WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2021.137161

ARCHIVES OF CIVIL ENGINEERING

Vol. LXVII ISSUE 2 2021

s. 165–180

Research paper

Variability of statistical parameters of resistance for reinforced concrete columns with circular cross-section

T. A. Lutomirski[1], M. Lutomirska[2]

Abstract: Columns perform a fundamental function in structures and studies on their reliability significantly impact structural safety. While the resistance and reliability models of rectangular reinforced concrete columns are addressed by many researchers, not much work has been done on the topic of columns with a circular cross-section. In this paper, a reliability model of resistance for circular reinforced concrete columns is formulated. A procedure for the representation of behaviour for short circular reinforced concrete eccentrically loaded columns is developed. It enables the consideration of many parameters including diameter of the column, concrete compressive strength, steel yielding strength, modulus of elasticity of steel, number of rebars, size of reinforcement, and position of bars in the cross-section given by the initial angle of rotation for the reinforcement. The representative design cases are selected for the most common four compressive strengths of concrete and five different reinforcement ratios. In total one hundred design cases are investigated. Statistical parameters of resistance, coefficient of variation and bias factor, are determined using the developed procedure and Monte Carlo simulations. A total of 10,000 full interaction diagrams of force and bending moment are generated for each design case. In each of the design cases, the failure zones are determined and the statistical parameters of resistance are calculated. The results are summarized in a table, presented in the forms of three-dimensional plots, and discussed. The study is performed based on American statistical data, materials and design codes.

Keywords: circular columns, reinforced concrete, statistical parameters, resistance, reliability, variability

[1] PhD., Gaz-System S.A., ul. Jana Kazimierza 578, 05-126 Nieporęt, Rembelszczyzna, Poland, e-mail: tomasz.lutomirski@gaz-system.pl, ORCID: https://orcid.org/0000-0001-6527-1045

[2] PhD., Warsaw University of Technology, Faculty of Civil Engineering, Al. Armii Ludowej 16, 00-637 Warsaw, Poland, e-mail: m.lutomirska@il.pw.edu.pl, ORCID: https://orcid.org/0000-0002-9673-0432

1. Introduction

The role of columns is crucial in the terms of the safety of structures. They transmit loads from the upper levels to the lower ones. The failure of one column in a critical localization can cause the progressive collapse of a whole structure, resulting in significant economic as well as human losses. Therefore, it is essential to provide designs of columns with adequate levels of safety. Reliability methods are a recognized approach to provide structural safety [10, 13, 16]. In order to perform calibration of safety factors, it is necessary to determine the reliability model and apply adequate statistical parameters of resistance [2, 6, 11, 12, 13, 15]. However, the statistical parameters of resistance can be subjected to changes with passage of time depending on the assumed material, fabrication and professional factors.

Columns with rectangular cross-sections, their resistance and the reliability model, are addressed by many researchers [3, 4, 9, 14, 17]. However, there is not much published research on the topic of reinforced concrete columns with a circular cross-section, especially on the usage of reliability theory in such columns. The reasoning behind this is that they are more complicated to describe mathematically. Therefore, there is a need to investigate the topic of circular RC columns, because such columns are commonly used by architects.

The scope of this paper is the derivation of statistical parameters of resistance for not slender circular reinforced concrete columns, which are necessary for calibration of safety factors and are not available in literature. One hundred design cases with different compressive strengths of concrete and reinforcement ratios are selected for analysis. Based on a developed analytical model for representation of behavior for circular reinforced concrete eccentrically loaded columns, the capacity for circular columns is determined. The interaction diagrams for force and bending moment in regards to the investigated columns are generated. The developed interaction diagram served as a basis for the Monte Carlo simulations. In each of the design cases, the failure zones are determined and the statistical parameters of resistance are calculated. The study is performed based on American statistical data [2, 6, 7, 8, 12], materials and design code ACI 318-19 [1].

2. Selection of design cases and scope of study

In order to perform the study, representative reinforced concrete columns with circular cross-section are chosen. The selected design cases include the most common four compressive strengths of concrete f_c': 27.6 MPa (4 ksi), 41.4 MPa (6 ksi), 55.2 MPa (8 ksi), 82.7 MPa (12 ksi) and five different reinforcement ratios: 1%, 2%, 3%, 4%, and 5%. For each reinforcement ratio there are five selected columns of different column diameters, number of bars, and bar diameter. In total one hundred design cases are investigated (Table 1).

Table 1. Investigated design cases

$f_c^{'}$	$\rho = 1\%$	$\rho = 2\%$	$\rho = 3\%$	$\rho = 4\%$	$\rho = 5\%$
27.6 MPa (4 ksi) 41.4 MPa (6 ksi) 55.2 MPa (8 ksi) 82.7 MPa (12 ksi	46cm 6Φ19mm (18in 6#6)	36cm 7Φ19mm (14in 7#6)	41cm 10Φ22mm (16in 10#7)	41cm 8Φ28mm (16in 8#9)	46cm 8Φ34.5mm (18in 8#11)
	66cm 9Φ22mm (26in 9#7)	51cm 8Φ25mm (20in 8#8)	56cm 9Φ31mm (22in 9#10)	46cm 13Φ25mm (18in 13#8)	51cm 10Φ34.5mm (20in 10#11)
	71cm 8Φ25mm (28in 8#8)	66cm 18Φ22mm (26in 18#7)	66cm 16Φ28mm (26in 16#9)	56cm 12Φ31mm (22in 12#10)	56cm 12Φ34.5mm (22in 12#11)
	76cm 9Φ25mm (30in 9#8)	76cm 18Φ25mm (30in 18#8)	71cm 12Φ34.5mm (28in 12#11)	61cm 12Φ34.5mm (24in 12#11)	61cm 13Φ34.5mm (24in 13#11)
	81cm 8Φ28mm) (32in 8#9)	81cm 16Φ28mm (32in 16#9)	81cm 19Φ31mm (32in 19#10)	76cm 18Φ34.5mm (30in 18#11)	66cm 15Φ34.5mm (26in 15#11)

Some limitations are necessary to narrow down the scope of the study and make it practical. The analysis is limited to not slender columns (second order effects are omitted), since such columns constitute a significant majority of erected columns. The effect of the slenderness of columns braced against sidesway is limited by the ACI 318-19 Code using Eq. (2.1) and Eq. (2.2):

$$\frac{kl_u}{r} \leq 34 - 12\left(M_1/M_2\right) \tag{2.1}$$

$$\frac{kl_u}{r} \leq 40 \tag{2.2}$$

where:

k - effective length factor for compression members, l_u - unsupported length of column or wall, r - radius of gyration of cross-section, M_1/M_2 - ratio of lesser to greater factored end moments on a compression member

Another limitation concerns the effect of the local buckling of longitudinal reinforcement, the contribution of the transverse reinforcement in the overall column capacity, and slippage between the reinforcement and the concrete, which are omitted.

3. Nominal resistance of circular columns

The interaction diagrams of force and bending moment are developed to define resistance of the circular columns. The schema of the interaction diagram is presented in Fig.1. The diagrams are divided into compression control and tension control zones. The characteristic points are labeled: axial load, point C, balance failure point and point T. The interaction diagrams are derived based on the developed incremental procedure considering the changing number of bars and their localization. Detailed explanation of the procedure may be found in [5].

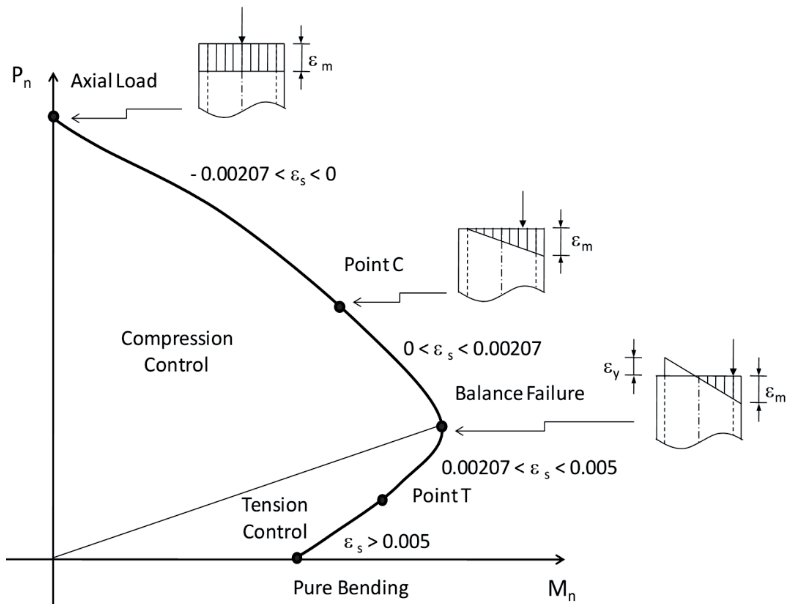

Fig. 1. Schema of P-M interaction diagram

The notations used in the procedure are the following:

a - depth of compression block of concrete

A_s - area of single reinforcement bar

c(a) - position of neutral axis due to changing depth of compression block

d - diameter of single reinforcement bar

i - number of i^{th} reinforcement bar in cross-section

n - number of bars

R - radius of cross-section

β_1 - reduction factor of compressive zone in concrete

$$\beta_1 = \begin{cases} 0.85 & \text{for} \quad f_c^{'} \le 4\text{ksi} \\ 1.05 - 0.05 f_c^{'} & \text{for} \quad 4\text{ksi} < f_c^{'} < 8\text{ksi} \\ 0.65 & \text{for} \quad f_c^{'} \ge 8\text{ksi} \end{cases}$$

ε_m - extreme compressive strain in concrete equal to 0.003

ψ - angle of initial rotation of the reinforcement

β - angle between the consecutive reinforcement bars

In the developed procedure, the existing eccentricity is described as a decreasing depth of the compression block of concrete, from the initial value a_I up to the point when the compression block does not exist ($a = 0$). The position of each bar in the cross-section is calculated with respect to the top of the cross-section (Fig.2) using Eq. (3.1). The distance from the top of the cross-section to the furthest reinforcing bar is defined by Eq. (3.2).

(3.1) $$D_i = R - \cos\left(\psi + \beta \cdot (i-1)\right) \cdot \left(R - \text{cover} - \frac{d}{2} \right)$$

(3.2) $$\varsigma = \text{maximum}(D_i)$$

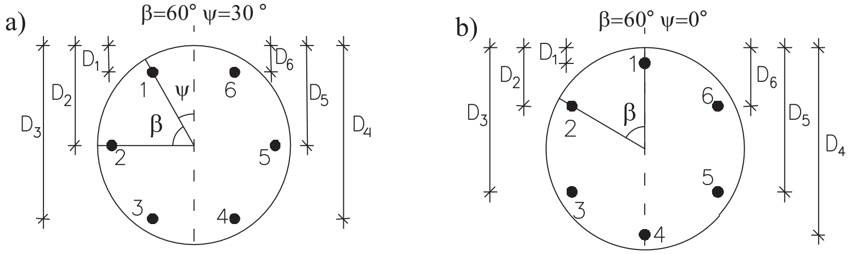

Fig. 2. Positioning of bars in the incremental procedure: a) ψ =30° b) ψ =0°

The first analyzed step is defined as the compression over the full cross-section and the lowest reinforcement bars yielding due to compression (Fig.3a,b). It means that strains in the concrete at the top of the cross-section reach the ultimate value ε_m, while the strains in the lowest reinforcement bar represent yielding strains due to compression in steel ε_y. The result for this step is theoretical, since the depth of the compression block is much bigger than the size of the cross-section. However, from this mathematical approach the position of the neutral axis for maximum axial loading can be found.

The depth of the initial compression block can be calculated as Eq. (3.3), and the position of the neutral axis in the initial step can be calculated from Eq. (3.4).

$$a_I = \beta_1 \frac{\varepsilon_m \cdot \zeta}{\varepsilon_m - \varepsilon_y} \tag{3.3}$$

$$c_I = \frac{a_I}{\beta_1} \tag{3.4}$$

The characteristic point C in the compression control zone is when the strain in the bottom reinforcement is equal to zero (Fig.3b). The depth of the compression block that represents the situation at point C can be calculated from Eq. (3.5). After passing this point, the bottom layers of the reinforcement are in tension, however the cross-section remains in compression control. The balance failure point (point B) represents the end of the compression control zone. The strains in the bottom layer or in the lowest bar of reinforcement reach the yielding strains for the steel (Fig.3c). The depth of the compression block in balance failure is represented by Eq. (3.6). The point T represents the end of the transition zone and from that point the tension control zone starts (Fig.3d). The strain in the lowest layer of reinforcement reaches and exceeds the value of 0.005 in tension. The depth of the compression block at point T is derived from Eq. (3.7).

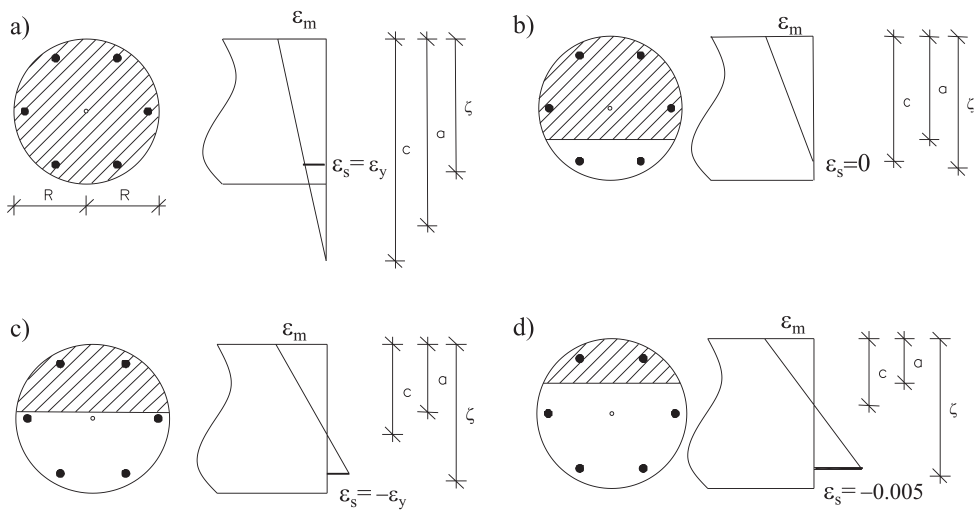

Fig. 3. Strains distribution in cross-section: a) axial loading, b) point C, c) point B, d) point T

(3.5)
$$a_C = \beta_1 \cdot \zeta$$

(3.6)
$$a_B = \beta_1 \frac{\varepsilon_m \cdot \zeta}{\varepsilon_m + \varepsilon_y}$$

(3.7)
$$a_T = \beta_1 \frac{\varepsilon_m \cdot \zeta}{\varepsilon_m + 0.005}$$

For each reinforcement bar and for each depth of the compression block the strains in reinforcement are computed from Eq. (3.8).

(3.8)
$$\varepsilon_s(i,a) = \varepsilon_m \cdot \left(1 - \frac{R - \cos\left(\psi + \beta(i-1)\right) \cdot \left(R - \text{cover} - \frac{d}{2} \right)}{c(a)} \right)$$

Having calculated strains in every reinforcement bar, it is possible to evaluate forces acting in the bars. Based on the mechanical properties of reinforcing steel defined by Eq. (3.9), the forces are computed for each reinforcement bar and each depth of the compression block from Eq. (3.10). The resultant force in the reinforcement is calculated using Eq. (3.11).

(3.9)
$$f_s = \begin{cases} -f_y & \text{for} \quad \varepsilon_s < -\varepsilon_y \\ E_s \cdot \varepsilon_s & \text{for} \quad -\varepsilon_y \le \varepsilon_s \le \varepsilon_y \\ f_y & \text{for} \quad \varepsilon_y < \varepsilon_s \le \varepsilon_m \\ 0 & \text{for} \quad \varepsilon_s > \varepsilon_m \end{cases}$$

(3.10)
$$P(i,a) = \begin{cases} -A_s \cdot f_y & \text{for} \quad \varepsilon_s(i,a) < -\varepsilon_y \\ A_s \cdot E_s \cdot \varepsilon_s(i,a) & \text{for} \quad -\varepsilon_y \le \varepsilon_s(i,a) \le \varepsilon_y \\ A_s \cdot f_y & \text{for} \quad \varepsilon_y < \varepsilon_s(i,a) \le \varepsilon_m \\ 0 & \text{for} \quad \varepsilon_s(i,a) > \varepsilon_m \end{cases}$$

(3.11)
$$P_{steel}(a) = \sum_i P(i,a)$$

To evaluate the resultant force in concrete, there is a need to analyze two cases: when the compressive block is smaller and larger than the radius of the column $a \le R$. For the first case $a \le R$ (Fig.4a), half of the central angle is calculated from Eq. (3.12). For the second case $R < a \le 2R$ $a \le R$ (Fig.4b), half of the exterior angle is calculated from Eq. (3.13) and Eq. (3.14).

(3.12) $$\theta(a) = \cos^{-1}\left(\frac{R-a}{R}\right)$$

(3.13) $$\theta(a) = 180 - \alpha(a)$$

(3.14) $$\alpha(a) = \cos^{-1}\left(\frac{a-R}{R}\right)$$

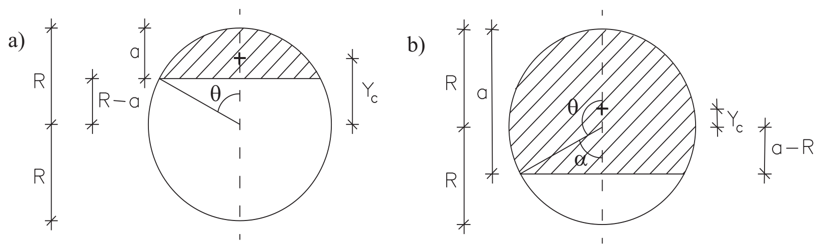

Fig. 4. Depth of the concrete compression block in the incremental procedure: a) θ ≤ 90° b) θ > 90°

After computing the area of the concrete compressive block from Eq. (3.15), and the distance of the centroid of the compressive block to the section centroid from Eq. (3.16), the concrete force components are calculated from Eq. (3.17), the resistance force of the cross-section is expressed as the sum of all forces acting in the cross-section by Eq. (3.18)., and the bending moment resistance by Eq. (3.19).

(3.15) $$A(a) = \begin{cases} (2R)^2 \cdot \dfrac{\theta(a) - \sin(\theta(a)) \cdot \cos(\theta(a))}{4} & \text{for} \quad a < 2R \\ \pi \cdot R^2 & \text{for} \quad a \ge 2R \end{cases}$$

(3.16) $$Y_c(a) = \begin{cases} \dfrac{(2R)^3 \cdot \dfrac{\sin^3(\theta(a))}{12}}{A(a)} & \text{for} \quad a < 2R \\ 0 & \text{for} \quad a \ge 2R \end{cases}$$

(3.17) $$P_c(a) = 0.85 \cdot A(a) \cdot f_c'$$

(3.18) $$P_{Total}(a) = \sum_i P(i,a) + P_c(a)$$

(3.19) $$M_{Total}(a) = \sum_i \left[P(i,a) \cdot \cos\left(\psi + \beta \cdot (i-1)\right) \cdot \left(R - cover - \frac{d}{2} \right) \right] + 0.85 \cdot A(a) \cdot f_c' \cdot Y_c(a)$$

4. Reliability model of resistance

Due to various categories of uncertainties, resistance R can be considered as a random variable and expressed by Eq. (3.1). Resistance is a product of nominal resistance R_n and three factors:
M - materials factor, F- fabrication factor and P - professional factor:

(3.1) $$R = R_n MFP$$

The materials factor represents material properties, in particular strength and modulus of elasticity. It is assumed based on values recommended by Nowak [12]. Statistical parameters for concrete compressive strength (bias factor and coefficient of variation) are the following: for 27.6 MPa (4 ksi) $\lambda = 1.24$ and $V = 0.150$, for 41.4 MPa (6 ksi) $\lambda = 1.15$ and $V = 0.125$, for 55.2 MPa (8 ksi) $\lambda = 1.11$ and $V = 0.11$, and for 68.9 MPa (10 ksi) $\lambda = 1.09$ and $V = 0.11$. Statistical Parameters for Reinforcing Steel, Grade 60 ksi, are the following: $\lambda = 1.13$ and $V = 0.03$. The past few decades have brought improvement in manufacturing and the quality control process, which has resulted in better materials properties. Therefore, use of the state–of–art data has a significant impact on reliability analysis. The fabrication factor represents the variations in the dimensions and geometry of the component. It is based on values recommended by Mirza [7] and Ellingwood [3]. Statistical Parameters for the dimensions of concrete column are: $\lambda = 1.005$ and $V = 0.04$, and for the dimensions of reinforcing steel bars: $\lambda = 1.000$ and $V = 0.015$. The professional factor represents the approximations involved in the structural analysis and idealized stress/strain distribution models, and it is defined as the ratio of the test capacity to analytically predicted capacity. It is based on values recommended by Ellingwood [3] for spiral columns: bias factor is $\lambda = 1.05$ and coefficient of variation is $V = 0.06$.

5. Results of simulations

Based on the generated force - moment interaction diagrams, a number of 10,000 Monte Carlo simulations are performed for each selected design case. An example of simulated interaction diagrams for one design case (diameter 36cm 7φ19mm, 27.6 MPa) is shown in Fig. 5. In the figure, beside simulated interaction diagrams, curved corresponding to mean values, nominal values, and standard deviations are presented. The results of the Monte Carlo simulations are used to establish statistical parameters of resistance for each design case.

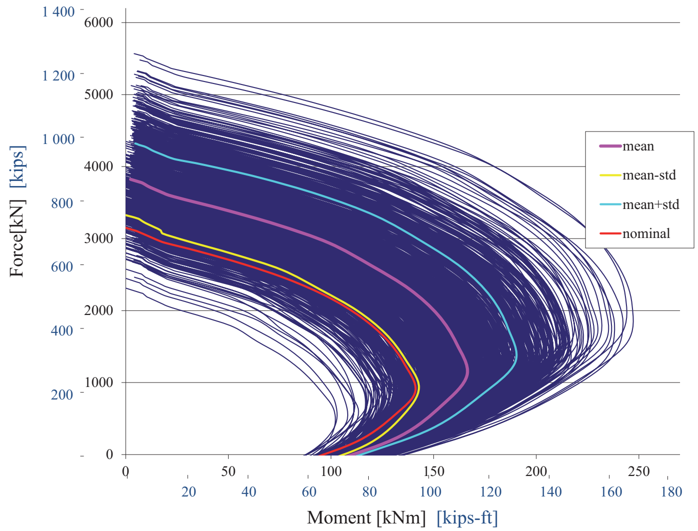

Fig. 5. Results of Monte Carlo simulations for one design case

The statistical parameters of resistance are derived for each eccentricity in the selected regions for all the analyzed design cases (Table 2). The equations Eq. (3.2) and Eq. (3.3) are applied. The bias factors λ_R (mean to nominal value) and coefficient of variations V_R are calculated based on two parameters describing resistance of columns: force and moment. The results are calculated as average values for each failure zone within the cases with the same reinforcement ratio and are presented in Table 2.

(3.2)
$$V_R = \frac{\sigma_R}{\mu_R}$$

$$\lambda_R = \frac{\mu_R}{D_R} \tag{3.3}$$

where:

σ_R – standard deviation of the resistance, μ_R – mean value of the resistance, D_R - nominal value of the resistance

Table 2. Statistical parameters of resistance for circular columns

f_c'	ρ_t	strains in steel reinforcement, ε							
		0.00207 to 0		0 to -0.00207		-0.00207 to -0.005		-0.005 to -0.01	
		V_R	λ_R	V_R	λ_R	V_R	λ_R	V_R	λ_R
27.6 MPa (4 ksi)	1%	0.15	1.23	0.12	1.23	0.11	1.22	0.12	1.24
	2%	0.13	1.22	0.11	1.21	0.11	1.21	0.11	1.21
	3%	0.12	1.20	0.10	1.20	0.10	1.20	0.11	1.20
	4%	0.11	1.19	0.09	1.19	0.11	1.18	0.10	1.18
	5%	0.10	1.18	0.09	1.17	0.10	1.17	-	-
41.4 MPa (6 ksi)	1%	0.13	1.16	0.10	1.15	0.09	1.15	0.10	1.15
	2%	0.12	1.15	0.09	1.15	0.09	1.14	0.11	1.13
	3%	0.10	1.15	0.08	1.14	0.09	1.14	0.10	1.13
	4%	0.10	1.14	0.08	1.14	0.09	1.13	0.11	1.12
	5%	0.09	1.15	0.08	1.14	0.09	1.13	0.09	1.12
55.2 MPa (8 ksi)	1%	0.12	1.11	0.09	1.11	0.09	1.10	0.10	1.10
	2%	0.11	1.12	0.08	1.11	0.08	1.11	0.10	1.09
	3%	0.10	1.12	0.08	1.11	0.08	1.10	0.08	1.10
	4%	0.09	1.12	0.07	1.11	0.08	1.09	0.09	1.09
	5%	0.09	1.12	0.07	1.11	0.08	1.10	0.09	1.08
82.7 MPa (10 ksi)	1%	0.12	1.09	0.09	1.09	0.08	1.09	0.09	1.08
	2%	0.11	1.09	0.08	1.09	0.08	1.09	0.10	1.07
	3%	0.11	1.10	0.08	1.09	0.08	1.08	0.09	1.07
	4%	0.10	1.10	0.08	1.09	0.08	1.08	0.11	1.02
	5%	0.10	1.10	0.07	1.09	0.08	1.08	0.09	1.06

The results presented in Table 2 and in three-dimensional plots of the statistical parameters of resistance show variability of the bias and coefficient of variation. The variability with respect to the compressive strength and steel strain is presented in Fig. 6 to Fig. 10, and with respect to the compressive strength and reinforcement ratio in Fig. 11 to Fig. 14. From the obtained results it can be concluded that the coefficient of variation of resistance is changing due to failure zones. With the increase of the reinforcement ratio, its variation decreases. For the case with reinforcement ratio equal to one percent, it is different for each failure zone. For the cases with reinforcement ratio equal to five percent it is almost constant throughout all failure zones. Another conclusion is that the bias and coefficient of variation of resistance for concrete circular columns are more consistent for the higher compressive strengths of concrete. It is due to better, from the statistical point of view, material parameters of higher strength concrete. However, the influence of the reinforcement ratio in the cross-section results is more significant.

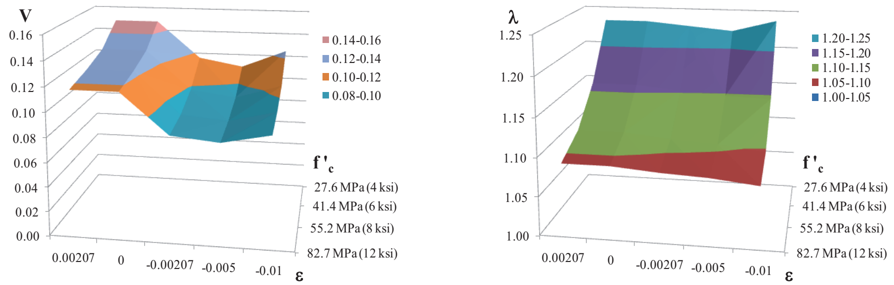

Fig. 6. Coefficient of variation and bias factor vs ε and f_c' for ρ_t=1%

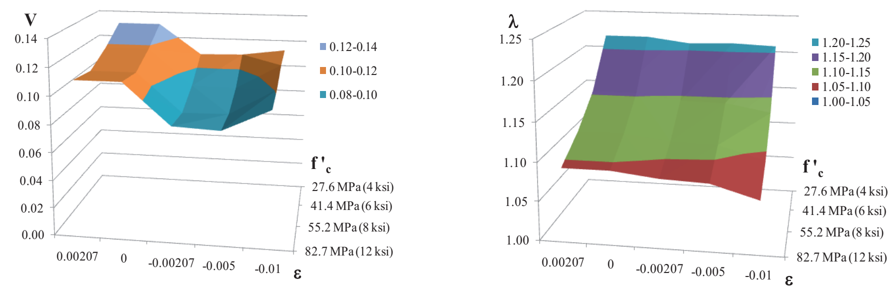

Fig. 7. Coefficient of variation and bias factor vs ε and f_c' for ρ_t=2%

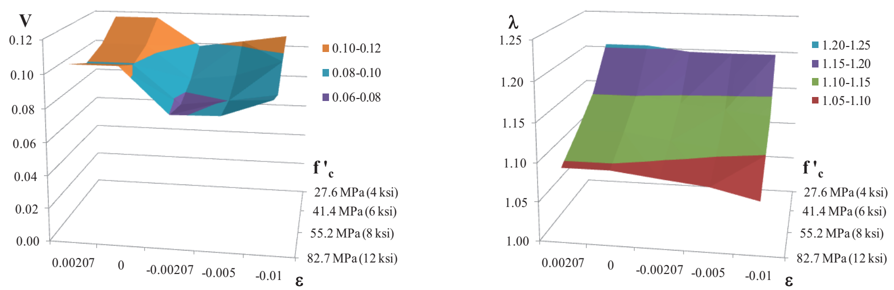

Fig. 8. Coefficient of variation and bias factor vs ε and f_c' for ρ_t=3%

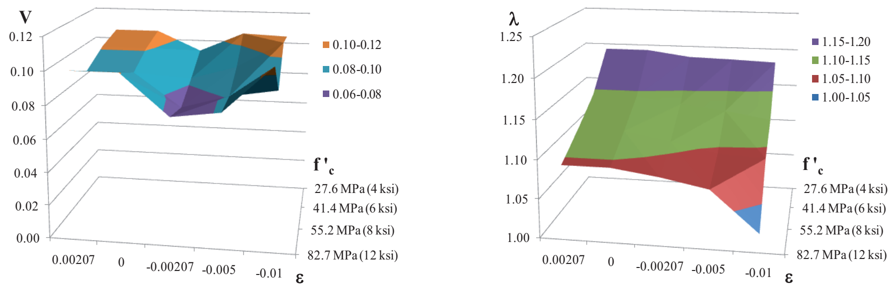

Fig. 9. Coefficient of variation and bias factor vs ε and f_c' for ρ_t=4%

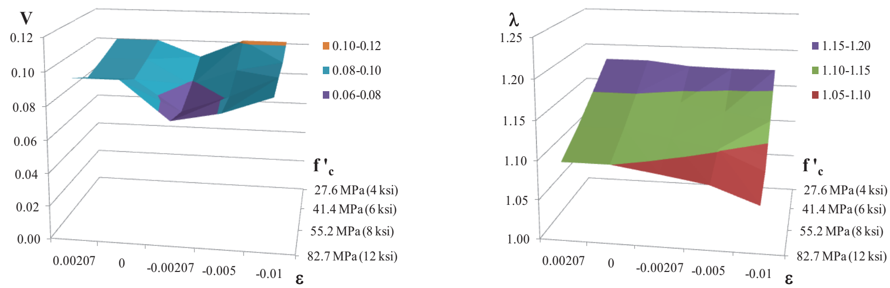

Fig. 10. Coefficient of variation and bias factor vs ε and f_c' for ρ_t=5%

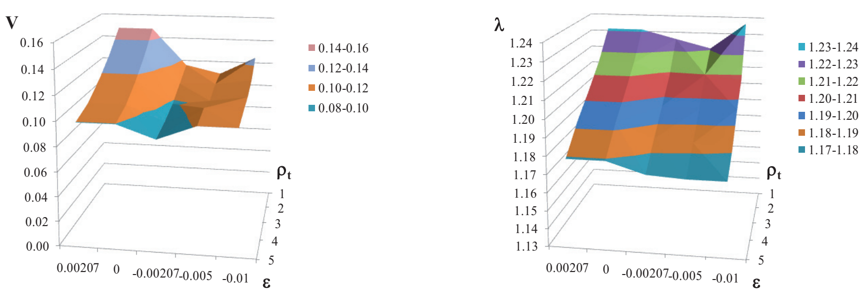

Fig. 11. Coefficient of variation and bias factor vs ε and ρ_t for f_c'= 4ksi

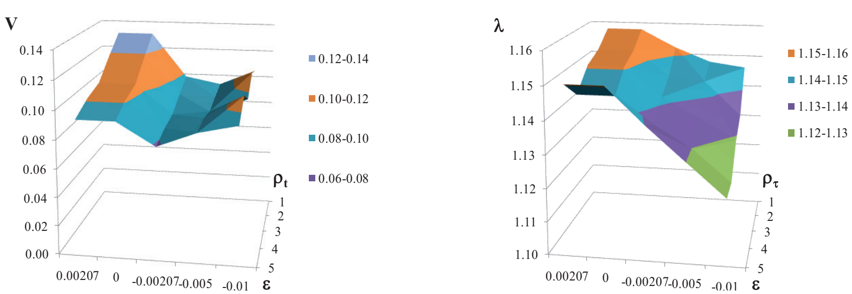

Fig. 12. Coefficient of variation and bias factor vs ε and ρ_t for f_c'= 6ksi

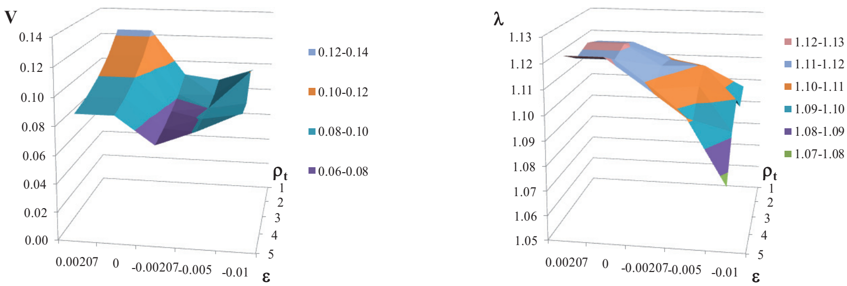

Fig. 13. Coefficient of variation and bias factor vs ε and ρ_t for f_c'= 8ksi

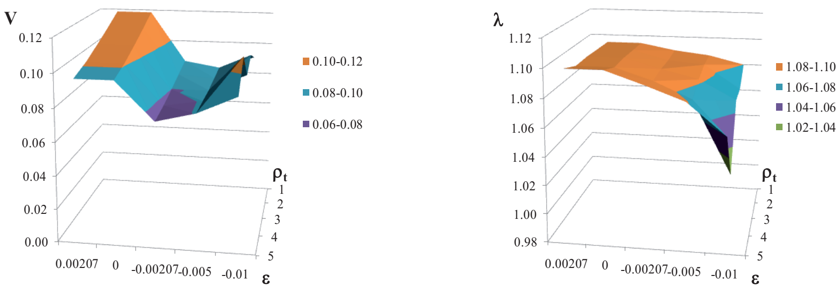

Fig. 14. Coefficient of variation and bias factor vs ε and ρ_t for f_c'= 12ksi

6. Summary and conclusions

The actual capacity of reinforced columns with circular cross-section is a random variable that varies from the nominal capacity. It is a function of variables such as compressive concrete strength, reinforcing steel yield strength, sectional geometry, the location of the bars, and approximations related to the calculation model. The objective of this research is to derive the statistical parameters of resistance, coefficient of variation, and bias factor for not slender circular RC columns. An incremental procedure based on the strain compatibility assumption is developed for calculation of the ultimate capacity of circular columns. The procedure allows for changes in the diameter of a column, the number and size of reinforcement bars, clear cover, different concrete compressive strengths and ultimate compressive concrete strains, steel yielding strength, the modulus of elasticity of steel and the initial angle of rotation for the reinforcement. As a result, full interaction diagrams of force and bending moment with characteristic points are generated for one hundred selected circular columns. The selected design cases include five different cases (different column dimensions and numbers of re-bar) for four compressive strengths of concrete (4, 6, 8 and 12 ksi), and for five reinforcement ratios (1, 2, 3, 4 and 5%). For each of the design cases, 10,000 Monte Carlo simulations are performed to determine the statistical parameters of resistance. The achieved scope of the paper, the derived coefficient of variation and bias factor are presented in the form of three-dimensional plots and summarized in Table 2, in respect to the compressive strength of concrete, steel strain and reinforcement ratio. It is observed that values of coefficient of variation change within the failure zone, and its higher variations occurs for the smaller reinforcement ratios. The bias and coefficient of variation of resistance for concrete circular columns are more stable for the higher compressive strengths of concrete, and the results are more sensitive to the reinforcement ratio than the compressive strength of concrete. The obtained data is needed for calibration of the strength reduction factor for reinforced concrete columns with circular cross-sections.

References

[1] ACI 318-19, Building Code Requirements for Structural Concrete and Commentary, American Concrete Institute Farmington Hills, USA, 2019.

[2] [2] B. Ellingwood, T.V. Galambos, J.G. MacGregor, and C.A. Cornell, "Development of a Probability Based Load Criterion for American National Standard A58," NBS Special Publication 577, Washington, DC, National Bureau of Standards, USA, 1980.

[3] L. Galan, A. Vignoli, "Strength and Ductility of HSC and SCC Slender Columns Subjected to Short-Term Eccentric Load," ACI Structural Journal, Title no.105-S25, pp 259-269, 2008.

[4] L.H. Grant, S.A. Mirza, J.G. MacGregor, "Monte Carlo Study of Strength of Concrete Columns," ACI Structural Journal, Title no.75-37, pp 348-358, 1978.

[5] T. Lutomirski, "Reliability models for circular concrete columns," Doctoral Dissertation, University of Nebraska, Lincoln, USA, 2009.

[6] J.G. MacGregor, S.A. Mirza, B. Ellingwood, "Statistical Analysis of Resistance of Reinforced and Prestressed Concrete Members," ACI Structural Journal, Title no.80-16, pp 167-176, 1983

[7] S.A. Mirza and J.G. MacGregor, "Variations in dimensions of reinforced concrete members," Journal of The Structural Division, ASCE, Vol. 105, No. ST4, pp 751-766, 1979. https://doi.org/10.1061/JSDEAG.0005132

[8] S.A. Mirza, J.G. MacGregor, "Variability of Mechanical Properties of Reinforcing Bars," Journal of The Structural Division, ASCE, Vol. 105, No. ST5, pp 921-937, 1979. https://doi.org/10.1061/JSDEAG.0005146

[9] S.A. Mirza, J.G. MacGregor, "Slenderness and Strength Reliability of Reinforced Concrete Columns," ACI Structural Journal, Title no.86-S40, pp 428-438, 1989.

[10] J. Murzewski, "Reliability of Engineering Structures" (in Polish), Arkady, Warsaw, Poland, 1989.

[11] A.S. Nowak, M. Szerszeń, E.K. Szeliga, A. Szwed, and P.J Podhorecki, "Reliability-Based Calibration for Structural Concrete," Phase 3, SN2849, Portland Cement Association, Skokie, Illinois, USA, 2008.

[12] A.S. Nowak, A.M. Rakoczy, and E.K. Szeliga, "Revised statistical resistance models for R/C structural components," American Concrete Institute, SP-284-6 1-16, USA, 2011.

[13] A. S. Nowak and K.R. Collins, "Reliability of Structures," CRC Press Taylor and Francis Group, Boca Raton, USA, 2013.

[14] M.M. Szerszeń, A. Szwed, A.S. Nowak, "Reliability Analysis for Eccentrically Loaded Columns," ACI Structural Journal, Vol. 102, No. 5, pp 676-688, 2005.

[15] M.M. Szerszeń, A.S Nowak, "Calibration of Design Code for Buildings (ACI 318) Part 2: Reliability Analysis and Resistance Factors", ACI Structural Journal, Vol. 100, No. 3, pp 383–391, 2003. https://doi.org/10.14359/12614

[16] Sz. Woliński, K. Wrobel, "Reliability of Engineering Structures" (in Polish), Publishers of Rzeszow Technical University (Oficyna Wydawnicza Politechniki Rzeszowskiej), 2001.

[17] W. Zhou, H.P. Hong, "Modeling Error of Strength of Short Reinforced Concrete Columns," ACI Structural Journal, Title no.97-S46, pp 427- 435, 2000.

Zmienność statystycznych parametrów nośności kolumn żelbetowych o przekroju kołowym

Słowa kluczowe: słupy okrągłe, żelbet, parametry statystyczne, nośność, niezawodność, zmienność

Streszczenie:

Celem niniejszej pracy jest wyprowadzenie paramentów statystycznych nośności dla krótkich okrągłych kolumn żelbetowych. Nośność okrągłych kolumn traktowana jako zmienna losowa, na którą wpływ ma zmienność materiałów, geometrii oraz niedoskonałości modeli obliczeniowych. Dla potrzeb pracy, sformułowany został analityczny model pracy mimośrodowo obciążonej okrągłej kolumny żelbetowej oraz opracowana została iteracyjna procedura numeryczna, przy pomocy której obliczana jest nośność kolumn oraz generowane są wykresy interakcji momentu i siły podłużnej. Procedura ta umożliwia uwzględnienie róźych średnic słupów, wytrzymałości betonu na ściskanie, granicy plastyczności stali, modułu sprężystości stali, różnej ilości prętów, ich romiarów i położenia. Zmienność mimośrodu siły definiowana jest poprzez zmianę wielkości strefy ściskanej betonu. Obliczenia przeprowadzono dla wybranch reprezentacyjnych słupów okrągłych. W sumie przeanalizowano sto przypadków projektowych, różniących się między sobą rozmiarem słupa, ilością i rozmiarami pretów zbrojeniowych, wytrzymałością betonu na ściskanie (27.6 MPa [4 ksi], 41.4 MPa [6 ksi], 55.2 MPa [8 ksi], 82.7 MPa [12 ksi]) oraz stopniem zbrojenia (1, 2, 3, 4 i 5%). Następnie, na podstawie wygenerowanych krzywych interakcji, wykonano symulacje metodą Monte Carlo. Dla każdego przypadku projektowego zostało przeprowadzonych 10 000 symulacji, na podstawie których określono parametry statystyczne nośności dla każdego z punktów charakterystycznych wykresu interakcji. Widoczna jest zależność parametrów statystycznych od położenia na wykresie interakcji. Otrzymane wartość wskaźników zmienności V_R oraz wartości stosunku wartości średniej do nominalnej λ_R zaprezentowano w formie w tabeli oraz trójwymiarowych wykresów. Wykresy obrazują zależność otrzymanych od wytrzymałości betonu na ściskanie, stopnia zbrojenia oraz odkszałceń w stali zbrojeniowej. Obserwuje się mniejszą zmienność wyników dla wyższych wartości wytrzymałości betonu. Czynnikiem mającym największy wpływ na wyniki jest stopień zbrojenia przekroju. Dla niskich stopni zbrojenia przekroju zmienność parametrów statystycznych wyraźnie rośnie, podczas gdy dla wysokch stopni zbrojenia przekroju wyniki stabilizują się dla różnych punktów wykresu interakcji. Praca została wykonana bazując amerykańskich danych statystycznych i materiałach oraz zgodnie z normą ACI 318-19.

Received: 30.09.2020, Revised: 13.01.2021