

# ORIGINAL MODEL FOR ESTIMATING THE WHOLE LIFE COSTS OF BUILDINGS AND ITS VERIFICATION

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The model for estimating the whole life costs of the building life cycle that allows the quantification of the risk addition lets the investor to compare buildings at the initial stage of planning a construction project in terms of the following economic criteria: life cycle costs (LCC), whole life costs (WLC), life cycle equivalent annual costs (LCEAC) and cost addition for risk ( $\Delta R_{LCC}$ ). The subsequent stages of the model development have been described in numerous publications of the authors, while the aim of this paper is to check the accuracy of the model in the case of changing the parameters that may affect the results of calculations. The scope of the study includes: comparison of the results generated by the model with the solutions obtained in the life cycle net present value method (LCNPV) for time and financial input data, not burdened with the risk effect; the analysis of the variability of results due to changes in input data; analysis of the variability of results as a consequence of changing the sets of membership functions for input data and methods for defuzzification the result.

*Keywords:* life cycle costs, whole life costs, life cycle equivalent annual costs, cost addition for risk, risk, building

## 1. INTRODUCTION

Striving to minimize the costs related to the implementation of buildings is the subject of interest to every investor, while limiting the costs related to their use, maintenance and withdrawal (for instance, decommissioning by demolition) is not an issue that has been discussed in the past as often as today, as discussed in publications [1, 3, 4, 5, 13, 14].

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The issue of life cycle costs is part of the Sustainable Development Strategy, which is the special focus of attention of the European Union, and the integrated product approach (also referring to buildings) is currently considered as the most effective way to implement the environmental dimension of this strategy. Today's requirement that construction projects be economically and environmentally effective in their whole life cycle makes the issue of life cycle costs of buildings become an increasingly common element of comprehensive analyses which may involve the impact on the environment, energy consumption, society or the impact of risk [2, 6, 8, 9, 11, 15, 16].

The authors have therefore attempted to develop a model for estimating the whole life costs of buildings enabling the quantification of cost addition for risk, which would allow the investor to compare buildings in terms of a number of economic criteria: life cycle costs (LCC – when the investor is not able to record incomes), whole life costs (WLC – when, in addition to bearing the costs during the life cycle of the building, the investor is able to record incomes), life cycle equivalent annual costs (LCEAC – when the durations of the operation phase differ) and cost addition for risk ( $\Delta R_{LCC}$  – expressed as a difference in currency units between the life cycle cost of a building which takes into account the impact of risk and the life cycle cost of a building that does not include this impact). The subsequent stages of the development of the model are described in numerous publications by the authors [10, 11, 12, 13, 17, 19].

The aim of this paper is, however, to check the accuracy of the model in estimating the whole life costs of buildings in the case of changing parameters that may affect the results of calculations. The scope of the research includes: comparison of the results generated by the model with the results obtained by the net present value in the life cycle (LCNPV) for the time and financial input data not influenced by risk; analysis of variability of results due to changes in input data; analysis of the variability of results as a result of changing sets of membership functions for input data and methods for defuzzification the result.

## **2. ORIGINAL MODEL FOR ESTIMATING THE WHOLE LIFE COSTS OF BUILDINGS**

In the model of estimating the whole life cost of the building that allows the quantification of the risk addition, the theory of fuzzy sets is combined with the most common, dynamic method used to analyse the economic effectiveness of construction projects on the basis of discounted cash flows (dCF). This is a net present worth method (NPW), also called net present value method (NPV), which in the model is applicable in the fuzzy version (fuzzy NPW method). The calculations in the

model are based on: the discounted cash flows (dCF) analysis at a specified discount rate ( $r$ ) value; the net present worth (value) indicator which is the difference between discounted cash flows (dCF) and initial investments; the concept of fuzzy numbers; the theorem on decomposition of a fuzzy set into  $\alpha$ -cuts; Zadeh's extension principle.

A set of calculation formulas, thanks to which it is possible to calculate economic indicators: LCC, WLC, LCEAC and  $\Delta R_{LCC}$ , was presented and discussed in detail in the following authors' publications [10, 11, 18, 19]. The examples below include only the most important equations. Equation (1.1) calculates the value of criterion LCC. Equation (1.2) is used to calculate incomes in the life cycle of the building (ILC). After the deduction of the LCC from ILC, the WLC criterion is established.

$$(1.1) \quad \overline{LCC}_i = \overline{C}_{in,i} + \overline{PWF}_{AC,i} \cdot \sum_{j=1}^{n_{AC,j}} \overline{C}_{opA,ij} + \sum_{k=1}^{n_{NAC,i}} \overline{C}_{opNA,ik} \cdot \overline{PWF}_{NAC,ik} + \overline{PWF}_{WD,i} \cdot \overline{C}_{wd,i}$$

where:

$\overline{LCC}_i$  – the amount of the life cycle costs of the  $i$ -th building,  $\overline{C}_{in,i}$  – the amount of initial costs,  $\overline{C}_{opA,ij}$  – the amount of the  $j$ -th annual operating cost,  $\overline{C}_{opNA,ik}$  – the size of the  $k$ -th periodic operational cost,  $\overline{C}_{wd,i}$  – the amount of decommission costs,  $n_{AC,i}$ ,  $n_{NAC,i}$  – the respective number (multiplicity) of operating costs of an annual and periodic nature,  $\overline{PWF}_{AC,i}$  – the value of the discount factor for annual operating costs,  $\overline{PWF}_{NAC,ik}$  – the value of the discount factor for the  $k$ -th periodic operational cost,  $\overline{PWF}_{WD,i}$  – the value of the discount factor for revenue achieved in the decommission phase; the overline in the formula means that the given value is a fuzzy number.

$$(1.2) \quad \overline{ILC}_i = \overline{PWF}_{AI,i} \cdot \sum_{l=1}^{n_{AI,i}} \overline{I}_{opA,il} + \sum_{m=1}^{n_{NAI,i}} \overline{I}_{opNA,im} \cdot \overline{PWF}_{NAI,im} + \overline{PWF}_{WDI,i} \cdot \overline{I}_{wd,i}$$

where:

$\overline{ILC}_i$  – the amount of incomes in the life cycle of the  $i$ -th building,  $\overline{I}_{opA,il}$  – the amount of the  $l$ -th annual income,  $\overline{I}_{opNA,im}$  – the amount of the  $m$ -th periodic income,  $\overline{I}_{wd,i}$  – the amount of income achieved during the decommission phase,  $n_{AI,i}$ ,  $n_{NAI,i}$  – the respective number (multiplicity) of operational incomes of an annual and periodic nature,  $\overline{PWF}_{AI,i}$  – the value of the discount factor for annual incomes,  $\overline{PWF}_{NAI,im}$  – the value of the discount factor for the  $m$ -th periodic income,  $\overline{PWF}_{WDI,i}$  – the value of the discount factor for income achieved in the decommission phase; the overline in the formula means that the given value is a fuzzy number.

The amount of the LCEAC criterion is the product of LCC and the AF factor (annual discount factor – annuity factor). The cost addition for risk ( $\Delta R_{LCC}$ ) criterion is calculated as the difference between the amount of life cycle costs of a building, which takes into account the impact of risk (LCC), and the value of life cycle costs of a building, which does not include this impact ( $LCC^{REF}$ ). Both amounts mentioned here ( $LCC$ ,  $LCC^{REF}$ ) are calculated with equation (1.1).

The input data for the model estimating the whole life costs of buildings that allows the quantification of the cost addition for risk are divided into the following parameters:

- temporal CG (of a global character), that is, the duration of the life cycle of the building  $T_i$ , where  $T_i$  equals the estimated service life of a building (ESLB),
- temporal CL (of a local character), that is, times  $t_{ik}$ ,  $t_{im}$ , after which the  $k$ -th periodic operating cost or  $m$ -th periodic income is calculated accordingly,
- financial FG (of a global character), in the form of a discount rate ( $r$ ), which is necessary to calculate the net present value of a given monetary amount based on its value determined in future time,
- financial FK (understood as costs that may occur in the life cycle of a building), among which the annual costs are distinguished – annual operating costs  $C_{opA,ij}$  and periodic – consecutively, initial costs  $C_{in,i}$ , periodic operating costs  $C_{opNA,ik}$  and decommission costs  $C_{wd,i}$ ,
- financial FP (understood as revenues that may occur in the life cycle of a building), namely incomes of annual character – annual  $I_{opA,il}$  and periodic – consecutively, periodic incomes obtained during the building operation phase  $I_{opNA,im}$  and the incomes obtained during the decommission phase  $I_{wd,i}$ .

To parameterize the above-mentioned input data in the model for estimating the whole life costs of buildings convex and normal fuzzy numbers were used. Fig. 1 presents the basic set of membership functions of fuzzy numbers (functions with piecewise linear charts) for input data not affected by risk (Fig. 1.a), discount rate (Fig. 1.b) and time parameters CG and CL, as well as financial FG and FL in the case when the impact of risk on the amount of a given parameter is taken into account (Fig. 1.c and Fig. 1.d).

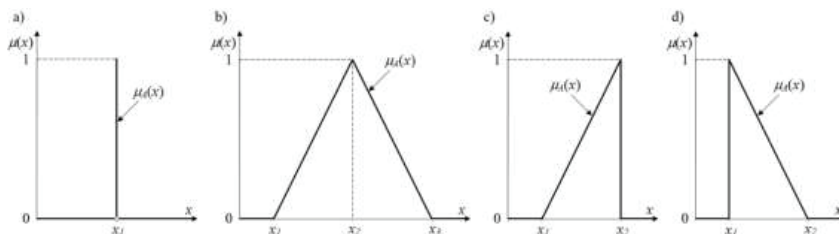


Fig. 1. Membership functions of fuzzy numbers for input parameters – basic set

### 3. VERIFICATION STUDIES AND SENSITIVITY ANALYSIS OF THE MODEL

#### 3.1. CALCULATION EXAMPLE (LIFE CYCLE COSTS ANALYSIS – LCCA)

The calculation example concerns the comparison of three alternative solutions of a multi-family building, which result from the need to take into account the impact of the identified risk factors in its life cycle. This example is also the basis for further verification tests and sensitivity analyses.

An economic life cycle analysis (LCCA) was performed for the investor (a Cracow housing cooperative) taking into account all the comparative criteria covered by the model's operation. Both the example and the results are discussed in detail in doctoral thesis [18].

After conducting an assessment of the impact of identified risk factors  $Z_1$  ("errors in designs") and  $Z_2$  ("incorrect assumptions for construction and material solutions") on the amount of the corresponding life cycle cost components of the building, life cycle scenarios have been defined for all alternative building solutions including the need for adoption of the way of reacting to identified and assessed risk factors and building management strategies during the operation phase due to the types of maintenance in accordance with the assumptions of the ISO standard 15686-5:2008 [7].

In three scenarios of the building's life cycle, the following three types of maintenance were proposed: prevention of structural deterioration (preventive maintenance in scenario  $i = 1$ ), repair-oriented maintenance (corrective maintenance in scenario  $i = 2$ ) and maintenance postponed until repairs are made, but only when the repair should be classified as urgent, that is, one that can have significant consequences for the life cycle cost of the building (deferred maintenance in scenario  $i = 3$ ). Moreover, scenarios  $i = 1$  and  $i = 2$  assume that total sale of residential space will take place in the first five years of building operation, and annual revenues will be those obtained from rents for

housing space and for renting service areas. In scenario  $i = 3$  the assumption of a long-term rental of the living space (with a constant rent indicator at the level of 85%) was adopted.

Table 1 presents the parameter values adopted for the LCCA analysis for each defined life cycle scenario of an alternative building solution. Fig. 2 illustrates an exemplary graph of results obtained for the economic criterion of life cycle costs (LCC).

Table 1. Data accepted for LCCA analysis

Parameters		Scenario $i = 0$	Scenario $i = 1$	Scenario $i = 2$	Scenario $i = 3$
Life cycle duration ( $T_i = \text{ESLB}$ )		50 yrs.	50 yrs.	no more than 60 yrs. ( $\Gamma$ class m.f., Fig. 1.c)	50 yrs.
Discount rate ( $r$ )		about 8 % (triangular m.f., Fig. 1.b)			
Initial costs ( $C_{in,i}$ )		7 206 100 PLN	no more than 7 427 400 PLN ( $\Gamma$ class m.f., Fig. 1.c)	no more than 7 333 700 PLN ( $\Gamma$ class m.f., Fig. 1.c)	7 206 100 PLN
Annual operating costs ( $C_{opA,ij}$ )		101 400 PLN	101 400 PLN	103 800 PLN	99 000 PLN
Periodic operating costs ( $C_{opNA,ik}$ ) after the time of ( $t_{ik} = \dots$ )	10	227 400 PLN	227 400 PLN	272 500 PLN	182 300 PLN
	20	227 400 PLN	227 400 PLN	272 500 PLN	182 300 PLN
	30	909 700 PLN	909 700 PLN	no more than 1 090 000 PLN ( $\Gamma$ class m.f., Fig. 1.c)	182 300 PLN
	40	227 400 PLN	227 400 PLN	272 500 PLN	no more than 1 270 300 PLN ( $\Gamma$ class m.f., Fig. 1.c)
	50	not applicable	not applicable	272 500 PLN	not applicable
Decommission costs ( $C_{wd,i}$ ) after ( $T_i = \text{ESLB}$ )		521 400 PLN	547 500 PLN	547 500 PLN	0 PLN
Annual incomes ( $I_{opA,ii}$ ) to and after the time of ( $t_{im} = \dots$ ) (rents)	30	not applicable	201 800 PLN	208 600 PLN	not applicable
		not applicable	208 600 PLN	215 400 PLN	not applicable
	40	not applicable	not applicable	not applicable	1 037 400 PLN
		not applicable	not applicable	not applicable	1 070 100 PLN
Periodic operational incomes ( $I_{opNA,im}$ ) after the time of ( $t_{im} = \dots$ ) (sale of flats)	1	not applicable	8 000 300 PLN	7 899 500 PLN	not applicable
	2	not applicable	2 461 600 PLN	2 430 600 PLN	not applicable
	3	not applicable	1 230 800 PLN	1 215 300 PLN	not applicable
	4	not applicable	492 300 PLN	486 100 PLN	not applicable
	5	not applicable	123 000 PLN	121 500 PLN	not applicable
Incomes ( $I_{wd,i}$ ) from the sale of the plot over time (ESLB)		not applicable	1 940 000 PLN	1 940 000 PLN	not applicable
Incomes ( $I_{wd,i}$ ) from the sale of the plot with the building over time (ESLB)		not applicable	not applicable	not applicable	10 297 000 PLN

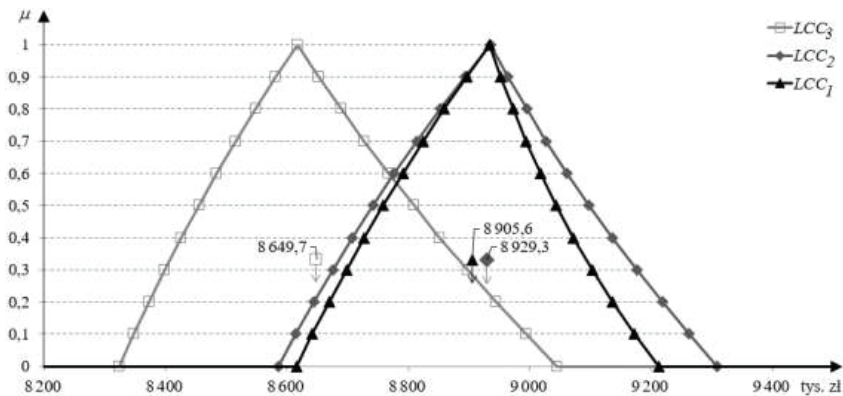


Fig. 2. Resulting distributions of membership for the LCC criterion – basic set

The LCCA analysis revealed that:

- the lowest (most favourable) value of the LCC is ensured by the implementation of scenario  $i = 3$  ( $LCC_3 = 8\,649\,700$  PLN), and the highest by scenario  $i = 2$  ( $LCC_2 = 8\,929\,300$  PLN),
- the lowest (most favourable) value of LCEAC is obtained for scenario  $i = 3$  ( $LCEAC_3 = 716\,900$  PLN), and the highest for scenario  $i = 1$  ( $LCEAC_1 = 737\,300$  PLN),
- the highest (most favourable) value of WLC is generated by the implementation of scenario  $i = 1$  ( $WLC_1 = 4\,611\,700$  PLN), and the lowest by scenario  $i = 2$  ( $WLC_2 = 4\,540\,400$  PLN),
- the lowest (most favourable) value of  $\Delta R_{LCC}$  is achieved by the implementation of scenario  $i = 3$  ( $\Delta R_{LCC,3} = -107\,900$  PLN), and the highest by scenario  $i = 2$  ( $\Delta R_{LCC,2} = 171\,700$  PLN).

In addition, it can be emphasized that in the case of scenario  $i = 3$ , there may be circumstances in the life cycle of the building that, due to the risk involved, will not burden the investor with additional costs (a loss, when  $\Delta R_{LCC}$  is greater than 0), but instead they will create an over-profit (benefit, when  $\Delta R_{LCC}$  is lower than 0).

### 3.2. MODEL VERIFICATION BY MEANS OF LIFE CYCLE NET PRESENT VALUE

The model of estimating the whole life costs of the building enabling the quantification of the risk addition was checked by means of a deterministic method, life cycle net present value (LCNPV), the generalized formula of which has the following form (3.1).

$$(3.1) \quad LCNPV = \sum_{i=0}^n \frac{CF_i}{(1+r)^i}$$

where:

$CF_i$  – cash flow in  $i$ -th year,  $n$  – number of years in the life cycle,  $i$  – subsequent year,  $r$  – discount rate.

The tests were performed in terms of the convergence of results generated by the model of estimating the whole life costs of the building with the results obtained by the LCNPV method. The following assumptions were adopted:

- calculating the amount of the economic comparison criteria LCC, LCEAC, WLC and  $\Delta R_{LCC}$  for the reference scenario and all the  $i$ -th building life cycle scenarios, which are presented in the calculation example (chapter 3.1),
- modelling time and financial input data as certain values (not affected by risk),
- applying singleton membership functions in the estimation model of the building whole life costs (Fig. 1.a), both for temporal and financial data,
- accepting a fixed value of the discount rate ( $r = 8\%$ ),
- adopting maximum values for financial data according to Table 1.

The results of the simulation are presented in Table 2.

Table 2. Results of the verification of the model for estimating the total cost of the building life using the LCNPV method

Comparative criterion	Model for estimating the whole life costs of buildings	LCNPV method	Differences	
LCC	$LCC_0 = 8\,712\,682$ PLN	$LCC_0 = 8\,712\,682$ PLN	0 PLN	0%
	$LCC_1 = 8\,934\,538$ PLN	$LCC_1 = 8\,934\,538$ PLN	0 PLN	0%
	$LCC_2 = 8\,935\,152$ PLN	$LCC_2 = 8\,935\,152$ PLN	0 PLN	0%
	$LCC_3 = 8\,617\,357$ PLN	$LCC_3 = 8\,617\,357$ PLN	0 PLN	0%
LCEAC	$LCEAC_1 = 730\,335$ PLN	$LCEAC_1 = 730\,335$ PLN	0 PLN	0%
	$LCEAC_2 = 721\,942$ PLN	$LCEAC_2 = 721\,942$ PLN	0 PLN	0%
	$LCEAC_3 = 704\,407$ PLN	$LCEAC_3 = 704\,407$ PLN	0 PLN	0%
WLC	$WLC_1 = 4\,522\,903$ PLN	$WLC_1 = 4\,522\,903$ PLN	0 PLN	0%
	$WLC_2 = 4\,476\,296$ PLN	$WLC_2 = 4\,476\,296$ PLN	0 PLN	0%
	$WLC_3 = 4\,303\,305$ PLN	$WLC_3 = 4\,303\,305$ PLN	0 PLN	0%
$\Delta R_{LCC}$	$\Delta R_{LCC,1} = 221\,856$ PLN	$\Delta R_{LCC,1} = 221\,856$ PLN	0 PLN	0%
	$\Delta R_{LCC,2} = 222\,471$ PLN	$\Delta R_{LCC,2} = 222\,471$ PLN	0 PLN	0%
	$\Delta R_{LCC,3} = -95\,325$ PLN	$\Delta R_{LCC,3} = -95\,325$ PLN	0 PLN	0%



### 3.3. INVESTIGATION ON THE EFFECT OF CHANGING THE VALUES OF TEMPORAL AND FINANCIAL DATA ON CALCULATION RESULTS

For the purpose of studying the impact of changes in the values of temporal and financial input data on the results of calculations generated by the model for estimating the whole life costs of the building that allows the quantification of the risk addition the ISO 15686-5:2008 standard proposal was used [7] and the results for economic comparative criteria were simulated (LCC, LCEAC, WLC and  $\Delta R_{LCC}$ ) for a selected scenario of the life cycle of an alternative building solution from the example shown in chapter 3.1. The following scope of changes was adopted related to the parameterization of temporal and financial data in relation to the data originally accepted for the LCCA analysis for scenario  $i = 2$ :

- the value of discount rate ( $r$ ) – change by  $\pm 1\%$  (from about 8% to about 7% and about 9%),
- life cycle duration ( $T_2$ ) – change by  $\pm 5$  years (from no more than 60 years to no more than 55 years and no more than 65 years),
- the amount of initial costs ( $C_{in,2}$ ) – change by  $\pm 10\%$  (from no more than PLN 7 333 700 to no more than PLN 6 600 330 and no more than PLN 8 067 070),
- the amount of operation costs ( $C_{opNA,230}$ ) – change by  $\pm 10\%$  (from no more than 1 090 000 PLN to no more than 981 000 PLN and no more than 1 199 000 PLN),

while it is assumed that the results will be generated only for the basic set of the membership function using the centre of gravity method (CoG) to defuzzification the resulting value.

Table 3 presents the results of the simulation of the impact of changing the value of input data on the calculation results. It should be noted that the changes in the value of temporal and financial data for the life cycle duration, the discount rate, initial costs and operating costs, were made assuming that combinations of changes of many parameters will not be taken into account. Thus, 8 combinations were considered, which are specified in Table 3.

Table 3. Results of the simulation of the impact of changing the value of temporal and financial input data on the calculation results

Parameters	Mod.	Result values for comparative criteria							
		LCC <sub>2</sub>		LCEAC <sub>2</sub>		WLC <sub>2</sub>		$\Delta R_{LCC,2}$	
		PLN	%	PLN	%	PLN	%	PLN	%
Reference values (scenario $i = 2$ )	not appl.	8 935 152	not appl.	735 000	not appl.	4 540 400	not appl.	171 700	not appl.
Life cycle duration ( $T_2$ )	-5 yr.	8 921 900	-0,08%	741 200	+0,84%	4 545 200	+0,11%	174 000	+1,34%
	+5 yr.	8 934 700	+0,06%	730 700	-0,59%	4 536 900	-0,08%	170 200	-0,87%
Discount rate ( $r$ )	-1%	9 185 800	+2,87%	799 300	+8,75%	4 821 300	+6,19%	193 000	+12,41%
	+1%	8 727 400	-2,26%	674 500	-8,23%	4 291 800	-5,48%	156 600	-8,79%

Initial costs ( $C_{in,2}$ )	-10%	8 200 200	-8,17%	675 400	-8,11%	5 269 500	+16,06%	163 200	-4,95%
	+10%	9 658 400	+8,17%	794 700	+8,12%	3 811 300	-16,06%	180 200	+4,95%
Operation costs ( $C_{opNA,230}$ )	-10%	8 918 500	-0,12%	734 100	-0,12%	4 551 300	+0,24%	170 700	-0,58%
	+10%	8 940 100	+0,12%	736 000	+0,14%	4 529 600	-0,24%	172 700	+0,58%

As the analysis of the results reveal, the model of estimating the whole life costs of the building enabling the quantification of the cost addition for risk generated the size of the LCC, LCEAC, WLC and  $\Delta R_{LCC}$  comparison criteria for a selected building life cycle scenario ( $i = 2$ ), which differ from the values obtained for reference values.

By changing the value of each input, the following results were obtained, according to which:

- shortening the life cycle duration ( $T_2$ ) by 5 years does not have a significant impact on the life cycle costs of the building ( $LCC_2$  – reduction by 0,08%), the life cycle equivalent annual cost of the building ( $LCEAC_2$  – increase by 0,84%) and the whole life costs ( $WLC_2$  – increase by 0,11%); also in the case of the cost addition for risk ( $\Delta R_{LCC,2}$ ), the increase in its value by 1,34% can be considered insignificant,
- lengthening the life cycle duration ( $T_2$ ) by 5 years does not have a significant impact on the value of  $LCC_2$  (increase by 0,06%),  $LCEAC_2$  (reduction by 0,59%) and  $WLC_2$  (reduction by 0,08%); also in the case of the  $\Delta R_{LCC,2}$  criterion, the reduction of its value by 0,87% can be considered insignificant,
- reduction of the discount rate ( $r$ ) by 1% does not have a significant impact on the value of  $LCC_2$  (increase by 2,87%); in the case of the criteria  $LCEAC_2$  (increase by 8,75%),  $WLC_2$  (increase by 6,19%) or  $\Delta R_{LCC,2}$  (increase by 12,41%), it should be concluded that the criteria listed here will be more sensitive if the discount rate changes more than  $\pm 1\%$ ,
- increase of the discount rate ( $r$ ) by 1% does not have a significant impact on the value of  $LCC_2$  (reduction by 2,26%); in the case of the criteria  $LCEAC_2$  (reduction by 8,23%),  $WLC_2$  (reduction by 5,48%) and  $\Delta R_{LCC,2}$  (reduction by 8,79%), it should be concluded that the criteria listed here will be more sensitive if the discount rate changes more than  $\pm 1\%$ ,
- reduction of initial costs ( $C_{in,2}$ ) by 10% causes a reduction in the value of  $LCC_2$  by 8,17%,  $LCEAC_2$  by 8,11% and  $\Delta R_{LCC,2}$  by 4,95%; in the case of the  $WLC_2$  criterion, its value increases by 16,06%; it should be concluded that all of the criteria listed here will be more sensitive if the value of initial costs changes to one greater than  $\pm 10\%$ ,
- increase of initial costs ( $C_{in,2}$ ) by 10% causes an increase in the value  $LCC_2$  by 8,17%,  $LCEAC_2$  by 8,12% and  $\Delta R_{LCC,2}$  by 4,95%; in the case of the  $WLC_2$  criterion, its value decreases by 16,06%; it should be concluded that all of the criteria listed here will be more sensitive if the value of initial costs changes to one greater than  $\pm 10\%$ ,

- reduction of operation costs ( $C_{opNA,230}$ ) by 10% causes a reduction in the value of  $LCC_2$  by 0,12%,  $LCEAC_2$  by 0,12% and  $\Delta R_{LCC,2}$  by 0,58%; in the case of the  $WLC_2$  criterion, its value increases by 0,24%; it should be concluded that all of the criteria listed here do not have a significant impact on the change in the resulting values,
- increase of operation costs ( $C_{opNA,230}$ ) by 10% causes an increase in the value  $LCC_2$  by 0,12%,  $LCEAC_2$  by 0,14% and  $\Delta R_{LCC,2}$  by 0,58%; in the case of the  $WLC_2$  criterion, its value decreases by 0,24%; it should be concluded that all of the criteria listed here do not have a significant impact on the change in the resulting values.

### 3.4. INVESTIGATION OF THE SENSITIVITY OF THE MODEL ON THE POSSIBILITY OF CHANGING PARAMETERS AFFECTING CALCULATION RESULTS

For temporal and financial data which in the calculation example (chapter 3.1) were modelled as uncertain values, the basic set of membership functions was changed to other alternative function sets:

- alternative I, which includes piecewise square functions,
- alternative II, created by harmonic functions,
- alternative III, made of the most complex Gaussian membership functions.

It was also assumed that the results should be verified by two defuzzification methods, that is, the centre of gravity method (CoG – the basic method) or the area compensation method (AC – the alternative method).

In the case of the parameterization of temporal and financial data for the alternative set of membership functions no. III (the Gaussian functions), the width of the curve distribution  $\sigma$  for fuzzy numbers was chosen in such a way that the degrees of membership  $\mu(x)$  in  $\alpha$ -cuts 0; 0,5 and 1 were equal or close to these values.

For each of the 84 combinations (8 cases for each comparative criterion LCC, LCEAC and WLC for the scenarios of building life cycle from  $i = 1$  to  $i = 3$  and 4 cases for the  $\Delta R_{LCC}$  criterion), sharp results were calculated. The results obtained were tabulated with the following values: arithmetic average ( $m$ ), standard deviation ( $s$ ) and coefficient of variation ( $V$ ), which was calculated by means of equation (3.2).

$$(3.2) \quad V = \frac{s}{|m|}$$

where:

m – arithmetic average, s – standard deviation.

Table 4 and table 5 reveal the results obtained for all cases of 84 combinations of membership function sets and methods for defuzzification the resulting value for the economic comparison criteria (LCC, LCEAC, WLC and  $\Delta R_{LCC}$ ). Table 5 also marks that in the case of the  $\Delta R_{LCC}$  the area compensation method (AC) is not possible to be applied to defuzzification the result, because part of the resulting values in the individual  $\alpha$ -cuts assume negative values, which disqualifies this method.

Table 4. Result values for the LCC and LCEAC criteria depending on the choice of the set of membership functions and the method of defuzzification

Comparative criterion			LCC			LCEAC		
Scenario of building's life cycle			i = 1	i = 2	i = 3	i = 1	i = 2	i = 3
	Set of membership functions	Defuzzific. method	PLN	PLN	PLN	PLN	PLN	PLN
1	Basic	CoG	8 905 600	8 929 300	8 649 700	737 300	735 000	716 900
2	Basic	AC	8 909 100	8 927 200	8 638 400	734 400	730 300	712 600
3	Alternative I	CoG	8 903 700	8 925 000	8 641 600	734 800	731 500	713 900
4	Alternative I	AC	8 905 500	8 923 700	8 635 550	733 300	728 900	711 600
5	Alternative II	CoG	8 903 900	8 925 600	8 642 600	735 200	732 000	714 300
6	Alternative II	AC	8 906 400	8 924 200	8 635 900	733 400	729 100	711 700
7	Alternative III	CoG	8 908 000	8 931 400	8 652 100	738 300	736 600	718 700
8	Alternative III	AC	8 908 000	8 927 000	8 640 100	735 000	731 200	713 700
	Arithmetic average	m	8 906 275	8 926 675	8 641 994	735 213	731 825	714 175
	Standard deviation	s	698	931	2 143	1 761	2 718	2 492
	Coefficient of variation	V	0,01%	0,01%	0,02%	0,24%	0,37%	0,35%

Table 5. Result values for the WLC and  $\Delta R_{LCC}$  criteria depending on the choice of the set of membership functions and the method of defuzzification

Comparative criterion			WLC			$\Delta R_{LCC}$		
Scenario of building's life cycle			i = 1	i = 2	i = 3	i = 1	i = 2	i = 3
	Set of membership functions	Defuzzific. method	PLN	PLN	PLN	PLN	PLN	PLN
1	Basic	CoG	4 611 700	4 540 400	4 543 800	147 800	171 700	-107 900
2	Basic	AC	4 582 200	4 517 300	4 428 150	not applic.	not applic.	not applic.
3	Alternative I	CoG	4 594 800	4 526 500	4 457 300	156 800	178 100	-105 400
4	Alternative I	AC	4 579 950	4 514 850	4 398 750	not applic.	not applic.	not applic.
5	Alternative II	CoG	4 597 200	4 528 600	4 469 800	155 500	177 300	-105 800
6	Alternative II	AC	4 580 350	4 515 300	4 403 050	not applic.	not applic.	not applic.
7	Alternative III	CoG	4 613 200	4 544 500	4 549 800	146 500	173 100	-111 300
8	Alternative III	AC	4 586 800	4 521 900	4 438 300	not applic.	not applic.	not applic.
	Arithmetic average	m	4 593 275	4 526 169	4 461 119	151 650	175 050	-107 600
	Standard deviation	s	13 435	11 261	58 138	2 625	1 565	1 350
	Coefficient of variation	V	0,29%	0,25%	1,30%	1,73%	0,89%	1,25%

The analysis of the coefficients of variation  $V$  value reveals that:

- the average value of the coefficient of variation  $V$  for all cases investigated is  $V_m = 0,58\%$ ,
- maximum value of the coefficient of variation  $V_{\max} = 1,73\%$  was obtained only in 1 out of 12 analysed cases, which constitutes approximately 8,33% of all examined cases,
- in 8 out of all 12 cases (66,67% of all examined cases), coefficient of variation  $V$  values were lower than the average value  $V_m = 0,58\%$ ,
- in 9 out of all 12 cases (75,00% of all examined cases), coefficient of variation  $V$  values were lower than 1%,
- in the case of the building life cycle cost criterion (LCC), coefficient of variation  $V$  values were only 0,01% and 0,02% (for the building life cycle scenarios  $i = 1$ ,  $i = 2$  and  $i = 3$ , respectively),
- in none of the cases the coefficient of variation  $V$  value exceeded 10%.

## 4. CONCLUSIONS

Based on the description of the verification study and sensitivity analyses performed in this paper, it can be concluded that the model for estimating the whole life costs of buildings enabling the quantification of the risk addition generates fully convergent output values of LCC, LCEAC, WLC and  $\Delta R_{LCC}$  comparative criteria for all life cycle scenarios with the results obtained by calculations of the life cycle net present value deterministic method (LCNPV). This is illustrated by the test results described in chapter 3.2 and presented in table 2.

Testing the sensitivity of the model to the possible change in the value of temporal and financial input data to the results of calculations (chapter 3.3) indicated financial parameters (discount rate –  $r$  and initial costs –  $C_{in}$ ) that if the value changes within a range exceeding  $\pm 1\%$  for ( $r$ ) and  $\pm 10\%$  for ( $C_{in}$ ) respectively, they will increasingly influence the value of results for all economic criteria.

In the case of testing the sensitivity of the model to the possibility of changes in the parameters affecting the results of calculations (change of the set of membership functions and the method of defuzzification the result), the maximum value of the coefficient of variation was  $V_{\max} = 1,73\%$  for one of the investigated cases. Thus, in none of the analysed cases the value of  $V$  exceeded 10%, which is mentioned by Zeliaś and Pawełek [20]. Basing on the results obtained and described in chapter 3.4 (table 4 and table 5), it was found that all parameters that may affect the results indicate insignificant differentiation of the resulting values for economic comparative criteria (LCC, LCEAC, WLC and  $\Delta R_{LCC}$ ). The study of the sensitivity of the model for estimating the whole life costs of the buildings, therefore, confirms the very high homogeneity of the obtained results.

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**AUTORSKI MODEL SZACOWANIA CAŁOŚCI KOSZTÓW ŻYCIA BUDYNKÓW I JEGO WERYFIKACJA**

*Słowa kluczowe:* cykl życia, całość kosztów życia, koszty cyklu życia, ryzyko, dodatek kosztowy za ryzyko, budynek

**STRESZCZENIE:**

Model szacowania całości kosztów życia budynków umożliwiający kwantyfikację dodatku kosztowego za ryzyko, pozwala inwestorowi porównać budynki na wstępnym etapie planowania przedsięwzięcia budowlanego pod względem następujących kryteriów ekonomicznych: kosztów cyklu życia budynku (LCC – gdy inwestor nie będzie mógł notować przychodów); całości kosztów życia budynku (WLC – gdy oprócz ponoszenia kosztów w fazach cyklu życia budynku, inwestor będzie mógł notować również przychody); ekwiwalentu rocznych kosztów cyklu życia budynku (LCEAC – gdy długości trwania fazy eksploatacji będą różne); dodatku kosztowego za ryzyko w cyklu życia budynku ( $\Delta R_{LCC}$  – wyrażonego jako różnica w jednostkach walutowych pomiędzy wielkością kosztu cyklu życia budynku, która uwzględnia wpływ ryzyka, a wielkością kosztu cyklu życia budynku, która nie uwzględnia tego wpływu). W modelu, teoria zbiorów rozmytych łączona jest z najbardziej powszechną, dynamiczną metodą służącą do analizy efektywności ekonomicznej przedsięwzięć budowlanych na podstawie zdyskontowanych przepływów pieniężnych. Jest to metoda wartości bieżącej netto, zwana pod akronimami NPW (ang. *net present worth*) lub NPV (ang. *net present value*), która w modelu ma zastosowanie w wersji rozmytej (ang. *fuzzy NPW* lub *fuzzy NPV*).

Dane wejściowe do modelu dzielą się na następujące parametry: czasowe CG (o charakterze globalnym), tj. czas trwania cyklu życia budynku  $T_i$ , przy czym  $T_i$  równy jest szacowanemu okresowi użytkowania budynku (ESLB – ang. *estimated service life of a building*); czasowe CL (o charakterze lokalnym), czyli czasy  $t_{ik}$ ,  $t_{im}$ , po których naliczane zostają odpowiednio  $k$ -ty okresowy koszt operacyjny lub  $m$ -ty okresowy przychód; finansowe FG (o charakterze globalnym), w postaci stopy dyskonta ( $r$ ), która jest niezbędna do obliczenia wartości bieżącej netto danej kwoty pieniężnej w oparciu o jej wartość określoną w czasie przyszłym; finansowe FK (rozumiane jako koszty mogące zaistnieć w cyklu życia budynku), wśród których wyróżnia się koszty o charakterze rocznym – roczne koszty operacyjne ( $C_{opA,ij}$ ) oraz okresowym – kolejno koszty początkowe ( $C_{m,i}$ ), okresowe koszty operacyjne ( $C_{opNA,ik}$ ) i koszty wycofania ( $C_{wd,i}$ ); finansowe FP (rozumiane jako przychody mogące zaistnieć w cyklu życia budynku), czyli przychody o charakterze rocznym – roczne ( $I_{opNA,im}$ ) oraz okresowym – kolejno okresowe przychody uzyskiwane w fazie eksploatacji budynku ( $I_{opNA,im}$ ) i przychody osiągnięte w fazie wycofania ( $I_{wd,i}$ ). Do parametryzacji wymienionych powyżej danych wejściowych w modelu użyto wypukłych i normalnych liczb rozmytych.

Celem niniejszego artykułu jest natomiast sprawdzenie poprawności działania modelu szacowania całości kosztów życia budynków umożliwiającego kwantyfikację dodatku kosztowego za ryzyko na ewentualność zmiany parametrów mogących wpływać na wyniki obliczeń. Zakres badań obejmuje: porównanie wyników wygenerowanych przez model z wynikami uzyskanymi przy wykorzystaniu deterministycznej metody wartości obecnej netto w cyklu życia (LCNPV – ang. *life cycle net present value*) dla danych wejściowych czasowych i finansowych nieobciążonych wpływem ryzyka; analizę zmienności wyników na skutek zmiany danych wejściowych; analizę zmienności wyników na skutek zmiany zestawów funkcji przynależności dla danych wejściowych oraz metody wyostrzenia wyniku.

W artykule zaprezentowano przykład obliczeniowy ekonomicznej analizy cyklu życia LCCA (rozdział 3.1). Przykład dotyczył porównania trzech alternatywnych rozwiązań budynku mieszkalnego wielorodzinnego, które wynikają z konieczności uwzględnienia wpływu oddziaływania zidentyfikowanych czynników ryzyka w jego cyklu życia ( $Z_1$  – błędy w projektach i  $Z_2$  – błędne założenia rozwiązań konstrukcyjno-materiałowych). Przykład ten stanowił również podstawę do przeprowadzenia szeregu badań weryfikacyjnych i analiz wrażliwości modelu.



W rozdziale 3.2 model został poddany sprawdzeniu deterministyczną metodą wartości obecnej netto w cyklu życia (LCNPV). Sprawdzenia dokonano pod kątem zbieżności wyników generowanych przez model z wynikami uzyskanymi przy wykorzystaniu metody LCNPV.

W rozdziale 3.3 omówiono wyniki analizy wrażliwości modelu poprzez badanie wpływu zmiany wartości danych wejściowych czasowych i finansowych na wyniki obliczeń generowanych przez model. W badaniu wykorzystano propozycję normy ISO 15686-5:2008 („Budynki i budowle. Planowanie okresu użytkowania. Część 5: Koszty cyklu życia”) i przeprowadzono symulację wyników dla ekonomicznych kryteriów porównawczych LCC, LCEAC, WLC i  $\Delta R_{LCC}$  dla wybranego scenariusza cyklu życia alternatywnego rozwiązania budynku z przykładu przedstawionego w rozdziale 3.1. Przyjęto następujący zakres zmian związanych z parametryzacją danych czasowych i finansowych w stosunku do danych przyjętych pierwotnie do analizy LCCA dla scenariusza  $i = 2$ . Wartość stopy dyskonta ( $r$ ) uległa zmianie o  $\pm 1\%$  (z około 8% do około 7% i około 9%). Długość cyklu życia ( $T_2$ ) zmieniono o  $\pm 5$  lat (z nie więcej niż 60 lat do odpowiednio nie więcej niż 55 lat i nie więcej niż 65 lat). Wielkość kosztów początkowych ( $C_{in,2}$ ) przyjęto ze zmianą o  $\pm 10\%$  (z nie więcej niż 7 333 700 PLN do nie więcej niż 6 600 330 PLN i nie więcej niż 8 067 070 PLN), a wielkość kosztów operacyjnych ( $C_{opNA,230}$ ) – również ze zmianą o  $\pm 10\%$  (z nie więcej niż 1 090 000 PLN do nie więcej niż 981 000 PLN i nie więcej niż 1 199 000 PLN).

W rozdziale 3.4, dla danych czasowych oraz finansowych, które w przykładzie obliczeniowym (rozdział 3.1) zostały zamodelowane jako wielkości niepewne, dokonano zmiany podstawowego zestawu funkcji przynależności na pozostałe alternatywne komplety funkcji, tj.: alternatywny I (złożony z funkcji odcinkowo kwadratowych; alternatywny II (utworzony przez funkcje harmoniczne); alternatywny III (zbudowany z funkcji Gaussa). Przyjęto również założenie o sprawdzeniu wyników dwoma metodami wyostrzenia, tj. metodą środka ciężkości (CoG) lub metodą równoważenia obszaru (AC – ang. *area compensation*). Dla każdej z 84 kombinacji (po 8 przypadków dla kryteriów LCC, LCEAC i WLC oraz po 4 przypadki dla kryterium  $\Delta R_{LCC}$ ) obliczono ostre wartości wynikowe. Otrzymane wyniki stabilizowano podając wartości: średniej arytmetycznej ( $m$ ); odchylenia standardowego ( $s$ ); współczynnika zmienności ( $V$ ).

We wnioskach (rozdział 4) wykazano, że model szacowania całości kosztów życia budynków umożliwiający kwantyfikację dodatku kosztowego za ryzyko wygenerował w pełni zbieżne wartości wynikowe dla kryteriów porównawczych LCC, LCEAC, WLC i  $\Delta R_{LCC}$ , dla wszystkich scenariuszy cyklu życia budynku z rezultatami otrzymanymi w drodze obliczeń deterministyczną metodą LCNPV. Badanie wrażliwości modelu na ewentualność zmiany wartości danych wejściowych czasowych i finansowych na wyniki obliczeń wskazało na te parametry finansowe – stopę dyskonta ( $r$ ) oraz koszty początkowe ( $C_{in}$ ), w przypadku których zmiany wartości w zakresie przewyższającym odpowiednio  $\pm 1\%$  dla ( $r$ ) oraz  $\pm 10\%$  dla ( $C_{in}$ ), będą wpływać istotnie na wartości wyników dla kryteriów ekonomicznych. W przypadku badania wrażliwości modelu na ewentualność zmiany parametrów wpływających na wyniki obliczeń, maksymalna wartość współczynnika zmienności wyniosła  $V_{max} = 1,73\%$ , a zatem w żadnym z analizowanych przypadków, wartość  $V$  nie przekroczyła 10%. Na podstawie uzyskanych wyników stwierdzono, że wszystkie parametry mogące mieć wpływ na wyniki, wskazują na nieistotne zróżnicowanie wartości wynikowych dla wszystkich ekonomicznych kryteriów porównawczych (LCC, LCEAC, WLC i  $\Delta R_{LCC}$ ). Badanie wrażliwości modelu szacowania całości kosztów życia budynków potwierdziło bardzo wysoką jednorodność generowanych wyników.