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Research Paper

The Single Degree of Freedom Simulation Model of Underwater Explosion Impact

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The hulls of naval ships are exposed to forces and moments coming from internal and external sources. Usually, these are interactions that can be described mathematically by harmonic and polyharmonic functions. The shock of UNDEX type (underwater explosion) works completely differently and its time waveform is difficult to describe with mathematical functions as pressure vs. time. The paper presents a simplification of physical and mathematical models of 1-D kickoff pressure whose aim is performance the simulation of the external force of the detonation wave. The proposed models were verified and tuned on naval, sea trials. The main goals of the proposed models are to perform simulation calculations of the detonation pressure for different explosion charge weights from different distances of the UNDEX epicentre for the design process of machine foundation. The effects of pressure are transformed as impulses exposed on shock absorber mounted at light shock machine.

Keywords: UNDEX; simulation; impact model; bubble.

1. Introduction

An essential criterion for the evaluation of the naval vessels project is ensuring survival in war conditions. As a vulnerability, the ship's ability to perform all war missions and protect the crew from damage or even death is usually defined. The ability to survive is the result of carefully thought out design and perfect production. The rules of all navies define vulnerability as the ability to perform tasks with a wide range of threats such as chemical, nuclear, fire, the breakthrough in the compartment and flood, and impact of underwater explosion (UNDEX). A large number of naval projects made in the XX century were considering the vulnerability in respect of UNDEX threats at the final level design stage of the Evans spiral. This means that protection against UNDEX impacts refers to a limited number of individual devices rather than a whole to naval and vital systems (Smith, Lee, 2017).

Contemporary designs of the naval vessels at the early stages of design analyse the vulnerability, including the survivability to an underwater explosion of a ship. The limitations are, in most designs, the tools used in the design process, because they are temporarily high in computation and the number of possible solutions seems to be unlimited. Therefore, optimization of design solutions should limit the number of initial conditions to the most likely in operational scenarios.

Publications on ship's vulnerability concern the modelling of detonation effects and their impact on the ship's hull structure. The works consider the effects of using various explosive materials, their shape and scale effects for the various mass of the explosive charge. The test results often refer to the UNDEX event in unlimited sea depth. The paper presents a more practical aspect of the design and calculation of shock absorbers. The work considers the technical data of a currently produced bottom mine however it refers to the small scale of explosive weights. The range of action of the detonator is defined as a distance from the epicentre to the ship's hull. The work aims to propose a model of the UNDEX impulse that whips the hull and foundation of the machine placed on the frames and side girders of the hull. The main task is to propose models in the form of function vs time, which allows their use in the solution of phenomenon dynamics differential equations.

2. UNDEX phenomena

The purpose of the sea mine is UNDEX events which effects are supposed to damage the ship or significantly reduce the combat mission. Moreover, the effect should force the user to return to shipyard or harbour for necessary repairs which disable the use of a ship for many days. UNDEX event consists of short-time whipping with a shock wave impulse and longer time pulses of waves reflected from the sea bottom and the free surface as well as from the pulsation of the gas bubble. Damage or destruction in the structure of the ship's hull and its mechanisms depend on many factors, of which the most important are the mass of explosive charge and standoff distance (Mair, 1996). Next are the chemical composition of the explosive charge, sea depth and type of bottom (sandy, gravel or rocky). Another important factor is the explosive charge location in sea depth, which is determined by the mines types, i.e. ground (bottom), drifting, contact or moored.

The detonation of the explosive charge creates a high-energy chemical reaction. It makes a gas bubble that produces a high speed wave. In the beginning, the shock wave moves at supersonic speed and after a short distance, its speed stays on the value of the sound speed in the water. The shockwave can be defined as a huge, discontinuous, compressive pressure wave over the outer surface of the growing up gas bubble (Brett, 1998). The UNDEX phenomenon is been typified as a near or far field (Cole, 1948). Both types can be classified internally as early or late-time what is shown in Table 1.

3. Near and Far Field Explosions

The potential damage that can be caused by UNDEX event classifies these phenomena at Near and Far Field Explosions (Dunbar, 2009). Near Field Explosion (NFE) is the most dangerous event for the crew, hull, superstructure and ship's mechanisms. It is assumed that NFE takes place when the standoff distance is less than 5 times the maximum diameter of the gas bubble. The maximum radius of the gas bubble was proposed in the empirical formula by Cole (1948) as:

$$R_{\max} = 1.53 \sqrt[3]{\frac{W}{1+0.1\cdot H}}\ [\mathrm{m}]. \tag{1}$$

NFE effects are plastic deformations, structural cracking of the hull and misalignment of rotating machines. The effects can also drastically affect the crew causing fractures of the extremities and spinal injuries. The NFE lasts a very short period, from the moment of detonation to about 0.5 seconds. It consists of the following effects: kickoff, shock-free surface reflection, bubble contact reloads (if standoff distance shorter than bubble radius), bubble pulse impact and global ship motion.

Far Field Explosion (FFE) occurs when the distance from the epicentre is so far away that the deformations of the hull structure are flexible or elastic – plastic in a limited range. The range and type of deformations depend on the standoff distance and the weight of the explosive charge. FFE consists essentially of the shock wave and subsequent bubble pulses generated during the expansion and compression of the gas bubble and reflected from sea bed and surface. On the littoral water, the effects of reflection impulses from the free surface, sea bottom and the impulse from the disappearance of the cavitation bubble are also created. The small depth of the sea less than the radius of the bubble throws to atmosphere lots of energy. All FFE effects affect the global ship motion but in a significantly smaller range than at the NFE event.

At the UNDEX event, it is assumed that all components of the explosive charge are consumed immediately and the wave extends in the water depth in the spherical form. Shock wave encounters three types of barriers, i.e. the free surface, the sea bottom and the ship's hull. The contact with the free surface of the sea causes that the wave is reflected as a tensile wave (Huang, Jiao, 2010). Part of the energy during contact with the surface is lost but a significant part of it

Table 1. Near and Far Field effects (Cole, 1948).

Process	Effects	Extent	Time scale
Shock wave	Kickoff, free surface reflection, bulk cavitation	Global	< 0.05 s
Shock wave	Machinery failure and rapture/fracture of foundation	Global and/or local	< 0.1 s
Shock wave	Rapture hull plating and plate deformation	Local	< 0.1 s
Gas bubble expansion	Lifting of the hull under added buoyancy (permanent deformation)	Global	0.1–0.5 s
Gas bubble collapse	Hull girder whipping leading to permanent deformation	Global	> 0.5 s
Gas bubble collapse	Misalignment of machinery shafts	Local	> 0.5 s
Gas bubble collapse	Dishing or rapture of hull plating		0.5–1.0 s

can react on the ship's hull. The sea bottom consumes a lot of wave energy but it depends on its type, i.e. sand, gravel or rocky. The sandy seabed suppresses the energy of UNDEX the most. The effect of the wave contact with the hull makes its global motion. The wave is almost completely reflected. The motion of the hull from kickoff pressure can cause a large acceleration of hull which will cause local cavitation. When collapsing the cavitation bubble, the hull's acceleration vector changes its direction, which causes a reload effect on the surface of the hull, superstructure and ship's mechanisms.

It is defined that the shockwave is a large, discontinuous, compressive pressure wave that is produced by a detonation wave which kicks the outer surface of the gas bubble. During bubble expansion, the pressure inside the bubble begins to have a lower pressure than the hydrostatic pressure of the sea. This is still a small period due to the kinetic energy of the bubble. After this moment the gas bubble collapse begins (compression) and then the renewed expansion process. The resulting wave during the expansion and compression creates new impulses. Moreover, the decreasing pressure of reflected wave at the free surface of the sea forms the cavitation. That phenomenon disappears in the form of a collapse of the steam area. During this phenomenon, a pulse effect is generated as well. During the expansion and compression, the gas bubble migrates towards the sea surface and has the shape of a flattened sphere as a Rayleigh-Taylor instability effect.

Submarines must be analysed taking into account all possible UNDEX events. This is due to its immersion and the directions of action of the detonation and reflected waves., Naval surface vessels have other shipping options; hence the range of analysed interactions can be reduced. Operation of sea mines igniters (excluding contact types) aims to minimize the distance between the hull and the epicentre of the mine detonation. It does not depend on the type of mine, because the purpose of their operation is the same. Variable depths, various types of seabed, weights and types of explosive charges make the main goal of vulnerability analysis determination of resistance to shock. Other factors, although high-energy ones are secondary, and their occurrence is stochastic.

4. Model of impact

The work considers the effect of a single UNDEX pulse. Subsequent impulses from reflecting or expanding and compressing gas bubbles are stochastic for littoral waters. Subsequent pulsations can cause a resonance effect for machines found on the ship. It depends, however, on the period between their operation, which cannot be calculated precisely.

Considering the total energy released during the underwater detonation, it is assumed in literature that about 53% of this energy is converted into a shockwave, while the remaining 47% is released during bubble pulsation. From 53% of the energy held by the shockwave, about 20% is used for it is spreading, while the remaining 33% can be transferred to the hull of the unit (Cole, 1948).

It is also possible to approximate the calculation of the other values describing the shockwave formulas (2)–(7). The use of "approximate" refers to the lack of a dynamic scale for explosive charges of different weights (Brett, 1998).

$$R = \frac{K_6 \cdot W^{1/3}}{(D+9.8)^{1/3}}, \tag{2}$$

$$T = \frac{K_6 \cdot W^{\frac{1}{3}}}{(D+9.8)^{5/6}}, \tag{3}$$

$$p(t) = \left(K_1 \cdot \left(\frac{W^{1/3}}{R}\right)^{A_1}\right) \cdot \frac{e^{-(t-t_0)}}{\theta}, \tag{4}$$

$$\theta = K_2 \cdot W^{1/3} \cdot \left(\frac{W^{1/3}}{R}\right)^{A_2}, \tag{5}$$

$$p_{\max} = K_1 \cdot \left(\frac{W^{1/3}}{R}\right)^{A_1}, \tag{6}$$

$$I = K_3 \cdot W^{1/3} \cdot \left(\frac{W^{1/3}}{R}\right)^{A_3}, \tag{7}$$

where $p(t)$ – changes of pressure vs time [MPa], t_0 – the time from the moment of first contact with the object of the pressure wave [ms], $p_{\max}$ – maximum pressure [MPa], θ – constant decay [ms], I – pressure impulse (kick off) [MPa·s], W – mass of charge (equivalent in TNT, see Table 2) [kg], R – standoff distance to the hull [m], D – depth of epicentre [m].

Table 2. Empirical coefficients.

Coefficient	HBX-1	TNT	PENT
K_1	53.52	52.12	56.21
A_1	1.44	1.18	1.194
K_2	0.092	0.092	0.086
A_2	−0.247	−0.185	−0.257
K_3	7.263	6.52	6.518
A_3	0.856	0.98	0.903
K_4	106.8	94.34	103.11
K_5	2.302	2.064	2.098
K_6	3.775	3.383	3.439

Four parameter's (weight of explosive charges) simulation of the kickoff pressure *vs* standoff distance is presented in Fig. 1.

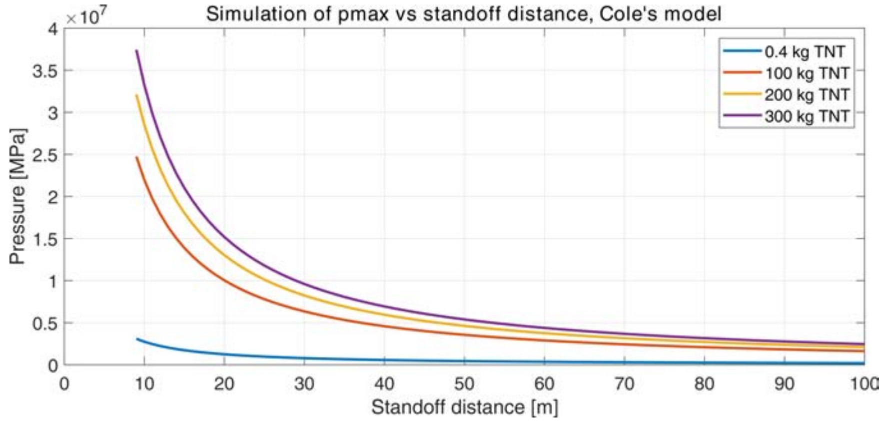

Fig. 1. Simulation of the kickoff pressure *vs* standoff distance for different weight of charges.

During the impact on the hull element with the surrounding seawater, the equation of motion of the hull can be expressed as follows:

$$\mathbf{M}x''(t) + \mathbf{C}x'(t) + \mathbf{K}x(t) = F(t), \tag{8}$$

where $\mathbf{M}$ is the inertia of the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, x is the displacement of the hull element, $F(t)$ is the time variable force affecting this element, what is defined as follow:

$$F(t) = -\mathbf{G}\mathbf{A}_f(p_i - p_s), \tag{9}$$

where $\mathbf{G}$ is a matrix related to the number of degrees of freedom of sea water surrounding the element, $\mathbf{A}_f$ is matrix containing the mesh areas representing the surrounding fluid, p_i is maximum pressure resulting from the underwater explosion, p_s is a pressure affecting the hull element.

Assuming that during the impact of pressure wave, the speed of water and hull element are the same, we can write that:

$$\mathbf{G}^{\mathrm{T}}x'(t) = v_i + v_s, \tag{10}$$

where v_i is instantaneous speed of the water caused by the UNDEX, v_s is dispersed velocity of the hull element.

It was assumed that water is incompressible and non-viscous. The pressure affecting the hull element and its speed are interrelated with the following dependence

$$p_s = \rho_w c_w v_s. \tag{11}$$

Taking into consideration Eqs (9) and (10) we obtained:

$$p_s = \rho_w c_w(\mathbf{G}^{\mathrm{T}}x' - v_i). \tag{12}$$

By substituting (12) to (9) we obtained:

$$F(t) = -\mathbf{G}\mathbf{A}_f\left[p_i + \rho_w c_w(\mathbf{G}^{\mathrm{T}}x'(t) - v_i 0)\right]. \tag{13}$$

After taking into account (8), the differential equation of movement of the hull element subjected to the impact of the shockwave can be presented as follows:

$$\mathbf{M}x''(t) + (C + \mathbf{G}\mathbf{A}_f\mathbf{G}^{\mathrm{T}}\rho_w c_w)x'(t) + \mathbf{K}x(t) = -\mathbf{G}\mathbf{A}_f(p_i + \rho_w c_w v_i). \tag{14}$$

$\rho_w c_w$ represents the additional damping caused by the dissipation of a part of the energy of the hull element into the sea water. The only unknown variable in Eq. (14) is the displacement, which can be calculated using the FEM (finite element method). The equation of motion (14) is correct until cavitation occurs, which has been neglected due to the littoral water effect at the sea surface. It should be noted that on the opposite side of the ship's hull to the UNDEX epicentre, the movement during the expansion of the gas bubble will cause hull cavitation delayed concerning the kickoff. Apart from the damping and nodal displacements during the UDEX impulse, it can be rewritten as:

$$mx''(t) + \rho_w c_w x'(t) = 2 \cdot P_m e^{-\frac{t}{\theta}}, \tag{15}$$

where m is a mass related to the unit of area of the considered hull element.

Introducing the initial conditions and dimensionless coefficient as $\psi_a = \rho_w c_w \theta / m$ and P_m maximum pressure, the pressure acting on the hull can be written as:

$$P_p(t) = 2p_m e^{-\frac{t}{\theta}} - \frac{2 \cdot P_m \psi_a}{\psi_a - 1}\left(e^{-\frac{t}{\theta}} - e^{-\frac{\psi_a t}{\theta}}\right). \tag{16}$$

The maximum pressure is saved by the formula:

$$P_p = 2P_m \psi_a^{\frac{1}{1-\psi_a}}, \tag{17}$$

while the maximum speed of the hull element is described as

$$V_{ma} = \frac{2P_m\theta}{m}\psi_a^{\frac{1}{1-\psi_a}}. \tag{18}$$

The energy of the shock wave transmitted to the element behind which the air is located is:

$$E_{pa} = \frac{mV_{ma}^2}{2} = \frac{2P_m^2\theta^2}{m}\psi_a^{\frac{2\psi_a}{1-\psi_a}}. \tag{19}$$

5. Sea trial tests

Several tests were carried out on a marine training ground with explosive charges weights from 75 g to 400 g TNT. The layout of the test bench is shown in Figs 2 and 3. During the tests, the kickoff pressure, acceleration of the hull and deformations of the hull plating were measured. The shock pressure gauge was attached to a flexible steel line in such position that in all tests the depth of gauge was the same as the UNDEX epicentre.

An exemplary result of the kickoff pressure measurements for detonation explosive charge W = 400 g TNT from a standoff distance of R = 40 m at a depth of D = 6 m is shown in Fig. 4.

Tests were aimed at acquiring the following information:

- the course of shock wave impacts,
- repeatability of measurement results for the same charges weights and standoff distances,
- analysis of possible pulsations for the shallow sea,
- analysis of the influence of sandy bottom on the measurement results for different immersion of epicentre.

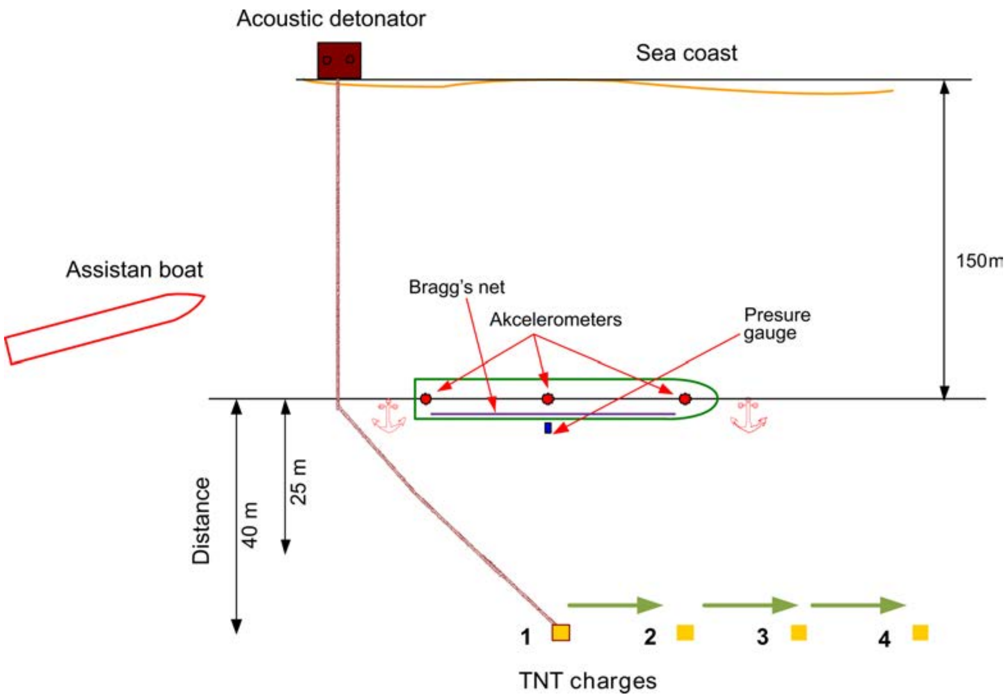

Fig. 2. Layout of the underwater detonation experiments.

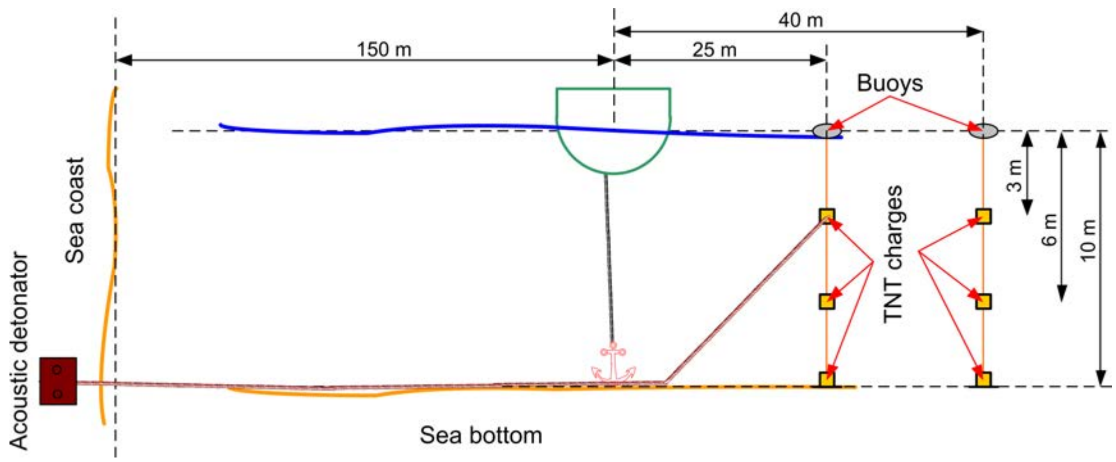

Fig. 3. Distance and depth of TNT charges placement.

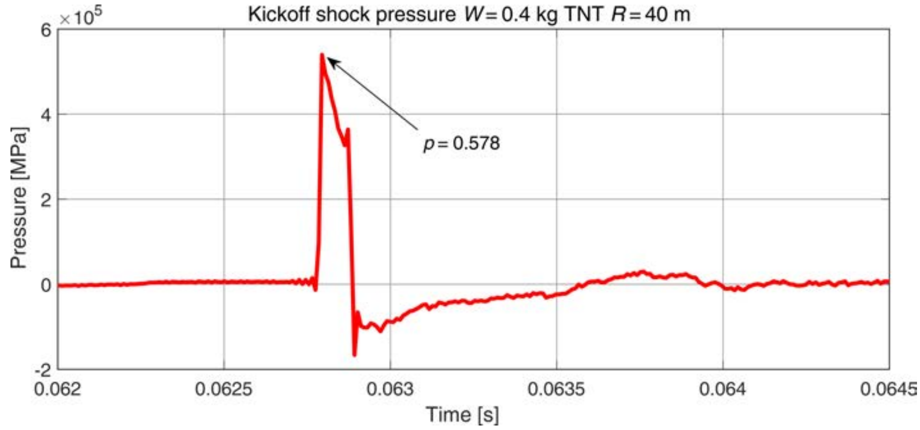

Fig. 4. The kickoff pressure measurements for detonation explosive charge W = 0.4 kg TNT from a distance of R = 40 m at a depth of D = 6 m.

6. Results of simulations

The main goal of the numerical simulations was to assess the adequacy of two mathematical models of kickoff pressure. The first model is the formula (4) representing Cole's formula (Cole, 1948). The second model is the result of research carried out by the authors of the work and it was called the AG model. The AG model presents itself as follows (Grządziela, 2011):

$$p_{AG}(t) = C_{SB} \left| \left(\frac{A^m}{R} \right) \left(\frac{\sqrt[3]{W}}{R} \right) \right| \cdot \sin \omega t \cdot e^{-st} \text{ [MPa]}, \quad (20)$$

where C_{SB} – coefficient of sea bed (for sandy bed C_{SB} = 0.95), A – coefficient of explosive charge type, for TNT $A = 8.5 \cdot 10^5$, W – mass of charge (equivalent in TNT) [kg], R – standoff distance to the epicentre [m], ω – 0.6, m – the exponent of the coefficient of explosive charge, for the small scale, $m = 0.00235 \cdot R + 0.935$, s – 150.

The coefficient of the explosive charge A was analyzed for immersion depth of explosive charge at least 2 times bigger than a maximum radius of the explosive bulb. For immersion less than 2 times of maximum radius of the bulb, the coefficient of explosive charge decreased its value from 8.5 10^5 up to 6.2 10^5 for detonation on the seabed.

The purpose of simulation was to assess the accuracy of the results obtained during research on the marine training ground. Due to the significant difference between the tested weights of explosive charges and weights of mines, the largest load, i.e. W = 400 g TNT, was selected for comparison. The simulation results were referenced to training ground's measurements. The simulation results using Cole's model (4) are shown in Figs 5 and 6, while the simulation results of the AG model (20) are presented in Figs 7 and 8.

Cole's model calculates the kickoff pressure in milliseconds while the AG model in seconds. For this reason, Fig. 9 presents a comparison of the simulation results of both models on the scale of milliseconds.

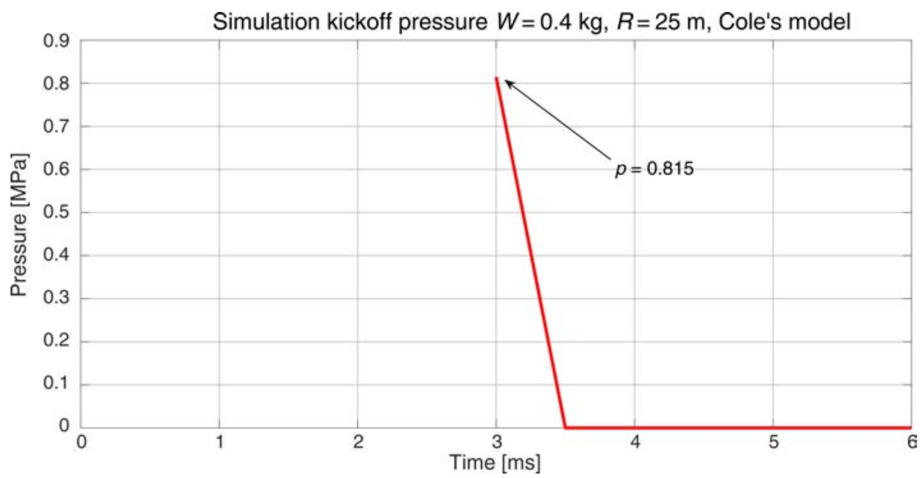

Fig. 5. Simulation of kickoff pressure for W = 0.4 kg TNT and standoff distance R = 25 m, Cole's model.

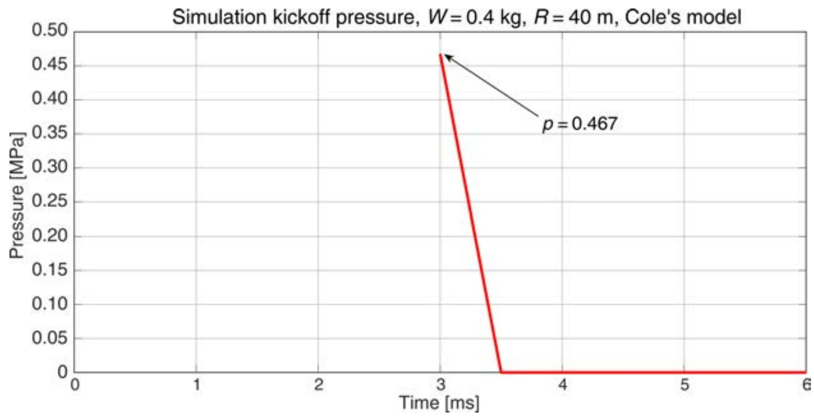

Fig. 6. Simulation of kickoff pressure for $W = 0.4$ kg TNT and standoff distance $R = 40$ m, Cole's model.

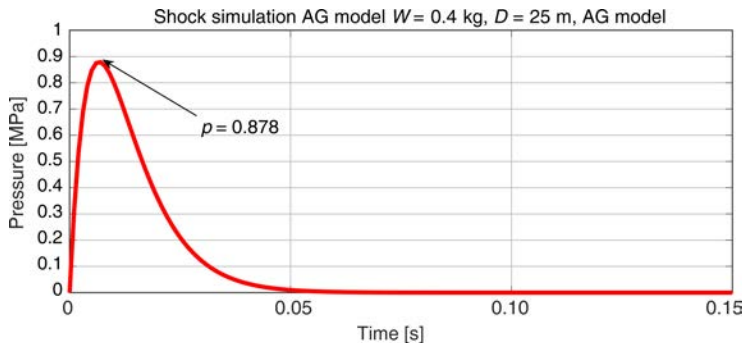

Fig. 7. Simulation of kickoff pressure for $W = 0.4$ kg TNT and standoff distance $R = 25$ m, AG model.

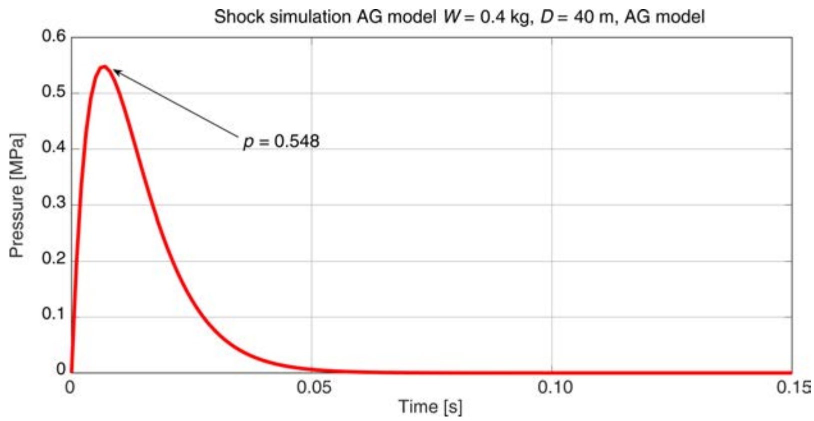

Fig. 8. Simulation of kickoff pressure for $W = 0.4$ kg TNT and standoff distance $R = 40$ m, AG model.

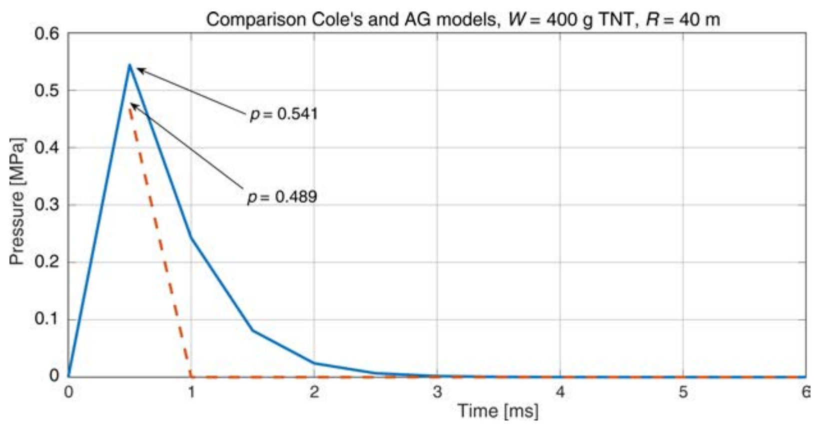

Fig. 9. Comparison of the results both simulations, Cole's (dotted line) and AG models (solid line), for $W = 0.4$ kg TNT and $R = 40$ m.

Table 3. The results of Cole's and AG models analyzes.

	$p_{\max}$ [MPa]	Standard deviation	Absolute error	Relative error	Energy [kJ]	Standard deviation	Absolute error	Relative error
W = 0.4 kg TNT, R = 25 m								
UNDEX	0.86	0.015	–	–	48.2	0.026	–	–
Cole's model	0.815	–	0.045	5.2	20.3	–	27.9	58.8
AG model	0.878	–	0.021	1.7	44.6	–	3.6	7.4
W = 0.4 kg TNT, R = 40 m								
UNDEX	0.57	0.012	–	–	29.2	0.039	–	–
Cole'smodel	0.476	–	0.094	16.4	11.7	–	17.5	60
AG model	0.548	–	0.022	3.8	17.9	–	11.3	38.7

The area under the surface of the kickoff pressure curve represents the energy transmitted by the shock wave per unit area. The work uses symbolic operations in MatLab to obtain comparable results from measurements and simulations. The results of the analyses are presented in Table 3.

The obtained results indicate a similar accuracy for standoff distance of R = 25 m. For R = 40 m, the AG model simulation gives more accurate results for $p_{\max}$ and energy transmitted per unit area.

7. Conclusions

Research on the UNDEX wave model was carried out on a small scale of weight explosive charge. Literature analysis indicates that experimental and numerical studies do not allow to clearly assign results for small and large detonations (An *et al.*, 2018; Brett, 1998). The problem is the lack of a dimensionless number for dynamic similitude for both processes. However, the obtained results for the AG model have the basic advantages for further research on UNDEX processes, i.e.:

- the AG model is a continuous function which facilitates the use in numerical dynamic models, unlike the Cole's model where the step function was used;
- simulation results for FFE indicate better similarity results for $p_{\max}$ and energy for AG model;
- the AG model contains a coefficient of sea bed, which in further research may allow better matching with mine bottom explosions for various types of seabed.

The studies are initiating research on a broader scale with the actual weight of the explosive charge of sea mines. Further research will be focused on a more precise determination coefficient of sea bed C_{SB}. The next objective will be a determination of the exponent of explosive charge m, which currently for small charges, has value at 1.

References

1. An F.J., Li X., Liu J., Wu C. (2018), Study of explosion loads of near-field underwater explosion, *International Journal of Multiphysics*, **12**(2): 117–130, doi: 10.21152/1750-9548.12.2.117.
2. Brett J.M. (1998), *Numerical modelling of shock wave and pressure pulse generation by underwater explosion*, DSTO Aeronautical and Marine Research Laboratory PO Box 4331, Melbourne, Australia.
3. Cole R.H. (1948), *Underwater explosions*, Princeton University Press, Princeton, doi: 10.5962/bhl.title.48411.
4. Dunbar T.E. (2009), *Modelling of close-proximity underwater explosion loads and structural response*, Defence R&D Canada – Atlantic, DRDC Atlantic CR 2008-272.
5. Grządziela A. (2011), Ship impact modelling of underwater explosion, *Journal of KONES Powertrain and Transport*, **18**(2): 145–152.
6. Huang H., Jiao Q.J. Nie J.X., Qin J.F. (2011), Numerical modelling of underwater explosion by one-dimensional ANSYS-AUTODYN, *Journal of Energetic Materials*, **29**(4): 292–325, doi: 10.1080/07370652.2010.527898.
7. Mair H.U. (1996), Preliminary compilation of underwater explosion benchmarks, *Proceedings of the 67th Shock and Vibration Symposium*, Volume I, SAVIAC, pp. 361–379.
8. Smith M.J., Lee J.J. (2017), *Ship-like target design for underwater explosion experiments*, Defence Research and Development Canada, Scientific Report, DRDC-RDDC-2017-R096, September 2017, https://cradpdf.drdc-rddc.gc.ca/PDFS/unc284/p805694_A1b.pdf.