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Thermal simulation of a continuous casting process subjected to water-sprays cooling

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Abstract As in many thermal processing technologies, there is a delicate balance between productivity and quality during ingot cooling process. Higher cooling velocities increase productivity but also create higher temperature gradients inside the ingot. Such a fast cooling does not leave sufficient time to establish the equilibrium within the solid, thus the final metal structure is strongly affected by the set up cooling mode throughout the liquid metal solidification. The first intention in this paper is to compare between three cooling modes in order to identify the required mode for a continuous casting process. Then, we study the influence of heat transfer coefficient on metal liquid-to-solid transition through the spray-cooled zone temperature and the metal latent heat of solidification. A gray iron continuous casting process subjected to water-sprays cooling was simulated using the commercial software for modeling and simulating multiphysics and engineering problems. The primary conclusions, from the obtained results, show the forcefulness of water spray cooling regarding standard cooling. Afterward, we highlight the great influence of heat transfer coefficient on the location of transition region as well as the relationship between heat transfer coefficient, wall outer temperature, latent heat dissipation, and the solidification time.

Keywords: Continuous casting; Shear region; Water-spray; Latent heat; Thermal optimization

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Nomenclature

C_p	– specific heat capacity, J/(kgK)
C_{p_l}	– specific heat of liquid metal, J/(kgK)
C_{p_s}	– specific heat of solid metal, J/(kgK)
ΔC_p	– specific heat variation, J/(kgK)
H_l	– enthalpy of liquid metal, J/kg
H_s	– enthalpy of solid metal, J/kg
ΔH	– enthalpy variation, J/kg
ΔH_f	– latent heat of fusion, J/kg
h	– heat transfer coefficient, W/(m ² K)
k	– thermal conductivity, W/(m K)
k_l	– liquid metal thermal conductivity, W/(m K)
k_s	– solid metal thermal conductivity, W/(m K)
Q	– heat source, W/m ³
q	– conduction heat flux, W/m ²
q_0	– convection heat flux, W/m ²
T_{ext}	– external fluid temperature, K
T_l	– liquidus temperature, K
T_m	– melting temperature, K
T_s	– solidus temperature, K
u	– velocity, m/s

Greek symbols

α_m	– mass fraction
ε	– emissivity
θ	– phase fraction before transition
ρ	– density, kg/m ³
ρ_l	– density of liquid metal, kg/m ³
ρ_s	– density of solid metal, kg/m ³
σ	– Stefan-Boltzmann constant, W/(m ² K ⁴)

1 Introduction

Continuous casting transforms molten metal into solid on a continuous basis and includes a variety of important commercial processes. These processes are the most efficient way to solidify large volumes of metal into simple shapes for subsequent processing. The molten metal solidifies against the mould walls while it is simultaneously withdrawn from the bottom of the mould at a rate which maintains the solid/liquid interface at a constant position with time. The process works best when all of its aspects operate in this steady-state manner.

Billets of different metals were heated and then subjected to cooling by water spray systems and immersion in water and mixtures of water and organic agents [1]. The temperature close to the surface was measured and

used for the calculation of the heat transfer coefficient by the implicit finite difference method. A means for analytically formulating the heat transfer coefficient was obtained, enabling easy calculation to be made of the temperature field during quenching or cooling in continuous casting.

Practical implications for controlling the effect of water cooling on direct chill casting are suggested. A newly developed technique for quantifying heat transfer coefficients and heat fluxes for direct chill water sprays and new results on the effect of composition, temperature and flow rate are reported [2].

The modeling of the convective heat transfer due to spray cooling using flat spray air-mist nozzles is the subject of the work conducted by Ramstorfer *et al.* [3]. Measurements of the spray pattern and air/water flows were carried out on a spray test stand in order to investigate the profiles of the water flux at different air/water pressures and different distances from the nozzle tip to the surface. The heat transfer coefficients due to spray cooling were measured using an experimental stand which allows spray cooling experiments up to a surface temperature of 1523 K (1250 °C).

Since 2004, research has been carried out on the nozzle technology and its use in cooling highly heated surfaces. Hydrodynamic and thermal studies have been the subject of several articles published to date [4–7]. Based on hydrodynamic and geometrical parameters, experiments lead to an empirical correlation defining the profile of fluid dispersion beneath liquid sprayers. Afterwards, thermal simulations have been achieved to estimate heat transfer coefficient in two-phase case. Moreover, a numerical simulation of twin overlapping sprays allows understanding the fluid behavior in the overlap region.

Investigation into the single water microjet surface cooling producing evaporating film was performed under steady state conditions. Experiments were conducted using the nozzle size of 70 μm and 100 μm . In the course of investigations obtained were experimental relations between heat flux and wall superheating. The spray cooling improves the boiling characteristic and reaching critical heat flux (CHF) higher value, which will fulfill the needs of high heat flux removal and protect the heating surface before unexpected failure [8,9].

Case studies of single-phase cooling process were also considered in the experimental investigation of single-phase microjet cooling of microelectronics [10] and the thermal parametric study of non evaporative spray cooling process [11].

Sengupta *et al.* [12,13] compare and contrast the water-cooling techniques used for casting steel and aluminum and discuss their implications in terms of final product quality based on fundamental studies and predictive mathematical models. Optimal practices for the control of cooling in casting processes for both steel and aluminum alloys are evaluated. The goal in each process was to match the rate of heat removal at the surface with the internal supply of latent and sensible heat, in order to lower the metal surface temperature monotonically, until cooling is complete.

Aiming to reduce the occurrence of the surface and internal defects in the products, several mathematical models are proposed for cooling a metal solidifying in a continuous casting mould. The purpose was to accurately predict and control temperature in real time during the continuous casting process in order to obtain the best possible metal quality, that is, minimizing the defects in the final product. More details about the history of the models are available in references [14–19].

Developments in spray cooling applied to the second cooling are reviewed and evaluated. The quantification of heat transfer performance of spray cooling in continuous casting is then studied and the leading correlations developed for the heat transfer coefficient related to major spray cooling parameters are selected and compared [20].

A temperature based 2D finite element model is developed to estimate the evolution of temperature, shell growth profiles and solidification length of a solidifying ingot in a twin-belt caster [21]. The model is extremely useful for industrial engineers to know the design aspects of the twin-belt caster such as final solidification length, belt, dam block and ingot dimensions and casting speed.

Water is used to cool the mould in the initial stages of solidification, and then below the mould, where it is in the direct contact with the newly solidified surface of the metal. A preliminary study presents a model of water-spray cooling effect on a continuous casting process where the effect of spraying heat transfer coefficient during the liquid-to-solid transition through the cooled zone temperature and the metal latent heat of solidification are highlighted [22]. The results obtained enables to locate the shear region.

Most of the above mentioned models have not taken into account the variation of thermophysical properties with temperature and having low casting speed. The present research work is a numerical investigation of the characteristics of molten metal, heat transfer and solidification during

the continuous casting process while being cooled by water-sprays. The simulation focuses on the effects of the heat transfer coefficient (h_{spray}) at the metal-die interface in the high pressure die casting process and the initial pouring temperature of liquid metal on solidification/fusion of gray cast iron.

2 Position of the problem and thermal modeling

In continuous casting, molten metal is poured into an open-ended mould that can be made of graphite or copper. Graphite moulds are widely used both in vertical and horizontal continuous casting. A steady state continuous casting process is modeled and simulated using the commercial Comsol Multiphysics 5.2. software, an interactive environment for modeling and simulating multiphysics and engineering problems [23]. The metal is first melted in a furnace and poured into a ladle. From the ladle the hot metal is transferred into the tundish. The hot metal is poured into the continuous casting machine from the tundish. The mould in the continuous casting process is water cooled; this helps speed up the solidification of the metal casting. The continuous casting does not completely harden in the mould; it does spend enough time in the water cooled mould to develop a protective solidified skin of an adequate thickness on the outside. The thin shell like solidified metal withdraws from the mould and passes through a straightening roller. In the chamber the strand is water sprayed, which prevents porosity. After solidification, predetermined lengths of strands are cut into pieces using mechanical shears or travelling oxyacetylene torches (Fig. 1).

By assumption, in order to simplify the problem, the flow field of the liquid metal was not considered and it was assumed that there was no volume change during solidification. It has also been assumed that the casting speed, of 0.0025 m/s, is constant and uniform throughout the studied domain. In this work we have supposed an axisymmetric two-dimensional geometry of the continuous casting domain composed of two rectangular sections. One represents the metal inside the mould and the second represents the spray cooled region outside the mould. In this region radiation cooling takes place. It is also assumed that the molten metal is in a hydrostatic state, and that the only movement in the fluid is due to the gravitational forces. This simplification allows the support of the mass motion in the domain.

The considered material is a gray cast iron with the thermal properties

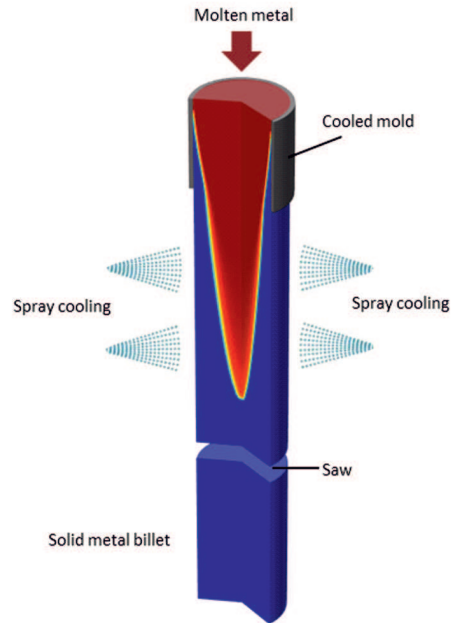


Figure 1: Schematization of the studied case.

Table 1: Gray cast iron thermophysical properties.

Parameter	Value	Unit
Latent heat of fusion	240×10^3	J/kg
Solidus temperature T_s	1356	K
Liquidus temperature T_l	1463	K
Specific heat at T_s	660	J/(kgK)
Specific heat at T_l	950	J/(kgK)
Thermal conductivity at T_s	29	W/(mK)
Thermal conductivity at T_l	26	W/(mK)
Density at T_s	6964	kg/m ³
Density at T_l	6368	kg/m ³
Viscosity at T_l	0.0143	Pa s

shown in Tab. 1. This material has been widely used for industrial and residential fields; therefore, learning about its property is very important. According to the different grades, the gray cast iron has different chemical

components, however, normally; all gray iron grades have the following chemical component range: Carbon (C) 2.8–3.9%, Silicon (Si) 1.1–2.6%, Manganese (Mn) 0.5–1.2%, P ≤ 0.3%, S ≤ 0.15%.

The heat conduction equation which describes the heat transfer of the studied case is written as follows:

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p u \nabla T + \nabla q = Q. \quad (1)$$

Equation in which u represents the casting speed and $q = -k \nabla T$. Hypothetically, C_{p_s} is supposed constant in solid phase, but in liquid phase C_{p_l} is evaluated using the following formula:

$$C_{p_l} = C_{p_s} + \frac{\Delta H}{T_m}, \quad (2)$$

where ΔH represents the enthalpy variation and T_m the melting temperature. Thus, the steady state heat equation of continuous casting process is merely written:

$$\rho C_p u \Delta T + \Delta (-k \Delta T) = 0. \quad (3)$$

The metal solidification begins when the liquid metal reaches the phase change temperature (T_{pc}). It is noted that the phase change occurs within a temperature interval between $T_{pc} - (\Delta T/2)$ and $T_{pc} + (\Delta T/2)$. Within this interval, the material phase is modeled by a smoothed function θ representing the phase fraction before transition, which is equal to one before $T_{pc} - (\Delta T/2)$ and equal to zero after $T_{pc} + (\Delta T/2)$.

The density and the specific enthalpy are respectively:

$$\rho = \theta \rho_s + (1 - \theta) \rho_l \quad (4)$$

and

$$H = \frac{1}{\rho} \{ \theta \rho_s H_s + (1 - \theta) \rho_l H_l \}. \quad (5)$$

During the liquid to solid transformation, a significant amount of latent heat is released and the total amount of heat released per unit mass during the transition is given by the enthalpy variation ΔH . Obviously, the specific heat varies before, during and after the transition:

$$\Delta C_p = \frac{\Delta H}{T}. \quad (6)$$

After some formal transformations:

$$C_p = \frac{1}{\rho} (\theta_1 \rho_s C_{p-s} + \theta_2 \rho_l C_{p-l}) + \Delta H_f \frac{\partial \alpha_m}{\partial T}, \quad (7)$$

where

$$\alpha_m = \frac{(1 - \theta) \rho_l - \theta \rho_s}{2 [\theta \rho_s + (1 - \theta) \rho_l]}. \quad (8)$$

It is equal to -1/2 before transformation and 1/2 after transformation. The specific heat capacity is the sum of an equivalent heat capacity

$$C_{eq} = \frac{1}{\rho} [\theta_1 \rho_s C_{p-s} + \theta_2 \rho_l C_{p-l}] \quad (9)$$

and the distribution of latent heat is

$$C_L(T) = (H_l - H_s) \frac{d\alpha_m}{dt}. \quad (10)$$

The latent heat distribution C_L is approximated by

$$C_L(T) = L \frac{d\alpha_m}{dt}. \quad (11)$$

So that the total heat per unit volume released during the phase transformation coincides with the latent heat

$$\int_{T_{pc} - \frac{DT}{2}}^{T_{pc} + \frac{DT}{2}} C_L(T) dT = L \int_{T_{pc} - \frac{DT}{2}}^{T_{pc} + \frac{DT}{2}} \frac{d\alpha_m}{dt} dT = L. \quad (12)$$

Finally, the apparent heat capacity used in the heat equation, is given by

$$C_p = \frac{1}{\rho} [\theta_1 \rho_s C_{p-s} + \theta_2 \rho_l C_{p-l}] + C_L. \quad (13)$$

The effective thermal conductivity reduces to

$$K = \theta_1 K_s + \theta_2 K_l. \quad (14)$$

The thermal conductivity varies allowing the expression

$$k = \theta k_s + (1 - \theta) k_l. \quad (15)$$

During the *primary cooling*, the convection heat flux transferred to the mould by the liquid metal was estimated using the Newton equation

$$q_0 = h (T_{ext} - T), \quad (16)$$

where T_{ext} and T are respectively external and mould wall temperatures, and h represent the mould heat transfer coefficient ($h = 800 \text{ W}/(\text{m}^2\text{K})$).

Regarding the secondary cooling, convection at the spray cooling zone is computed using the same equation Eq. (16). In this case, T_{ext} and T are respectively external and steel temperatures, and h denotes spray heat transfer coefficient. As mentioned before, radiation occurs during the continuous casting process, and it is described by Stefan-Boltzmann equation in which the temperature T is expressed in Kelvins

$$q_0 = \varepsilon\sigma \left(T_{ext}^4 - T^4 \right), \quad (17)$$

where σ symbolize the Stefan-Boltzmann constant and ε the material emissivity. In our calculation, $\varepsilon = 0.8$.

3 Simulation and discussion of the results

3.1 Cooling mode selection

The first test consists of simulating natural convection cooling with air at atmospheric pressure and a temperature of 300 K. The simulation of this cooling mode, illustrated in Fig. 2a, shows the transition zone, which exceeds the cutting level and, therefore, a risk of obtaining a final product solid on the outside but still liquid at the core.

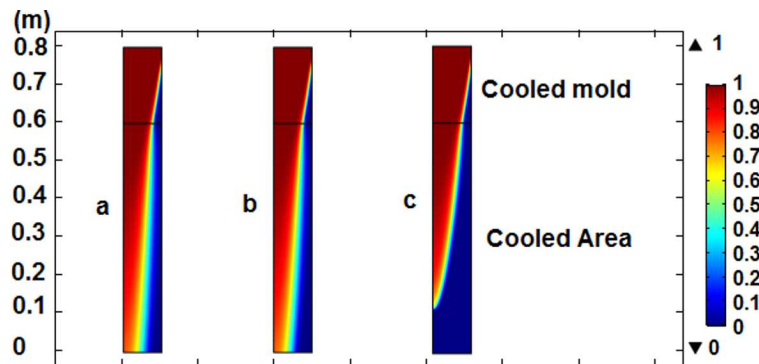


Figure 2: Solidification configuration under three cooling modes.

The second test is a simulation of forced convection cooling with an air wind velocity of 3 m/s at atmospheric pressure and a temperature of 293 K.

The obtained result in Fig. 2b is similar to the previous free convection; nevertheless there is a slight difference that cannot be observed with the naked eye. The third and final test in this study consists of water-spray cooling simulation. Figure 2c illustrates the result of this mode of cooling. One can distinguish a complete solidification of the liquid metal before reaching the cutting zone, which is sought in continuous casting processes.

More convincing comparisons of the three cooling modes described in Fig. 2 are illustrated in Figs. 3, 4, and 5. Regarding the temperature within the cooled zone, Fig. 3, it does not vary so much for natural and forced convection, in fact the temperature drops by a few degrees. The minimum temperature recorded for these two cooling modes is about 1318 K, which remains above the required value for a complete solidification. This analysis is confirmed by Fig. 4, showing the change evolution from liquid state to solid state.

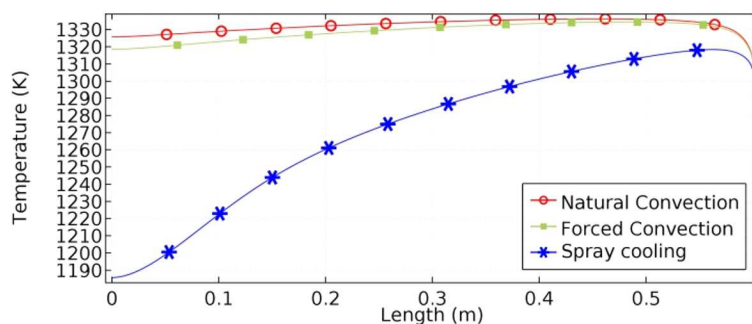


Figure 3: Evolution of the cooled area temperature under three cooling modes.

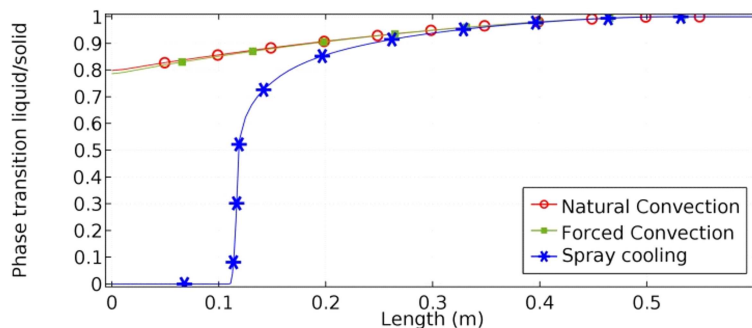


Figure 4: Liquid to solid transition under three cooling modes.

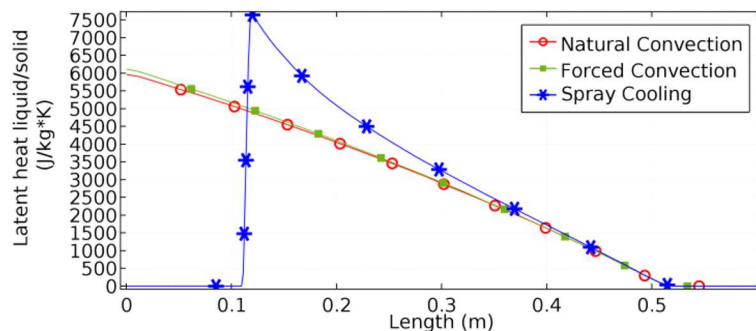


Figure 5: Evolution of solidification latent heat.

The transition values are taken at the symmetry axis for a measurement interval of 0–0.6 m. For convection cooling, the metal solidification reaches only 20%, whereas the aim is to achieve 0% of liquid at $L = 0$ m. On the other hand, as it can be seen through the graphs (Figs. 3 and 4), spray cooling allows a complete solidification (liquid 0%) from point $L = 0.1$ m. This is due to a relatively large drop in temperature, in fact the temperature $T = 1186$ K is recorded at the point $L = 0$ m.

Obviously, the wall temperature (cooled zone) has a direct influence on the solidification of the liquid metal. This temperature and the dissipation interval of solidification latent heat are intimately linked, and this is clearly visible graphically (Figs. 3 and 5). It is noted that, when the outside wall temperature is high, the latent heat released is low, less than 5000 J/(kgK).

In conclusion, the spray cooling process is very efficient since it accelerates the process through a large amount of heat evacuation without water wasting, which imply time and energy saving.

3.2 Spray cooling analysis

The efficiency of the spray cooling being established and with a view to a parametric optimization of this type of cooling during continuous casting processes, in this section we study the influence of heat transfer coefficient (h_{spray}) on the cooling rate of the metal. Even if the values adopted during the simulation are below the actual values [9], the obtained results (Fig. 6) show very significantly the effectiveness of water spray cooling. Indeed, the increase in h_{spray} accelerates heat transfer. If we consider the level $L = 0.3$ m, the wall temperature drops to 35% corresponding to h_{spray} value of 1500 W/(m²K). If h_{spray} value is higher, namely of the order of

6000 W/(m²K), the decrease in wall temperature is almost 70%.

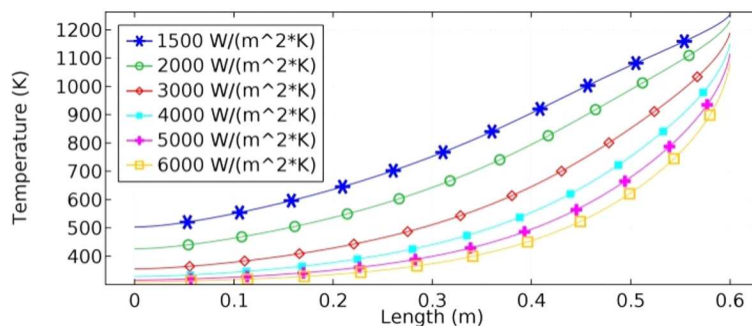


Figure 6: Influence of spray heat transfer coefficient on wall temperature.

The computed values of latent heat of solidifying are obtained showing different sites of heat evacuation depending on the initial value of h_{spray} (Fig. 7). Visibly, the dissipation of the latent heat is slower when h_{spray} decreases, since the wall temperature remains relatively high.

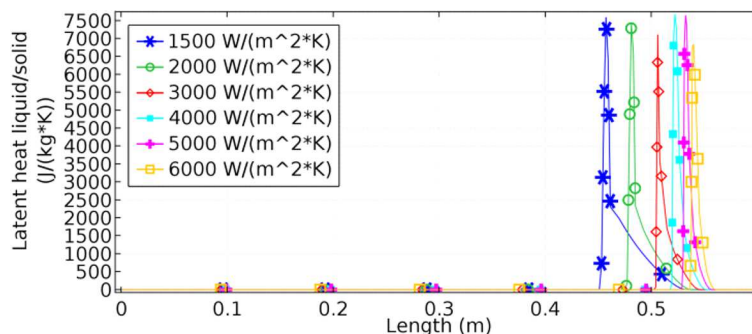


Figure 7: Influence of spray heat transfer coefficient on latent heat.

The solidification is faster for higher heat transfer coefficients, e.g., for $h_{spray} = 6000$ W/(m²K) metal solidification begins at $L = 0.5615$ m and finish at $L = 0.537$ m, whereas, for $h_{spray} = 1500$ W/(m²K) the process begins at the level $L = 0.532$ m and finishes at $L = 0.451$ m (Fig. 8).

This investigation is an attempt to take account of the evolution of latent heat and to determine its effect on the rate of solidification which implicitly has a remarkable influence on the location of the transition region

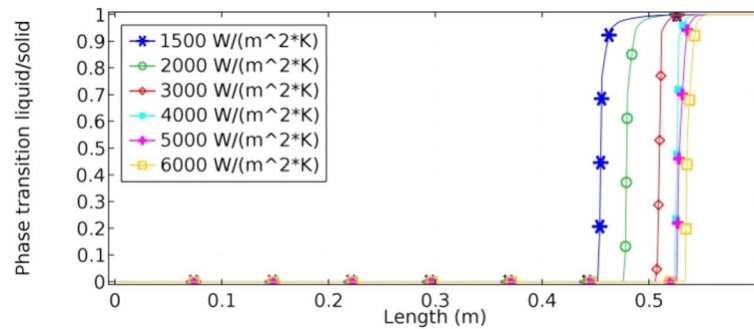


Figure 8: Influence of spray heat transfer coefficient on phase transition.

(Fig. 9). The metal solidifies gradually from the outer walls, and the rate of advance of crystallization is thought to play an important part in the segregation of impurities from the liquid metal.

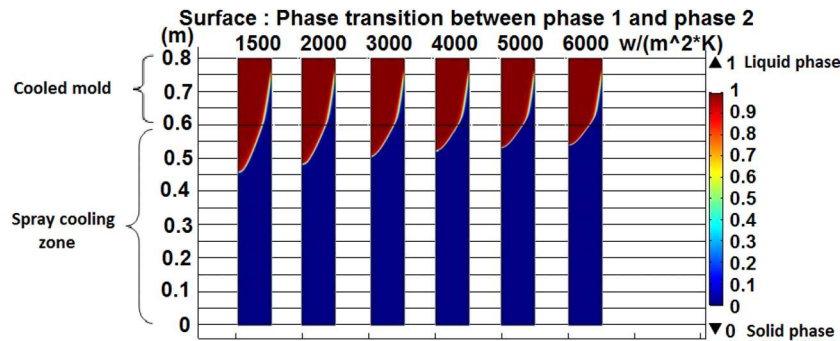


Figure 9: Influence of spray heat transfer coefficient on transition localization.

4 Final conclusions and perspectives

Water-cooled metal solidification in conventional foundry and continuous casting processes was simulated using identical horizontal nozzles. The main conclusions that emerge from this paper can be summarized as follows:

- Undeniably the spray cooling technology is the optimal cooling mode for processes where very high temperatures occur.
- In the spray cooling zone, the metal surface temperature has a clear influence on the metal liquid-to-solid transition. In fact, this allows

us to determine an inversely proportional relation between the surface temperature and the latent heat dissipation range.

- The increase of spraying heat transfer coefficient (h_{spray}), leads to a significant drop in metal temperature. Therefore, we deduce a proportional relationship between the heat transfer coefficient and the dissipation of the latent heat.
- The localization of the liquid-to-solid transition zone is strongly affected by spraying heat transfer coefficient.
- Difference between latent heats of melting and solidifying for the considered metal is probably due to different heat capacities for under-cooled melt and solid state.

To avoid negative effects of cracking and remelting, it is necessary to determine the heat flux as a function of the influencing parameters and to control the cooling in order to obtain a maximal productivity with the required quality. The cooling may be characterized by the heat transfer coefficients measured in different boiling regimes on the surface or directly by the heat flux.

It is assumed that the performed simulations will be possible to use in optimization of real casting process, but it would be very useful also, due to an insufficient experimental and theoretical data in the accessible literature in case of phase transitions, to perform more significant experiments.

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