WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2020.134420
ARCHIVES OF CIVIL ENGINEERING
Vol. LXVI ISSUE 3 2020

PROBABILITY-BASED CRITICAL TEMPERATURE ASSESSMENT FOR SIMPLE STEEL BEAM EXPOSED TO FIRE

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An advanced evaluation technique, helpful in the fire resistance assessment of a simple steel structure exposed to fire is presented and discussed in detail on the example of an unrestrained and uniformly heated steel beam. The proposed design methodology deals with the generalised probability-based approach in which the most probable failure point is formally identified. The random nature of all variables considered in the detailed analysis is taken into account. The critical temperature of the steel from which the considered beam is made of is accepted here as the authoritative safety measure. This temperature value is associated with the fire resistance limit state defined for the maximum acceptable value of failure probability. When forecasting the failure probability, not only the risk of a potential fire being initiated but also not being effectively extinguished is included in the calculation. Various levels of the target failure probability may be assumed in such the analysis, depending on the selected reliability class. They are specified in general by setting an appropriate value of the required reliability index β_{req}^{fire}. In the presented design algorithm no representative values of the considered random variables are specified. The critical temperature estimates obtained from these calculations are always less restrictive in comparison with the corresponding solutions computed after applying the conventional standard procedure.

Keywords: fire, critical temperature, safety requirements, failure probability, reliability index.

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1. INTRODUCTION

The verification whether a given structural element is able to safely transfer the load applied to it is usually based on a comparison of the representative value E of the authoritative action effect with the corresponding value R of the resistance of member in question. The safety condition is met if for the random pair of numbers (E, R), assigned to the random load implementation, the relationship $E < R$ is true. Due to the random nature of these two quantities compared each other, the failure probability p_f is calculated for the structure under consideration to determine the safety level guaranteed to the user of such the structure at a given load combination. The value of this probability should be appropriately small and limited to the acceptable level $p_{f,ult}$, which means that:

(1.1) $$p_f = P(E \geq R) < p_{f,ult}$$

Let us notice that failure in this formal model is the case when random value of the reliable action effect E becomes at least equal (or higher) to suitable random member resistance R, associated with such the effect. This specification leads to the conclusion that even if the random value of the conclusive action effect E is relatively high but in the same pair of numbers selected for analysis the random member resistance R compared to this effect still remains higher, this situation should be treated as fully acceptable in context of the structural safety analysis and, as a consequence, no failure is identified in the considered structure. By analogy, the same interpretation may be made for member resistance not high enough if only the corresponding action effect still remains sufficiently small.

If the structural safety analysis is to be carried out for the accidental design situation of a fully developed fire then the failure probability must relate only to cases caused by its impact. It is therefore necessary to exclude from consideration all types of failures generated by any other causes. For this reason the conditional probability $P(failure / fire)$ becomes meaningful for detailed inference. Consequently, the Eq. (1.1) is conventionally formulated as follows [1]:

(1.2) $$p_f = P(failure) = P(fire) \cdot P(failure / fire) < p_{f,ult}$$

In this equation $P(fire)$ is the annual probability that the fire in a given building compartment was initiated and developed and also, importantly, was not effectively extinguished while $P(failure/fire)$ - the annual conditional probability of a failure caused by this fire. The interpretation of Eq. (1.2) is unambiguous if one refers it to the fault tree shown in Fig. 1 [2].

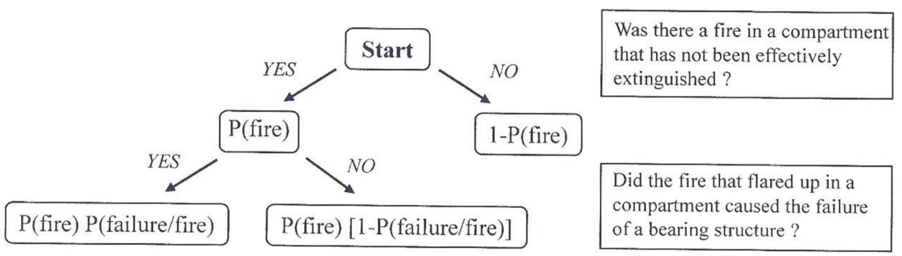

Figure 1: Fault tree formally used to interpret the Eq. (1.2).

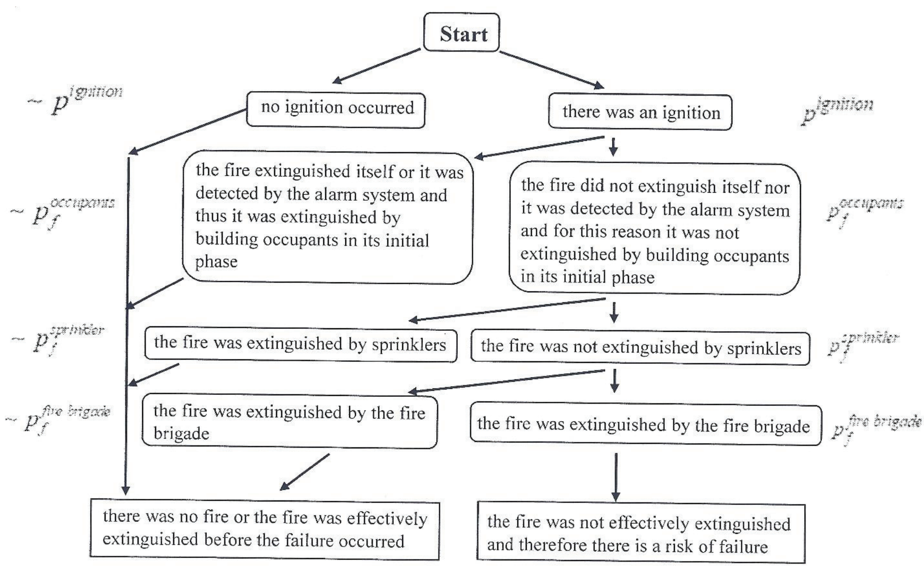

Figure 2: The Fitzgerald type diagram illustrating the sequence of events analysed in the article (see Eq.1.3)

The probability $P(fire)$ is generally estimated as the probability of a random event occurring in the following sequence: a fire has been initiated AND the fire which had been initiated has not been effectively extinguished, because it has not been NEITHER successively detected by the alarm system in its initial phase and due to that stopped by building occupants, NOR fully suppressed by the sprinkler installations (if any sprinkler exists in the considered compartment), AND NOT effectively extinguished by the action of the public fire brigade. This concept, presented in detail on a Fitzgerald type diagram [3, 4] given in Fig. 2, leads to the following estimation:

$$P(fire) = p^{ignition} \cdot p_f^{extinguish} = p^{ignition} \cdot \left(p_f^{occupants} \cdot p_f^{sprinkler} \cdot p_f^{fire\,brigade} \right) \tag{1.3}$$

in which $p^{ignition}$ is the probability of a fire ignition, whereas $p_f^{extinguish}$ - the probability that this fire, which has been previously initiated, has not been effectively extinguished. In such the approach $p_f^{occupants}$, $p_f^{sprinkler}$ and $p_f^{fire\,brigade}$ are the annual failure probabilities of the occupants stopping the fire, of the sprinkler installation in fire suppression and of the public fire brigade in fire extinguishing, respectively.

2. ESTIMATING THE TARGET VALUE OF FAILURE PROBABILITY

Initiation of a fire in a given building compartment is in general a rare event and therefore the probability of its occurrence $p^{ignition}$ is estimated based on the assumptions of the classical model of a Poisson process [5]. In this article its value is interpreted as the annual probability $p^{ignition} = p_{1\,year}^{ignition}$ and assumed at the level $p_{1\,year}^{ignition} = 10 \cdot 10^{-6} \left[\text{fire events}/\left(\text{m}^2 \cdot \text{year}\right)\right]$, based on many statistical estimates. The reliable values of other probabilities identified above should be estimated experimentally; however, in this example they are taken directly from the recommendations given by *Fontana* et al. [6], [7]. Particularly, it is assumed that: $p_f^{occupants} = 0{,}40$ and $p_f^{fire\,brigade} = 0{,}10$. Furthermore, it is set that $P_f^{sprinkler} = 1{,}0$ which means that no sprinklers are installed in the considered building compartment.

Let us assume that the area of the considered building compartment is $A_f = 40\,\text{m}^2$. Consequently:

(2.1) $$p_{1\ \text{year}}^{ignition} = 10 \cdot 10^{-6} \cdot 40 = 0,0004$$

which gives:

(2.2) $$P(fire) = 0,0004 \cdot 0,40 \cdot 1.0 \cdot 0,10 = 1,6 \cdot 10^{-5}$$

Table 1. Minimum (target) values of the reliability index and corresponding maximum acceptable level of the annual failure probability depending on the selected reliability class (based on data taken from [8]).

Reliability class	Failure consequences	β_{req}^{1year}	$p_{f,ult}^{1year}$
RC 1	Low consequences (for example warehouses, temporary buildings)	4,2	$13,35 \cdot 10^{-6}$
RC 2	Medium consequences (for example office buildings, residential buildings, industrial halls)	4.7	$1,301 \cdot 10^{-6}$
RC 3	High consequences (for example hospitals, museums, high rise buildings, power plants)	5,2	$0,996 \cdot 10^{-7}$

In the next step of the procedure used to specify the target value of failure probability associated with fire conditions it is postulated that the annual probability $P(failure)$ be quantitatively equal to the acceptable level of the annual probability $p_{f,ult} = p_{f,ult}^{1year}$ taken directly from the standard EN 1990 [8] and varied depending on the appropriate reliability class, i.e. RC1, RC2 or RC3, respectively (see Table 1) [9]. As is well known, the representative values of this limit probability, gathered in [8], have been formally calibrated for persistent design situation, so without taking into account the impact of a fire. Thanks to such an assumption; however, the level of safety guaranteed to the building user in the event of a fire, and referred to one year reference period, turns out to be identical to the one previously assumed for him/her for ordinary design requirements. The authoritative values of the probability $p_{f,ult}^{1year}$ as well as the corresponding target values of the reliability index β_{req}^{1year} are shown in detail in Table 1. Such the specification was proposed first by Weilert and Albrecht [10] and next incorporated in the German code [11].

Due to the fact that in Eq. (1.2) $P(failure/fire) \leq 1$ the inequality $P(fire) \geq P(failure)$ must always be true. In many practically important design cases, however, a very low value of the probability $P(fire)$ is estimated and simultaneously the acceptable value of the probability $P(failure)$ is set at a level high enough that the condition mentioned above cannot be met. This is generally the case when a comprehensive active fire protection measures are used in the considered building compartment. In such a situation the Eq. (1.2) cannot be used for calculations in a direct way. Nevertheless, it is not a situation that gives cause for concerns because in this case the estimated probability $P(failure)$ never reaches the acceptable level set for the building user on the basis of the relevant regulations.

Setting the value of the probability $P(failure)$ allows calculating the sought value of the probability $P(failure/fire)$. Simple transformation of formula (1.2) gives:

(2.3)
$$P(failure/fire) = \frac{P(failure)}{P(fire)}$$

Let the beam considered in the example be part of the structure for which the calculations carried out as for the RC1 reliability class are authoritative. In this case, basing on (2.3):

(2.4)
$$P(failure/fire) = \frac{13{,}35 \cdot 10^{-6}}{1{,}6 \cdot 10^{-5}} = 0{,}8344$$

The knowledge this value allows to identify the target value of the reliability index β_{req}^{fire}, appropriate for fire situation. If random variables E_{fi} and R_{fi} are described by normal or log-normal probability distribution, then:

(2.5)
$$\beta_{req}^{fire} = -inv\Phi\left(P(failure/fire)\right)$$

Symbol $\Phi(\cdot)$ denotes here the cumulative distribution function (*cdf*) of standardized normal probability distribution. Its values are easy to find in ordinary statistical tables. Notation $inv\Phi$ is understood as an inverse function of Φ . Considering the fact that, for persistent design situation, the classical evaluation $p_{f,ult} = \Phi(-\beta_{req})$ is usually applied in structural safety analysis, the Eq. (2.5) can be rearranged to the form:

$$\beta_{req}^{fire} = -inv\Phi\left[\frac{P(failure)}{P(fire)}\right] = -inv\Phi\left[\frac{\Phi(-\beta_{req}^{1\,year})}{P(fire)}\right] \tag{2.6}$$

Inserting to this equation the values taken from the top row of Table 1 yields the minimum value of the reliability index equal to $\beta_{req}^{fire} = -inv\Phi(0,8344) \cong -0,97$ which is required to meet the global safety condition.

Similarly, assuming the RC2 reliability class, the following result is obtained:

$$P(failure\,/\,fire) = \frac{1,301\cdot 10^{-6}}{1,6\cdot 10^{-5}} = 0,08131 \tag{2.7}$$

This means that $\beta_{req}^{fire} = -inv\Phi(0,08131) \cong 1,40$.

Finally, if the reliability class RC3 is considered in the analysis then:

$$P(failure\,/\,fire) = \frac{0,996\cdot 10^{-7}}{1,6\cdot 10^{-5}} = 0,00623 \rightarrow \beta_{req}^{fire} \cong 2,50 \tag{2.8}$$

It should be emphasized that the higher minimum required value of the reliability index is tantamount to higher requirements needed to meet the global safety condition. In further analysis, only the case for which $\beta_{req}^{fire} = 1,40$ occurs is analysed in detail (corresponding to the assumption of the reliability class RC2).

3. DESCRIPTION OF THE STEEL BEAM CONSIDERED IN THE EXAMPLE

The new probability-based approach proposed by the author to verify the structural safety level related to the fire situation is presented in this article on the example of an unrestrained and simply supported steel beam exposed to a direct fire action. This beam is made of the S235 steel grade with the IPE300 cross-section (belonging to class 1 according to EN 1993-1-1 [12]), for which $W_{pl} = 628\,\text{cm}^3$. Its span length is set at $L = 6\,\text{m}$. The permanent load g and only one variable load q, both uniformly distributed, are applied to the beam. Let us assume that all considered external loads are random variables with normal probability distribution. In the presented example one has the mean values: $m_G = 5\ \text{kN/m}$, $m_Q = 12\ \text{kN/m}$ as well as the standard deviations: $\sigma_G = 0{,}3\,\text{kN/m}$, $\sigma_Q = 1{,}5\,\text{kN/m}$ [13]. Furthermore, $\psi_2 = 0{,}3$ (ψ_2 is the factor reducing the characteristic value of variable action to its quasi-permanent equivalent [8]). Resistance of the beam cross-section is proportional to the steel yield point, which is also the random variable but described by log-normal probability distribution. Its median value μ_f is estimated assuming that the log-normal coefficient of variation is set at $\upsilon_f = 0{,}08$ - in accordance with numerous statistical data. This means that:

(3.1) $$\mu_f = f_{y,k} \exp\left(2\upsilon_f\right) = 235 \exp\left(2 \cdot 0{,}08\right) = 275{,}8\ \text{MPa}$$

The steel yield point must be reduced if the considered beam is subject to a fire exposure. The appropriate reduction factors $k_{y,\Theta}$ are then used with values depending on the steel temperature Θ, according to the standard EN 1993-1-2 [14], which gives:

(3.2) $$f_{y,k,\Theta} = k_{y,\Theta} f_{y,k} = k_{y,\Theta} \mu_f \exp\left(-2\upsilon_f\right)$$

The factor $k_{y,\Theta}$ is also a random variable. In this analysis it is described by normal probability distribution. According to [15] its nominal values gathered in [14] should be interpreted as the appropriate mean values $m_{k\Theta}$. Furthermore, the variability of this factor is the higher the higher temperature of structural steel. In [16] Van Coile recommends the following estimations regarding the coefficient of variation $v_{k\Theta}$: $v_{k20} = 0$ and $v_{k500} = 0{,}052$, respectively. If $20^\circ\text{C} < \Theta < 500^\circ\text{C}$ the linear interpolation may be applied. If $\Theta > 500^\circ\text{C}$ the value of the considered factor is maintained at a constant level $v_{k\Theta} = 0{,}052$. This means that:

(3.3) $$v_{k\Theta} = \frac{v_{k500}}{480}\left(\Theta - 20\right) \qquad \text{for} \qquad 20^\circ\text{C} < \Theta < 500^\circ\text{C}$$

A list of selected values of the coefficient $v_{k\Theta}$ calculated in this way is given in Table 2.

Table 2. Selected mean values $m_{k\Theta}$ taken from [14] and corresponding coefficients of variation $v_{k\Theta}$ determined from Eq. (3.3) depending on the steel temperature Θ.

$m_{k20} = 1,00$	$v_{k20} = 0$	$m_{k600} = 0,47$	$v_{k600} = 0,0520$
$m_{k100} = 1,00$	$v_{k100} = 0,0087$	$m_{k700} = 0,23$	$v_{k700} = 0,0520$
$m_{k200} = 1,00$	$v_{k200} = 0,0195$	$m_{k800} = 0,11$	$v_{k800} = 0,0520$
$m_{k300} = 1,00$	$v_{k300} = 0,0303$	$m_{k900} = 0,06$	$v_{k900} = 0,0520$
$m_{k400} = 1,00$	$v_{k400} = 0,0412$	$m_{k1000} = 0,04$	$v_{k1000} = 0,0520$
$m_{k500} = 0,78$	$v_{k500} = 0,0520$	$m_{k1100} = 0,02$	$v_{k1100} = 0,0520$

In further analysis it is also assumed that the steel temperature monotonically rises during fire; nevertheless, at each particular fire moment the member temperature distribution is always uniform both in the considered cross-section and along the whole beam length. Such simplification allows to conclude that a value of the plastic bending modulus W_{pl} does not change under fire exposure. Consequently, the median value of beam resistance $\mu_{R\Theta}$ can be calculated as follows (for simplicity it is assumed that $m_{k\Theta} \approx \mu_{k\Theta}$):

(3.4) $$\mu_{R\Theta} = W_{pl}\mu_{k\Theta}\mu_f = 628\cdot 10^{-6}\cdot \mu_{k\Theta}\cdot 275,8\cdot 10^3 = 173,20\mu_{k\Theta} \text{ kNm}$$

As one can see, the bending modulus W_{pl} is treated in this formula as a deterministic parameter; however, its variability is added to the global coefficient of variance $\upsilon_{R\Theta}$ calculated for beam resistance. Let the symbol υ_f^2 denote variance of the steel yield point, υ_A^2 - variance of the beam geometrical dimensions, $\upsilon_{k\Theta}^2 \approx v_{k\Theta}^2$ - variance resulting from the uncertainty of a model describing mechanical properties of steel under fire conditions. In this example it is set that $\upsilon_f = 0,08$ and $\upsilon_A = 0,06$ which is also consistent with many statistical estimates, made also in Poland. Consequently, for such the initial data one obtains:

(3.5) $$\upsilon_{R\Theta} = \sqrt{\upsilon_f^2 + \upsilon_A^2 + \upsilon_{k\Theta}^2} = \sqrt{0,08^2 + 0,06^2 + \upsilon_{k\Theta}^2} \approx \sqrt{0,01 + v_{k\Theta}^2}$$

Recalculation the log-normal beam resistance parameters to corresponding Gaussian characteristics gives:

(3.6) $$m_{R\Theta} = \mu_{R\Theta} \exp\left(\frac{\upsilon_{R\Theta}^2}{2}\right) \quad \text{and} \quad \sigma_{R\Theta} = m_{R\Theta}\sqrt{\exp\left(\upsilon_{R\Theta}^2\right)-1}$$

The detailed results obtained after using Eqs. (3.4-3.6) for the data considered in the example are shown in Table 3. Similarly, for the action effect one has:

(3.7) $$m_E = \left(m_G + \psi_2 m_Q\right)\frac{L^2}{8} = \left(5 + 0.3\cdot 12\right)\frac{6^2}{8} = 38{,}7 \text{ kNm}$$

(3.8) $$\sigma_E = \sqrt{\sigma_G^2 + \psi_2^2\sigma_Q^2}\,\frac{L^2}{8} = \sqrt{0{,}3^2 + 0{,}3^2\cdot 1{,}5^2}\,\frac{6^2}{8} = 2{,}43 \text{ kNm}$$

Table3: The mean values and the Gaussian standard deviations calculated for the resistance of the beam considered in the example.

$\Theta\ [^\circ\text{C}]$	Log-normal		Gaussian	
	$\mu_{R\Theta}$ [MPa]	$\upsilon_{R\Theta}$	$m_{R\Theta}$ [MPa]	$\sigma_{R\Theta}$ [MPa]
20	173,20	0,100	174,07	17,45
100	173,20	0,100	174,07	17,45
200	173,20	0,102	174,10	17,80
300	173,20	0,104	174,14	18,16
400	173,20	0,108	174,21	18,87
500	135,10	0,113	135,96	15,41
600	81,40	0,113	81,93	9,29
700	39,84	0,113	40,09	4,54

4. EVALUATION OF BEAM CRITICAL TEMPERATURE BY THE CONVENTIONAL STANDARD APPROACH

At the beginning of the analysis it is necessary to show that the beam considered in the example is capable to safely transfer the loads applied to it for the conditions of persistent design situation (i.e. without taking into account the impact of a fire). In fact, according to the conventional standard approach one can obtain that [8]:

(4.1) $$g_k = m_G + 1,64\sigma_G = 5 + 1,64 \cdot 0,3 = 5,49 \ \text{kN/m}$$

(4.2) $$q_k = m_Q + 1,64\sigma_Q = 12 + 1,64 \cdot 1,5 = 14,46 \ \text{kN/m}$$

(4.3) $$E_d = \left(\gamma_G g_k + \gamma_Q q_k\right)\frac{L^2}{8} = \left(1,35 \cdot 5,49 + 1,5 \cdot 14,46\right)\frac{6^2}{8} = 130,96 \text{ kNm}$$

(4.4) $$E_d < R_d = W_{pl}\frac{f_{y,k}}{\gamma_M} = 628 \cdot 10^{-6} \cdot \frac{235 \cdot 10^3}{1,0} = 147,58 \text{ kNm}$$

The design value of the action effect specified for an accidental fire situation is significantly smaller:

(4.5) $$E_{fi,d} = \left(g_k + \psi_2 q_k\right)\frac{L^2}{8} = \left(5,49 + 0,3 \cdot 14,46\right)\frac{6^2}{8} = 44,23 \text{ kNm}$$

Let us notice that, according to Eq. (4.4) with the equivalence $\gamma_{M,fi} = \gamma_M = 1.0$:

(4.6) $$R_{fi,\Theta,d} = 147.58 k_{y,\Theta} \text{ kNm}$$

Table 4: Evaluation of the critical temperature Θ_{cr} for the beam considered in the example by conventional standard approach [13].

$\Theta\left[{}^\circ C\right]$	$k_{y,\Theta}$	$E_{fi,d}\left[kNm\right]$	$R_{fi,\Theta,d}\left[kNm\right]$	$\rho(\Theta)$
400	1,00	44,23	147,58	0,300
500	0,78	44,23	115,11	0,384
600	0,47	44,23	69,36	0,638
700	0,23	44,23	33,94	1,303
650	0.35	44,23	51,65	1,167
660	0,33	44,23	48,70	1,101
670	0,30	44,23	44,27	1.001

The ratio $\rho(\Theta) = E_{fi,d}/R_{fi,\Theta,d}$ is selected here as a measure helpful in the calculation of critical temperature Θ_{cr}. It is simply the temperature value of the steel beam considered in the example for which the equality $\rho(\Theta_{cr}) = 1,0$ is achieved. The results of the analysis are shown in Table 4.

5. APPLICATION OF THE PROPOSED PROBABILITY-BASED APPROACH

In the presented analysis only the simple case is discussed in detail for which both the action effect E_{fi} and the member resistance $R_{fi,\Theta}$ are random variables described by normal probability distribution [17]. To obtain the value of the reliability index $\beta_{fi,\Theta}$ considered random variables must be transformed into the standardized space $(u_E, u_{R\Theta})$. It is sufficient to take:

$$(5.1) \qquad u_E = \frac{E_{fi} - m_E}{\sigma_E} \quad \text{and} \quad u_{R\Theta} = \frac{R_{fi,\Theta} - m_{R\Theta}}{\sigma_{R\Theta}}$$

which means that:

$$(5.2) \qquad E_{fi} = m_E + u_E \sigma_E \quad \text{and} \quad R_{fi,\Theta} = m_{R\Theta} + u_{R\Theta}\sigma_{R\Theta}$$

As a consequence of such specification the limit state condition may be expressed by linear equation:

$$(5.3) \qquad R_{fi,\Theta} - E_{fi} = 0 \rightarrow m_{R\Theta} - m_E + u_{R\Theta}\sigma_{R\Theta} - u_E\sigma_E = 0$$

To standardize this formula the quantity $\Xi_\Theta = \sqrt{\sigma_E^2 + \sigma_{R\Theta}^2}$ is previously defined. As a result of its application one obtains:

$$(5.4) \qquad (m_{R\Theta} - m_E)/\Xi_\Theta + (\sigma_{R\Theta}/\Xi_\Theta)u_{R\Theta} - (\sigma_E/\Xi_\Theta)u_E = \beta_{fi,\Theta} + \alpha_{R\Theta}u_{R\Theta} - \alpha_{E\Theta}u_E = 0$$

Let us notice that in formula (5.4) not only the reliability index equal to $\beta_{fi,\Theta} = (m_{R\Theta} - m_E)/\Xi_\Theta$ is included but also the coefficients $\alpha_{E\Theta} = \sigma_E/\Xi_\Theta$ and $\alpha_{R\Theta} = \sigma_{R\Theta}/\Xi_\Theta$ interpreted here as the direction cosines of this straight line if it is drawn in $(u_E, u_{R\Theta})$ Cartesian coordinate system. In this approach the value of critical temperature of the beam considered in the example Θ_{cr} is understood as the lowest steel temperature for which the beam failure probability p_f becomes large enough that it may not be accepted. This means that the ultimate limit state is associated with the equation $\beta_{fi,\Theta} = \beta_{req}^{fire}$.

Table 5: The safety level guaranteed to the beam user at various steel temperature values.

$\Theta\left[{}^{\circ}C\right]$	Ξ_Θ [kNm]	$\alpha_{E\Theta}$	$\alpha_{R\Theta}$	$\beta_{fi,\Theta}$
20	17,618	0,138	0,990	7,684
100	17,618	0,138	0,990	7,684
200	17,965	0,135	0,991	7,537
300	18,322	0,133	0,991	7,392
400	19,026	0,128	0,992	7,122
500	15,600	0,158	0,988	6,235
600	9,603	0,253	0,967	4,502
700	5,149	0,472	0,882	0,270

Table 6: Estimation of the critical temperature Θ_{cr} value for the beam considered in the example using the probability-based approach described in this article.

Θ $\left[{}^{\circ}C\right]$	$\mu_{k\Theta} =$ $k_{y,\Theta}$	$\mu_{R\Theta}$ [MPa]	$\upsilon_{R\Theta}$	$m_{R\Theta}$ [MPa]	$\sigma_{R\Theta}$ [MPa]	Ξ_Θ [kNm]	$\alpha_{E\Theta}$	$\alpha_{R\Theta}$	$\beta_{fi,\Theta}$
650	0,35	60,62	0,113	61,01	6,92	7,334	0,331	0,944	3,042
680	0,28	48,50	0,113	48,81	5,53	6,040	0,402	0,916	1,674
685	0,27	46,76	0,113	46,97	5,32	5,849	0,415	0,910	1,414
686	0,26	45,03	0,113	45,32	5,14	5,685	0,427	0,904	1,164

The results obtained after applying the numerical data determined previously for the beam considered in the example are presented in detail in Table 5. It is clearly visible that the critical temperature of the beam in question, for which the equality $\beta_{fi,\Theta} = \beta_{req}^{fire} = 1,40$ occurs, is within the range $600\,^{\circ}\mathrm{C} < \Theta_{cr} < 700\,^{\circ}\mathrm{C}$. More detailed evaluation is presented in Table 6. As can be seen, the critical temperature of this beam, determined in this way, has a value $\Theta_{a,cr} = 685\,^{\circ}\mathrm{C}$. It is therefore 15 degrees higher than the corresponding temperature $\Theta_{a,cr} = 670\,^{\circ}\mathrm{C}$ determined previously, after applying the simplified standard approach (see Table 4). This means that this conventional estimate was safe in general but too conservative.

In the Fig. 3 a graphical interpretation of the results related to the probability-based calculations carried out in the considered example is illustrated in detail. In the subsequent calculation steps, for each selected value of the temperature Θ, a straight line is drawn precisely in the $\left(u_E, u_{R\Theta}\right)$ coordinate system. These lines, described previously by the Eq. (5.4), are associated with the fire resistance limit state because they delimit the safe domain from failure domain. The distance of each of these straight lines from the origin of the coordinate system is a measure of the reliability index $\beta_{fi,\Theta}$. This means that the point lying on the line and located closest to such the origin is simply the most probable failure point.

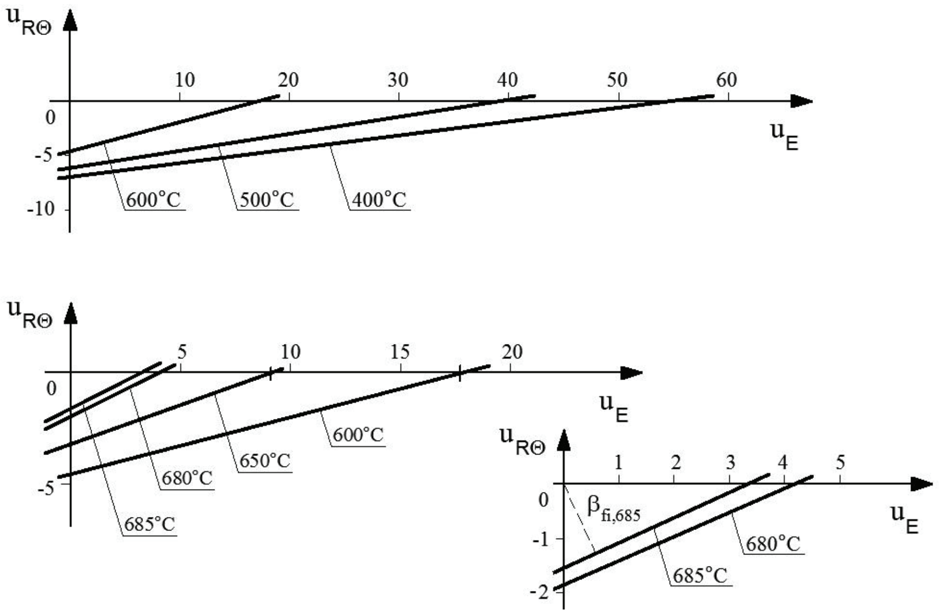

Figure 3: Graphical interpretation of the critical temperature evaluation for the beam considered in the example using the proposed probability-based approach.

6. CONCLUDING REMARKS

The example presented in this paper confirms that the probability-based approach, alternative to the traditional simplified standard design procedure [14] and recommended by the author for application in engineering practice, can be effectively used to assess the reliable value of critical temperature of steel structural members exposed to fire. Solutions obtained in this way may be treated as better justified than those calibrated traditionally because no representative values of considered random variables are specified. Moreover, the failure seems to be defined more rationally. The fact that all evaluations of critical temperature Θ_{cr} which are the results of practical use of the proposed methodology are less restrictive in comparison to the corresponding solutions obtained owing to the application of classical standard design technique must be underlined. This means that such the standard approach, commonly used by professionals and designers, gives in this field the safety assessments which are safe in general but too conservative. The main advantage of the algorithm presented in this article seems to be; however, the ability to adapt it to the differentiated safety requirements identified, for example, by assigning the appropriate reliability class to the considered structure. A more detailed discussion on this topic is conducted in [18]. In further analysis, it seems advisable to formally link the calculations of this type with the quantitative and qualitative risk analysis. To do this, it would be necessary to make the safety requirements dependent on the anticipated failure consequence. In the author's opinion, it would also be more correct, in a mathematical sense, to replace the mean values $m_{k\Theta} \approx \mu_{k\Theta} = k_{y,\Theta}$ used in the conventional standard design procedure with the corresponding characteristic values $k_{y,k,\Theta} = \mu_{k\Theta} \exp\left(-2\upsilon_{k\Theta}\right)$ calibrated as the appropriate quantiles of the log-normal probability distribution [19].

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PROBABILISTYCZNA OCENY TEMPERATURY PROSTEJ BELKI STALOWEJ EKSPONOWANEJ NA ODDZIAŁYWANIE POŻARU

Słowa kluczowe:

pożar, temperatura krytyczna, wymagania bezpieczeństwa, prawdopodobieństwo zawodu, wskaźnik niezawodności.

STRESZCZENIE

Zaproponowano i szczegółowo przedyskutowano nowe podejście do szacowania temperatury krytycznej eksponowanej na warunki pożarowe stalowej konstrukcji nośnej. Opiera się ono na rozważaniach w pełni probabilistycznych i może stanowić alternatywę w stosunku do tradycyjnych obliczeń normowych. W opinii autora rezultaty uzyskane dzięki zastosowaniu prezentowanej metodyki można uznać za bardziej wiarygodne i lepiej uzasadnione w zestawieniu z odpowiadającymi im wynikami otrzymanymi metodami konwencjonalnymi. Taki wniosek można uzasadnić faktem uwzględnienia w analizie losowego charakteru zarówno przyłożonych do konstrukcji obciążeń zewnętrznych jak i miarodajnej nośności przekroju poprzecznego, redukowanej wskutek oddziaływania na elementy konstrukcyjne wysokiej temperatury pożarowej. Ponadto wymagania co do gwarantowanego poziomu bezpieczeństwa ustalono w sposób bardziej racjonalny, przez specyfikację granicznej wartości prawdopodobieństwa zniszczenia, maksymalnej możliwej do zaakceptowania w warunkach wyjątkowej sytuacji projektowej kojarzonej z pożarem rozwiniętym potencjalnie zainicjowanym w rozważanej strefie pożarowej. Oszacowana w ten sposób temperatura krytyczna stalowej konstrukcji nośnej, kojarzona z osiągnięciem przez tę konstrukcję stanu granicznego nośności ogniowej, jest zawsze wyższa niż odpowiadająca jej temperatura wyliczona przy zastosowaniu konwencjonalnej procedury normowej. Wynika stąd

wniosek, że tradycyjny algorytm normowy, opierający się najpierw na specyfikacji a następnie na porównywaniu ze sobą reprezentatywnych, obliczeniowych wartości miarodajnego efektu obciążenia i odpowiadającej temu efektowi nośności elementu, daje oszacowania wprawdzie bezpieczne ale niepotrzebnie nazbyt konserwatywne. W przedstawionym w pracy przykładzie obliczeniowym dotyczącym równomiernie ogrzanej i swobodnie podpartej belki stalowej uzyskana z obliczeń różnica ilościowa osiągnęła wartość $15^{\circ}\mathrm{C}$, co w praktyce, w warunkach pożaru, przekłada się na dodatkowe minuty bezpiecznego przenoszenia obciążeń. Podstawową zaletą proponowanej przez autora procedury obliczeniowej wydaje się być możliwość różnicowania wymogów bezpieczeństwa. Czyni się to przez odpowiednią specyfikację docelowej, wymaganej, wartości wskaźnika niezawodności, dobieranej na ogół tak aby odpowiadała ona wybranej do analizy klasie niezawodności RC o parametrach szczegółowo opisanych w normie EN 1990 [8]. Zasadą jest jednak aby globalny warunek bezpieczeństwa formułować przy tego typu rozważaniach zgodnie z regułami wyjątkowej sytuacji projektowej, a więc w sposób specyficzny, nieco różniący się od tego który stosowany jest powszechnie w analogicznych zadaniach odniesionych do sytuacji trwałej. Różnica ta dotyczy w szczególności przyjętej do analizy wartości wskaźnika β_{req}^{fire}, kalibrowanej w oparciu o odpowiednie prawdopodobieństwa jednoroczne, interpretowane przy tym jako prawdopodobieństwa warunkowe. W obliczeniach prezentowanych w przykładzie do kwantyfikacji wartości zależnego od temperatury stali współczynnika zmienności $v_{k\Theta}$, będącego miarą losowej zmienności współczynnika redukcyjnego granicy plastyczności stali $k_{y,\Theta}$ a więc swego rodzaju parametrem niepewności modelu obliczeniowego, wykorzystano oszacowanie pochodzące z badań eksperymentalnych dotyczących stalowych prętów używanych do zbrojenia betonu. Jest to w zasadzie jedyne dostępne w literaturze oszacowanie tego typu, uzyskane na reprezentatywnej i odpowiednio licznej próbie statystycznej. Nie musi jednak okazać się w pełni miarodajne również w odniesieniu do tradycyjnych stalowych elementów konstrukcyjnych. Niemniej jednak wpływ tego typu zmienności na oszacowanie temperatury krytycznej elementu nośnego, co wynika z obliczeń prezentowanych w niniejszym artykule, nie jest bardzo znaczący, a zatem nie prowadzi do istotnych błędów ilościowych. Wydaje się jednak, że formalnie bardziej prawidłowe byłoby zastąpienie zebranych w normie EN 1993-1-2 [14] nominalnych wartości współczynnika redukcyjnego $k_{y,\Theta}$, potraktowanych w niniejszych rozważaniach jako odpowiednie wartości średnie $m_{k\Theta} \approx \mu_{k\Theta} = k_{y,\Theta}$, skojarzonymi z nimi wartościami charakterystycznymi $k_{y,k,\Theta} = \mu_{k\Theta} \exp\left(-2\upsilon_{k\Theta}\right)$, interpretowanymi jako kwantyle log-normalnego rozkładu prawdopodobieństwa.

Received: 05.05.2020 Revised: 27.06.2020