BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 68, No. 4, 2020
DOI: 10.24425/bpasts.2020.134193

Optimal selection of inner-branches parameters of matching circuits under the minimum branch-current RMS values and zero active-power conditions

M. JARACZEWSKI*

Politechnika Krakowska, ul. Warszawska 24, 31-155 Kraków, Poland

Abstract. The article is a continuation of a study on the synthesis of matching multi-terminal networks, also known as compensators. The reactive four-terminal-network compensators for linear loads were introduced in previous publications, but it appeared that they operate effectively with nonlinear loads too. The methods to create a compensator for a mono-harmonic source, which allows complete independence of input from output waveforms, ensuring optimal operating conditions for the source, are presented herein. The work for the first time presents the optimal four-terminal-network compensator applied to a nonlinear load.

Key words: multi-terminal-network compensators, optimization, nonlinear load.

1. Introduction

A multi-terminal matching circuit that connects power sources to a load, in contrast to a simple two-terminal compensator [1–4], ensures high independence between input (source) voltages and currents and the output (load) voltages and currents. In the case when power source has an internal impedance, a commonly used reactive current compensation with one capacitor or a shunt RLC filters, does not minimize transmission losses and the RMS value of the source currents. If the known parameters of an equivalent power source circuit are taken into account, then the minimization of the source current and transmission losses cannot be accomplished by the means of a two-terminal passive compensator. This paper presents a four-terminal circuit synthesis for source-current compensation that ensures optimal operating conditions for the source, nominal operating conditions for the load and is implemented only by means of passive LC branches.

2. Reactive four-terminal compensators

The source optimal operation condition is usually assumed to be a source's current minimum RMS value (transmission losses) transmitting given active power to the load. It can be formulated in Hilbert space as [5–8]:

$$\begin{gathered}\left(\boldsymbol{i}_e, \boldsymbol{i}_e\right) \rightarrow \min, \\ \left(\boldsymbol{e}, \boldsymbol{i}_e\right) - \left(\boldsymbol{R}\,\boldsymbol{i}_e, \boldsymbol{i}_e\right) = \mathrm{P},\end{gathered} \tag{1}$$

where:

$\boldsymbol{i}_e$, $\boldsymbol{e}$ – source current and EMF that can be expressed e.g. as complex numbers or as vectors of samples or as any representation in terms of an orthogonal base;

$\boldsymbol{R}\,\boldsymbol{i}_e$ is in general a convolution of i_e with source's inner-loss operator $\boldsymbol{R}$;

$(.,.)$ – scalar product in Hilbert space.

To provide optimal (minimal) source current at unchanged load voltage and current, one needs to build a matching circuit (a four-terminal network) that will ensure these voltages and currents at its ends. Because finding parameters of such a four-terminal network, based only on its I/O signals is not unique, an additional optimization criterion should be formulated as follows

$$\begin{gathered}\sum_{k=1}^{g} (i_k, i_k) \rightarrow \min, \\ \sum_{k=1}^{g} (u_k, i_k) = 0,\end{gathered} \tag{2}$$

where:

g – number of inner branches of a four-terminal network,

$\boldsymbol{i}_k$, $\boldsymbol{u}_k$ – current and voltage of a k-th inner branch.

2.1. Four-terminal matching network. In the case of a four-terminal network, we have one input and one output port. The condition for a lossless four-terminal network is that the sum of the active power of ports is zero i.e. $\mathrm{P_e} + \mathrm{P_o} = 0$ (Fig. 1). The internal topology of a four-terminal network was assumed to be a ladder structure. An example of such a structure, with nine internal branches, is shown in Fig. 2.

If all branch currents and voltages are determined by a set of N orthogonal components, usually harmonics or samples, then

*e-mail: jaracz@pk.edu.pl

Manuscript submitted 2019-07-13, revised 2020-03-02, initially accepted for publication 2020-03-23, published in August 2020

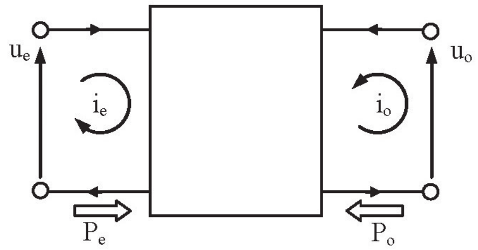

Fig. 1. Four-terminal network connecting power source with a load

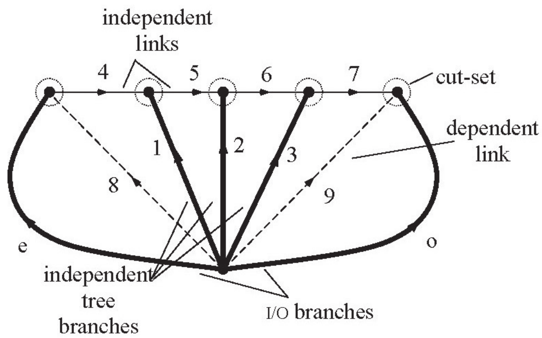

Fig. 2. An example of a directed graph of a four-terminal network and its cut-sets

for all internal voltages $\boldsymbol{U}$ and currents $\boldsymbol{I}$ (Fig. 2) it holds that:

$$\boldsymbol{U} = \mathbf{P}\begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{u} \end{bmatrix}, \qquad \boldsymbol{I} = \mathbf{C}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix}, \tag{3}$$

where

$\boldsymbol{P}$, $\boldsymbol{C}$ – cut-set and loop matrices,

$\boldsymbol{v}$, $\boldsymbol{j}$ – vectors of known or desired voltages and currents of I/O branches,

$\boldsymbol{u}$, $\boldsymbol{i}$ – voltages of independent inner tree branches and currents of independent links,

$\boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$ – branch signal (current or voltage),

N – number of orthogonal component of each branch signal (e.g. samples).

The sum of squared RMS values of inner-branch currents is given by the formula

$$\sum_{n=1}^{N}\sum_{k=1}^{g} i_{k,n}^2 = \sum_{k=1}^{g}\left[\boldsymbol{I}^T \ \boldsymbol{I}\right]_k, \tag{4}$$

where for each k-th branch:

$$\boldsymbol{I}^T\boldsymbol{I} = \left[\boldsymbol{j}^T \ \boldsymbol{i}^T\right]\boldsymbol{C}^T\boldsymbol{C}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix} = \left[\boldsymbol{j}^T \ \boldsymbol{i}^T\right]\boldsymbol{A}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix} =$$
$$= \left[\boldsymbol{j}^T \ \boldsymbol{i}^T\right]\begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix}. \tag{5}$$

The vector of active power of all inner branches is given by the formula:

$$\boldsymbol{P}_a = \begin{bmatrix} \boldsymbol{u}_1^T\boldsymbol{i}_1 \\ \vdots \\ \boldsymbol{u}_g^T\boldsymbol{i}_g \end{bmatrix} \tag{6}$$

where

$$\begin{bmatrix} \boldsymbol{u}_1^T\boldsymbol{i}_1 \\ \vdots \\ \boldsymbol{u}_g^T\boldsymbol{i}_g \end{bmatrix} = \left[\diagdown\right][\boldsymbol{I}] = \left[\diagdown\right]\mathrm{C}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix} = \mathrm{B}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix}$$
$$= \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 \end{bmatrix}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix} \tag{7}$$

and

$$\left[\diagdown\right] = \begin{bmatrix} \boldsymbol{u}_1^T & & \\ & \ddots & \\ & & \boldsymbol{u}_g^T \end{bmatrix} = \mathrm{diag}\left(\boldsymbol{P}\begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{u} \end{bmatrix}\right),$$

g – number of inner branches.

Optimization task that yields inner-current values takes the form:

$$\sum_{k=1}^{g}\left[\boldsymbol{I}^T \ \boldsymbol{I}\right]_k \to \min, \tag{8}$$

$$\begin{bmatrix} \boldsymbol{u}_1^T\boldsymbol{i}_1 \\ \vdots \\ \boldsymbol{u}_g^T\boldsymbol{i}_g \end{bmatrix} = 0. \tag{9}$$

The functional for the above optimization task takes the form:

$$\mathbf{F} = \frac{1}{2}\sum_{k=1}^{g}\left[\boldsymbol{I}^T\boldsymbol{I}\right]_k + \boldsymbol{P}_a^T\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{j}^T & \boldsymbol{i}^T \end{bmatrix}$$
$$\left(\frac{1}{2}\begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_1^T \\ \boldsymbol{B}_2^T \end{bmatrix}\begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_g \end{bmatrix}\right) \to \min, \tag{10}$$

where

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_g \end{bmatrix} \quad \text{– vector of Lagrange coefficients.}$$

This leads to the equations for minimum solution with respect to i:

$$\frac{\mathrm{d}F}{\mathrm{d}\boldsymbol{i}} = \begin{bmatrix} \mathbf{0} & d\boldsymbol{i}^T \end{bmatrix}\begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & d\boldsymbol{i}^T \end{bmatrix}\begin{bmatrix} \boldsymbol{B}_1^T \\ \boldsymbol{B}_2^T \end{bmatrix}\boldsymbol{\Lambda} =$$
$$= \begin{bmatrix} \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}\begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix} + \boldsymbol{B}_2^T\boldsymbol{\Lambda} = 0 \tag{11}$$

or otherwise:

$$\begin{bmatrix} \boldsymbol{A}_{22} & \boldsymbol{B}_2^T \end{bmatrix} \begin{bmatrix} \boldsymbol{i} \\ \boldsymbol{\Lambda} \end{bmatrix} = -\boldsymbol{A}_{21}\boldsymbol{j}$$

subject to zero active power of each internal branch:

$$\frac{\mathrm{d}F}{\mathrm{d}\boldsymbol{\Lambda}} = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix} = \boldsymbol{0} \tag{12}$$

or $\boldsymbol{B}_2\,\boldsymbol{i} = -\boldsymbol{B}_1\,\boldsymbol{j}$.

A general system of equations for independent currents and $\boldsymbol{\Lambda}$ takes the form

$$\begin{bmatrix} \boldsymbol{A}_{22} & \boldsymbol{B}_2^T \\ \boldsymbol{B}_2 & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{i} \\ \boldsymbol{\Lambda} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{A}_{21} \\ \boldsymbol{B}_1 \end{bmatrix} \boldsymbol{j}. \tag{13}$$

3. Four-terminal Π network for monoharmonic signals

Let us assume a four-terminal Π matching circuit depicted in Fig. 3.

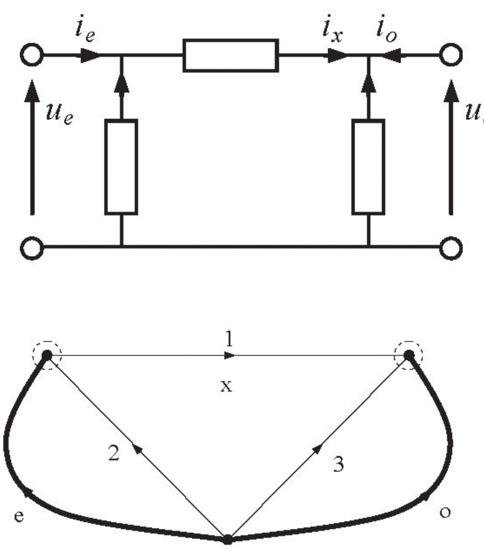

Fig. 3. Diagram and directed graph of the four-terminal Π network

For this structure we define:

$\boldsymbol{j} = \begin{bmatrix} \boldsymbol{i}_e \\ \boldsymbol{i}_o \end{bmatrix}$ – currents of I/O branches;

$\boldsymbol{i} = \boldsymbol{i}_x$ – independent current of the internal link;

$\boldsymbol{v} = \begin{bmatrix} \boldsymbol{u}_e \\ \boldsymbol{u}_o \end{bmatrix}$ – I/O branch voltages;

$\boldsymbol{u} = [\ ]$ – no internal tree branches.

The full incidence matrix (cut-set matrix) of the graph is:

$$\begin{array}{cc} \begin{array}{cc} \mathrm{e} & \mathrm{o} \end{array} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}\right. & \left.\begin{bmatrix} \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \end{bmatrix}\right] \end{array} = \begin{bmatrix} \boldsymbol{P}_1^T & \boldsymbol{P}^T \end{bmatrix} \tag{14}$$

thus

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \quad \text{and}$$

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \\ \boldsymbol{u}_3 \end{bmatrix} = \boldsymbol{P}\,\boldsymbol{v} = \begin{bmatrix} \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_\mathrm{e} \\ \boldsymbol{v}_\mathrm{o} \end{bmatrix},$$

where:

e, o – source and load branches,

1 2 3 – inner branch index,

1 – unity matrix, **0** – zero matrix.

The full loop matrix is:

$$\begin{array}{cc} \begin{array}{cc} \mathrm{e} & \mathrm{o} \end{array} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\right. & \left.\begin{bmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{1} & \mathbf{-1} \end{bmatrix}\right] \end{array} = \begin{bmatrix} \boldsymbol{C}_1^T & \boldsymbol{C}^T \end{bmatrix} \tag{15}$$

thus

$$\boldsymbol{C} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{-1} & \mathbf{-1} \end{bmatrix} \quad \text{and}$$

$$\boldsymbol{I} = \begin{bmatrix} \boldsymbol{i}_x \\ \boldsymbol{i}_2 \\ \boldsymbol{i}_3 \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} \boldsymbol{i}_e \\ \boldsymbol{i}_o \\ \boldsymbol{i}_x \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{i} \end{bmatrix}.$$

Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ take the values:

$$\boldsymbol{A} = \boldsymbol{C}^T\,\boldsymbol{C} = \left[\begin{array}{cc|c} \mathbf{1} & \mathbf{0} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \hline \mathbf{-1} & \mathbf{1} & \mathbf{3} \end{array}\right] = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}, \tag{16}$$

$$\boldsymbol{B} = \mathrm{diag}\left(\boldsymbol{P}\,\boldsymbol{v}^T\right)\boldsymbol{C} = \left[\begin{array}{cc|c} \mathbf{0} & \mathbf{0} & -\boldsymbol{u}_e^T + \boldsymbol{u}_o^T \\ -\mathbf{u}_e^T & \mathbf{0} & \boldsymbol{u}_e^T \\ \mathbf{0} & -\mathbf{u}_o^T & -\boldsymbol{u}_o^T \end{array}\right] =$$

$$= \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 \end{bmatrix}. \tag{17}$$

Because the Π-compensator is entirely passive, one of its inner branches (e.g. 1) should not be considered in power balance. We can achieve this using a permutation matrix $\boldsymbol{P}_m$:

$$\boldsymbol{P}_m \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 \end{bmatrix} = \left[\begin{array}{cc|c} -\boldsymbol{u}_e^T & \mathbf{0} & \boldsymbol{u}_e^T \\ \mathbf{0} & -\boldsymbol{u}_o^T & -\boldsymbol{u}_o^T \end{array}\right], \tag{18}$$

where

$$P_m = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}.$$

Equation (10) takes now the form:

$$\begin{bmatrix} 3\cdot\mathbf{1} & \boldsymbol{u}_e & -\mathbf{u}_o \\ \boldsymbol{u}_e^T & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{u}_o^T & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_x \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = - \begin{bmatrix} -\mathbf{1} & \mathbf{1} \\ -\boldsymbol{u}_e^T & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{u}_o^T \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_e \\ \boldsymbol{i}_o \end{bmatrix}. \tag{19}$$

Now we can apply Gaussian elimination once e.g. by left multiplying equation by matrix $\begin{bmatrix} 3\cdot\mathbf{1} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{u}_e^T & -3 & 0 \\ -\boldsymbol{u}_o^T & 0 & -3 \end{bmatrix}$ to get

$$\begin{bmatrix} 9\cdot\mathbf{1} & 3\boldsymbol{u}_e & -3\boldsymbol{u}_o \\ \mathbf{0} & \boldsymbol{u}_e^T\boldsymbol{u}_e & -\boldsymbol{u}_e^T\boldsymbol{u}_o \\ \mathbf{0} & -\boldsymbol{u}_e^T\boldsymbol{u}_o & \boldsymbol{u}_o^T\boldsymbol{u}_o \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_x \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = - \begin{bmatrix} -3\boldsymbol{i}_e + 3\boldsymbol{i}_o \\ 2\boldsymbol{u}_e^T\boldsymbol{i}_e + \boldsymbol{u}_e^T\boldsymbol{i}_o \\ 2\boldsymbol{u}_o^T\boldsymbol{i}_o + \boldsymbol{u}_o^T\boldsymbol{i}_e \end{bmatrix} \tag{20}$$

where:

$\mathbf{1}$ – unity matrix, $\mathbf{0}$ – zero matrix,

$\boldsymbol{u}_e^T\,\boldsymbol{u}_e = |\boldsymbol{u}_e|^2$, $\boldsymbol{u}_o^T\,\boldsymbol{u}_o = |\boldsymbol{u}_o|^2$,

$\boldsymbol{u}_e^T\,\boldsymbol{i}_e = -\boldsymbol{u}_o^T\boldsymbol{i}_o = p_e$ – active power given by the source branch.

The solution for the independent current $\boldsymbol{i}_x$ can be obtained from the above matrix equation.

If the internal branches of the compensator had active power losses, we could modify the equation (20) adding these losses to the right side of the equation (active sign convention):

$$\begin{bmatrix} 9\cdot\mathbf{1} & 3\boldsymbol{u}_e & -3\boldsymbol{u}_o \\ \mathbf{0} & \boldsymbol{u}_e^T\boldsymbol{u}_e & -\boldsymbol{u}_e^T\boldsymbol{u}_o \\ \mathbf{0} & -\boldsymbol{u}_e^T\boldsymbol{u}_o & \boldsymbol{u}_o^T\boldsymbol{u}_o \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_x \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = - \begin{bmatrix} -3\boldsymbol{i}_e + 3\boldsymbol{i}_o \\ 2\boldsymbol{u}_e^T\boldsymbol{i}_e + \boldsymbol{u}_e^T\boldsymbol{i}_o \\ 2\boldsymbol{u}_o^T\boldsymbol{i}_o + \boldsymbol{u}_o^T\boldsymbol{i}_e \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ p_2 \\ p_3 \end{bmatrix}, \tag{21}$$

where: p_2, p_3 – active powers of brunch 2 and 3.

4. Numerical example

Let us consider the optimization of the source current for the source-load circuit depicted in Fig. 4 [7, 9] We only take fundamental harmonic into consideration (this approach is even effective for nonlinear load) thus before optimization:

$$i_o = -E/(Z_E + Z_o) = -0.1 + 0.1j \rightarrow \boldsymbol{i}_o = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$

$$|i_o| = 0.141,$$

$$u_o = i_o Z_o = 0.8 \ \rightarrow \ \boldsymbol{u}_o = \begin{bmatrix} 0.8 \\ 0 \end{bmatrix}.$$

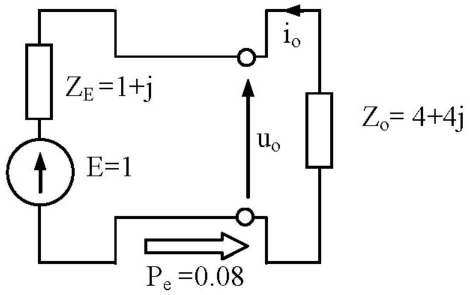

Fig. 4. Power source and load circuit before optimization

Let us decide that the optimal operation condition for the source meets the (1) and (2) condition [1, 4–6].

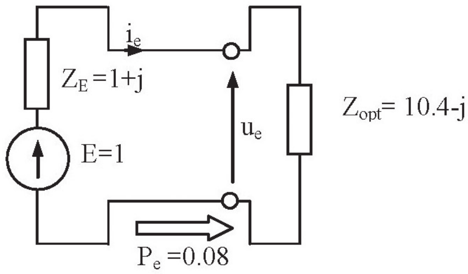

Fig. 5. Power source and its optimal load

From active power condition $P_e = \dfrac{R_{opt}E^2}{(R_E + R_{opt})^2}$ we get

$$R_{opt} = \frac{1}{2}\frac{E^2}{P_e} - R_E + \frac{1}{2}\frac{\sqrt{E^4 - 4E^2P_eR_E}}{P_e} = 10.4. \tag{22}$$

Thus the EMF optimal conductance is

$$G_e = \frac{1}{1 + 10.4039} = 0.087.$$

After optimization:

$$i_e = |E|G_e = 0.087 \ \rightarrow \ \boldsymbol{i}_e = \begin{bmatrix} 0.087 \\ 0 \end{bmatrix},$$

$$u_e = i_e\ Z_{opt} = 0.916 - 0.087j \ \rightarrow \ \boldsymbol{u}_e = \begin{bmatrix} 0.916 \\ -0.087 \end{bmatrix},$$

$$|u_e| = \sqrt{0.84}.$$

4.1. Π-compensator synthesis. Taking previous formulas into account the equation (20) now is:

$$\begin{bmatrix} 9 & 0 & 2.736 & -2.400 \\ 0 & 9 & -0.263 & 0 \\ 0 & 0 & 0.840 & -0.729 \\ 0 & 0 & -0.729 & 0.640 \end{bmatrix} \begin{bmatrix} i_{x1} \\ i_{x2} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = - \begin{bmatrix} -0.563 \\ 0.300 \\ 0.060 \\ -0.089 \end{bmatrix} \tag{23}$$

which yields

$$\begin{bmatrix} i_{x1} \\ i_{x2} \end{bmatrix} = \begin{bmatrix} 0.100 \\ 0.128 \end{bmatrix}.$$

In this case all the impedances of inner branches are supposed to be reactances and can be calculated by determining the reactive power from a formula which imitates a complex values formula (active sign convention) as follows:

$$Q = -\begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}. \tag{24}$$

Thus, using the above formula we get

$$b_k = \frac{Q_k}{|u_k|^2} \tag{25}$$

for any k-th internal branch, the resulting susceptances of Π compensator are

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1.140 \\ -0.140 \\ 0.285 \end{bmatrix}.$$

The circuit diagram for this solution is depicted in Fig. 6.

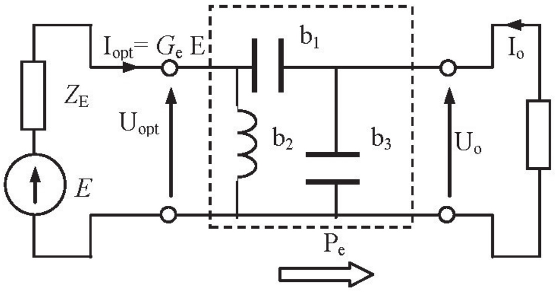

Fig. 6. Solution 1

Because we can independently shift output signals by any time interval the other values of the four-terminal-network susceptances can be calculated by changing output current and voltage phases by a given angle φ, i.e. by multiplying the four-terminal-network chain matrix by $e^{-j\varphi}$:

$$\begin{bmatrix} u_{opt} \\ i_{opt} \end{bmatrix} = \begin{bmatrix} \frac{b_1+b_3}{b_1} & -\frac{j}{b_1} \\ \frac{j(b_1b_2+b_1b_3+b_2b_3)}{b_1} & \frac{b_2+b_1}{b_1} \end{bmatrix} e^{-j\varphi} \begin{bmatrix} 1 & 0 \\ 1/Z_o & 1 \end{bmatrix} e^{j\varphi} \begin{bmatrix} u_o \\ -i_o \end{bmatrix}.$$

The new four-terminal-network chain matrix is:

$$\begin{bmatrix} \frac{b'_1+b'_3}{b'_1} & -\frac{j}{b'_1} \\ \frac{j(b'_1b'_2+b'_1b'_3+b'_2b'_3)}{b'_1} & \frac{b'_1+b'_2}{b'_1} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{b_1+b_3}{b_1} & -\frac{j}{b_1} \\ \frac{j(b_1b_2+b_1b_3+b_2b_3)}{b_1} & \frac{b_2+b_1}{b_1} \end{bmatrix} (\cos(\varphi) - j\cdot\sin(\varphi)).$$

Hence solving the above equations we get the new susceptances' values:

$$b'_1 = b_1(\cos(\varphi) + j\sin(\varphi)),$$
$$b'_2 = b_2 + b_1(1 - \cos(\varphi) - j\sin(\varphi)),$$
$$b'_3 = b_3 + b_1(1 - \cos(\varphi) - j\sin(\varphi)).$$

From the above formula it results that only for $\varphi = 0$ or π it is possible for the susceptances b to have real values, thus the second solution is:

$$\begin{bmatrix} b'_1 \\ b'_2 \\ b'_3 \end{bmatrix} = \begin{bmatrix} -1.140 \\ 2.140 \\ 2.566 \end{bmatrix}$$

and the corresponding circuit diagram is depicted in Fig. 7.

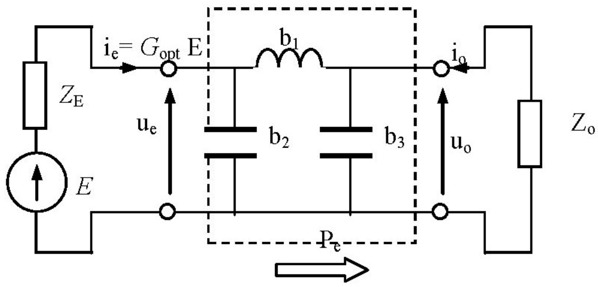

Fig. 7. Solution: 2

Thus, the four-terminal lossless networks that completely compensate the reactive current of the source are feasible and their structures are shown in Figs. 6 and 7.

4.2. Π-compensator for nonlinear load. This very procedure for determining the internal voltages and currents of a four-terminal compensator can also be applied to a non-linear load, because the essence of the approach lies in determining only the required voltages and currents at the inputs and outputs of the four-terminal compensator, which is feasible for any even nonlinear load.

Taking into account circuit with rectifier (Fig. 8) with parameters in Table 1, and assuming 20 samples by period we get the

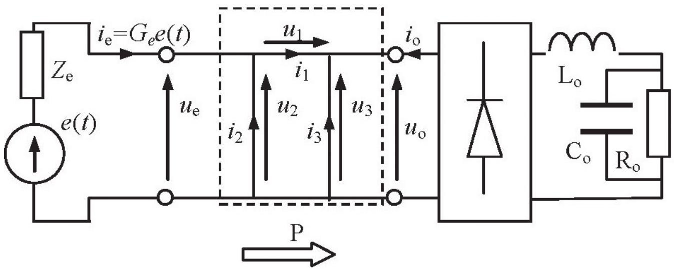

Fig. 8. Π-losless compensator with nonlinear load

Table 1
Circuit parameters

Source	$e_{RMS} = 100$ V $\omega = 314.16$ $R_e = 1\,\Omega$ $L_e = 0.0063662$ H
Load	$R_o = 20\,\Omega$ $L_o =$ 1e-4 H $C_o =$ 1e-3 F

following I/O signals Fig. 9 with optimal source current as follows

$$i_e = G_e e(t) \;\rightarrow\; \boldsymbol{i}_e = 0.058\begin{bmatrix} \mathrm{e}_1 & \cdots & \mathrm{e}_{20} \end{bmatrix}^T = \begin{bmatrix} \mathrm{i}_{e,1} & \cdots & \mathrm{i}_{e,20} \end{bmatrix}^T.$$

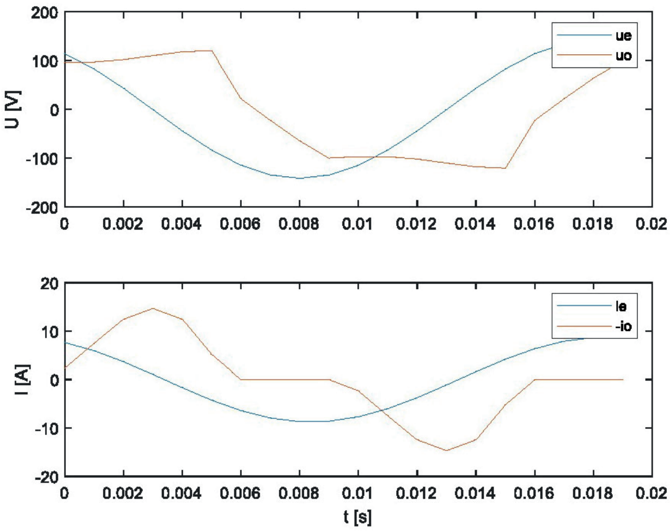

Fig. 9. Input u_e, i_e and output u_o, i_o signals of the compensator (Similink simulation)

Subsequently we use (21), with $p_2 = p_3 = 0$ condition, to get three inner-branch signals (Fig. 10) of minimal RMS values and zero active branch-power, which can be easily accomplished using voltage source inverters.

It is worth noting that signals of branches 1 and 2 resemble, to some extent, sinusoids.

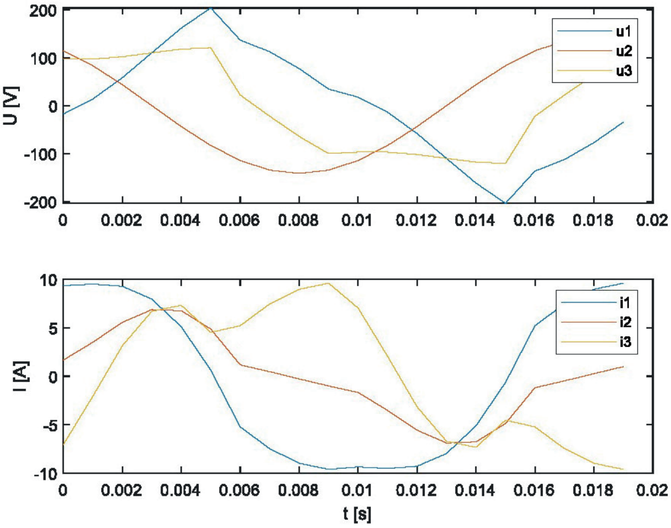

Fig. 10. Inner signals of the compensator

Because of the generality of this procedure the method can also be applied to multiphase systems and simplicity is its asset.

5. Conclusion

The lossless four-terminal compensator, much more than a two-terminal one, makes the source (input) circuit independent of the load (output) circuit, which results in ensuring optimal operating conditions for source without the need to change the receiver's operating conditions (voltage, current) which is unavoidable when parallel or serial compensation is applied. In addition, due to the fact that all branches of the four-terminal matching system are electrically passive, they can be implemented with the help of so-called active filters. I am not aware of any study similar to this work that has been carried out by other teams.

References

[1] L.S. Czarnecki, "Currents' Physical Components (CPC) concept: a fundamental for power theory", *Przegląd Elektrotechniczny* 84 (6), 28–37 (2008).

[2] Z.M. Ali, F.Q. Alenezi, S.S. Kandil, and S. Aleem, "Practical considerations for reactive power sharing approaches among multiple-arm passive filters in non-sinusoidal power systems", *Int. J. Electr. Power Energy Syst.* 103, 660–675 (2018).

[3] H.E. Shady, A. Abdel, M.E. Balci, and S. Selcuk, "Effective utilization of cables and transformers using passive filters for non-linear loads", *Int. J. Electr. Power Energy Syst.* 71, 344–350 (2015).

[4] H.E. Shady, A. Abdel, A.F..Zobaab, and M.E. Balci, "Optimal resonance-free third-order high-pass filters based on minimization of the total cost of the filters using Crow Search Algorithm", *Electr. Power Syst. Res.* 151, 381–394 (2017).

[5] M. Siwczyński, and M. Jaraczewski, "Electric circuit analysis by means of optimization criteria part II – Complex circuits", *Bull. Pol. Ac.: Tech.* 54 (1), 107–124 (2006).

[6] M. Siwczyński and M. Jaraczewski, "Electric circuit analysis by means of optimization criteria Part I – The simple circuits", *Bull. Pol. Ac.: Tech.* 52 (4), 359–367 (2004).

[7] M. Jaraczewski, "Four-terminal network reactive compensators for sinusoidal signals", *Przeglad Elektrotechniczny* 93 (5), 44–47 (2017) [in Polish].

[8] M. Jaraczewski, "Reactive T-topology four-terminal-network compensator for multiharmonic current", *Technical Transactions* 12, 117–126 (2018).

[9] M. Siwczyński and K. Hawron, "Optimum tasks and solutions for energy transmission from the source to the receiver", *Bull. Pol. Ac.: Tech.* 66 (5), 655–663 (2018).