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Metaheuristic approach to optimal power flow using mixed integer distributed ant colony optimization

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Abstract: This paper presents the application of an improved ant colony optimization algorithm called mixed integer distributed ant colony optimization to optimize the power flow solution in power grids. The results provided indicate an improvement in the reduction of operational costs in comparison with other optimization algorithms used in optimal power flow studies. The application was realized to optimize power flow in the IEEE 30 and the IEEE 57 bus test cases with the objective of operational cost minimization. The optimal power flow problem described is a non-linear, non-convex, complex and heavily constrained problem.

Key words: ant colony optimization, IEEE 30 bus, IEEE 57 bus, metaheuristic algorithm, mixed integer distributed ant colony optimization, optimal power flow

1. Introduction

The optimal power flow (OPF) problem is important to many stakeholders with regard to their respective objectives. For the utilities, the OPF problem represents a method to reduce costs involved [1–6] in order to maximize profit along with optimal reallocation of generating resources. Alternatively, it is also used by the utilities in order to reduce transmission line losses [3, 7, 8]. The society in its drive towards a sustainable future has driven research into optimizing the power system with an objective to minimize emissions [9] thereby maximizing social benefits.

The OPF problem can be defined as a non-linear, non-convex, complex, heavily constrained optimization problem with an objective function that is non-differentiable [5]. There are many approaches to finding a solution and they can be classified as conventional and heuristic/intelligent approaches. The conventional approaches include the interior point method [10, 11], non-linear programming [12, 13], quadratic programming [14, 15], gradient based method, Newton method [16], sequential unconstrained minimization technique [17] and the alternating direction method of multipliers [8]. The conventional methods though very reliable can sometimes be entrapped in the local minimum of the solution thereby not realizing the global minimum value of the objective function [6]. Moreover, gradient-based methods are ineffective in obtaining the solution when the problem is non-continuous and non-differentiable [18].

The intelligent methods overcome the drawbacks of the conventional methods. Additionally, they are reliable global optimization solution finders.

These intelligent methods which can be described as swarm based or population-based methods very often use a nature-inspired methodology to find the optimal solution. Each of these methods have their own advantages and disadvantages [19–21].

In article [1] the OPF problem has been solved using a glow worm optimization technique described by Krishnanad and Ghose [22]. This method takes advantage of the behavior amongst glow worms during the night. The interaction amongst glow worms depends on the amount of light emitted by each member which in turn depends on the amount of luciferin that each glow worm produces. Every iteration results in the change of luciferin produced by each glow worm and a subsequent movement towards the glow worm emitting the most light and hence eventually converging towards a solution. The application of this method has been tested on the IEEE 30 bus test case and an Indian 75 bus system.

Artificial bee colony optimization is used to solve the economic load dispatch problem in article [23]. The artificial bee colony optimization which is based on the foraging behavior of bees is described in [24, 25]. In this case food sources represent solutions to the objective function and the fitness of every food source is dependent on the amount of nectar within that food source. An initial population of bees is created, an initial solution vector is created and then it is updated based on the search pattern of the bees. The approach is tested using 4 cases consisting of different numbers of generating units.

The teaching-learning based Levy mutation strategy for OPF is described in paper [5]. It is a population-based evolutionary algorithm which mimics the teaching and learning process in a classroom [26, 27]. The initial population is represented by the students in a classroom and parameters are represented by the different courses' students participate in and the fitness value would be the student's overall grade. The best fitness value within the population would be then chosen as the teacher. The teacher would then guide the learning in order to improve the fitness value and also simultaneous learning amongst the students is allowed in order to eventually obtain the final solution. The approach is tested on the IEEE 30 and the IEEE 7 bus system.

The tabu search algorithm for OPF is described in [6] and the algorithm is described more in detail in [28]. The methodology involves creating an initial solution vector and then looks for better solutions in the neighborhood of the initial solution. These other possible solutions are called as moves and certain moves are forbidden (those that do not improve the solution) and these forbidden moves are updated in a list called the tabu list. This is so done in order to prevent the algorithm from revisiting worsening solutions repeatedly. The process of searching for a better solution continues until an acceptable solution is reached. The approach is validated using the IEEE 30 bus system. A similar approach with the gravity search algorithm is described in [29].

The remainder of the paper is organized as follows: Section 2 presents the mathematical foundation behind the OPF problem and the test case systems. Section 3 presents the mixed integer distributed ant colony optimization (MIDACO) methodology and how it is applied to solve the OPF problem. Section 4 describes the results and analysis followed by conclusions of solving the OPF for the IEEE 30 bus and the IEEE 57 bus test cases.

2. Optimal power flow and test cases

2.1. Optimal power flow mathematical foundation

The objective function in this study is the minimization of cost which is modelled as a second-order Lagrangian function [3, 4] and is presented in Equation (1).

$$\text{Minimize } F_{\text{cost}} = \sum_{i=1}^{N_{\text{gen}}} F_i\left(P_i\right), \tag{1}$$

where $F_i(P_i)$ represents the operational also called fuel cost functions of the individual generators present in the network.

Here, i represents the sources of power, P_i is the output of the source (MW), F_i is the operating cost of the source in \$/hr, whereas α, β, γ represent the cost coefficients in \$/hr.

The power balance equations for active and reactive power in the system are presented in Equations (2) and (3).

$$P_{gi} - P_{di} - V_i \sum_{k=1}^{n} V_j \left[G_{ij} \cos\left(\delta_i - \delta_j\right) + B_{ij} \sin\left(\delta_i - \delta_j\right)\right] = 0, \tag{2}$$

$$Q_{gi} - Q_{di} - V_i \sum_{k=1}^{n} V_j \left[G_{ij} \sin\left(\delta_i - \delta_j\right) + B_{ij} \cos\left(\delta_i - \delta_j\right)\right] = 0, \tag{3}$$

where (2) and (3) represent the equality constraints. V_i and V_j represent voltage values at i-th and j-th buses respectively. P_{gi}, Q_{gi} are the active and reactive powers generated at bus i, P_{di} and Q_{di} are the active and reactive power demand at bus i, δ_i represents the voltage angle, G_{ij} and B_{ij} are the conductance and susceptance between buses i and j.

The mathematical formulation of the OPF problem is simplified if the variables involved are appropriately classified. The classification is as below:

Dependent variables: these along with the independent variables fulfil the load flow equations and the dependent variables are represented by vector X. Such variables are:

– voltage magnitude at PQ nodes,
– voltage angle at PQ nodes,
– voltage angle at PV nodes.

Independent variables, represented by vector Y are:

– voltage magnitude and angle at slack node,
– active and reactive power at PQ nodes,
– active power and voltage magnitude at PV nodes.

The inequality constraints for the OPF problem are as stated below:

– active power output limits for all sources – Equation (4),

$$P_{gi(\min)} \leq P_{gi} \leq P_{gi(\max)}, \tag{4}$$

– reactive power output limits for all sources – Equation (5),

$$Q_{gi(\min)} \leq Q_{gi} \leq Q_{gi(\max)}, \tag{5}$$

– voltage limits – Equation (6),

$$V_{i(\min)} \leq V_i \leq V_{i(\max)}, \tag{6}$$

where: P_{gi} and Q_{gi} represent the active and reactive power at each bus of the network, P_{di} and Q_{di} represent the active and reactive power demand, V_i represents the voltage magnitude. The transformer tap settings and shunt VAR compensator constraints are taken as hard constraints where their value remains constant equal to their value provided in the IEEE 30 and IEEE 57 bus data.

Hence with regard to a typical optimization problem, Equations (2) and (3) represent $g(x, u) = 0$ and inequalities in Equations (4), (5) and (6) represent $h(x, u) \leq 0$, and subject to both these type of constraints of Equation (1) is minimized.

2.2. IEEE 30 and IEEE 57 bus test cases

Table 1 presents the cost coefficients of the 6 generators present within the IEEE 30 bus test case system at buses 1, 2, 5, 8, 11, 13 to be optimized. The number of variables for the optimization problem to be solved for this system are 72 variables in total (state and control variables).

Table 1. Cost coefficients for IEEE 30 bus system

Generator bus	γ	β	α
1	0.004	2	0
2	0.018	1.75	0
5	0.063	1	0
8	0.008	3.25	0
11	0.025	3	0
13	0.025	3	0

The IEEE 57 bus test case system has a total of generators at bus numbers 1, 2, 3, 6, 8, 9 and 12. The cost coefficients for the system are provided in Table 2. The reason for introducing another test case for the algorithm is to measure its performance when the size of the system is increased. The number of variables for the optimization problem to be solved for this system are 128 variables in total (state and control variables).

Table 2. Cost coefficients for IEEE 30 bus system

Generator bus	γ	β	α
1	0.077	20	0
2	0.010	40	0
3	0.250	20	0
6	0.010	40	0
8	0.022	20	0
9	0.010	40	0
12	0.032	20	0

3. Mixed integer distributed and colony optimization

The ant colony optimization (ACO) algorithm was introduced first as a result of a PhD thesis by Marco Dorigo [1]. Further detailed explanation of the ant colony optimization can be found in Dorigo and Stuetzle [2].

This study uses an extension of the ACO called mixed integer distributed ant colony optimization (MIDACO). The advantage of this optimization approach is that it is capable of taking in integer values along with operating in a continuous domain. The methodology of how it is done along with the oracle penalty method which is a novel way of penalising the objective function is described in [37–40]

This paper presents an approach to how the MIDACO can be applied to solving non-convex, non-linear and complex optimal power flow problem with active and reactive power equality constraints along with inequality constraints imposed by operational parameters and hardware of power systems

The methodology which is based on the foraging behaviour of ants works in the following manner. Initially ants are spread out randomly around their colony searching for food. The ant which locates a food source brings it back to the colony and while doing so lays down a pheromone trail which can now be used by other ants in order to locate food. The pheromone trails fade over time due to evaporation of pheromones and hence the trails that are updated repeatedly with more pheromones remain the most attractive. The food sources that are located closest to the colony will be more frequently visited and their trails updated compared to the ones that are located far off because of the distance covered by the ants. This makes the shortest trail most attractive for the ants and subsequently will be chosen by all ants as the route to a food source [36–38].

The traditional ACO utilizes a weighted graph often called a construction graph GC (V, E), where V represents the vertices of the graph and possible solutions to the optimization problem whereas E represents the edges of the graph and is associated with the pheromone and heuristic values.

The pheromone values lead to the modelling of the probability distribution of all components of the solution. Artificial ants now traverse this graph moving from one vertex to another creating an optimal path while using the pheromone values of the edges as a guide. This way they incrementally construct a solution and while doing so also update the pheromone values associated with the path they have utilized.

The pheromone values that they give to a path depends on the quality of the solution that they have reached. This information is then further used by subsequent ants [37]. While construction of the solution and pheromone updates are two important components of the ACO, daemon actions further bias the search process so that the solution moves towards a global maximum.

These actions are those that cannot be performed by individual ants but based on the individual solutions that are created by the ants, certain conditions (such as a local optimization problem) can be applied to the solutions and pheromone values changed depending on the fitness of the solution [38]. This way global optimal solutions can be obtained while satisfying local constraints. The MIDACO in its framework uses a mixed integer sequential quadratic programming (MISQP) as a local solver. The stopping criterion for the search can either be the maximum time of searching or the maximum number of solutions constructed. The algorithm for the process is shown in Fig. 1.

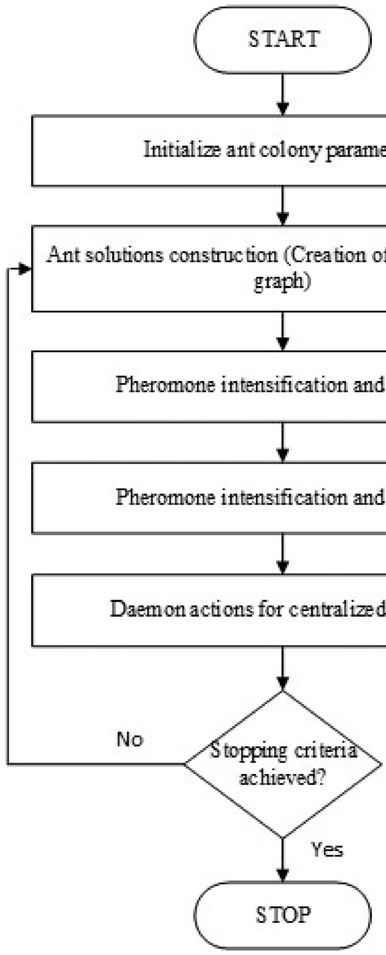
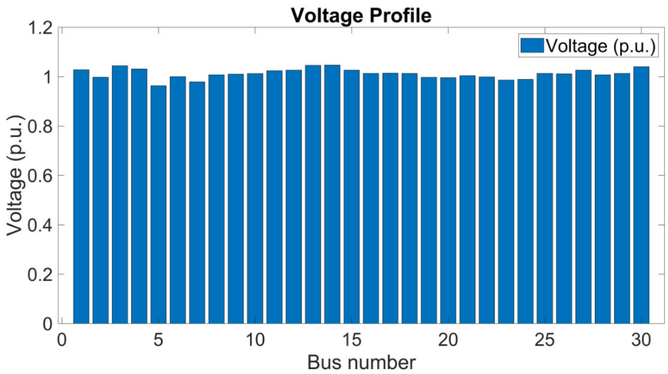


Fig. 1. ACO metaheuristic algorithm

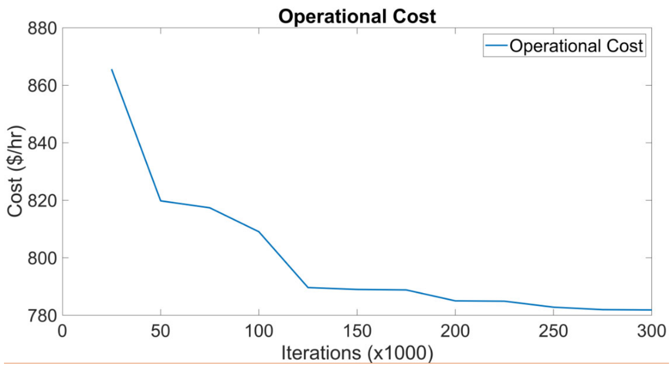
4. Results and analysis

The optimization using MIDACO on the IEEE 30 bus test case system was performed on an intel core i5 system with 8 GB RAM. One iteration of the MIDACO for this system takes about 24 ms.

The voltage profile for all the buses is shown in Fig. 2(a) in per unit values at each bus and for this study, all the voltage values satisfy their constraint bounds. The maximum voltage magnitude in p.u. is 1.049 seen in bus 25 and the minimum is 0.958 in bus 30 t.



(a)

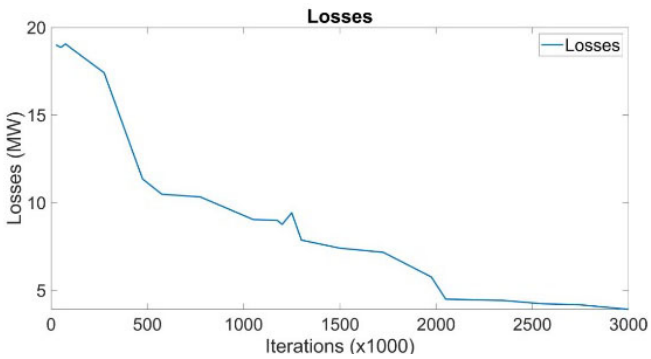


(b)

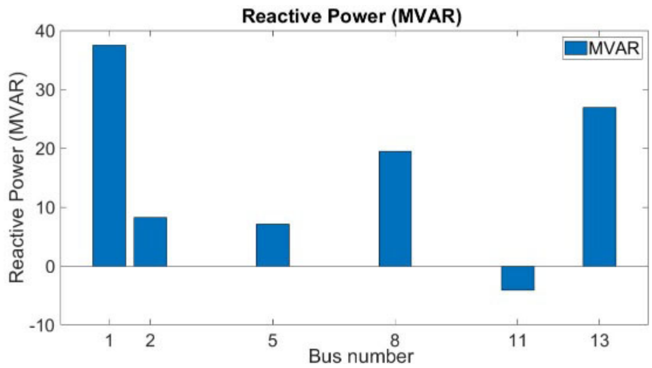
Fig. 2. Voltage profile (30 – bus) (a); cost minimization (30 – bus) (b)

The minimization of operational cost for the IEEE 30 bus system is depicted in Fig. 2(b). The curve is obtained by plotting the best solution achieved by the algorithm every 25 000 iterations. The minimization of the cost is achieved in 3 000 000 iterations converging to a final value of 790.863 $/hr. The stopping criteria used is maximum evaluations performed.

Fig. 3(a) represents how the losses are reduced by the optimization algorithm with increasing iterations. It can be seen that the reduction of line losses in fact contributes to the overall reduction of cost for the system. The characteristics of Fig. 2(b) and Fig. 3(a) though not identical are quite similar.



(a)



(b)

Fig. 3. Losses reduction (30 – bus) (a); reactive power at generating buses (30 – bus) (b)

Fig. 3(b) provides information regarding the reactive power that is produced/consumed at every generating bus at the end of the optimization process. It can be noticed that while for most generators the reactive power is produced, for the generator at bus 11 it is consumed.

Table 3 presents a comparison of different optimization algorithms used for optimal power flow on the IEEE 30 bus test case system. While some of the algorithms have been discussed in the introduction section, the rest of the algorithms and their performance have been taken from relevant research articles.

In general, it can be concluded that all heuristic algorithms have been able to minimize the cost to an amount lower than the gradient method. Amongst the heuristic approaches, it can be seen that the MIDACO has minimized the cost value to 790.863 $/hr which is lower than all other heuristic algorithms.

The algorithms that the comparison has been made with and shown in the Table 3 are: the genetic algorithm (GA), glowworm swarm optimization (GSO), particle swarm optimization (PSO), biogeography-based optimization (BBO), differential evolution algorithm (DE), gradient method, modified differential algorithm (MDE), faster evolutionary algorithm (FEA) and improved genetic algorithm.

The results for the IEEE 57 test case system are also presented in Fig. 4 and Fig. 5.

Table 3. Comparison of control variables for different algorithms for the IEEE 30 bus system

Control variable values	**GA [1]**	**GSO [1]**	**PSO [41]**	**BBO [42]**	**Gradient method [43]**	**MDE [44]**	**FEA [19]**	**MIDACO**
PG1 (MW)	176.11	174.92	176.96	177.018	187.219	175.974	177.285	**165.351**
PG2 (MW)	49.098	44.151	48.98	48.641	53.781	48.884	48.93	**51.792**
PG5 (MW)	21.718	21.76	21.3	21.239	16.955	21.51	21.29	**19.454**
PG8 (MW)	21.086	25.73	21.19	21.136	11.288	22.24	20.49	**10.032**
PG11 (MW)	11.83	11.12	11.97	11.944	11.287	12.251	11.93	**24.143**
PG13 (MW)	12.218	13.81	12	12.054	13.355	12	12.23	**16.549**
V1 (p.u.)	1.1	1.092	1.086	1.1	1.1	1.05	1.098	**1.033**
V2 (p.u.)	1.08	1.074	1.065	1.088	1.08	1.038	1.08	**1.042**
V5 (p.u.)	1.064	1.043	1.033	1.061	1.03	1.011	1.053	**1.019**
V8 (p.u.)	1.066	1.068	1.039	1.07	1.04	1.019	1.062	**1.026**
V11 (p.u.)	1.06	1.077	1.085	1.098	1.08	1.095	1.08	**0.967**
V13 (p.u.)	1.087	1.083	1.051	1.1	1.08	1.084	1.078	**1.017**
Losses (MW)	8.66	–	9.216	8.63	10.486	9.459	8.755	**3.923**
Generation cost ($/hr)	**799.52**	**800.805**	**800.41**	**799.112**	**804.853**	**802.376**	**799.56**	**790.863**

Fig. 4(a) presents the minimization of operational cost for the IEEE 57 bus test case system. The system is bigger in size and it includes a higher number of variables compared to the IEEE 30 bus system that took a much higher number of iterations before a minimal value was attained. The stopping criteria in this case as in the previous was the maximum number of evaluations. It was set at 10 000 000 iterations. The final minimum value, in this case, is 41576.70 ($/hr). The voltage profile at the buses for the system is provided in Fig. 4(b). The voltage at its lowest point in p.u is observed at buses 4 and 55 at 0.964. The highest is seen at bus 57 at 1.049 p.u.

The voltage profile at the buses for the system is provided in Fig. 4(b). The voltage at its lowest point in p.u is observed at buses 4 and 55 at 0.964. The highest is seen at bus 57 at 1.049 p.u.

Fig. 5(a) is a representation of how losses are minimized through the iterations. It can be seen here as in the case for the IEEE 30 bus system that overall cost minimization is also dependent on the minimization of losses. The characteristic while not identical has a pathway similar to that of the cost minimization curve described in Fig. 3(a).

Fig. 5(b) presents the reactive power generation at all the generating buses at the final iteration and it is evident that at bus 1 maximum reactive power is produced whereas bus 8 is least.

Table 4 presents a comparison of the control variables for different algorithms. The algorithms are the artificial bee colony (ABC), gravity search algorithm (GSA), evolving ant direction differential evolution (EADDE), imperialist competitive algorithm (ICA), teaching learning algorithm

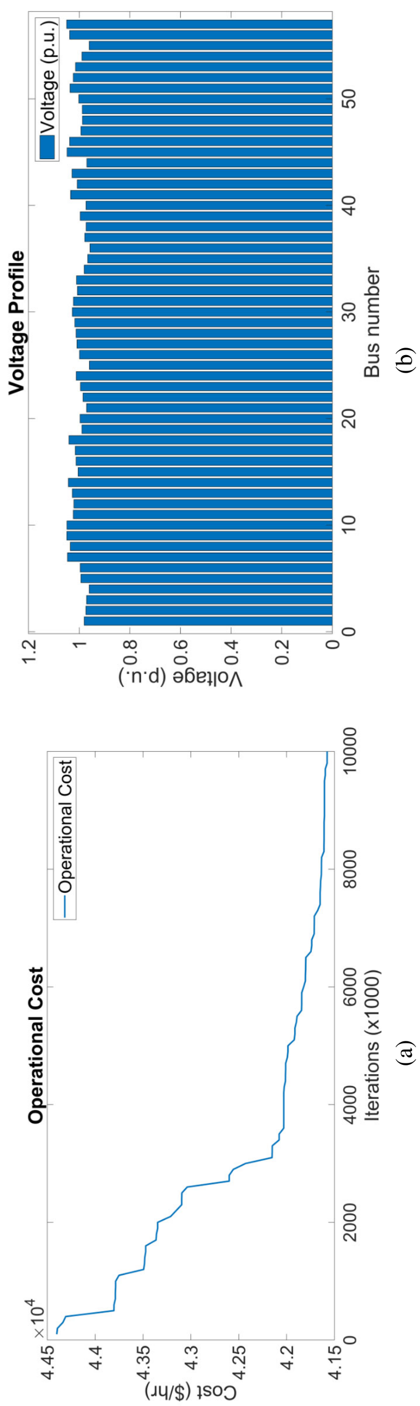


Fig. 4. Cost minimization (57 – bus) (a); voltage profile at buses (57 – bus) (b)

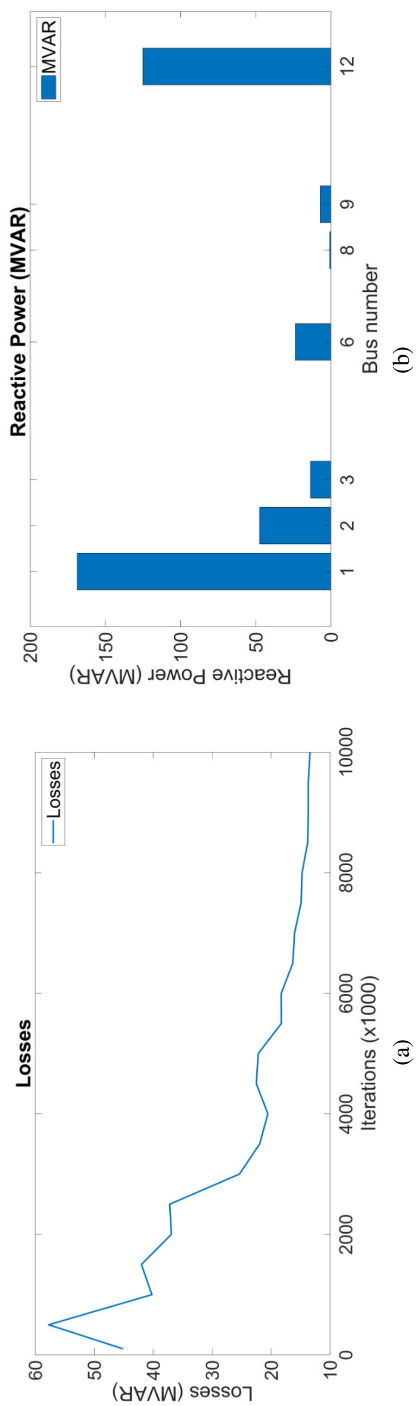


Fig. 5. Losses reduction (57 – bus) (a); reactive power at generating buses (57 – bus) (b)

(TLA), modified imperialist competitive algorithm (MICA) and a hybrid MICA-TLA. It can be seen that the final cost in ($/hr) is the lowest for the MIDACO at 41576.70 and the associated line losses are also the lowest at 13.417 (MW).

Table 4. Comparison of control variables for different algorithms for the IEEE 57 bus system

Control variables	**ABC [26]**	**GSA [26]**	**EADDE [26]**	**TLA [26]**	**MICA–TLA [26]**	**MIDACO**
PG1 (MW)	142.811	142.369	143.15	143.53	142.77	**133.809**
PG2 (MW)	90.033	92.63	95.29	88.058	89.217	**77.996**
PG3 (MW)	44.515	45.318	45.32	44.818	44.959	**45.917**
PG6 (MW)	74.2	72.355	73.6	76.473	71.479	**81.621**
PG8 (MW)	454.848	464.743	464.85	458.51	459.485	**481.101**
PG9 (MW)	96.885	84.999	83.44	91.948	96.989	**98.075**
PG12 (MW)	362.772	363.951	361.24	362.699	360.918	**345.698**
VG1 (p.u.)	1.042	1.059	1.05	1.044	1.05	**1.034**
VG2 (p.u.)	1.041	1.058	1.048	1.047	1.054	**1.043**
VG3 (p.u.)	1.039	1.06	1.041	1.034	1.042	**1.042**
VG6 (p.u.)	1.055	1.06	1.049	1.048	1.053	**1.035**
VG8 (p.u.)	1.064	1.06	1.056	1.06	1.06	**1.026**
VG9 (p.u.)	1.037	1.06	1.034	1.031	1.034	**0.968**
VG12 (p.u.)	1.041	1.046	1.041	1.032	1.034	**0.97**
Power losses (MW)	–	–	16.09	15.24	15.015	**13.417**
Cost ($/h)	**41693.95**	**41695.87**	**41713.62**	**41686.79**	**41675.05**	**41576.7**

5. Conclusions

The paper discusses in-depth the mixed Integer distributed ant colony optimization algorithm and its application to optimal power flow studies. The performance of the algorithm has been tested against two standard bus cases of the IEEE 30 and the IEEE 57 buses. Then a comparison of its performance has been made with other optimization algorithms (conventional and heuristic).

The idea is to also develop the same for real time applications as demonstrated in [45–48], in comparison with the real time OPF. The method has to be developed further so that the convergence time is significantly reduced. Moreover, there is also the question of handling the uncertainty concerned with renewable energy sources. The approach for including uncertainties will be in terms of using forecasted values through statistical analysis or machine learning.

Furthermore, future work is also focused upon extending the application of the algorithm to consider both the prohibited zones and the valve point effect and use a piecewise quadratic fuel cost function. To consider a multi-objective function such as reducing operational costs along with reducing emissions.

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