^{(0)} = \sin\varphi_\xi^{(0)}$, $c_\xi^{(0)} = \cos\varphi_\xi^{(0)}$ for $\xi \in \{x, y, z\}$.

Connected to the central body are n open kinematic chains. Each of the chains can consists of $\kappa^{(p)}$ flexible beam-like links connected by means of rotary joints. The modification of the rigid finite element method [1, 3, 9, 11, 12] is used in order to discretised flexible links. As a result, flexible link (p, l) is divided into $m^{(p,l)} + 1$ rigid finite elements (rfe) connected by $m^{(p,l)}$ non-dimensional and massless spring-damping elements (sde).

The motion of rfe(p, l, i) with respect to the preceding rfe$(p, l, i - 1)$ is defined by the following vector of generalized coordinates:

$$\tilde{\mathbf{q}}^{(p,l,i)} = \left[\begin{array}{ccc} \varphi_x^{(p,l,i)} & \varphi_y^{(p,l,i)} & \varphi_z^{(p,l,i)} \end{array} \right]^T \tag{3}$$

where: $p = 1, \ldots, n, l = 1, \ldots, \kappa^{(p)}, i = 1, \ldots, m^{(p,l)}$.

Fig. 1. Kinematic chain p with flexible link (p, l)

The respective transformation matrices from the system $\text{rfe}(p, l, i)$ to the system $\text{rfe}(p, l, i - 1)$ for $i = 1, \ldots, m^{(p,l)}$ take the form:

$$
\tilde{\mathbf{B}}^{(p,l,i)} = \begin{bmatrix} c_z^{(p,l,i)} c_y^{(p,l,i)} & c_z^{(p,l,i)} s_y^{(p,l,i)} s_x^{(p,l,i)} - s_z^{(p,l,i)} c_x^{(p,l,i)} & c_z^{(p,l,i)} s_y^{(p,l,i)} s_x^{(p,l,i)} + s_z^{(p,l,i)} s_x^{(p,l,i)} & l^{(p,l,i-1)} \\ s_z^{(p,l,i)} c_y^{(p,l,i)} & s_z^{(p,l,i)} s_y^{(p,l,i)} s_x^{(p,l,i)} + c_z^{(p,l,i)} c_x^{(p,l,i)} & s_z^{(p,l,i)} s_y^{(p,l,i)} c_x^{(p,l,i)} - c_z^{(p,l,i)} s_x^{(p,l,i)} & 0 \\ -s_y^{(p,l,i)} & c_y^{(p,l,i)} s_x^{(p,l,i)} & c_y^{(p,l,i)} c_x^{(p,l,i)} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$(4)$$

The element $(p, l, 0)$ has only one degree of freedom, which is a rotation of the kinematic pair:

$$
\tilde{\mathbf{q}}^{(p,l,0)} = \left[\varphi_z^{(p,l,0)} \right]
$$

$$(5)$$

The vector of generalised coordinates describing the motion of element (p, l, i) with respect to the inertial system can be written as:

$$
\mathbf{q}^{(p,l,i)} = \begin{bmatrix} \mathbf{q}^{(0)} \\ \breve{\mathbf{q}}^{(p,l-1)} \\ \tilde{\mathbf{q}}^{(p,l,0)} \\ \vdots \\ \tilde{\mathbf{q}}^{(p,l,i)} \end{bmatrix}
$$

$$(6)$$

where: $\breve{\mathbf{q}}^{(p,l-1)}$ – vector of generalised coordinates of links preceding link l in kinematic chain p,

$$
\breve{\mathbf{q}}^{(p,l-1)} = \left[\begin{array}{ccc} \tilde{\mathbf{q}}^{(p,1)^T} & \cdots & \tilde{\mathbf{q}}^{(p,l-1)^T} \end{array} \right]^T,
$$

$\tilde{\mathbf{q}}^{(p,k)}$ – vector of generalised coordinates of the flexible link (p, k), $k = 1, \ldots, l - 1$,

$$\tilde{\mathbf{q}}^{(p,k)} = \left[\begin{array}{cccc} \tilde{\mathbf{q}}^{(p,k,0)^T} & \tilde{\mathbf{q}}^{(p,k,1)^T} & \ldots & \tilde{\mathbf{q}}^{(p,k,m^{(p,k)})^T} \end{array} \right]^T .$$

The transformation matrix from the system $\{p, l, i\}$ to the inertial coordinate system $\{\}$ can be calculated as a product of transformation matrices of the preceding bodies in the kinematic chain:

$$\mathbf{B}^{(p,l,i)} = \mathbf{B}^{(0)}\breve{\mathbf{B}}^{(p,l-1)} \prod_{j=0}^{i} \tilde{\mathbf{B}}^{(p,l,j)} \tag{7}$$

where: $\displaystyle \breve{\mathbf{B}}^{(p,l-1)} = \prod_{k=0}^{l-1} \tilde{\mathbf{B}}^{(p,k)}$,

$$\tilde{\mathbf{B}}^{(p,k)} = \prod_{j=0}^{m^{(p,k)}} \tilde{\mathbf{B}}^{(p,k,j)}.$$

The equations of motion of the satellite are derived from the Lagrange equations:

$$\frac{d}{dt}\frac{\partial E}{\partial \dot{\mathbf{q}}} - \frac{\partial E}{\partial \mathbf{q}} + \frac{\partial V}{\partial \mathbf{q}} = \mathbf{0} \tag{8}$$

where: E, V – kinetic and potential energies respectively,

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}^{(0)} \\ \tilde{\mathbf{q}}^{(1)} \\ \tilde{\mathbf{q}}^{(2)} \\ \vdots \\ \tilde{\mathbf{q}}^{(n)} \end{bmatrix} - \text{vector of the generalised coordinates of the system,}$$

$$\tilde{\mathbf{q}}^{(p)} = \begin{bmatrix} \tilde{\mathbf{q}}^{(p,1)} \\ \vdots \\ \tilde{\mathbf{q}}^{(p,\kappa^{(p)})} \end{bmatrix} - \text{vector of the generalised coordinates of the } p\text{-th}$$

kinematic chain.

The kinetic energy of the satellite is a sum of the kinetic energies of the central body and all flexible links, and it takes the following form:

$$E = \tilde{E}^{(0)} + \sum_{p=1}^{n} \sum_{l=1}^{\kappa^{(p)}} \sum_{i=0}^{m^{(p,l)}} \tilde{E}^{(p,l,i)} \tag{9}$$

where:

$\tilde{E}^{(0)} = \dfrac{1}{2}\text{tr}\{\dot{\mathbf{B}}^{(0)}\mathbf{H}^{(0)}\dot{\mathbf{B}}^{(0)^T}\}$ – kinetic energy of the central body,

$\tilde{E}^{(p,l,i)} = \frac{1}{2}\text{tr}\{\dot{\mathbf{B}}^{(p,l,i)}\mathbf{H}^{(p,l,i)}\dot{\mathbf{B}}^{(p,l,i)^T}\}$ – kinetic energy of rfe(p,l,i),

$\mathbf{H}^{(0)}$, $\mathbf{H}^{(p,l,i)}$ – pseudo-inertial matrices of the central body and rfe(p,l,i) with elements defined in [11].

For further considerations, the potential energy of the forces of gravity is omitted. The potential energy of the system is the sum of spring deformation energy of all sdes of the system, which is described as:

$$V = \sum_{p=1}^{n} \sum_{l=1}^{\kappa^{(p)}} \sum_{i=1}^{m^{(p,l)}} \tilde{V}^{(p,l,i)} \tag{10}$$

where:

$\tilde{V}^{(p,l,i)} = \frac{1}{2}\tilde{\mathbf{q}}^{(p,l,i)^T} \mathbf{C}^{(p,l,i)} \tilde{\mathbf{q}}^{(p,l,i)}$ – potential energy of s$de(p,l,i)$,

$\mathbf{C}^{(p,l,i)}$ – stiffness matrix of bending and torsional coefficients of sde(p,l,i),

$$\mathbf{C}^{(p,l,i)} = \begin{bmatrix} c_x^{(p,l,i)} & 0 & 0 \\ 0 & c_y^{(p,l,i)} & 0 \\ 0 & 0 & c_z^{(p,l,i)} \end{bmatrix}.$$

Having substituted (9) and (10) into the Lagrange equations and performed the necessary transformation as in [11], we can present the equations of motion in the form:

$$\mathbf{A}\ddot{\mathbf{q}} + \mathbf{C}\mathbf{q} = -\mathbf{h} \tag{11}$$

where:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{0,0} & \mathbf{A}_{0,1} & \mathbf{A}_{0,2} & \dots & \mathbf{A}_{0,n} \\ \mathbf{A}_{1,0} & \mathbf{A}_{1,1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{A}_{2,0} & \mathbf{0} & \mathbf{A}_{2,2} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{n,0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}_{n,n} \end{bmatrix}$$ – mass matrix of the system,

$$\mathbf{C} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{C}_n \end{bmatrix}$$ – stiffness matrix of the system,

$\mathbf{C}^{(k)}$ – stiffness matrices of the flexible link in the k-th kinematic chain, $k = 1, \dots, n$

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}_0 \\ \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_n \end{bmatrix} \quad \text{– vector of centrifugal, gyroscopic and Coriolis forces.}$$

It is assumed that the motion of the arms of the system is realised by means of the kinematic input. This means that the rotation of the kinematic pair is a known function of time, which can be written as follows:

$$\varphi_z^{(p,l,0)} = \alpha^{(p,l)}(t) \tag{12}$$

where: $\alpha^{(p,l)}(t)$ – is a given rotary angle of rfe$(p, l, 0)$ in relative motion with respect to the preceding element.

Having integrated equations (11) twice with respect to time and included them in the equations of motion (10), one obtains the set of differential equations in the form:

$$\begin{aligned} \mathbf{A\ddot{q}} + \mathbf{DM} &= -\mathbf{h} \\ \mathbf{D}^T \mathbf{\ddot{q}} \quad\quad &= -\mathbf{\Gamma} \end{aligned} \tag{13}$$

where: $\mathbf{D}$ – matrix of coefficients after double integration of constraint equations (11),

$\mathbf{\Gamma}$ – vector of second derivatives of drive functions $\alpha^{(p,l)}(t)$,

$\mathbf{M}$ – vector of unknown moments which realise the kinematic input.

3. Dynamic analysis of the deployment process

For numerical simulations, it has been assumed that the satellite consists of the central body and two kinematic chains, each of which consists of two flexible links. The scheme of the system with denotations is presented in Fig. 2.

Data for the central body and flexible links are the same as in [4], where the results of indirect verification of the model, by means of comparison with MSC.Adams, were also presented. Good correspondence between the results proved the correctness of the model formulated using the rigid finite element method. In this paper, the influence of different methods of extending the arms on the behavior of the system is analysed. Three variants of kinematic input are considered. For all cases it is assumed that all the arms should be extended to the final position in 4 s (Fig. 2). Courses of kinematic input for all the variants are presented in Fig. 3.
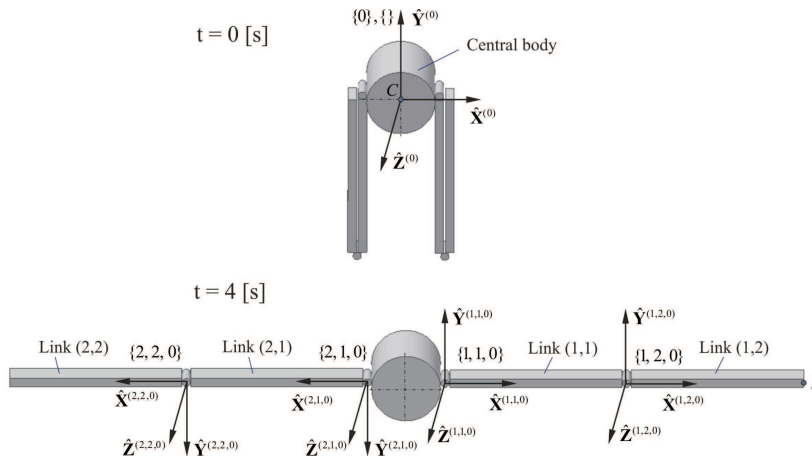
Fig. 2. Initial and final configurations of the satellite with the coordinate systems assumed
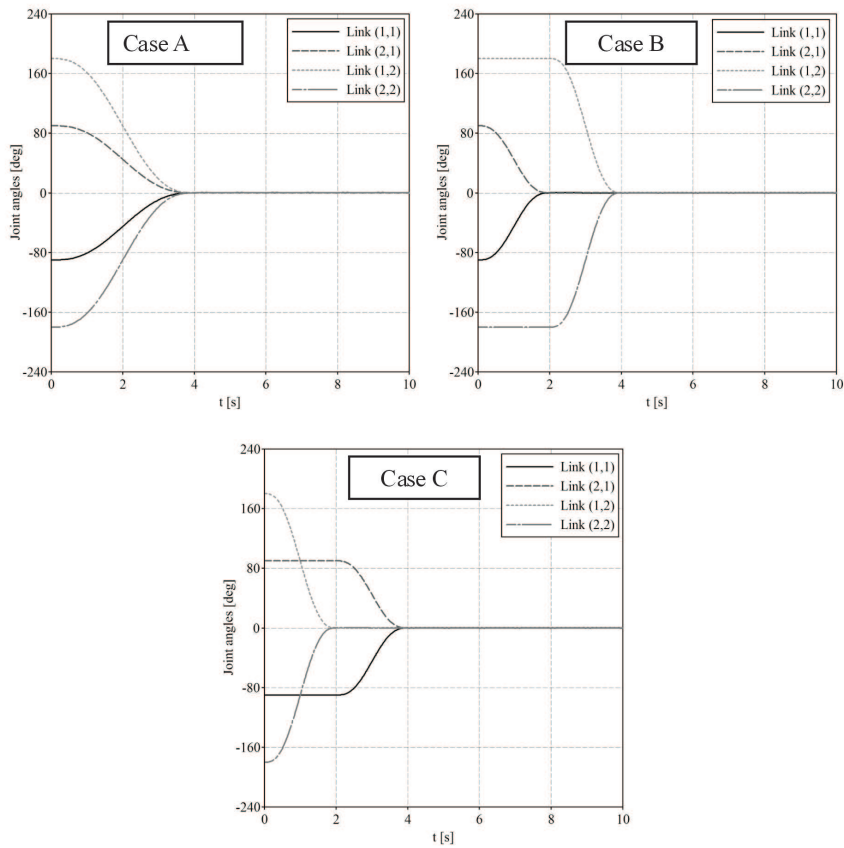

Fig. 3. Variants of kinematic inputs analysed

For case A, it is assumed that all the arms extend simultaneously in 4 s. Case B describes the situation in which the motion of the arms is performed in two phases. During the first 2 s, links (1, 1) and (2, 1) extend and then, in the next 2 s, the remaining links extend. Case C is similar but the arms extend in the reverse order – first the external (1, 2) and (2, 2) links and then the internal ones. The displacements of the central body and the end of link (1, 2) for all cases are presented in Fig. 4 and Fig. 5 respectively.
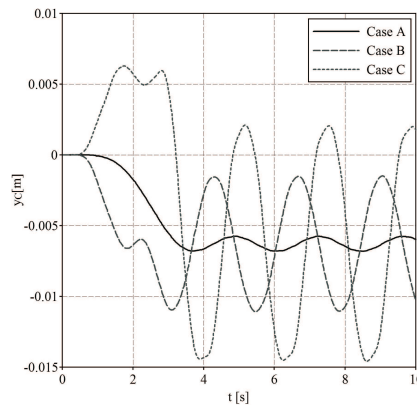

Fig. 4. Displacements of the centre of mass of the central body with respect to the Y axis
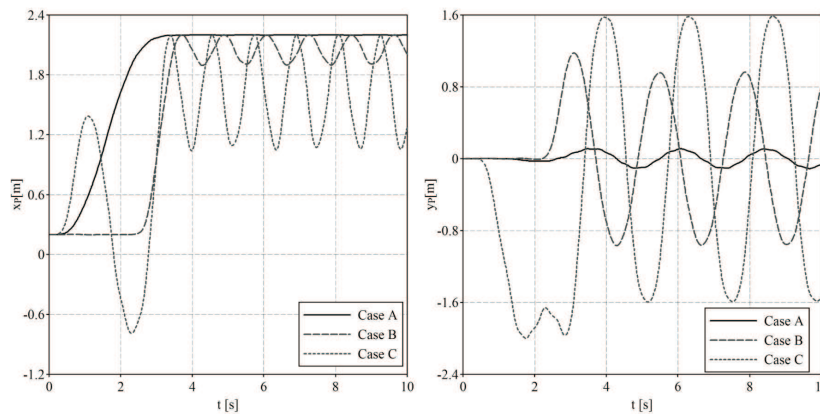

Fig. 5. Displacements of the end of link (1, 2) along X, Y axes in the system {0}

The results of numerical simulations show that the largest vibrations of the central body and flexible links occur in cases B and C. The amplitude of vibrations of the end of the external links reaches 1 or 1.6 meters, respectively, for cases B and C. This shows that the drive functions used for the deployment process have a considerable influence on the behavior of

the system causing, in some cases, large displacements of the flexible arms. It can be seen that the smallest influence of the centrifugal forces on the systems is in case A, when all the links extend simultaneously.

4. Final remarks

The paper presents the application of the modification of the rigid finite element method together with joint coordinates and homogenous transformations to modelling a satellite antenna with flexible arms. This approach enables us to take into account large base motion, large displacements of flexible links and mutual interactions of base motion and vibrations of flexible links. The analysis of different variants of extending the arms of the satellite is presented. It is shown that inadequate realisation of the deployment process can cause additional vibrations in the system, and can even lead to large displacements of the arms. For numerical simulations, kinematic input has been assumed. In fact, because of the flexibility and clearance which can occur in joints, the motion of the arms may not be carried out simultaneously. Thus, the moments realising the drive functions have to be controlled and this will be a subject of further research.

REFERENCES

[1] Adamiec-Wójcik I.: Modelling dynamics of multibody systems using homogenous transformations. Wydawnictwo ATH, Rozprawy Naukowe, Nr 3, Bielsko-Biała 2003.

[2] Adamiec-Wójcik I., Wittbrodt E., Wojciech S.: Modelling of large deflections of beam-like links by means of the rigid finite element method. Multibody Dynamics 2007, ECCOMAS Thematic Conference, Milano, Italy.

[3] Adamiec-Wójcik I., Maczyński A., Wojciech S.: Zastosowanie metody przekształceń jednorodnych w modelowaniu urządzeń offshore. Wydawnictwa Komunikacji i Łączności, 2008.

[4] Augustynek K., Adamiec-Wójcik I., Wittbrodt E.: Analiza dynamiczna anteny satelitarnej z uwzględnieniem podatności członów. Teoria Maszyn i Mechanizmów. Wydawnictwo ATH, Bielsko Biała, 2008.

[5] Cai G.-P., Lim C.W.: Optimal tracking control of a flexible hub-beam system with time delay. Multibody System Dynamics, 16, 2006, pp.331-350.

[6] Chaolan Y., Jaizhen H., Guoping C.: Modeling study of a flexible hub-beam system with large motion and with consideration the effect of shear deformation. Journal of Sound and Vibration, 295, 2006, pp.282-293.

[7] Cheng Y., Dai S.L., Ren G.X.: A hybrid finite segment/finite element modeling and experiment on a flexible deployment system. Journal of Sound and Vibration, 258(5), 2002, pp. 931-949.

[8] Hamdan M.N., EL-Sinawi A.H.: On the non-linear vibrations of an inextensible rotating arm with setting angle and flexible hub. Journal of Sound and Vibration, 281, 2005, pp.375-398.

[9] Kruszewski J., Gawroński W., Wittbrodt E., Najbar F., Grabowski S.: Metoda sztywnych elementów skończonych. Arkady, Warszawa 1975.

[10] Liu J-Y., Lu H.: Rigid flexible coupling Dynamics of three-dimensional hub-beam system. Multibody System Dynamics, 18, 2007, pp.487-510.

[11] Wittbrodt E., Adamiec-Wójcik I., Wojciech S.: Dynamics of flexible multibody systems, Rigid finite element method. Springer, 2006.

[12] Wojciech S.: Dynamika płaskich mechanizmów dźwigniowych z uwzględnieniem podatności ogniw oraz tarcia i luzów w węzłach. Rozprawy Naukowe, Nr 66, Łódź 1984.

Analiza dynamiczna satelity o podatnych członach

S t r e s z c z e n i e

W artykule przedstawiono model przestrzenny satelity złożonego z dowolnej liczby podatnych ramion. Układ taki jest przykładem otwartego łańcucha kinematycznego o strukturze drzewa. Do dyskretyzacji członów podatnych zastosowano modyfikację metody sztywnych elementów skończonych. Równania ruchu wyprowadzono z równań Lagrange'a II rodzaju, stosując do opisu ruchu ciał współrzędne złączowe oraz przekształcenia jednorodne. W symulacjach numerycznych analizowano wpływ różnych sposobów rozkładania ramion na dynamikę układu.