BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 69(2), 2021, Article number: e136718
DOI: 10.24425/bpasts.2021.136718

SPECIAL SECTION
Machine modelling and simulations

Theoretical study of heat conduction in the multi-disc brake integrated into the drive wheel AGV during braking

Daniel VARECHA*, Róbert KOHÁR, and Michal LUKÁČ

University of Žilina, Faculty of Mechanical Engineering, Department of Design and Mechanical Elements,
Univerzitná 8215/1, 010 26 Žilina, Slovakia

Abstract. This paper is focused on the theoretical study of heat conduction in the multi-brake system of the automated guided vehicle (AGV). The study aims to compare the amount of heat generated during braking from 10 m/s until a stop in a brake system based on organic and ceramic friction material. The theoretical study of heat conduction is solved in Matlab computational software using a derived Fourier partial differential equation for nonstationary heat conduction. The results of the simulation of the heat conduction are shown in the diagrams and indicate not only the temperature dependence in the period during braking from a speed of 10 m/s to a stop but also the amount of heat accumulated in the steel disc during braking. The simulation results show that braking in both brake systems generates approximately the same amount of heat. The difference occurs in the period of thermal activity, which was influenced by the length of the braking distance. This is caused by a coefficient of friction that significantly affects the final braking result. Finally, it can be stated that the brake system based on organic material must be equipped with a steel disc with a minimum thickness of 8 mm. This is because the brake system based on organic friction material has a set temperature limit of 160 degrees Celsius. The results presented in this study will help an engineer constructor to choose the right procedures and parameters of geometry for designing the mentioned braking system for the considered AGV.

Key words: simulation; braking systems; one-dimensional heat conduction; multi-disc brake; AGV; ceramic vs. organic brake pads.

1. Introduction

In this paper, the authors present their theoretical study on heat conduction during braking of the integrated multi-disc brake concept (Fig. 1).

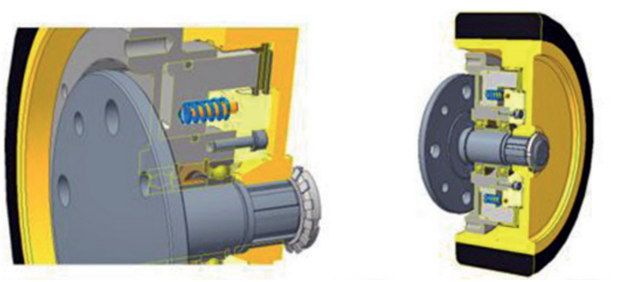

Fig. 1. Concept of drive wheel with integrated multi-disc brake [4, 6]

The brake system is integrated into the free space drive wheel of the automated guided vehicle (AGV). This type of multi-disc brake is currently the subject of research at the Department of Design and Mechanical Elements of the University of Zilina in Zilina. The present market offers AGVs whose working speeds are at the level of 2 m/s [1]. The AGV is powered by an electric motor, which is supplied with electricity from a battery [2, 3]. The regulation of kinetic energy is ensured by the electric engine brake. However, as the speed increases, so does the amount of force of inertia applied to the AGV during braking [4, 5]. The current electric engine brake is not designed for braking at high speeds, and the vehicle is not able to stop at a safe distance in crises (sudden braking) [6]. During braking, the drive wheels are locked, and the vehicle tends to slide a few tens of centimetres forward. Currently, with the advent of the 4th Industrial Revolution, manufacturing plants are introducing progressive control and organization systems. The fourth industrial revolution is defined as a new level of organization and control over the entire product life cycle value chain [7]. The philosophy is increasingly focused on individual customer requirements [7]. The stated AGV is operated by computer control on an electronic data interchange platform [8]. The AGV operation is planned in the so-called virtual logistics corridors between the production line and warehouse, and they can be tens of meters apart [9]. The considered transport speed of the AGV in the said corridor is set up to 10 m/s ($\sim$28 km/h). Transport must be carried out with powerful towing AGVs. The braking must be smooth in the form of minimizing sharp braking and accelerating because driving and braking style has a direct impact on the range of the AGV [10, 11]. The vehicle does not pollute the air in the production and warehouse areas [11].

2. Multi-disc brake

In the late 1980s, automatic planetary gearboxes were equipped with friction discs with organic friction pads [12]. The friction discs formed part of the clutches and brakes in the listed gearbox

*e-mail: daniel.varecha@fstroj.uniza.sk

Manuscript submitted 2020-08-21, revised 2020-11-04, initially accepted for publication 2020-12-02, published in April 2021

[12–14]. The organic brake pad of the discs has some properties that predispose it to the construction of brake systems [14]. In the first place, the organic material has an almost constant course of the coefficient of friction depending on the slip speed of the friction discs compared to the ceramic or metallic friction material for common industrial use. In the case of ceramic friction material, the value of the coefficient of friction reaches a maximum after the start of the abrasion process (braking) and

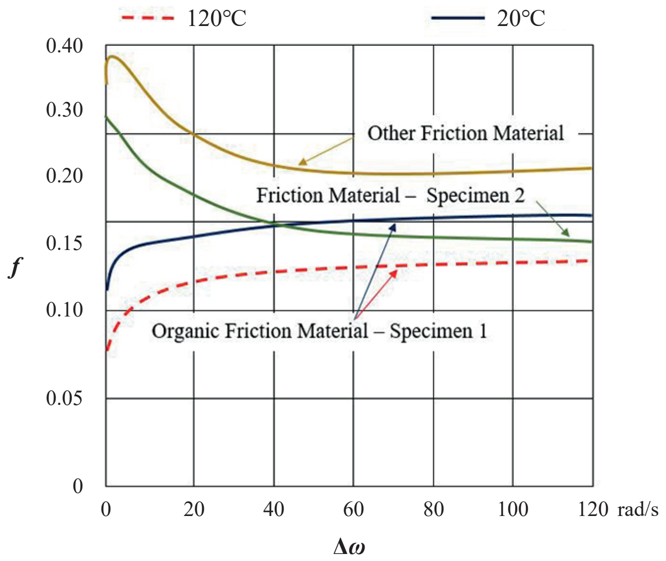

Fig. 2. Coefficient of friction of different friction materials [12]

then decreases and stabilizes [15] (Fig. 2). This statement is made in several publications [16].

With organic friction material in the region of low slide speeds, the coefficient of friction even decreases with the decreasing slide speed. The static coefficient of friction is lower than the dynamic. These organic brake pad properties have led to better gear shifting in planetary gearboxes. The organic friction brake pads serve as good damping of torsional vibrations, which arose at the beginning of the gearshift, or during a sudden increase in the pressure in the brake system and thus the friction torque. This statement only applies if the organic friction brake pad is in continuous contact with the disc during braking. In terms of operating conditions, the angular velocities of the planetary gearbox are higher than the considered speed of the AGV during operation. Ultimately, it follows from the above text that it is appropriate to consider the use of a multi-disc brake based on organic friction material. The disadvantage of brake pads based on organic friction material is their very poor thermal conductivity, which forms a thermally insulating layer from the layer of organic (paper) brake pad. The brake system based on organic material uses oil for its function to increase adhesion during braking. However, with an increasing temperature, the oil loses its properties (degrades), and the brake system loses its functionality. Ultimately, the oil quality affects the efficiency of the brake system.

2.1. The concept of a multi-disc brake. The construction of the multi-disc brake consists of two types of discs (spacers & washers). The first type of disc is made of a friction material (Fig. 3a) and the second of steel for brake systems (Fig. 3b).

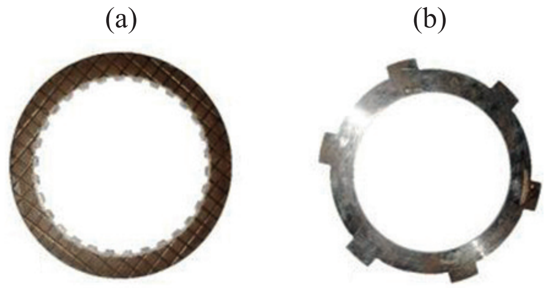

Fig. 3. Friction and steel disc [12]

The inner space of the drive wheel is limited in size. The size of the friction surface is selected, which is the most dimensionally suitable for the inner space of the wheel (Table 1). Four types of steel discs are proposed ranging from 1.75 mm to 14.2 mm in thickness. The aim is to investigate the effect of the thickness of the steel disc on the amount of heat absorbed.

Table 1
Dimensional parameters of the multi-disc brake

Symbol	Value	Units	Description
D_1	148	mm	Outside diameter
d_2	106	mm	Inside diameter
$h_{\text{friction disc}}$	2.6	mm	Thickness (friction)
$h_{\text{steel disc}}$	$1.75 \div 14.2$	mm	Thickness (steel)

The right functionality of a multi-disc brake based on organic friction material depends on the operating temperature of the oil. During frequent braking, the temperature in the brake system must not exceed 160°C. It is desirable that the thickness of the steel disc is cooled fast enough. For ceramic friction material, the maximum braking temperature may exceed 160°C because the brake system is dry (no oil used). The parameters of the materials used in the multi-disc brakes are given in Table 2.

Table 2
Thermal parameters of the multi-disc brake

Description	Symbol	Ceramic	Steel	Organic
Specific heat [J/kg]	[c]	420	477	1210
Mass destiny [kg/m^3]	[ρ]	5500	7850	700
Thermal conductivity coefficient [W/m]	[λ]	42	42	0.092
Thermal diffusivity [m^2/s]	[a]	1.82×10^{-5}	1.12×10^{-5}	10.8×10^{-8}

3. Simulation braking AGV

The basic weight of the AGV is 520 kg (Table 3). The design of the AGV allows transporting 3020 kg of cargo (including weight AGV) at a speed of up to 10 m/s. The braking simulation was performed for two friction materials, considering all the resistance that acts on the AGV during the operation (rolling resistance, air resistance, etc.). The first friction material is based on an organic basis with a coefficient of friction $f = 0.12$ and the second type of ceramic friction material with a coefficient of friction $f = 0.31$.

Table 3
Parameters of considered AGV

Symbol	Value	Unit	Description	Note
v	10	m/s	Speed	Max speed
M	520	kg	Weight	AGV weight
M_{Full}	3020	kg	Weight	Shipping weight

Using the simulation of braking, the braking distance is detected and total time to stop for both types of friction materials at the considered two driving modes and a speed of 10 m/s. In the first operating mode, the AGV transports a load of 2020 kg and in the second a load of 3020 kg, including its own weight. The brake system is located on one drive axle. The resulting data of the braking distance and total time to stop are needed to simulate the heat conduction on the surface of the steel disc during braking. The simulation of AGV braking was made in the Matlab calculation software using Newton equations of motion for deceleration (1), speed (2), and distance (3). All driving resistances, such as resistance of rolling, the resistance of air, and resistance of inertia, were considered in the calculations. Figure 4 shows the acting forces on the AGV in consideration.

$$a = -\frac{F_B}{m}, \tag{1}$$

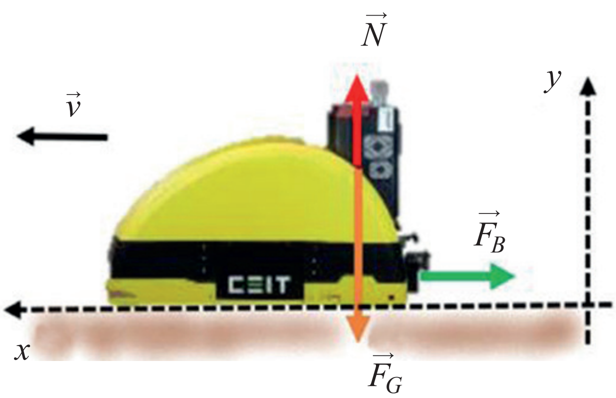

Fig. 4. Forces acting on the AGV [authors]

$$v(t) = \int a\,dt = -\frac{F_B}{m} \times t + c_1, \tag{2}$$

$$x(t) = \int v(t)\,dt = -\frac{F_B}{2 \cdot m} \times t^2 + v_0 \times t + c_2. \tag{3}$$

The results of the braking simulation are shown in Table 4 below.

Table 4
Parameters for braking with friction material
$f = 0.12$ and $f = 0.31$

Load	Coefficient of friction: $f = 0.12$		Coefficient of friction: $f = 0.31$	
[kg]	Braking time [s]	Braking route [m]	Braking time [s]	Braking route [m]
2020	1.062	5.33	0.59	3.18
3020	1.24	6.20	0.76	4.07

4. Theoretical study

In the braking system based on a ceramic friction material, the kinetic energy is transformed into thermal energy by using dry friction effects and, after that, dissipated into and absorbed to a greater extent by the steel discs, and a small percentage is passed into the surrounding space and floor [17]. The brake system based on organic friction material is based on a wet brake (in oil). In this case, the heated oil is used in braking, which has the manufacturer's prescribed recommended operating temperature.

4.1. Conversion of kinetic energy into thermal energy. During braking from the operating speed ($v = 10$ m/s) to a stop ($v = 0$ m/s), the kinetic energy (E_K) with the rotational energy (E_R) of four wheels acts on the AGV. The sum of these energies forms the total braking energy (E_B), which is needed for the safety stop of the AGV (4).

$$E_B = \left[\left(\frac{1}{2} \times M \times v^2\right) + 4 \times \left(\frac{1}{2} \times I \times \omega^2\right)\right], \tag{4}$$

where: E_K is the kinetic energy [J]; E_R is the rotational energy of the wheel [J]; I is the moment of inertia of the wheel [kg.m^2]; ω is the angular velocity [Rad/s]; M is the weight of AGV [kg].

The angular velocity (ω) from Eq. (4) is a replaced relationship of (5).

$$v = R \times \omega \rightarrow \omega = \frac{v}{R}. \tag{5}$$

The moment of inertia of the wheels (6) is also added to Eq. (4).

$$I = \frac{1}{2} \times m \times R^2, \tag{6}$$

where: R is the diameter of the drive wheel [m]; m is the weight of the wheel [kg].

The equation for total braking energy (4) is rewritten in the following mathematical Eq. (7) and subsequently adjusted (7.1).

$$E_B = \frac{M \times v^2}{2} + 4 \times \left(\frac{1}{2} \times m \times R^2\right) \times \left(\frac{v}{R}\right)^2, \tag{7}$$

$$E_B = \frac{v^2}{2} \times \left(M + (2 \times m)\right). \tag{7.1}$$

The heat conduction (8) that acts on the surface of one brake disc is defined as the proportion of the total brake energy with the effective friction surface of the disc and their number.

$$q = \frac{E_B}{j \times S_p}. \tag{8}$$

The heat conduction (Fig. 5) is assumed perpendicular to the friction surface in the direction of the disc, where: q is heat conduction $[\mathrm{J/m^2}]$; S_P is the effective area of one side of the disc $[\mathrm{mm^2}]$; j is the number of friction surfaces $[-]$.

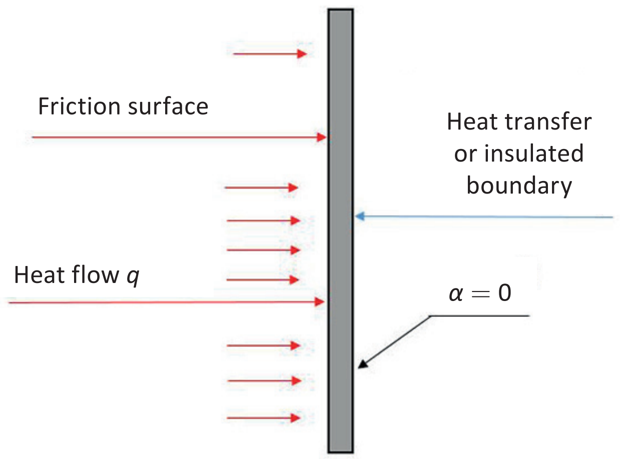

Fig. 5. Spatial boundary conditions for organic material [12]

4.2. Heat conduction simulation results. The classical theory of heat conduction is based on the Fourier law [18]. The basic heat equation is solved using the Fourier transform. Four thicknesses from 1.75, 4.2, 8.2 to 14.2 mm steel disc are selected for the simulation of heat conduction. The braking time calculated from the previous simulation is important and will be used in the next simulation of heat conduction. A reference temperature corresponding to the evenly heated steel disc is calculated (9).

$$\vartheta_0 = \frac{E_{\bar{B}}}{c \times \varrho \times S_p \times h} = \frac{q}{c \times \varrho \times h}, \tag{9}$$

where: c is the specific heat capacity [J/kg]; ϱ is the mass destiny of the steel disc $[\mathrm{kg/m^3}]$; h is the thickness of the steel disc [mm].

From several works about non-stationary, one- dimensional heat conduction, a dimensionless Fourier number is known, which contains the characteristic values of the described process (10) [19, 20]

$$F_0 = \frac{a \times t_b}{h^2}, \tag{10}$$

where: a is the thermal diffusivity $[\mathrm{m^2/s}]$; t_b is the braking time [s].

To show the results in a dimensionless representation, it is more advantageous to introduce the reciprocity of the Fourier number (11)

$$G = \frac{1}{F_0} = \frac{h^2}{a \times t_b}. \tag{11}$$

Heat conduction is also possible to describe the use of non-integer order models [20, 21]. By introducing the reference temperature (9) into Eq. (12), the dimensionless numbers in the diagram are changed to the course of temperature during braking. The end of the curve in the diagram represents the temperature that the steel disc accumulated during the braking from speed 10 until coming to a complete stop. Using Eq. (12) below, the time course of the heat conduction on the surface of the considered discs is calculated. While $t = t^*$ for $t \le t_b$, and $t^* = t_b$ if $t > t_b$,

$$\begin{aligned} \frac{\vartheta_p}{\vartheta_0} = {} & \frac{2 \times t^*}{t_b} \times \left(1 - \frac{t^*}{2 \times t_b}\right) + \frac{4}{\pi^2} \times G \times \sum_{m=1} \frac{1}{m^2} \times \left[\left(1 + \frac{G}{m^2 \times \pi} - \frac{t^*}{t_b}\right) \times e^{-\frac{m^2 \times \pi^2}{G} \times \left(\frac{t}{t_b} - \frac{t^2}{t_b}\right)} - \right. \\ & \left. - \left(1 + \frac{G}{m^2 \times \pi}\right) \times e^{-\frac{m^2 \times \pi^2}{G}} \times \frac{t}{t_b}\right], \end{aligned} \tag{12}$$

where: ϑ_p is the initial temperature [°C]; ϑ_o is the temperature of an evenly heated disc [°C]; t^* is the observed time [s]; t is the computing time [s].

As can be seen from the diagrams (Figs. 5a–5b), the thinner disc thicknesses absorb a large amount of heat. It appears that when designing the said brake system, the disc thicknesses from 8.2 mm will suffice. When considering a brake system based on a ceramic friction material, the braking distances are shorter compared to the previous brake system based on organic friction material (Figs. 6a–6b).

The intensity of the heat conduction seems to be the same for both brake systems, and the difference occurs during the conduction. However, the difference between the absorbed heat between

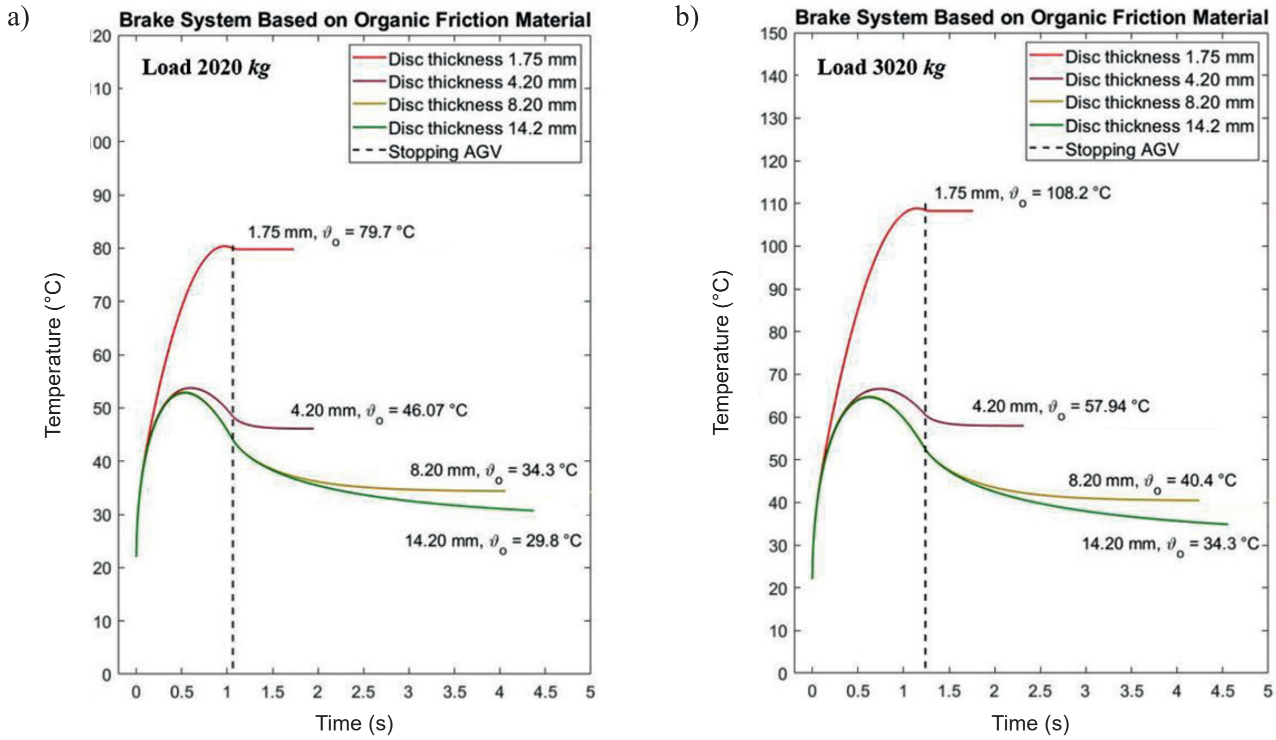

Fig. 6. Temperature course on the disc surface during braking (organic friction material)

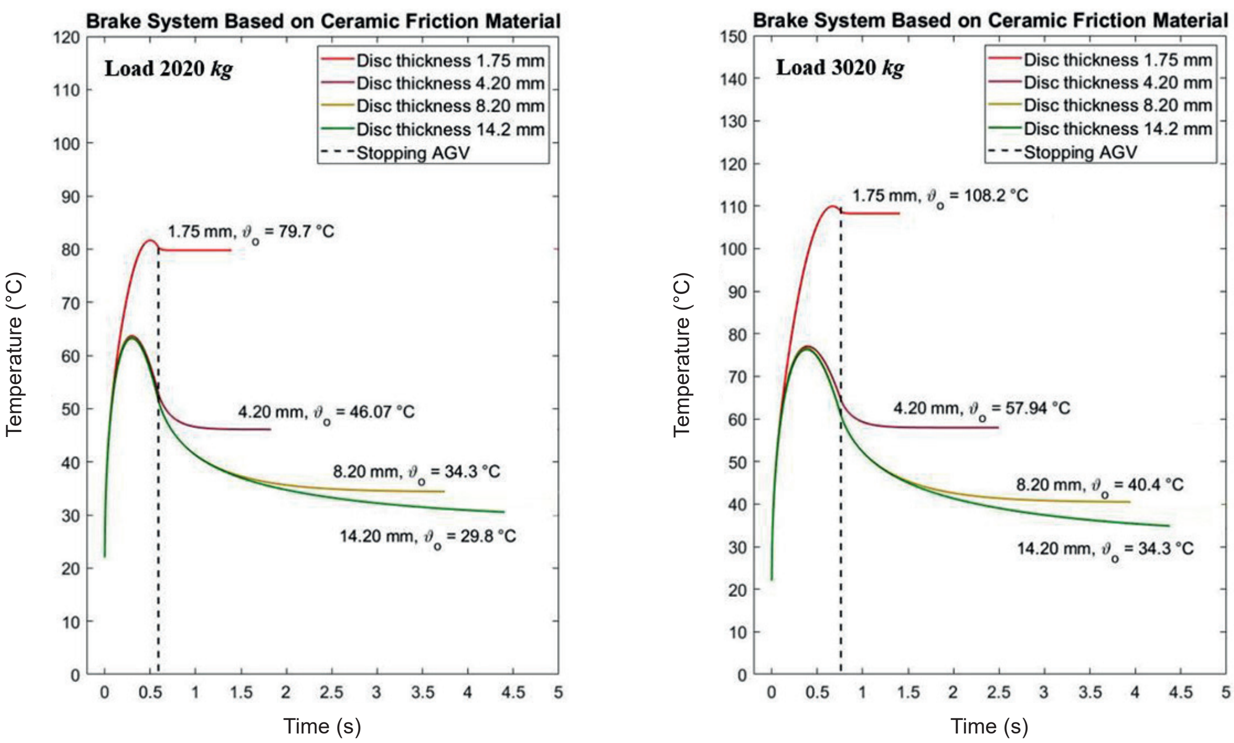

Fig. 7. Temperature course on the disc surface during braking (ceramic friction material)

the thicknesses of the discs 8.2 to 14.2 mm is almost negligible. It is necessary to verify this fact and further investigate the difference in the absorbed heat between the said steel discs.

The heat conduction simulation is performed only for the brake system based on an organic material because the temperature limit of its operation activity is limited up to 160°C. This verification will help the designer to choose the correct steel disc thickness for the brake system.

5. Heat conduction in a specific cross-section

In the following lines, it is verified how large the difference in the absorbed heat is between the thicknesses of 8.2 and 14.2 mm from the surface to the specific cross-section area of the disc. The calculation of the temperature into the specific cross-section area of the disc is solved according to the Fourier partial differential Eq. (13). Equation (13) is coming on the equation for calculating the disc surface temperature (12). The symbol "x" expresses the distance of the specific cross-section area from the surface steel disc, where the value of the accumulated heat is investigated. The symbol "h" expresses the thickness of the monitored steel disc (Fig. 8). Here, the same numerical conditions apply as for the mathematical relationship from Eq. (12). In this way, it is possible to find out at what distance from the surface the steel disc heats up during braking.

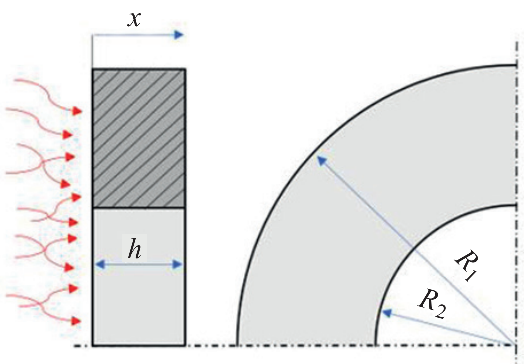

Fig. 8. Schematic view of steel disc [authors]

$$\begin{aligned}\frac{\vartheta_{(x,t)}}{\vartheta_0} &= \frac{2\times t^*}{t_b}\times\left(1-\frac{t^*}{2\times t_b}\right)+\frac{4}{\pi^2}\times G\times\\ &\times\sum_{m=1}\frac{\cos\left(m\times\pi\times\frac{x}{h}\right)}{m^2}\times\left[\left(1+\frac{G}{m^2\times\pi}-\frac{t^*}{t_b}\right)\times\right.\\ &\left.\times\, e^{-\frac{m^2\times\pi^2}{G}\times\left(\frac{t}{t_b}-\frac{t^*}{t_b}\right)}-\left(1+\frac{G}{m^2\times\pi}\right)\times e^{\frac{m^2\times\pi^2}{G}\times\frac{t}{t_b}}\right],\end{aligned} \tag{13}$$

where: $t = t^*$ for $t \le t_b$, $t^* = t_b$ if $t > t_b$.

The heat conduction in the brake system is only monitored when braking from a speed of 10 m/s to a full stop. Since the AGV operates in two operating modes, for each type of braking system, two diagrams of the heat conduction on the surface of the steel disc and also at a specific distance from the surface steel disc are given.

5.1. Heat conduction for a disc 8.2 mm thick. The diagrams in Fig. 9 and Fig. 10 show the time course of the temperature in

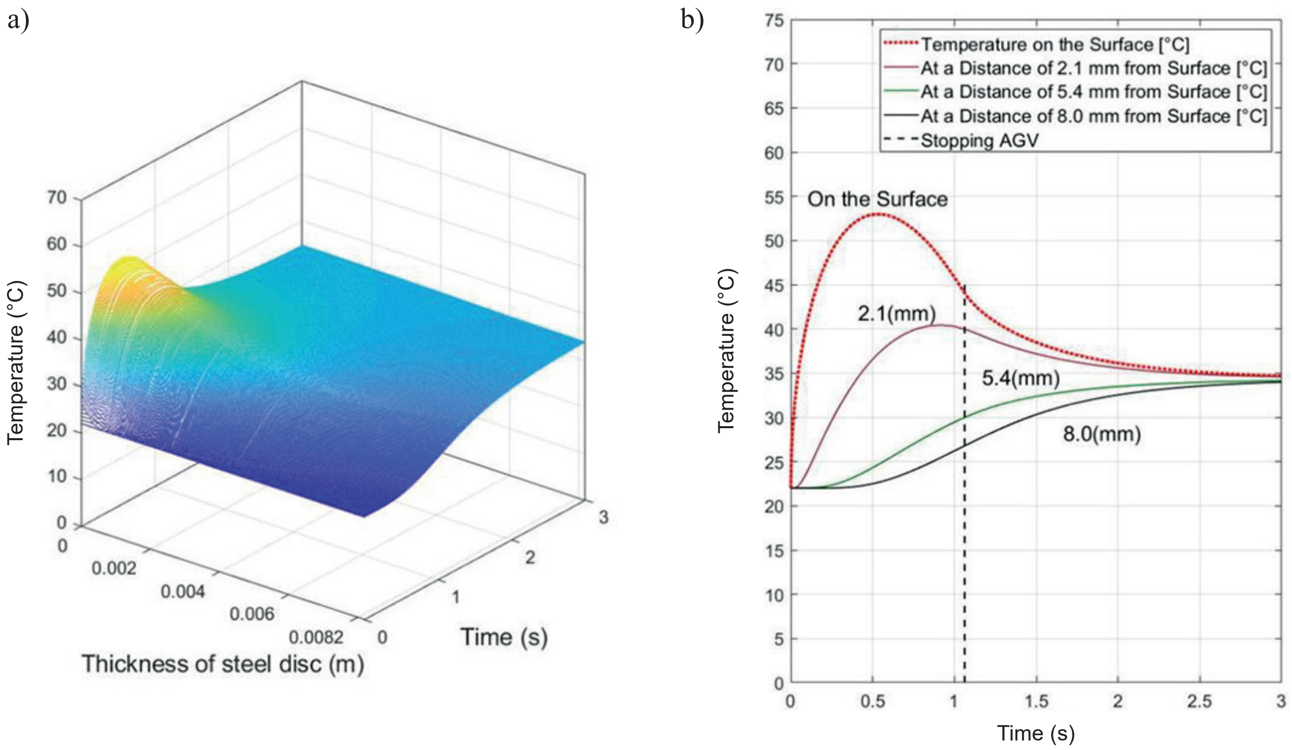

Fig. 9. Organic friction material and steel disc in the thickness of 8.2 mm, load 2020 kg

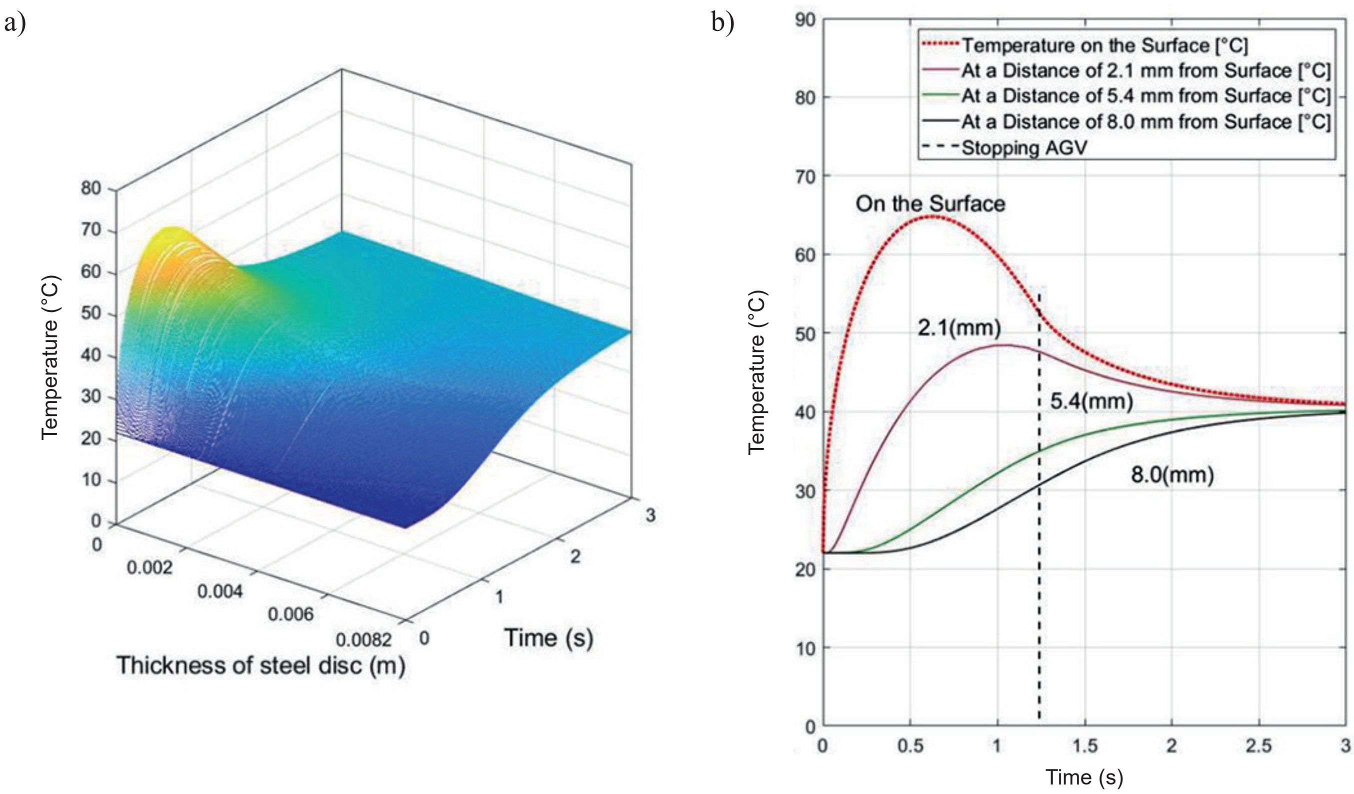

Fig. 10. Organic friction material & steel disc in the thickness of 8.2 mm, load 3020 kg

the brake system based on the organic material with a steel disc thickness of 8.2 mm for the first and second operating modes.

The three-dimensional diagram (Fig. 9a) shows the time course of the temperature during braking on the surface and in the complete body of the steel disc. The second diagram (Fig. 9b) shows the time course of the temperature on the surface of the steel disc and in a specific cross-section area from the surface. As in the previous case, Figs. 10a-10b show diagrams that depict the time course of the temperature in a brake system based on organic friction material with a steel disc 8.2 mm thick but using the second operating mode. As can be seen from the diagrams (Figs. 9 and 10), in both operating modes, the steel discs received a small amount of heat. In the case of 3020 kg weight, the temperature was from 10 to 15°C more than in the first operating mode (2020 kg). When evaluating the results of the heat conduction, it must be borne in mind that the brake system based on an organic material uses oil to its function. Therefore, by repeated braking, the temperature of the oil cannot exceed 160°C. Otherwise, the oil degrades, and the brake system would lose its effectiveness very quickly. It is also very important to note in the diagrams how much heat the investigated thicknesses of the steel discs absorb and, last but not least, how fast the temperature drops after braking. It appears that in the case of the first operating mode, the repeated braking of the braking system would probably not absorb a large amount of heat and the braking system would cool down quickly enough.

5.2. Heat conduction for a disc 14.2 mm thick. Like the simulation of the heat conduction for a disc with a thickness of 8 mm, a heat conduction simulation was created for a 14.2 mm thick steel disc (Figs. 11 and 12). When the brake system is equipped with a 14.2 mm thick steel disc, it absorbs a very small amount of heat and also cools down very quickly. It can be seen in Figs. 11 and 12 that in the first operating mode, the steel disc heats up only to two-thirds of its thickness from the surface of the disc. In the second operating mode, the time course of the temperature results was very similar. It seems this setting would be the most suitable for a brake system that is based on organic friction material. During repeated braking, the brake system would not absorb a large amount of heat and would be sufficient to maintain a permissible operating temperature level up to 160°C. This statement applies only under the given operating conditions and the use of the considered dimensions of the braking system.

6. Conclusion and discussion

The subject of the theoretical study was to compare the influence of friction materials on the resulting braking temperature, which rises during AGV braking. As can be seen from the diagrams, when the weight increases, the braking distance and the temperature in the braking system begin to increase as expected. The results of the simulation of the heat conduction also indicate that the resulting temperature on the surface of the steel disc did not differ significantly in both operating modes (65～78°C). The only difference that occurs is the period of the heat conduction, which depends on the braking distance of

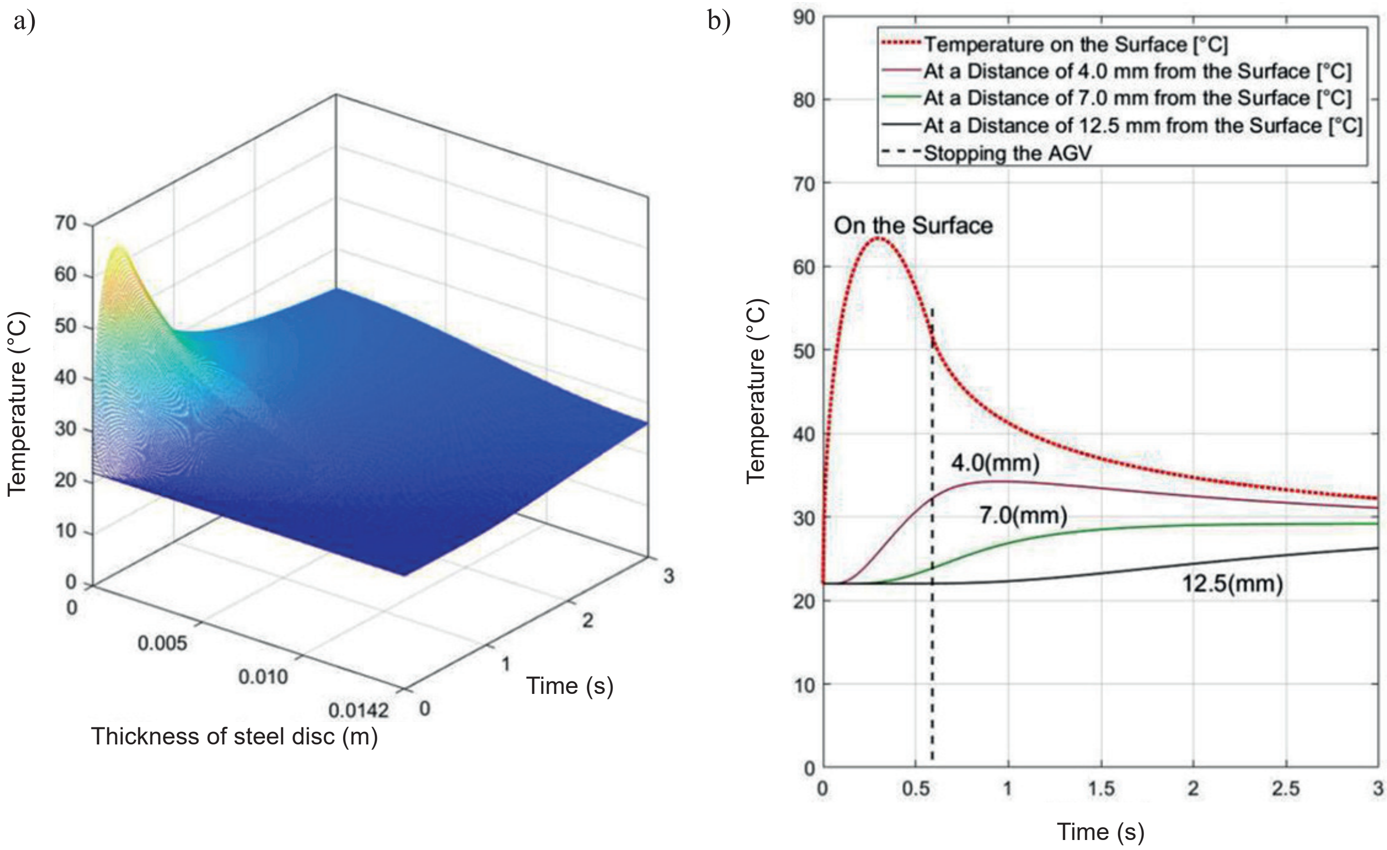

Fig. 11. Organic friction material and steel disc in the thickness of 14.2 mm, load 2020 kg

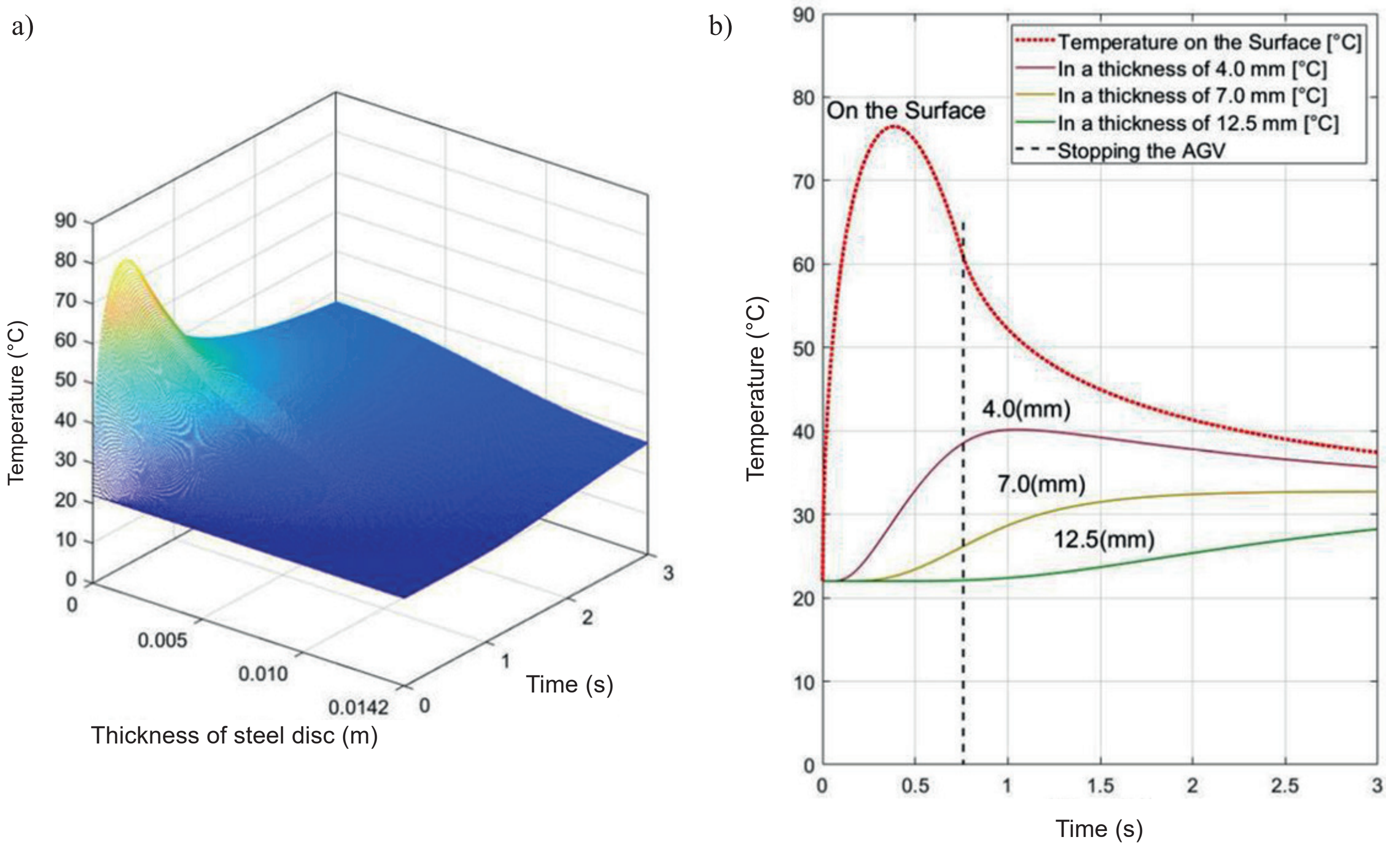

Fig. 12. Organic friction material and steel disc in the thickness of 14.2 mm, load 3020 kg

the AGV. Larger thicknesses of steel discs do not absorb large amounts of heat during braking from a speed of 10 m/s to a full stop. It is assumed that they will not become very heated up. The results of the simulation of the heat conduction show that it is necessary to equip the brake system with discs of at least a minimum thickness of 8 mm and more. Increase in the number of discs. However, adding more friction and steel discs into the brake system causes an increase in the braking torque and, ultimately, a shortening of the braking distance.

The brake system based on a ceramic friction material is not limited by the maximum operating temperature of 160°C as with the braking system based on an organic material with regard to the same considered operating terms of the AGV. The design of the said brake is dry and a cleaner operation is expected compared to a brake system based on organic friction material (in oil). The only question is the resulting comfort during braking which is verifiable only by experiment. The disadvantage is that the field studies, which are perceived as the most credible, provide information about the brake system until after the brake has been manufactured [22]. The advantage of organic materials is the almost constant coefficient of friction, which will ultimately have a positive effect on the comfort of braking. AGV braking should be smooth, with no torsional vibrations. For this reason, organic friction material was selected for comparison. Furthermore, the publication [22] states that when using an automatic brake system, there are no sudden changes in the friction properties of the brake pads when the friction material is heated. However, this only applies to the ceramic dry brake. It would certainly be appropriate for future research to focus on how to dissipate heat more quickly from the wheel body. For example, by incorporating a fin heat exchanger to dissipate the heat between the wheel body and the air. Several highly qualified publications address this issue, for example [23]. Many of the mathematical equations used in this theoretical study were taken from the habilitation work of our colleague. The habilitation thesis was published in a book form in the 1980s and there is no online access to it. All literary sources used by our colleague come from foreign (German, Austrian) and also domestic professional literature. Nevertheless, these mathematical formulas (theory) are traceable and used to this day. They are mentioned in the literature in [24], for instance.

The habilitation thesis is focused on the research of a brake system of an epicyclic gear train (also known as a planetary gear) for commercial vehicles and trucks. The research was focused not only on the design but also on the heat conduction in the brake system during shifting gears.

The effect of the size and number of friction discs on the resulting temperature during gear shifting was also investigated in the thesis. It is worth mentioning the results achieved during the development, which were used at that time in the design of planetary gearboxes. Although the technology of planetary gearboxes has advanced to a higher level, the theory described in the habilitation thesis from the 1980s is still relevant. That is why my colleagues and I decided to create a multi-disc brake for the AGV, based on our elder colleague's research. We have taken over the technology of the multi-disc brake system of the epicyclic gear train (planetary gear) and incorporated it into the current requirements in the field of logistic vehicles (AGV). In a theoretical study, controlling the brake system using an electric servo motor, not hydraulic energy, is considered. In the future, it is planned to create an experiment of braking which considers the braking system and verify the data found by the simulation.

Last but not least, the quality of the simulation in this theoretical study is several times higher at a numerical and graphic level than in the habilitation thesis from the 1980s. This is due to the advances in the fields of computer simulation and modelling. At present, it can be stated that using the computer simulation method we can solve much more complex mathematical issues than in the past.

Acknowledgements. The research was supported by the Slovak Research and Development Agency under contract No. APVV-18–0450, and by the project of the Research & Development Operational Programme funded by the ERDF: "Competency Centre for Knowledge technologies applied in Innovation of Production system in Industry and Services", ITMS: 26220220155.

References

[1] D. Varecha, R. Kohar, and F. Brumercik, "AGV brake system simulation", *LOGI – Scientific Journal on Transport and Logistics* 10(1), p. 9 (2019).

[2] G. Kovács, "Novel supply chain concepts and optimization of virtual enterprises to reduce cost, increase productivity and boost competitiveness", *Bull. Pol. Acad. Sci. Tech. Sci.* 66(6), 973–980 (2018).

[3] P. Piotrowski, D. Baczyński, S. Robak, M. Kopyt, M. Piekarz, and M. Polewaczyk, "Comprehensive forecast of electromobility mid-term development in Poland and its impacts on power system demand", *Bull. Pol. Acad. Sci. Tech. Sci.* 68(4), 697–709 (2020).

[4] M. Belorit et al., "Description of the bearing check program for countershaft gearboxes", *Proceding of 58th International Conference of Machine Design Departmens (ICDM)*, Prague, 2017, pp. 32–35.

[5] M. Jacyna, R. Jachimowsky, E. Szczepański, and M. Izdebski, "Road vehicle sequencing problem in a railroad intermodal terminal – simulation research", *Bull. Pol. Acad. Sci. Tech. Sci.* 68(5), 1135–1148 (2020).

[6] D. Varecha, R. Kohar, and T. Gajdosik, "Optimizing the braking system for handling equipment", *IOP Conf. Ser.: Mater. Sci. Angl.* 659, 012062 (2019).

[7] S. Devansh, P. Sahil, and R.S. Aravind, "Industry 4:0: Tools and Implementation", *Manag. Prod. Angl. Rev*. 10(3), 3–13 (2019).

[8] K. Chwesiuk, "Integrated computer system of management in logistics", *Arch. Transp.* 23(2), 153–163 (2011).

[9] T. Kornuta, C.C. Zieliński, and T. Winiarski, "A universal architectural pattern and specification method for robot control system design", *Bull. Pol. Acad. Sci. Tech. Sci.* 68(1), 3–29 (2020).

[10] L. Kucera, T. Gajdosik, I. Gajdac, M. Mruzek, and M. Tomasikova, "Simulation of real driving cycles of electric cars in laboratory conditions", *Communications – Scientific Letters of the University of Zilina*, 19(2A), 42–47 (2017).

[11] M. Mruzek, I. Gajdac, L. Kucera, and T. Gajdosik, “The possibilityies of increasing the electric vehicle range”, *TRANSCOM – International Scientific Conference on Sustainable, Modern and Safe Transprt, Procedia Engineering,* 192, 621–625, (2017).
[12] V. Kraus, *Výpočet teplôt radiacích lamelových spojok a bŕzd* (Calculation of temperature multi-disc shifting brake and shifting clutches), Habilitation thesis, p. 70, Žilina (1980), [in Slovak].
[13] M. Lukac, F. Brumercik, L. Krzywonos, and Z. Krzysiak, “Transmission system power flow model”, *Communications – Scientific Letters of the University of Zilina*, 19(2), 27–31, (2017).
[14] A. Estevez-Torres et. al., “Fourier analysis to measure diffusion coefficients and resolve mixtures on a continuous electrophoresis chip”, *Anal. Chem*. 79(21), 8222–8231, (2007).
[15] A.W. Orlowic, M. Mróz, G. Wnuk, O Markowska, W, Homik, and B. Kolbusz, “Coefficient of friction of a brake disc-brake pad friction couple”, *Arch. Foundry Eng*. 16, 196–200 (2016).
[16] F. Talati and S. Jalalifar, “Analysis of heat conduction in disc brake system”, *Heat Mass Transfer* 45, 1047 (2009).
[17] D.P. Milenković et al., “The influence of brake pads thermal conductivity on passenger car brake system efficiency”, *Therm. Sci.* 14, 221–230 (2010).
[18] U. Siedlecka,”Heat conduction in the finite medium using the fractional single-phase-lag model”, *Bull. Pol. Acad. Sci. Tech. Sci.* 67(2), 401–407 (2019).
[19] M. Lenarczyk and R. Domański, “Investigation of non-fourier thermal waves interaction in a solid material”, *Arch. Thermodyn.* 40(1), 115–126 (2019).
[20] D. Spałek, “Two relations for generalized discrete fourier transform coefficients”, *Bull. Pol. Acad. Sci. Tech. Sci.* 66(3), 275–281 (2018).
[21] K. Oprzędkiewicz, W. Mitkvski, E. Gawin, and K. Dziedzic, “The caputo vs. caputo-fabrizio operators in modeling of heat transfer process”, *Bull. Pol. Acad. Sci. Tech. Sci.* 66(4), 501–507 (2018).
[22] Y. Slavchev, L. Dimitrov, and Y. Dimitrov, “3-D computer research and comparative analysis of dynamic aspect of drum brakes and caliper disc brakes”, *Arch. Mech. Eng.* 65(2), 253–276 (2018).
[23] T. Muszyńsky and S. Kozieł, “Parametric study of fluid flow and heat transfer over louvered fins of air heat pump evaporator”, *Arch. Thermodyn.* 37(3), 45–62 (2016).
[24] K. Wolf, *Integral transforms in science and engineering*, pp. 255–378, 1st Edition, Springer US, Boston, 1979.