archives
of thermodynamics
Vol. **41**(2020), No. 2, 147–168
DOI: 10.24425/ather.2020.133626

Generalized thermal microstretch elastic solid with harmonic wave for mode-I crack problem

KHALED LOTFY[*a,c]
ALAA ABD EL-BARY[b]
MOHAMED ALLAN [b]
MARWA H. AHMED[a,c]

[a] Department of Mathematics, Faculty of Science, P.O. Box 44519, Zagazig University, Zagazig, Egypt

[b] Arab Academy for Science,Technology and Maritime Transport, P.O. Box 1029, Alexandria, Egypt

[c] Department of Mathematics, Faculty of Science, Taibah University, Madinah, Saudi Arabia

Abstract A general model of the equations of generalized thermo-microstretch for an infinite space weakened by a finite linear opening mode-I crack is solved. Considered material is the homogeneous isotropic elastic half space. The crack is subjected to a prescribed temperature and stress distribution. The formulation is applied to generalized thermoelasticity theories, using mathematical analysis with the purview of the Lord-Şhulman (involving one relaxation time) and Green-Lindsay (includes two relaxation times) theories with respect to the classical dynamical coupled theory (CD). The harmonic wave method has been used to obtain the exact expression for normal displacement, normal stress force, coupled stresses, microstress and temperature distribution. Variations of the considered fields with the horizontal distance are explained graphically. A comparison is also made between the three theories and for different depths for the case of copper crystal.

Keywords: Mode-I Crack; L-S theory; GL theory; Thermoelasticity; Microrotation; Microstretch

*Corresponding Author. Email: khlotfy_1@yahoo.com, khlotfy@zu.edu.eg

Nomenclature

C_E	–	specific heat at constant strain ($C_E = 383.1$ J/kg/k)
e	–	dilatation
e_{ij}	–	components of strain tensor
J	–	current density vector
K	–	thermal conductivity ($K = 386$ N/Ks)
m_{ij}	–	couple stress tensor
t	–	time
T	–	absolute temperature
T_0	–	reference temperature chosen so that $\left\lvert \frac{T-T_0}{T_0} \right\rvert < 1$ ($T_0 = 293$ K)
u_1	–	components of displacement vector

Greek symbols

α_{t}	–	coefficient of linear thermal expansions ($\alpha_t = 1.78 \text{ x } 10^{-5}$ N/m^2)
$\alpha_0,\ \lambda_0, \lambda_1$	–	microstretch elastic constants
δ_{ij}	–	Kronecker delta
$\varepsilon_{\mathrm{ijr}}$	–	alternate tensor
$\lambda,\ \mu$	–	Lame's constants ($\lambda = 7.76\ \times 10^{10}$ N/m^2, $\mu = 3.86 \times 10^{10}$ N/m^2)
λ_{i}^*	–	first moment tensor
ν	–	Poisson's ratio
ρ	–	density ($\rho = 8954$ kg/m^3)
σ_{ij}	–	components of stress tensor
τ_0, ν_0	–	relaxation times ($\tau_0 = 0.005\,\mathrm{s},\ \nu_0 = 0.007\,\mathrm{s}$)
φ	–	rotation vector
φ^*	–	the scalar microstretch
$\mathrm{k}, \alpha, \beta, \gamma$	–	micropolar constants

1 Introduction

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the most common engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g., concrete, wood and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the clastomer and polymer type, e.g., polymethyl-methacrylate (Perspex), polyethylene and polyvinyl chloride. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves, i.e., for the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant. This influence of microstructure results in the development of new type of waves which are not in the classical theory of elasticity. Metals, polymers, composites, solids, rocks, concrete are typical media with microstructures. More generally, most of the natural and manmade materials including engineering, geological and biological media possess a microstructure. Eringen and Şuhubi [1] and Eringen [2] developed a linear theory of micropolar elasticity. Othman [3] studied the relaxation effects on thermal shock problems in elastic half space of generalized magneto-thermoelastic waves under three theories. Othman [4] construct a model of the two-dimensional equations of generalized magneto-thermoelasticity with two relaxation times in an isotropic elastic medium with the modulus of elasticity being dependent on the reference temperature. Eringen [5] introduced the theory of microstretch of elastic solids. This theory is a generalization of the theory of micropolar elasticity [2,6] and a special case of the micromorphic theory. The material points of microstretch elastic solids can stretch and contract independently of their translations and rotations. The micro-stretch continua are used to characterize composite materials and various porous media [7]. The basic results in the theory of micro stretch elastic solids were obtained in the literature [8-11].

The theory of thermo-microstretch elastic solids was introduced by Eringen [7]. In the frame-work of the theory of thermos-microstretch solids Eringen established a uniqueness theorem for the mixed initial-boundary value problem. The theory was illustrated through the solution of one dimensional waves and compared with lattice dynamical results. The asymptotic behavior of solutions and an existence result were presented by Bofill and Quintanilla [12]. A reciprocal theorem and a representation of Galerkin type were presented by De Cicco and Nappa [13]. De Cicco and Nappa [14] extended a linear theory of thermal microstretch elastic solids that permits the transmission of heat as thermal waves at finite speed. The theory is based on the entropy production inequality proposed by Green and Laws [15]. In [14], the uniqueness of the solution of the mixed initial-boundary-value problem is also investigated. The basic results and an extensive review on the theory of thermo-microstretch elastic solids can be found in the book of Eringen [8]. The coupled theory of thermoelasticity has been extended by including the thermal relaxation time in the constitutive equations by Lord and Shulman [16] and Green and Lindsay [17]. These theories eliminate the paradox of infinite velocity of heat propagation and are termed the generalized theories of thermoelasticity.

Lotfy [18] studied a novel solution of fractional order heat equation for photothermal waves in a semiconductor medium with a spherical cavity. Lotfy [19] Response of a semiconducting infinite medium under the two temperature theory with photothermal excitation due to laser pulses. Othman and Lotfy [20] studied the plane waves in generalized thermo-microstretch elastic half-space by using a general model of the equations of generalized thermo-microstretch for a homogeneous isotropic elastic half space. Othman and Lotfy [21] studied the generalized thermo-microstretch elastic medium with temperature dependent properties for different theories. Othman and Lotfy [22] studied the effect of magnetic field and inclined load in micropolar thermoelastic medium possessing cubic symmetry under three theories. The normal mode analysis was used to obtain the exact expression for the temperature distribution, thermal stresses, and the displacement components.

In recent years, considerable efforts have been devoted the study of failure and cracks in solids. This is due to the application of the latter generally in industry and particularly in the fabrication of electronic components. Most of the studies of dynamical crack problem are done using the equations of coupled or even uncoupled theories of thermoelasticity [23–31]. This is suitable for most situations where long time effects are sought. However, when short time are important, as in many practical situations, the full system of generalized thermoelastic equations must be used [16].

The purpose of the present paper is to obtain the normal displacement, temperature, normal force stress, and tangential couple stress in a microstretch elastic solid. The normal mode analysis used for the problem of generalized thermo-microstretch for an infinite space weakened by a finite linear opening mode-I crack is solving for the considered variables. The distributions of the considered variables are represented graphically. A comparison is carried out among the temperature, stresses and displacements as calculated from the generalized thermoelasticity (Lord-Şhulman, L-S), (Green-Lindsay, G-L) and (classical dynamical coupled theory, CD) theories for the propagation of waves in semi-infinite microstretch elastic solids.

2 Formulation of the problem

Following Eringen [3], Green and Lindsay [15] and Lord and Şhulman [16], the constitutive equations and field equations for a linear isotropic generalized thermo-microstretch elastic solid in the absence of body forces are obtained. We consider Cartesian coordinate system (x, y, z) having origin on the surface $y = 0$ and z-axis pointing vertically into the medium, the region G given by G = $\{(x, y, z)| -\infty < x < \infty, -\infty < z < \infty\}$, with a crack on the x-axis, $|\mathbf{x}| \leq a$, is considered. The crack surface is subjected to a known temperature and normal stresses distributions. There are many types of crack and this study will be devoted to mode-I shown in Fig. 1.

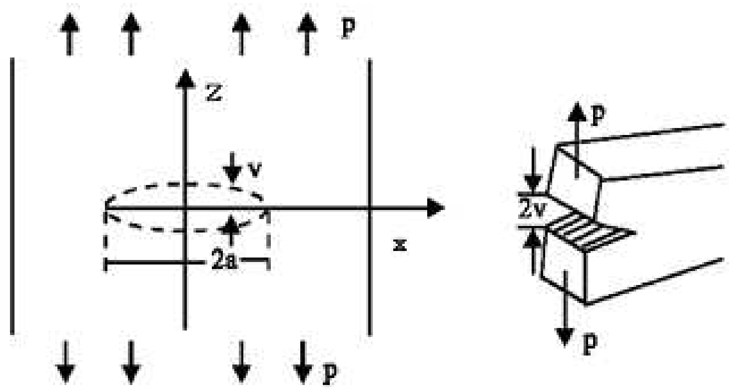

Figure 1: Displacement of an external mode-I crack.

The fundamental system of field equations consists of the equations of motion for a linear, isotropic generalized thermo-microstretch elastic soiled medium are given by:

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+(\mu+k)\nabla^2\vec{u}$$
$$+k(\nabla\vec{\varphi})+\lambda_0\nabla\varphi^*-\hat{\gamma}\left(1+-\nu_0\frac{\partial}{\partial t}\right)\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2}\,, \tag{1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\varphi})-\gamma\nabla\times(\nabla\times\vec{\varphi})+k(\nabla\times\vec{u})-2k\vec{\varphi}=j\rho\frac{\partial^2\vec{\varphi}}{\partial t^2}\,, \tag{2}$$

$$\alpha_0\nabla^2\varphi^*-\frac{1}{3}\lambda_1\varphi^*-\frac{1}{3}\lambda_0(\nabla\cdot\vec{u})+\frac{1}{3}\hat{\gamma}_1\left(1+\nu_0\frac{\partial}{\partial t}\right)T=\frac{3}{2}\rho j\frac{\partial^2\varphi^*}{\partial t^2}\,. \tag{3}$$

The equation of heat conduction

$$K\nabla^2T=\rho C_E\left(n_1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+\hat{\gamma}T_0\left(n_1+n_0\tau_0\frac{\partial}{\partial t}\right)\dot{e}+\hat{\gamma}_1T_0\frac{\partial\varphi^*}{\partial t}\,. \tag{4}$$

The constitutive law for the theory of generalized thermoelasticity with two relaxation times

$$\sigma_{il} = (\lambda_0\varphi^* + \lambda\, u_{r,r})\delta_{il} + (\,\mu + k\,)\, u_{l,i} + \mu\, u_{i,l} - k\,\varepsilon_{ilr}\varphi_r - \hat{\gamma}\left(1 + \nu_0\frac{\partial}{\partial\, t}\right) T\,\delta_{il}\ . \tag{5}$$

The field equations and constitutive relations for thermo-microstretch generalized thermoelastic medium read

$$m_{il} = \alpha\varphi_{r,r}\delta_{il} + \beta\varphi_{i,l} + \gamma\varphi_{l,i}\ , \tag{6}$$

$$\lambda_i = \alpha_0\,\varphi_i^*\ . \tag{7}$$

The relation between strain-displacement

$$e = \frac{\partial\, u}{\partial\, x} + \frac{\partial\, v}{\partial\, y} + \frac{\partial\, w}{\partial\, z}. \tag{8}$$

The state of plane strain parallel to the xz-plane is defined by

$$\begin{gathered} u_1 = u(x,z,t)\ ,\quad u_2 = 0\ ,\quad u_3 = w(x,z,t)\ ,\quad \varphi_1 = \varphi_3 = 0\ , \\ \varphi_2 = \varphi_2(x,z,t)\ ,\quad \varphi^* = \varphi^*(x,z,t)\ . \end{gathered} \tag{9}$$

The field Eqs. (1)–(4) reduce to

$$\begin{aligned} &(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x\partial z}\right) + (\mu+k)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \\ &\quad -k\frac{\partial\varphi_2}{\partial z} + \lambda_0\frac{\partial\varphi^*}{\partial x} - \hat{\gamma}\left(1+\nu_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}\ , \end{aligned} \tag{10}$$

$$\begin{aligned} &(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x\partial z} + \frac{\partial^2 w}{\partial z^2}\right) + (\mu+k)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) \\ &\quad +k\frac{\partial\varphi_2}{\partial x} + \lambda_0\frac{\partial\varphi^*}{\partial z} - \hat{\gamma}\left(1+\nu_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2}\ , \end{aligned} \tag{11}$$

$$\gamma\left(\frac{\partial^2\varphi_2}{\partial x^2} + \frac{\partial^2\varphi_2}{\partial z^2}\right) - 2k\varphi_2 + k\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = j\,\rho\frac{\partial^2\varphi_2}{\partial\, t^2}\ , \tag{12}$$

$$\begin{aligned} &\alpha_0\left(\frac{\partial^2\varphi^*}{\partial x^2} + \frac{\partial^2\varphi^*}{\partial z^2}\right) - \frac{1}{3}\lambda_1\varphi^* \\ &\quad -\frac{1}{3}\lambda_0\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + \frac{1}{3}\hat{\gamma}_1\,(1+\nu_0\frac{\partial}{\partial t})T = \frac{3}{2}\,\rho\, j\frac{\partial^2\varphi^*}{\partial\, t^2}\ , \end{aligned} \tag{13}$$

$$K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho C_E\left(n_1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}$$
$$+\hat{\gamma}T_0\left(n_1+n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial e}{\partial t}+\hat{\gamma}_1 T_0\frac{\partial \varphi^*}{\partial t}\,, \tag{14}$$

where

$$\hat{\gamma}=(3\lambda+2\mu+k)\,\alpha_{t_1}\,,\quad \hat{\gamma}_1=(3\lambda+2\mu+k)\alpha_{t_2},\quad \text{and}\quad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\,. \tag{15}$$

The constants $\hat{\gamma}$ and $\hat{\gamma}_1$ depend on mechanical as well as thermal properties of the body and the dot denotes the partial derivative with respect to time.

Equations (10)–(14) are the field equations of the generalized thermo-microstretch elastic solid, applicable to the (L-S) theory, the (G-L) theory, as well as the classical coupled theory (CD), as follows:

1. The equations of the coupled thermo-microstretch (CD) theory, when

$$n_0=0,\quad n_1=1,\quad \tau_0=\nu_0=0 \tag{16}$$

Equations (10), (11), (13), and (14) have the form:

$$\rho\,\ddot{u}=(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x\partial z}\right)$$
$$+(\mu+k)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)-k\frac{\partial \varphi_2}{\partial z}+\lambda_0\frac{\partial \varphi^*}{\partial x}-\hat{\gamma}\frac{\partial T}{\partial x}\,, \tag{17}$$

$$\rho\,\ddot{w}=(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial z^2}\right)$$
$$+(\mu+k)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)+k\frac{\partial \varphi_2}{\partial x}+\lambda_0\frac{\partial \varphi^*}{\partial z}-\hat{\gamma}\frac{\partial T}{\partial z}\,, \tag{18}$$

$$c_3^2\left(\frac{\partial^2 \varphi^*}{\partial x^2}+\frac{\partial^2 \varphi^*}{\partial z^2}\right)-c_4^2\varphi^*-c_5^2\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+c_6^2 T=\frac{\partial^2 \varphi^*}{\partial t^2}\,, \tag{19}$$

$$K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho C_E\frac{\partial T}{\partial t}+\hat{\gamma}T_0\frac{\partial e}{\partial t}+\hat{\gamma}_1 T_0\frac{\partial \varphi^*}{\partial t}\,. \tag{20}$$

The constitutive relation can be written as:

$$\sigma_{xx} = \lambda_0 \varphi^* + (\lambda + 2\mu + k)\,\frac{\partial u}{\partial x} + \lambda \frac{\partial w}{\partial z} - \hat{\gamma}\,T\ , \tag{21}$$

$$\sigma_{zz} = \lambda_0 \varphi^* + (\lambda + 2\mu + k)\,\frac{\partial w}{\partial z} + \lambda \frac{\partial u}{\partial x} - \hat{\gamma}\,T\,, \tag{22}$$

$$\sigma_{xz} = \mu \frac{\partial u}{\partial z} + (\mu + k)\,\frac{\partial w}{\partial x} + k\varphi_2\,, \tag{23}$$

$$\sigma_{zx} = \mu \frac{\partial w}{\partial x} + (\mu + k)\,\frac{\partial u}{\partial z} + k\varphi_2\,, \tag{24}$$

$$m_{xy} = \gamma\,\frac{\partial \varphi_2}{\partial x}\ , \tag{25}$$

$$m_{zy} = \gamma\,\frac{\partial \varphi_2}{\partial z}\ , \tag{26}$$

where

$$c_3^2 = \frac{2\alpha_0}{3\rho\,j}\ ,\quad c_4^2 = \frac{2\lambda_1}{9\rho\,j}\ ,\quad c_5^2 = \frac{2\lambda_0}{9\rho\,j}\ ,\quad c_6^2 = \frac{2\hat{\gamma}_1}{9\rho\,j}\ . \tag{27}$$

2. Lord-Şhulman (L-S) theory, when

$$n_1 = n_0 = 1, \quad \nu_0 = 0, \quad \tau_0 > 0\ . \tag{28}$$

Equations (10), (11), and (13) are the same as Eqs (17), (18), and (19) and Eq. (14) has the form

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial\,t^2}\right)(\rho\,C_E T + \hat{\gamma}\,T_0\,e) + \hat{\gamma}_1 T_0 \frac{\partial \varphi^*}{\partial\,t}\ . \tag{29}$$

3. Green-Lindsay (G-L) theory, when

$$n_1 = 1, \quad n_0 = 0, \qquad \nu_0 \geq\ \tau_0 > 0 \tag{30}$$

Equations (10), (11), and (13) remain unchanged and Eq. (14) has the form

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho\,C_E\left(1 + \tau_0 \frac{\partial}{\partial\,t}\right)\frac{\partial T}{\partial\,t} + \hat{\gamma}\,T_0\,\frac{\partial e}{\partial\,t} + \hat{\gamma}_1 T_0 \frac{\partial \varphi^*}{\partial\,t}\,. \tag{31}$$

4. The corresponding equations for the generalized micropolar thermoelasticity without stretch can be obtained from the above mentioned cases by taking:

$$\alpha_0 = \lambda_0 = \lambda_1 = \varphi^* = 0\ .$$

For convenience, the following non-dimensional variables are used:

$$\begin{aligned}
&\bar{x}_i = \tfrac{\omega*}{Co}\,x_i\,,\quad \bar{u}_i = \tfrac{\rho C2\,\omega*}{\hat{\gamma}To}u_i\,,\ \bar{t} = \omega*t\,,\ (\bar{\tau}_0\,\bar{\nu}_0 = \omega*(\tau_0\,,\nu_0),\\
&\bar{T} = \tfrac{T}{To}\,,\ \bar{\sigma}_{ij} = \tfrac{\sigma_{ij}}{\hat{\gamma}To}\,,\ (\bar{m}_{ij}\,,\bar{\lambda}_3) = \tfrac{\omega*}{C_2\hat{\gamma}T_0}(m_{ij},\,\lambda_3)\,,\\
&(\bar{\varphi}_2\,,\bar{\varphi}) = \tfrac{\rho C_2^2}{\hat{\gamma}To}\,,(\varphi_2\,,\varphi),\omega* = \tfrac{\rho C_E\,C_2^2}{K}\,,\ C_2^2 = \tfrac{\mu}{\rho}\,.
\end{aligned}\tag{32}$$

Using Eqs. (32), Eqs. (10)–(14) become (dropping the dashes for convenience)

$$\begin{aligned}
\frac{\partial^2 u}{\partial t^2} = \frac{(\mu+k)}{\rho C_2^2}\nabla^2 u + \frac{(\mu+\lambda)}{\rho C_2^2}\frac{\partial e}{\partial x}\\
-\frac{k}{\rho C_2^2}\frac{\partial\varphi_2}{\partial z} + \frac{\lambda_0}{\rho C_2^2}\frac{\partial\varphi^*}{\partial x} - \left(1+\nu_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}\,,
\end{aligned}\tag{33}$$

$$\begin{aligned}
\frac{\partial^2 w}{\partial t^2} = \frac{(\mu+k)}{\rho C_2^2}\nabla^2 w + \frac{(\mu+\lambda)}{\rho C_2^2}\frac{\partial e}{\partial z}\\
+\frac{k}{\rho C_2^2}\frac{\partial\varphi_2}{\partial x} + \frac{\lambda_0}{\rho C_2^2}\frac{\partial\varphi^*}{\partial z} - \left(1+\nu_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}\,,
\end{aligned}\tag{34}$$

$$\frac{j\,\rho C_2^2}{\gamma}\frac{\partial^2\varphi_2}{\partial\,t^2} = \nabla^2\varphi_2 - \frac{2kC_2^2}{\gamma\,w^*}\varphi_2 + \frac{kC_2^2}{\gamma\,w^{*2}}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\,,\tag{35}$$

$$\left(\frac{c_3^2}{C_2^2}\nabla^2 - \frac{c_4^2}{w^{*2}} - \frac{\partial^2}{\partial t^2}\right)\varphi^* - \frac{c_5^2}{w^{*2}}e + a_9\left(1+\nu_0\frac{\partial}{\partial t}\right)T = 0\tag{36}$$

$$\nabla^2 T - \left(n_1 + \tau_0\frac{\partial}{\partial\,t}\right)\frac{\partial T}{\partial\,t} - \frac{\hat{\gamma}^2\,T_0}{\rho K w^*}\left(n_1 + n_0\tau_0\frac{\partial}{\partial\,t}\right)\frac{\partial e}{\partial\,t} = \frac{\hat{\gamma}\,\hat{\gamma}_1 T_0}{\rho K w^*}\frac{\partial\varphi^*}{\partial\,t}.\tag{37}$$

Assuming the scalar potential functions $\varphi(x,\,z,\,t)$ and $\psi(x,\,z,\,t)$ defined by the relations in the non-dimensional form:

$$u = \frac{\partial\varphi}{\partial x} + \partial\psi\partial z\,,\quad w = \frac{\partial\varphi}{\partial z} - \partial\psi\partial x\,.\tag{38}$$

Using (38) in Eqs. (33)–(37), we obtain:

$$\left[\nabla^2 -\ a_0\frac{\partial^2}{\partial t^2}\right]\varphi\ -\ a_0\,(1+\nu_0\,\frac{\partial}{\partial t})T +\ a_1\varphi^*\ =\ \ 0,\tag{39}$$

$$\left[\nabla^2 -\ a_2\frac{\partial^2}{\partial t^2}\right]\psi\ -\ a_3\varphi_2 = 0,\tag{40}$$

$$\left[\nabla^2 - 2\,a_4 - \ a_5\,\frac{\partial^2}{\partial t^2}\right]\varphi_2 - \ a_4\,\nabla^2\psi = \ 0, \tag{41}$$

$$\left[a_6\nabla^2 \ - a_7 - \frac{\partial^2}{\partial\, t^2}\right]\varphi^* - a_8\nabla^2\varphi + a_9(1+\nu_0\frac{\partial}{\partial\, t}\,)\,T \ = \ 0, \tag{42}$$

$$\left[\nabla^2 - \left(n_1\frac{\partial}{\partial t} + \tau_0\,\frac{\partial^2}{\partial t^2}\right)\right]T \ - \ \varepsilon\;(\,n_1\frac{\partial}{\partial t} + \ n_0\tau_0\,\frac{\partial^2}{\partial t^2}\,)\,\nabla^2\varphi - \ \varepsilon_1\frac{\partial\varphi^*}{\partial t} = 0, \tag{43}$$

where

$$\begin{gathered}
c_1^2 = \frac{\lambda+2\mu+k}{\rho}\ ,\ a_0 = \frac{C_2^2}{C_1^2}\ ,\ a_1 = \frac{\lambda_0}{\lambda+2\mu+k}\ ,\ a_2 = \frac{\rho C_2^2}{\mu+k}\ ,\\
a_3 = \frac{k}{\mu+k}\,,\ a_4 = \frac{k\,C_2^2}{\gamma w^{*2}}\ ,\ a_5 = \frac{\rho j C_2^2}{\gamma}\,,\ a_6 = \frac{C_3^2}{C_2^2}\,,\ a_7 = \frac{C_4^2}{w^{*2}}\ ,\\
a_8 = \frac{C_5^2}{w^{*2}}\ ,\ a_9 = \frac{2\hat{\gamma}_1 C_2^2}{9\hat{\gamma} j w^{*2}}\ ,\ \varepsilon = \frac{\hat{\gamma}^2\,T_0}{\rho\,\omega^* K}\ ,\ \varepsilon_1 = \frac{\hat{\gamma}\,\hat{\gamma}_1 T_0}{\rho\,\omega^* K}\ .
\end{gathered} \tag{44}$$

3 Harmonic wave method

The solution of the considered physical variables can be decomposed in terms of normal mode as the following form:

$$\begin{aligned}
&[\varphi\,,\psi\,,\varphi^*\,,\varphi_2\,,\sigma_{il}\,,m_{il}\,,T](x\,,z\,,t)\\
&\quad = [\,\bar{\varphi},\bar{\psi},\bar{\varphi}^*,\ \bar{\varphi}_2\,,\bar{\sigma}_{il},\bar{m}_{il},\bar{T}]\,(x)e^{(\omega\,t+i\,a\,z)}\ ,
\end{aligned} \tag{45}$$

where $[\,\bar{\varphi},\bar{\psi},\bar{\varphi}^*,\ \bar{\varphi}_2\,,\bar{\sigma}_{il},\bar{m}_{il},\bar{T}](x)$ are the amplitude of the functions ω is a complex and a is the wave number in the z-direction.
Using Eq. (45), then Eqs. (39)–(43) become respectively

$$(\,D^2 - A_1)\bar{\varphi} - A_2\,\bar{T} + a_1\,\bar{\varphi}^* = 0\ , \tag{46}$$

$$(\,D^2 - A_3)\bar{\psi} - a_3\,\bar{\varphi}_2 = 0\ , \tag{47}$$

$$(\,D^2 - A_4)\,\bar{\varphi}_2 - a_4(D^2 - a^2)\bar{\psi} = 0\ , \tag{48}$$

$$(\,a_6 D^2 - A_5)\bar{\varphi}^* - a_8(D^2 - a^2)\bar{\varphi} + A_8\bar{T} = 0\ , \tag{49}$$

$$\left[(D^2 - a^2) - A_6\right]\bar{T} - A_7(D^2 - a^2)\bar{\varphi} - \varepsilon_1\omega\bar{\varphi}^* = 0\ , \tag{50}$$

where $D = \frac{d}{dx}$,

$$A_1 = a^2 + a_0\omega^2\ , \tag{51}$$

$$A_2 \;=\; a_0 \;\; (1+\nu_0\omega)\,, \tag{52}$$

$$A_3 = a^2 + a_2\omega^2 \;, \tag{53}$$

$$A_4 = a^2 + 2a_4 + a_5\omega^2 \;, \tag{54}$$

$$A_5 = a^2 a_6 + a_7 + \omega^2 \;, \tag{55}$$

$$A_6 = \omega \;\; (n_1 + \tau_0\omega) \;\;, \tag{56}$$

$$A_7 = \varepsilon\omega \;\; (n_1 + n_0\tau_0\omega) \;\;, \tag{57}$$

$$A_8 \;=\; a_9 \;\; (1+\nu_0\omega) \;\; . \tag{58}$$

Eliminating $\bar{\varphi}_2, \bar{\psi}$ between Eqs. (47) and (48), we get the following fourth order ordinary differential equation satisfied by $\bar{\varphi}_2$ and $\bar{\psi}$

$$\Big[\,D^4 - A\,D^2 + B\Big] \;\Big\{\bar{\varphi}_2(x), \bar{\psi}(x)\Big\} \;= 0 \;\; . \tag{59}$$

Eliminating $\bar{\varphi}$, $\bar{T}$, and $\bar{\varphi}^*$ between Eqs. (46), (49), and (50) we obtain the following sixth order ordinary differential equation satisfied by $\bar{\varphi}^*(x), \bar{\varphi}(x)$ and $\bar{T}(x)$

$$\Big[\,D^6 - C\,D^4 + E\,D^2 - H\Big] \;\Big\{\bar{\varphi}^*(x), \bar{\varphi}(x), \bar{T}(x)\Big\} \;= 0\,, \tag{60}$$

where

$$A = A_3 + A_4 + \;\; a_3 a_4 \;, \tag{61}$$

$$B = A_3 A_4 + a^2 a_3 a_4 \;, \tag{62}$$

$$C = \frac{g_3(g_7+g_8) - g_5}{g_3} \;, \tag{63}$$

$$E = \frac{a^2 g_3 g_8 + \varepsilon_1\omega g_1 A_2 - g_6 - g_5 g_7 - g_4 g_8}{g_3} \;, \tag{64}$$

$$H = \frac{a^2 g_4 g_8 + g_6 g_7 - \varepsilon_1\omega g_2 A_2}{g_3} \;, \tag{65}$$

$$\begin{gathered} g_1 = A_8 - a_8 A_2\,, \quad g_2 = a^2 a_8 A_2 - A_1 A_8\,, \quad g_3 = a_6\,A_2\,, \\ g_4 = a_1\,A_8 + A_5 A_2\,, \quad g_5 = g_4 - A_1\,g_3 - \;a_1 g_1\,, \\ g_6 = A_1\,g_4 + \;a_1 g_2\,, \quad g_7 = a^2 A_6 + A_7\,, \quad g_8 = \; A_2 A_8\,. \end{gathered} \tag{66}$$

The solution of Eqs. (59) and (60), has the form:

$$\bar{\psi}(x) = \sum_{j=1}^{2} M_j(a,\omega)\ e^{-k_j x}, \tag{67}$$

$$\bar{\varphi}_2(x) = \sum_{j=1}^{2} M'_j(a,\omega)\ e^{-k_j x}, \tag{68}$$

$$\bar{\varphi}(x) = \sum_{n=3}^{5} M_n(a,\omega)\ e^{-k_n x}, \tag{69}$$

$$\bar{\varphi}^*(x) = \sum_{n=3}^{5} M'_n(a,\omega)\ e^{-k_n x}, \tag{70}$$

$$\bar{T}(x) = \sum_{n=3}^{5} M''_n(a,\omega)\ e^{-k_n x}, \tag{71}$$

where $M_j(a,\omega)$, $M'_j(a,\omega)$, $M_n(a,\omega)$, $M'_n(a,\omega)$ and $M''_n(a,\omega)$ are some parameters depending on a *and* ω. Subsequently k_j^2, ($j = 1, 2$) are the roots of the characteristic equation of Eq. (59) and k_n^2, ($n = 3, 4, 5$) are the roots of the characteristic equation of Eq. (60). Using Eqs. (67)- (71) into Eqs. (46) and (50) we get the following relations

$$\bar{\varphi}_2(x) = \sum_{j=1}^{2} a_j^* M_j(a,\omega)\ e^{-k_j x}, \tag{72}$$

$$\bar{\varphi}^*(x) = \sum_{n=3}^{5} b_n^* M_n(a,\omega)\ e^{-k_n x}, \tag{73}$$

$$\bar{T}(x) = \sum_{n=3}^{5} c_n^* M_n(a,\omega)\ e^{-k_n x}. \tag{74}$$

where,

$$a_j^* = \frac{k_j^2 - A_3}{a_3}, \qquad j = 1, 2\ , \tag{75}$$

$$b_n^* = \frac{g_1 k_n^2 + g_2}{g_3 k_n^2 + g_4}, \qquad n = 3, 4, 5\ , \tag{76}$$

$$c_n^* = \frac{g_3 k_n^4 + g_5 k_n^2 - g_6}{A_2\,(g_3 k_n^2 + g_4)}, \qquad n = 3, 4, 5\ . \tag{77}$$

4 Application

The plane boundary subjects to an instantaneous normal point force and the boundary surface is isothermal, the boundary conditions at the vertical plan $y = 0$ and in the beginning of the crack at $x = 0$ are

$$\begin{gathered}\sigma_{zz} = -p(x)\,, \quad |x| < a\,,\\ T = f(x)\,, \quad |x| < a \ \text{ and } \ \frac{\partial T}{\partial z} = 0\,, \ \ |x| > a\,,\\ \sigma_{xz} = 0\,, \qquad -\infty < x < \infty\,,\\ m_{xy} = 0\,, \qquad -\infty < x < \infty\,,\\ \lambda_z = 0\,, \qquad -\infty < x < \infty\,.\end{gathered} \tag{78}$$

Using (32), (38), (39)–(43) with the non-dimensional boundary conditions and using (67), (69), (72)–(74), we obtain the expressions of displacement components, force stress, coupled stress and temperature distribution for microstretch generalized thermoelastic medium as follows:

$$\begin{aligned}\bar{u}(x) = {} & ia(M_1 e^{-k_1 x} + M_2 e^{-k_2 x})\\ & -k_3 M_3 e^{-k_3 x} - k_4 M_4 e^{-k_4 x} - k_5 M_5 e^{-k_5 x}\,,\end{aligned} \tag{79}$$

$$\begin{aligned}\bar{w}(x) = {} & k_1 M_1 e^{-k_1 x} + k_2 M_2 e^{-k_2 x}\\ & + i\,a(M_3 e^{-k_3 x} + k_4 M_4 e^{-k_4 x} + k_5 M_5 e^{-k_5 x})\,,\end{aligned} \tag{80}$$

$$\begin{aligned}\bar{\sigma}_{zz}(x) = {} & s_1 M_1 e^{-k_1 x} + s_2 M_2 e^{-k_2 x}\\ & + s_3 M_3 e^{-k_3 x} + s_4 M_4 e^{-k_4 x} + s_5 M_5 e^{-k_5 x}\,,\end{aligned} \tag{81}$$

$$\begin{aligned}\bar{\sigma}_{xz}(x) = {} & r_1 M_1 e^{-k_1 x} + r_2 M_2 e^{-k_2 x}\\ & + r_3 M_3 e^{-k_3 x} + r_4 M_4 e^{-k_4 x} + r_5 M_5 e^{-k_5 x}\,,\end{aligned} \tag{82}$$

$$\bar{m}_{xy}(x) = q_1 M_1 e^{-k_1 x} + q_2 M_2 e^{-k_2 x}\,, \tag{83}$$

$$\bar{T}(x) = c_3^* M_3 e^{-k_3 x} + c_4^* M_4 e^{-k_4 x} + c_5^* M_5 e^{-k_5 x}\,, \tag{84}$$

$$\lambda_z = f_8(b_3^* M_3 e^{-k_3 x} + b_4^* M_4 e^{-k_4 x} + b_5^* M_5 e^{-k_5 x})\,, \tag{85}$$

where

$s_1 = iak_1(f_2 - f_3)$, $\quad s_2 = iak_2(f_2 - f_3)$,

$s_3 = f_1 b_3^* - a^2 f_2 + f_3 k_3^2 - c_3^*(1 + \nu_0 \omega)$,
$s_4 = f_1 b_4^* - a^2 f_2 + f_3 k_4^2 - c_4^*(1 + \nu_0 \omega)$,
$s_5 = f_1 b_5^* - a^2 f_2 + f_3 k_5^2 - c_5^*(1 + \nu_0 \omega)$,
$r_1 = a_1^* f_6 - a^2 f_4 - f_5 k_1^2, \quad r_2 = a_2^* f_6 - a^2 f_4 - f_5 k_2^2$,
$r_3 = -\, i a k_3 (f_4 + f_5), \quad r_4 = -\, i a k_4 (f_4 + f_5), \quad r_5 = i a k_5 (f_4 + f_5)$,
$q_1 = -f_7 a_1^* k_1, \quad q_2 = -f_7 a_2^* k_2, \quad f_1 = \frac{\lambda_0}{\rho c_2^2}, \quad f_2 = \frac{\lambda + 2\mu + k}{\rho\, c_2^2}, \quad f_3 = \frac{\lambda}{\rho\, c_2^2}$,
$$f_4 = \frac{\mu}{\rho\, c_2^2}, \quad f_5 = \frac{\mu + k}{\rho\, c_2^2}, \quad f_6 = \frac{k}{\rho\, c_2^2}, \quad f_7 = \frac{\gamma {\omega^*}^2}{\rho\, c_2^4}, \text{ and } f_8 = \frac{\alpha_0 {\omega^*}^2}{\rho\, c_2^4}\,. \tag{86}$$

Applying the boundary conditions (78) at the surface $x = 0$ of the plate, we obtain a system of five equations. After applying the inverse of matrix method, we obtain the values of the five constants M_j, $j = 1, 2$, and M_n, $n = 3, 4, 5$. Hence, we obtain the expressions for displacements, force stress, couple stress and temperature distribution for microstretch generalized thermoelastic medium.

5 Numerical results and discussions

In order to illustrate our theoretical results obtained in preceding section and to compare these in the context of various theories of thermoelasticity, we now present some numerical results. In the calculation process, we take the case of copper crystal as the material subjected to mechanical and thermal disturbances for numerical calculations. Since, ω is the complex constant then we taken $\omega = \omega_0 + i\,\zeta$. The other constants of the problem are taken as $\omega_0 = -2$; $\zeta = 0.01$ and $a = 1$.

The results are shown in Figs. 2–15. The graph shows the three curves predicted by different theories of thermoelasticity. In these figures, the solid lines represent the solution in the coupled theory, the dotted lines represent the solution in the generalized Lord and Şhulman theory and dashed lines represent the solution derived using the Green and Lindsay theory. We notice that the results for the temperature, the displacement and stresses distribution when the relaxation time is including in the heat equation are distinctly different from those when the relaxation time is not mentioned in the heat equation, because the thermal waves in the Fourier's theory of heat equation travel with an infinite speed of propagation as opposed to finite speed in the non-Fourier case. This demonstrates clearly the difference between the coupled and the generalized theories of thermoelasticity.

For the value of z, namely $z = 0.1$, were substituted in performing the computation. It should be noted (Fig.2) that in this problem, the crack's size, x is taken to be the length in this problem so that $0 \leq x \leq 3$, $z = 0$ represents the plane of the crack that is symmetric with respect to the z-plane. It is clear from the graph that T has a maximum value at the beginning of the crack ($x = 0$), it begins to fall just near the crack edge ($x = 3$), where it experiences sharp decreases (with maximum negative gradient at the crack's end). The value of temperature quantity converges to zero with increasing distance x.

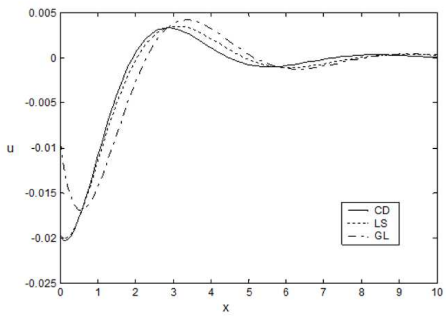

Figure 2: Variation of temperature distribution T with different theories.

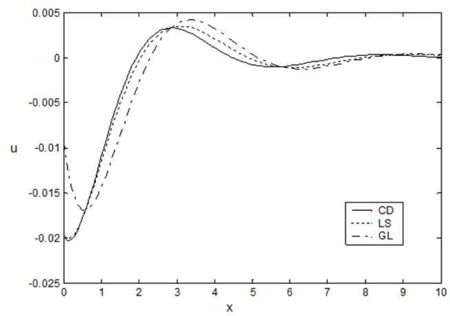

Figure 3: Variation of displacement distribution u with different theories.

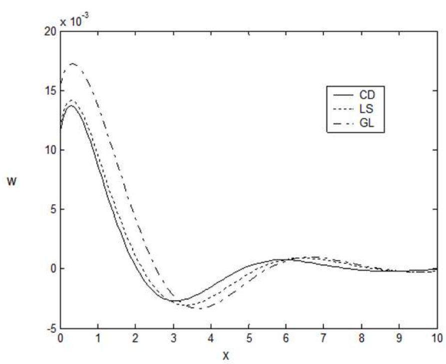

Figure 4: Variation of displacement distribution w with different theories.

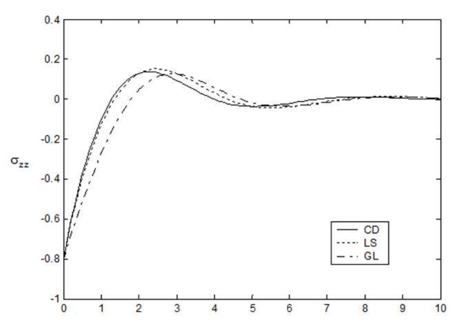

Figure 5: Variation of stress distribution σ_{xz} with different theories

In Fig. 3, the horizontal displacement, u, begins with the descent followed by the smooth increase to reach its maximum just at the crack end. Beyond it u falls again to try to retain zero at infinity. Figure 4 presents the vertical displacement w. We see that the displacement component w always starts from the zero value and terminates at the zero value. Also, at the crack end to reach minimum value, beyond reaching zero at the double

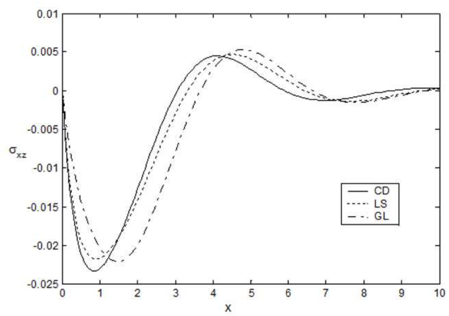

Figure 6: Variation of stress distribution σ_{xz} with different theories.

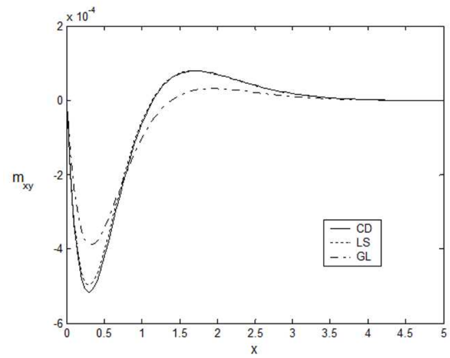

Figure 7: Variation of tangential couple stress m_{xy} with different theories.

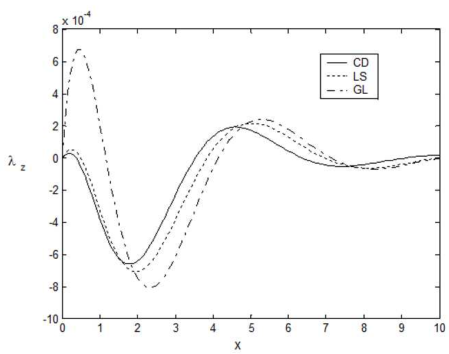

Figure 8: Variation of microstress λ_z with different theories.

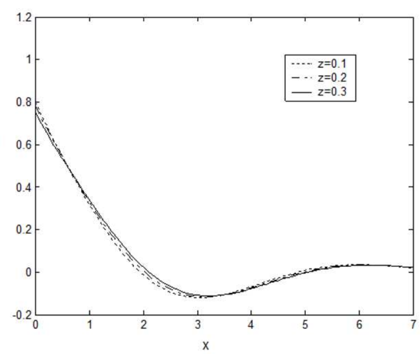

Figure 9: Variation of temperature distribution T for different vertical distances, under GL theory.

of the crack size (state of particles equilibrium).

The displacements u and w show different behaviours, because of the elasticity of the solid tends to resist vertical displacements in the problem under investigation. Both of the components show different behaviours, the former tends to increase to maximum just before the end of the crack. Then it falls to a minimum with a highly negative gradient. Afterwards it rises again to a maximum beyond about the crack end. The stress component, σ_{zz} reaches coincidence with negative value (Fig. 5) and satisfies the boundary condition at $x = 0$. Then it reaches the maximum value near the end of crack ($x \approx 3$) and converges to zero with the increasing distance

x. Figure 6, shows that the stress component σ_{xz} satisfies the boundary condition at $x = 0$ and exhibits a different behaviour. It initially decreases followed by increasing to reach the maximum in the context of the three theories until reaching the crack end. These trends obey elastic and thermoelastic properties of the solid under investigation.

Figure 7 shows the tangential coupled stress m_{xy} which satisfies the boundary condition at $x = 0$. It decreases in the start and start increasing to reach the maximum in the context of the three theories until reaching the crack end. The values of microstress for λ_z satisfy the boundary condition at $x = 0$. The distribution begins with increase followed by decrease to reach its minimum magnitude just near the crack end, beyond reaching zero at the double of the crack size (state of particles equilibrium), as depicted in Fig. 8.

Figures 9–15 show the comparison between the temperature T, displacement components u, w, force stresses components σ_{zz}, σ_{xz}, tangential coupled stress m_{xy} and the microstress λ_z, for the case of three different values of z, (namely $z = 0.1$, $z = 0.2$ and $z = 0.3$) under GL theory. It should be noted (Fig. 9) that in this problem it is clear from the graph that T reaches the maximum value at the beginning of the crack ($x = 0$), it begins to fall just near the crack edge ($x = 3$), where it experiences sharp decreases (with maximum negative gradient at the crack's end). Graph lines for both values of y show different slopes at crack ends according to y-values. In other words, the temperature line for $z = 0.1$ has the highest gradient when compared with that of $z = 0.2$ and $z = 0.3$ at the first of the range. In addition, all lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach the reference temperature of the solid. These results obey physical reality for the behaviour of copper as a polycrystalline solid.

Figure 10 presents the horizontal displacement u, despite the peaks (for different vertical distances $z = 0.1$, $z = 0.2$, and $z = 0.3$) which occur at equal value of x. The magnitude of the maximum displacement peak strongly depends on the vertical distance y. It is also clear that the rate of change of u increases with increasing y as we go farther apart from the crack. On the other hand, Fig. 11 shows atonable increase of the vertical displacement, w, near the crack end to reach minimum value beyond $x = 3$ reaching zero at the double of the crack size(state of particles equilibrium).

Figure 12 shows the vertical stresses σ_{zz}. Graph lines for both values of z show different slopes at crack ends according to z-values. In other words, the σ_{zz} component line for $z = 0.1$ has the highest gradient when compared with that of $z = 0.2$ and $z = 0.3$ at the edge of the crack. In addition, all lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach zero after their relaxations at infinity. Variation of y has a serious effect on both magnitudes of mechanical stresses. These trends obey elastic and thermoelastic properties of the solid under investigation. Figure 13, shows that the stress component σ_{xz} satisfies the boundary condition, the line for $z = 0.3$ has the highest gradient when compared with that of $z = 0.2$ and $z = 0.1$ in the range $0 \leq x \leq 2.5$, the line for $z = 0.1$ has the highest gradient when compared with that of $z = 0.2$ and $z = 0.3$ in the range $2.5 \leq x \leq 5$ and converge to zero when $x > 5$. These trends obey elastic and thermoelastic properties of the solid.

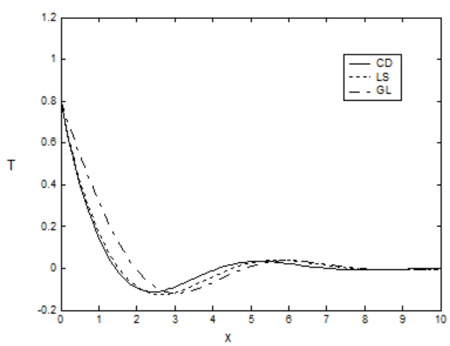

Figure 10: Variation of displacement distribution u for different vertical distances, under GL theory.

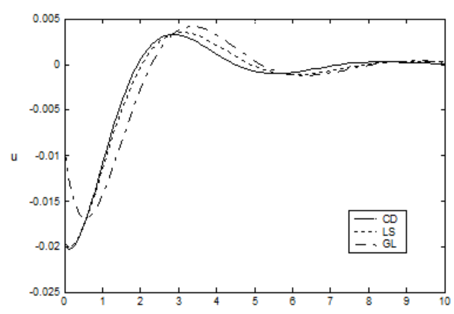

Figure 11: Variation of displacement distribution w for different vertical distances, under GL theory.

Figure 14 presents the tangential coupled stress m_{xy} which decreases at the start followed by increasing to attain maximum in the context of the three values of z until reaching the crack end. For $z = 0.3$ it has the highest gradient when compared with that of $z = 0.2$ and $z = 0.1$ at the edge of the crack. All lines begin to coincide when the horizontal distance x is beyond the edge of the crack. In Fig. 15 shown are the values of microstress for λ_z which increases at the beginning followed by the decrease to reach the minimum in the context of the three values of z until reaching nearly the crack end. For $z = 0.3$ it has the highest gradient when compared with that of $z = 0.2$ and $z = 0.1$ at the edge of the crack. All lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach zero after their relaxations at infinity.

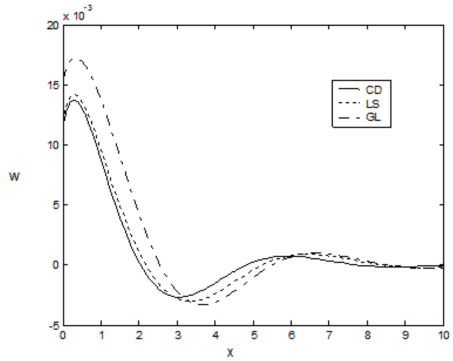

Figure 12: Variation of stress distribution σ_{zz} for different vertical distances, under GL theory.

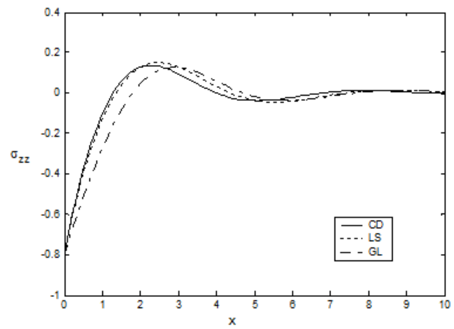

Figure 13: Variation of stress distribution σ_{xz} for different vertical distances, under GL theory.

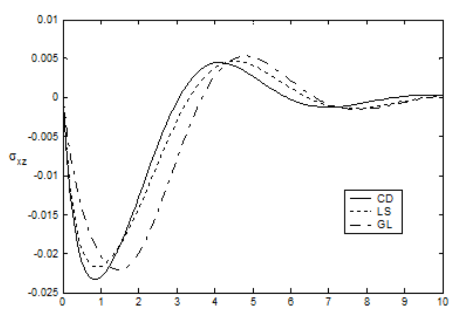

Figure 14: Variation of tangential couple stress m_{xy} for different vertical distances, under GL theory.

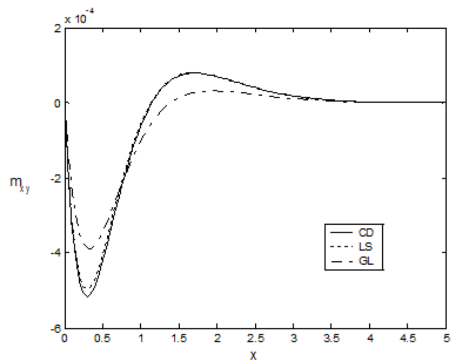

Figure 15: Variation of microstress λ_z for different vertical distances, under GL theory.

6 Conclusions

The models of generalized thermo-microstretch for an infinite space weakened by a finite linear opening mode-I crack is solved. The physical quantities are given analytically and illustrated graphically by the normal mode method. The effects of the thermal relaxation time (three theories), the case of different three values of the depth are discussed. The following conclusions can be drawn:

1. The curves in the context of (CD), (L-S) and (G-L) theories decrease exponentially with increasing x, this indicate that the thermoelastic

waves are unattenuated and nondispersive, where purely thermoelastic waves undergo both attenuation and dispersion.

2. The presence of microstretch plays a significant role in all the physical quantities.
3. The curves of the physical quantities with (L-S) theory in most of figures are lower in comparison with those under (G-L) theory, due to the relaxation times.
4. Analytical solutions based upon normal mode analysis for themoelastic problem in solids have been developed and utilized.
5. A linear opening mode-I crack has been investigated and studied for copper solid
6. Temperature, radial and axial distributions were estimated at different distances from the crack edge.
7. The stresses distributions, the tangential coupled stress and the values of microstress were evaluated as functions of the distance from the crack edge.
8. Crack dimensions are significant to elucidate the mechanical structure of the solid.
9. Cracks are stationary and external stress is demanded to propagate such cracks.
10. It can be concluded that a change of volume is attended by a change of the temperature while the effect of the deformation upon the temperature distribution is the subject of the theory of thermoelasticity.
11. The value of all the physical quantities converges to zero with an increase in distance y and all functions are continuous.

Received 1 March 2018

References

[1] Eringen A.C., Suhubi E.S.: *Non linear theory of simple micropolar solids.* Int. J. Engng. Sci. **2**(1964), 1–18.

[2] Eringen A.C.: *Linear theory of micropolar elasticity.* J. Math. Mech. **15**(1966), 909–923.

[3] Othman M.I.A.: *Relaxation effects on thermal shock problems in an elastic half-space of generalized magneto-thermoelastic waves*. Mechanics Mechanical Eng. **7**(2004), 165–178.

[4] Othman M.I.A, Lotfy Kh. and Farouk R.M.: *Transient disturbance in a half-space under generalized magneto-thermoelasticity with internal heat source*. Acta Phys. Pol. A **116(**2009), 2, 185–192.

[5] Eringen A.C.: *Micropolar elastic solids with stretch*. Ari Kitabevi Matbassi, Istanbul **24**(1971), 1–18.

[6] Eringen A.C.: *Theory of micropolar elasticity*. In: , Fracture, Vol. II, (H. Liebowitz, Ed.) Academic Press, New York 1968.

[7] Eringen A.C.: *Theory of thermo-microstretch elastic solids*. Int. J. Engng. Sci. **28**(1990), 291–1301.

[8] Eringen A.C.: *Microcontinuum Field Theories I: Foundation and Solids*. Springer-Verlag, New York, Berlin, Heidelberg 1999.

[9] Iesau D., Nappa L.: *On the plane strain of microstretch elastic solids*. Int. J. Engng. Sci. **39**(2001), 1815–1835.

[10] Iesau D., Pompei A.: *On the equilibrium theory of microstretch elastic solids*. Int. J. Engng. Sci. **33**(1995), 399–410.

[11] De Cicco S.: *Stress concentration effects in microstretch elastic bodies*. Int. J. Engng. Sci. **41**(2003), 187–199.

[12] Bofill F., Quintanilla R.: *Some qualitative results for the linear theory of thermo-microstretcli elastic solids*. Int. J. Engng. Sci. **33**(1995), 2115–2125.

[13] De Cicco S., Nappa L.: *Some results in the linear theory of thermo-microstretch elastic solids*. J. Math. Mech. **5**(2000), 467–482.

[14] De Cicco S., Nappa L.: *On the theory of thermomicrostretch elastic solids*. J. Therm. Stresses **22**(1999), 565–580.

[15] Green A.E., Laws N.: *On the entropy production inequality*. Arch. Ration. Mech. Anal. **45**(1972), 47–59.

[16] Lord H.W., Şhulman Y.: *A generalized dynamical theory of thennoelasticity*. J. Mech. Phys. Solid **15**(1967), 299–306.

[17] Green A.E., Lindsay K.A.: *Thermoelasticity*. J. Elasticity **2**(1972), 1–7.

[18] Lotfy Kh.: *A novel solution of fractional order heat equation for photothermal waves in a semiconductor medium with a spherical cavity*. Chaos Soliton Fract. **99**(2017), 233–242.

[19] Lotfy Kh., Gabr M.E.: *Response of a semiconducting infinite medium under two temperature theory with photothermal excitation due to laser pulses*. Opt. Laser Technol. **97**(2017), 198–208.

[20] Othman M.I.A., Lotfy Kh.: *On the plane waves in generalized thermo- microstretch elastic half-space*. Int. Commun. Heat Mass **37**(2010), 192–200.

[21] Othman M.I.A.., Lotfy Kh.: *Effect of magnetic field and inclined in micropolar thermoelastic medium possessing cubic symmetry*. Int. J. Industrial Math. **1**(2009), 2, 87–104.

[22] Othman M.I.A., Lotfy Kh.: *Generalized thermo-microstretch elastic medium with temperature dependent properties for different theories*. Eng. Anal. Bound. Elem. **34**(2010), 229–237.

[23] Dhaliwal R.: *External Crack due to Thermal Effects in an Infinite Elastic Solid with a Cylindrical Inclusion*. In: Thermal Stresses in Server Environments, Springer-Verlag, 1980, 665–692.

[24] Lotfy Kh., Abo-Dahb S.: *Two-dimensional problem of two temperature generalized thermoelasticity with normal mode analysis under thermal shock problem*. J. Comput. Theor. Nanos. **12**(2015), 8, 1709–1719.

[25] Abo-Dahb S., Lotfy Kh., Gohaly A.: *Rotation and magnetic field effect on surface waves propagation in an elastic layer lying over a generalized thermoelastic diffusive half-space with imperfect boundary.* Math. Probl. Eng. 2015.

[26] Lotfy Kh., Abo-Dah S.: *Generalized Magneto-thermoelasticity with fractional derivative heat transfer for a rotation of a fibre-reinforced thermoelastic*. J. Comput. Theor. Nanos. **12**(2015), 8, 1869–1881.

[27] Lotfy Kh.: *The elastic wave motions for a photothermal medium of a dual-phase-lag model with an internal heat source and gravitational field*. Can. J. Phys. **94**(2016), 400–409.

[28] Lotfy Kh.: *Mode-I crack in a two-dimensional fibre-reinforced generalized thermoelastic problem*. Chin. Phys. B. **2**(2012), 1, 014209.

[29] Lotfy Kh.: *The effect of a magnetic field on a 2D problem of fibre-reinforced thermoelasticity rotation under three theories*. Chin. Phys. B. **21**(2012), 064214.

[30] Sarkar N., Lahiri A.: *A three-dimensional thermoelastic problem for a half-space without energy dissipation.* Int. J. Eng. Sci. **51**(2012), 310–325.

[31] Abd-alla A.M.: *Influences of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space*. Appl. Math. Sci. **4**(2010), 2, 91–108.