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Output tracking control of an aircraft subject to additive state dependent disturbance: an optimal control approach

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In this paper, model reference output feedback tracking control of an aircraft subject to additive, uncertain, nonlinear disturbances is considered. In order to present the design steps in a clear fashion: first, the aircraft dynamics is temporarily assumed as known with all the states of the system available. Then a feedback linearizing controller minimizing a performance index while only requiring the output measurements of the system is proposed. As the aircraft dynamics is uncertain and only the output is available, the proposed controller makes use of a novel uncertainty estimator. The stability of the closed loop system and global asymptotic tracking of the proposed method are ensured via Lyapunov based arguments, asymptotic convergence of the controller to an optimal controller is also established. Numerical simulations are presented in order to demonstrate the feasibility and performance of the proposed control strategy.

Key words: optimal control, aerospace applications, nonlinear systems, mechanical/mechatronics applications, robust control

1. Introduction

Designing optimal controllers is the focus of some of the recent works on control of aircraft. Optimal control is mostly preferred when a performance index, usually a quadratic function of error and control input, is required to be minimized [6, 8, 14, 17, 21, 27]. In [22], an optimal controller design was introduced for trajectory tracking of a helicopter model having known dynamics.

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Optimal control of a twin rotor system was presented in [23]. In [26], robust optimal control of a quadrotor model was performed in the presence of parametric uncertainty and measurement noise. In [1], adaptive-critic-based neural networks were utilized for optimal control of longitudinal dynamics of an aircraft. In [35], a finite horizon optimal guaranteed cost control was fused with a neural network term for the entry guidance problem of the Mars lander. In [36], multilayer perceptron neural networks were utilized for vertical take-off and landing of an aircraft. In [15], a nonlinear programming based formulation was proposed to achieve optimal control of a quadrotor. In [7], a tracking control based on linear optimal control theory was designed for a small scale helicopter model. In most optimal control designs, including some of the above past works, the aircraft model is considered to be fully or partially available. In the presence of parametric uncertainties in the aircraft dynamics adaptive control methods are common [11, 12, 16, 30], and when dealing with unstructured uncertainties robust methods are preferred [5, 10, 19, 31, 32, 34].

Designing an optimal controller when the aircraft dynamics is uncertain is a challenging research problem. This work aims to provide a solution to this control problem. Specifically, the aircraft dynamics is considered to be unavailable and only the output is measured for control design. The control problem is further complicated by the non-symmetric nature of the input gain matrix. The design is initiated by temporarily considering the aircraft dynamics as known and system states being available. Then an auxiliary term fused with an optimal controller that minimizes a performance index is proposed. Next, an uncertainty estimator is designed by aiming to converge to the auxiliary term that is temporarily assumed known and available. The stability of the closed-loop system is investigated via Lyapunov-type methods and global boundedness and asymptotic tracking is ensured. Next, the convergence of the estimator to the auxiliary uncertain term is shown. Thus concluding the optimality analysis. The result is a robust controller, that does not require aircraft dynamics, which asymptotically converges to an optimal controller minimizing a performance index. When compared with the relevant past works on optimal control of an aircraft, the proposed controller does not depend on the aircraft dynamics via the design of an uncertainty estimator. Extensive numerical simulations are presented for proof of concept.

2. Aircraft model

The nonlinear aircraft model considered in this work is represented by a linear state space model fused with a nonlinear disturbance term as follows [29]:

$$\dot{x} = Ax + f + Bu, \quad (1)$$

$$y = Cx, \quad (2)$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, $A \in \mathbb{R}^{n \times n}$ is the constant state matrix, $f(x, t) \in \mathbb{R}^n$ includes time- and state-dependent terms such as gravity, inertial coupling, nonlinear gust modeling effects and other disturbances, $B \in \mathbb{R}^{n \times m}$ is the constant input matrix, $u(t) \in \mathbb{R}^m$ is the control input, $C \in \mathbb{R}^{m \times n}$ is the output matrix, and $y(t) \in \mathbb{R}^m$ is the output of the aircraft with $n > m$. In the above dynamics, C is considered to be known and measurements of y are available, and A , B , f are uncertain where state measurements are unavailable. Similar to [18] and [32], $f(x, t)$ is assumed to be divided into only state-dependent terms $f_1(x)$ and only time-dependent terms $f_2(t)$ in the sense that $f = f_1 + f_2$ where f_2 , \dot{f}_2 , $f_1(x)$, $\partial f_1(x)/\partial x$ are bounded for all $x(t)$.

3. Error system development and control design

The tracking control objective is to ensure that the output of the aircraft tracks the output of a subsequently introduced reference model. Optimality is targeted via minimizing a quadratic performance index which is a function of output tracking error and control input. Guaranteeing the boundedness of the signals under the closed-loop operation is also aimed.

The reference model is represented as:

$$\dot{x}_m = A_m x_m + B_m u_m, \quad (3)$$

$$y_m = C x_m, \quad (4)$$

where $x_m(t) \in \mathbb{R}^n$ is the reference state vector, $A_m \in \mathbb{R}^{n \times n}$ is the reference state matrix, $B_m \in \mathbb{R}^{n \times m}$ is the reference input matrix, $u_m(t) \in \mathbb{R}^m$ is the reference input, and $y_m(t) \in \mathbb{R}^m$ is the reference output. To ensure smoothness of reference state and output vectors and their time derivatives, A_m is required to be chosen Hurwitz along with the reference input $u_m(t)$ and its time derivative being bounded.

Following Lemma from [25, 33] is essential for the subsequent derivations.

Lemma 1 *An $m \times m$ real matrix Ω with non-zero leading principal minors can be factored as $\Omega = SDU$ where S is symmetric positive definite, D is diagonal with entries ± 1 , and U is unity upper triangular.*

Remark 1 *We would like to note that for the aircraft model we used in our simulation studies (and also for most aircraft models we found in the literature) the decomposition of CB according to Lemma 1 results in the matrix D being equal to the identity matrix. However, for the completeness of the presentation, the subsequent controller design will assume the general case for the matrix D .*

Since the aircraft model in (1), (2) includes uncertainties, a two step control design will be performed. First, after temporarily assuming exact knowledge

of aircraft dynamics (*i.e.*, A , B and f are considered known and available) and availability of all system states (*i.e.*, x is considered to be measurable), an optimal controller will be designed. Next, since aircraft dynamics is uncertain and only the output of the aircraft is available, an uncertainty estimator fused with the optimal controller will be designed.

To quantify the tracking control objective, an output tracking error, denoted by $e(t) \in \mathbb{R}^m$, is defined as:

$$e \triangleq y - y_m. \quad (5)$$

After making use of (1)–(4), the time derivative of $e(t)$ can be written as:

$$\dot{e} = CAx + Cf + SDUu - CA_mx_m - CB_mu_m, \quad (6)$$

where Lemma 1 was utilized with $CB = SDU$. Premultiplying (6) with the symmetric and positive definite matrix $M \triangleq S^{-1}$ yields:

$$M\dot{e} = w + Du \quad (7)$$

with $w(t) \in \mathbb{R}^m$ defined as¹:

$$w \triangleq MCAx + D(U - I_m)u + MCf - MCA_mx_m - MCB_mu_m. \quad (8)$$

It should be noticed that since the aircraft dynamics is uncertain and system states are unavailable, then $w(t)$ includes uncertain and/or unmeasurable terms. Provided the temporary assumption that $w(t)$ is available, the control input is designed as:

$$u = -D(w - \bar{u}), \quad (9)$$

where $\bar{u}(t) \in \mathbb{R}^m$ is an auxiliary control input that will be designed subsequently. It is noted that, due to the structure of D from Lemma 1, $D^{-1} = D$. The controller in (9) is the one that is designed by temporarily considering the aircraft dynamics as known and system states being available. It is highlighted that as a consequence of U being unity upper triangular and thus $(U - I_m)$ being a strictly upper triangular matrix there is no singularity in (8) and (9). Aside from that, in the subsequently designed final form of the controller, w will not be utilized. By substituting (9) into (7), the time derivative of the tracking error can be represented in standard state space form as:

$$\dot{e} = \bar{A}e + \bar{B}\bar{u}, \quad (10)$$

where $\bar{A} \triangleq 0_{m \times m}$ and $\bar{B} \triangleq M^{-1}$. A quadratic performance index $J(\bar{u}) \in \mathbb{R}$ is chosen as:

$$J(\bar{u}) \triangleq \int_0^{\infty} L(e, \bar{u}) dt, \quad (11)$$

¹Throughout the paper, I_{n_1} and $0_{n_1 \times n_2}$ will be used to represent an $n_1 \times n_1$ standard identity matrix and an $n_1 \times n_2$ zero matrix, respectively.

where $L(e, \bar{u}) \in \mathbb{R}$ is defined as:

$$L(e, \bar{u}) \triangleq \frac{1}{2} e^T(t) Q e(t) + \frac{1}{2} \bar{u}^T(t) R \bar{u}(t) \quad (12)$$

with $Q, R \in \mathbb{R}^{m \times m}$ being constant, positive definite, symmetric weighting matrices. Given the performance index $J(\bar{u})$, the control objective is to find the auxiliary control input $\bar{u}(t)$ that minimizes (11) subject to the differential constraint imposed by (10). The optimal controller that achieves this objective will be denoted by $\bar{u}^*(t)$. A necessary and sufficient condition for $\bar{u}(t)$ to minimize (11) subject to (10) is that there exists a positive definite value function $V_{opt}(e) \in \mathbb{R}$ satisfying the Hamilton Jacobi Bellman equation [17]:

$$\min_{\bar{u}} \left[e^T Q e + \bar{u}^T R \bar{u} + \frac{\partial V_{opt}}{\partial e} \dot{e} + \frac{\partial V_{opt}}{\partial t} \right] = 0. \quad (13)$$

The value function V_{opt} is chosen as:

$$V_{opt} \triangleq e^T \bar{K} e \quad (14)$$

with $\bar{K} \in \mathbb{R}^{m \times m}$ being a constant, symmetric, positive definite matrix. Substituting partial derivatives of V_{opt} into (13) yields:

$$\min_{\bar{u}} \left[e^T Q e + \bar{u}^T R \bar{u} + 2e^T \bar{K} \bar{A} e + 2e^T \bar{K} \bar{B} \bar{u} \right] = 0. \quad (15)$$

To minimize (15), its partial derivative with respect to \bar{u} is evaluated to yield a solution for \bar{u} as:

$$\bar{u} = -R^{-1} \bar{B}^T \bar{K} e. \quad (16)$$

Evaluation of \bar{K} in (14) can be achieved from the following Riccati equation [17]:

$$\bar{K} \bar{A} + \bar{A}^T \bar{K} - \bar{K} \bar{B} R^{-1} \bar{B}^T \bar{K} + Q = 0 \quad (17)$$

which is obtained by substituting the solution of \bar{u} in (16) into (15). The design of the value function V_{opt} for the state space form in (10) is finalized by choosing $\bar{K} = M$ as:

$$V_{opt} = e^T M e \quad (18)$$

which yields the optimal controller \bar{u}^* that minimizes (11) to be found as:

$$\bar{u}^* = -R^{-1} M^{-T} M e = -R^{-1} e. \quad (19)$$

By using \bar{A} and \bar{B} introduced in (10), from the Riccati equation in (17), it can be found that $Q = R^{-1}$. This concludes the design of the optimal part of the controller. The controller in (9) is the preliminary step of the subsequently designed final

form of the robust controller which is obtained by fusing the auxiliary term in (8) with the optimal controller in (19) that minimizes the performance index in (11). We also would like to note that the selection of \bar{K} in (17) is to remove the model dependency of the controller formulation yet to be designed.

In view of (9) and (19) the control input is designed as $u = -D(w + R^{-1}e)$ based on the temporary assumption that $w(t)$ is available. In the subsequent development, an uncertainty estimator will be designed and then will be fused with the optimal controller in (19).

The error system of the second part of the design will be based on an auxiliary error, denoted by $r(t) \in \mathbb{R}^m$, which is defined as:

$$r \triangleq \dot{e} + \Lambda e, \quad (20)$$

where $\Lambda \in \mathbb{R}^{m \times m}$ is a constant, positive definite, diagonal control gain matrix. After substituting (6) into (20), following expression can be obtained:

$$r = CAx + SDUu + Cf - CA_m x_m - CB_m u_m + \Lambda e. \quad (21)$$

Premultiplying the time derivative of (21) with M gives:

$$M\dot{r} = M \left[CA\dot{x} + C\dot{f} - CA_m \dot{x}_m - CB_m \dot{u}_m + \Lambda \dot{e} \right] + DU\dot{u}. \quad (22)$$

Motivated by the subsequent stability analysis, the control input $u(t)$ is designed as:

$$u = -D\hat{w} - DR^{-1}e, \quad (23)$$

where $\hat{w}(t) \in \mathbb{R}^m$ is the uncertainty estimator designed to be updated according to:

$$\hat{w}(t) = K \left[e(t) - e(0) + \Lambda \int_0^t e(\tau) d\tau \right] + \beta \int_0^t \text{Sgn}(e(\tau)) d\tau, \quad (24)$$

where $\beta \in \mathbb{R}^{m \times m}$ is a constant, positive definite, diagonal control gain matrix, $\text{Sgn}(\cdot)$ denotes the vector signum function, and $K \in \mathbb{R}^{m \times m}$ is a constant, positive definite, diagonal control gain matrix designed as:

$$K = I_m + k_g I_m + \text{diag}\{k_{d,1}, k_{d,2}, \dots, k_{d,m-1}, 0\} \quad (25)$$

with $k_g, k_{d,1}, \dots, k_{d,m-1} \in \mathbb{R}$ being positive gains. $\hat{w}(t)$ in (23) is designed specifically to estimate $w(t)$ in (9) and when this estimation is achieved (*i.e.*, $\hat{w}(t) \rightarrow w(t)$) then the control input in (23) converges to the optimal controller in (9). Via the design of the estimator of the auxiliary term in (8) that was temporarily assumed known and available, the design of the controller is completed. It is

highlighted that the proposed estimator is different from the disturbance observer in [9, 20] in the sense that the estimated variable is not required to be constant or slowly time-varying.

Via substituting the time derivative of (23) into (22), following closed-loop error system can be obtained:

$$M\dot{r} = N - e - DUD\beta\text{Sgn}(e) - D(U - I_m)DKr - Kr, \quad (26)$$

where $N(x, \dot{x}, t) \in \mathbb{R}^m$ is defined as:

$$N \triangleq M \left[CA\dot{x} + Cf - CA_m\dot{x}_m - CB_m\dot{u}_m + \Lambda\dot{e} \right] + e - DUDR^{-1}\dot{e} \quad (27)$$

which is partitioned as:

$$N = N_d + \tilde{N}, \quad (28)$$

where $N_d(t) \triangleq MCA\dot{x}_m + MC\frac{\partial f_1}{\partial x}\dot{x}_m + MCf_2 - MCA_m\dot{x}_m - MCB_m\dot{u}_m \in \mathbb{R}^m$ which includes terms that are bounded by constants in the sense that:

$$|N_{d,i}| \leq \zeta_{d,i} \quad \forall i = 1, \dots, m \quad (29)$$

where $N_{d,i}(t) \in \mathbb{R}$ is the i -th entry of N_d , $\zeta_{d,i} \in \mathbb{R}$ are positive bounding constants, and $\tilde{N}(x, \dot{x}, e, \dot{e}) \triangleq MCA(\dot{x} - \dot{x}_m) + MC\frac{\partial f_1}{\partial x}(\dot{x} - \dot{x}_m) + M\Lambda(r - \Lambda e) + e - DUDR^{-1}(r - \Lambda e) \in \mathbb{R}^m$ contains functions that can be bounded by error terms as:

$$|\tilde{N}_i| \leq \rho_i \|z\| \quad \forall i = 1, \dots, m, \quad (30)$$

where $\tilde{N}_i(t) \in \mathbb{R}$ is the i -th entry of \tilde{N} , $\rho_i \in \mathbb{R}$ are positive bounding constants and $z(t) \triangleq \begin{bmatrix} e^T & r^T \end{bmatrix}^T \in \mathbb{R}^{2m}$. In obtaining the bound of $(\dot{x} - \dot{x}_m)$, the time derivatives of (2), (4), (5) were utilized along with (20) and a pseudo inverse of C .

4. Stability and optimality analysis

In this section, the stability of the closed-loop will be investigated first and the optimality analysis will be performed afterwards.

Theorem 1 *The controller in (23) with the uncertainty estimator in (24) ensures asymptotic tracking in the sense that $\|e(t)\| \rightarrow 0$ as $t \rightarrow +\infty$ provided that the control gain matrices K and β are selected by using the following procedure:*

1. For $i = m$, β_m is selected according to:

$$\beta_m \geq \zeta_{d,m} \left(1 + \frac{\kappa_2}{\Lambda_m} \right) \quad (31)$$

and from $i = m - 1$ to $i = 1$, β_i are selected according to:

$$\beta_i \geq \left(\zeta_{d,i} + \sum_{j=i+1}^m \zeta_{U_{i,j}} \beta_j \right) \left(1 + \frac{\kappa_2}{\Lambda_i} \right), \quad (32)$$

where $\zeta_{U_{i,j}}$ are positive bounding constants that satisfy $|U_{i,j}| \leq \zeta_{U_{i,j}}$.

2. Control gain k_g is chosen big enough to decrease the constant $\sum_{i=1}^m \frac{\rho_i^2}{4k_g}$.
3. Choose $k_{d,i}$, $i = 1, \dots, (m - 1)$ to decrease the constant $\sum_{i=1}^{m-1} \frac{\zeta_{\Phi_i}^2}{4k_{d,i}}$ where ζ_{Φ_i} are positive bounding constants.

Proof. A highlight of the proof is provided for simplicity reasons. Specifically, first, boundedness of all the signals under the closed-loop operation will be presented by choosing $V_b \triangleq \frac{1}{2}e^T e + \frac{1}{2}r^T M r$ as a Lyapunov function. Taking its time derivative and substituting (20) and (26), and performing straightforward mathematical manipulations yield $\dot{V}_b \leq -\gamma_1 V_b + \gamma_2$ where γ_1 and γ_2 are some positive constants. In view of these, it is clear that $V_b(z)$ and thus $e(t)$, $r(t)$ are bounded functions of time. The definition of $r(t)$ in (20) can be utilized to prove that $\dot{e}(t) \in \mathcal{L}_\infty$. By using (5) and its time derivative, along with the assumption that the reference model signals being bounded, it can be proven that $y(t)$, $\dot{y}(t)$, $x(t)$, $\dot{x}(t) \in \mathcal{L}_\infty$. The above boundedness statements can be utilized along with (1) to prove that $u(t) \in \mathcal{L}_\infty$. From the time derivative of (23), it is easy to see that $\dot{u}(t) \in \mathcal{L}_\infty$. After utilizing the above boundedness statements along with (22), it is clear that $\dot{r}(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can then be utilized to demonstrate boundedness of all the signals under the closed-loop operation.

Following Lemma from [28] is essential for the rest of the stability analysis.

Lemma 2 *Provided that $e(t)$ and $\dot{e}(t)$ are bounded, the following expression for the upper bound of the integral of the absolute value of the i -th entry of $\dot{e}(t)$ can be obtained:*

$$\int_{t_0}^t |\dot{e}_i(\tau)| d\tau \leq \kappa_1 + |e_i(t)| + \kappa_2 \int_{t_0}^t |e_i(\tau)| d\tau, \quad (33)$$

where $\kappa_1, \kappa_2 \in \mathbb{R}$ are some positive bounding constants.

Lemma 4.3 of [2] is the next step of the stability analysis.

Lemma 3 *Let an auxiliary function $L(t) \in \mathbb{R}$ be defined as:*

$$L \triangleq r^T (N_d - DUD\beta \text{Sgn}(e)). \quad (34)$$

If the entries of β are selected to satisfy the conditions in (31) and (32), then it can be concluded that the following auxiliary function $P(t) \in \mathbb{R}$:

$$P \triangleq \zeta_b - \int_0^t L(\tau) d\tau \quad (35)$$

is non-negative where $\zeta_b \in \mathbb{R}$ is a positive bounding constant.

Let $V_o(t) \in \mathbb{R}$ be a Lyapunov function defined as:

$$V_o \triangleq V_b + P. \quad (36)$$

Time derivative of V_o is obtained as:

$$\dot{V}_o = e^T \dot{e} + r^T M \dot{r} + \dot{P} \quad (37)$$

Substituting (20), (26) along with (25) and (28), and the time derivative of (35) into (37) results in:

$$\begin{aligned} \dot{V}_o = & -e^T \Lambda e + r^T (N_d + \tilde{N}) - r^T DUD\beta \text{Sgn}(e) - r^T [\Phi^T \ 0]^T \\ & - r^T r - k_g r^T r - \sum_{i=1}^{m-1} k_{d,i} r_i^2 - r^T (N_d - DUD\beta \text{Sgn}(e)), \end{aligned} \quad (38)$$

where $\Phi(t) \in \mathbb{R}^{m-1}$ is obtained from $[\Phi^T, 0]^T = D(U - I_m)DKr$ and its entries are bounded as follows:

$$|\Phi_i| \leq \zeta_{\Phi_i} \|z\| \quad (39)$$

with ζ_{Φ_i} , $i = 1, \dots, (m-1)$ being positive bounding constants. After utilizing simplifications on (38), the time derivative of the Lyapunov function is rewritten as:

$$\dot{V}_o = -e^T \Lambda e + r^T \tilde{N} - r^T [\Phi^T \ 0]^T - r^T r - k_g r^T r - \sum_{i=1}^{m-1} k_{d,i} r_i^2. \quad (40)$$

After utilizing (30) and (39) in (40), time derivative of the Lyapunov function can be upper bounded as:

$$\dot{V}_o \leq -e^T \Lambda e - r^T r + \sum_{i=1}^m \rho_{z,i} |r_i| \|z\| - k_g r^T r + \sum_{i=1}^{m-1} \zeta_{\Phi_i} |r_i| \|z\| - \sum_{i=1}^{m-1} k_{d,i} |r_i|^2. \quad (41)$$

The right hand side of (41) can be upper bounded as:

$$\dot{V}_o \leq - \left[\min\{\lambda_{\min}\{\Lambda\}, 1\} - \sum_{i=1}^m \frac{\rho_{z,i}^2}{4k_g} - \sum_{i=1}^{m-1} \frac{\zeta_{\Phi_i}^2}{4k_{d,i}} \right] \|z\|^2, \quad (42)$$

where the following were utilized:

$$\rho_{z,i}|r_i|\|z\| - k_g r_i^2 \leq \frac{\rho_{z,i}^2}{4k_g} \|z\|^2, \quad (43)$$

$$\zeta_{\Phi_i}|r_i|\|z\| - k_{d,i} r_i^2 \leq \frac{\zeta_{\Phi_i}^2}{4k_{d,i}} \|z\|^2. \quad (44)$$

Provided that the control gains Λ , k_g , $k_{d,1}$, \dots , $k_{d,m-1}$ are selected sufficiently high, the below expression can be obtained for the time derivative of the Lyapunov function:

$$\dot{V}_o \leq -\gamma_3 \|z\|^2, \quad (45)$$

where γ_3 is some positive bounding constant. From (36) and (45), it is clear that $V_o(t)$ is non-increasing and bounded. After integrating (45), it can be concluded that $z(t) \in \mathcal{L}_2$. Since $z(t) \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\dot{z}(t) \in \mathcal{L}_\infty$, from Barbalat's Lemma [13], $\|z(t)\| \rightarrow 0$ as $t \rightarrow \infty$, thus meeting the tracking control objective. Since no restrictions with respect to the initial conditions of the error signals were imposed on the control gains, the result is global. \square

Now, the optimality analysis for the proposed controller in (23) is presented.

Theorem 2 *The controller given in (23) with the uncertainty estimator in (24) is optimal in the sense that it minimizes the performance index in (11).*

Proof. By substituting (20) into (7), following expression can be obtained:

$$Mr = w + Du + M\Lambda e \quad (46)$$

and substituting the controller in (23) results in:

$$Mr = w - \hat{w} - R^{-1}e + M\Lambda e. \quad (47)$$

Since, $e(t)$ and $r(t)$ go asymptotically to zero, then, in (47), Mr , $R^{-1}e$, $M\Lambda e$ will go to zero, as a result, \hat{w} will asymptotically converge to w . Thus the control input in (23) will asymptotically converge to the optimal controller in (9). \square

This concludes the optimality analysis of the proposed controller.

5. Simulation results

The model of Osprey fixed wing aerial vehicle in [18, 19] was used in the numerical simulations. The system matrices $A \in \mathbb{R}^{8 \times 8}$, $B \in \mathbb{R}^{8 \times 4}$ and $C \in \mathbb{R}^{4 \times 8}$ are given as:

$$A = \begin{bmatrix} A_{lon} & 0_{4 \times 4} \\ 0_{4 \times 4} & A_{lat} \end{bmatrix}, \quad B = \begin{bmatrix} B_{lon} & 0_{4 \times 2} \\ 0_{4 \times 2} & B_{lat} \end{bmatrix}, \quad C = \begin{bmatrix} C_{lon} & 0_{2 \times 4} \\ 0_{2 \times 4} & C_{lat} \end{bmatrix}, \quad (48)$$

where $A_{lon}, A_{lat} \in \mathbb{R}^{4 \times 4}$, $B_{lon}, B_{lat} \in \mathbb{R}^{4 \times 2}$, $C_{lon}, C_{lat} \in \mathbb{R}^{2 \times 4}$ are system matrices for the longitudinal and lateral subsystems. The state vector is $x(t) = \begin{bmatrix} x_{lon}^T & x_{lat}^T \end{bmatrix}^T \in \mathbb{R}^8$ with $x_{lon} = \begin{bmatrix} v & \alpha & q & \theta \end{bmatrix}^T$, $x_{lat} = \begin{bmatrix} \gamma & p & \mu & \phi \end{bmatrix}^T$ where the state variables $v(t)$, $\alpha(t)$, $q(t)$, $\theta(t)$, $\gamma(t)$, $p(t)$, $\mu(t)$ and $\phi(t)$ are velocity, angle of attack, pitch rate, pitch angle, side slip angle, roll rate, yaw rate and bank angle, respectively. The control input is $u(t) \triangleq \begin{bmatrix} u_{lon}^T & u_{lat}^T \end{bmatrix}^T \in \mathbb{R}^4$ where $u_{lon} = \begin{bmatrix} u_e & u_t \end{bmatrix}$, $u_{lat} = \begin{bmatrix} u_a & u_r \end{bmatrix}$ where the control inputs $u_e(t)$, $u_t(t)$, $u_a(t)$ and $u_r(t)$ are elevator deflection angle, control thrust, aileron deflection angle and rudder deflection angle, respectively. The output vector consisted of pitch rate and forward velocity for the longitudinal subsystem, and roll rate and yaw rate for the lateral subsystem where tracking control of these states is considered.

Following system matrices of the Osprey aircraft, are based on experimentally determined data at a cruising velocity of 25 [m/s] and at an altitude of 60 [m]:

$$\begin{aligned}
 A_{lon} &= \begin{bmatrix} -0.15 & 11.08 & 0.08 & 0 \\ -0.03 & -7.17 & 0.83 & 0 \\ 0 & -37.35 & -9.96 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, & A_{lat} &= \begin{bmatrix} -0.69 & -0.03 & -0.99 & 0 \\ -3.13 & -12.92 & 1.1 & 0 \\ 17.03 & -0.10 & -0.97 & 0 \\ 0 & 1 & -0.03 & 0 \end{bmatrix}, \\
 B_{lon} &= \begin{bmatrix} 3 \times 10^{-3} & 0.06 \\ 10^{-5} & 10^{-4} \\ 0.98 & 0 \\ 0 & 0 \end{bmatrix}, & B_{lat} &= \begin{bmatrix} 0 & 0 \\ 1.5 & -0.02 \\ -0.09 & 0.17 \\ 0 & 0 \end{bmatrix}, \\
 C_{lon} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, & C_{lat} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.
 \end{aligned} \quad (49)$$

The disturbance term $f(x, t) \triangleq \begin{bmatrix} f_{lon}^T(x, t) & f_{lat}^T(x, t) \end{bmatrix}^T$ with $f_{lon}(x, t)$, $f_{lat}(x, t) \in \mathbb{R}^4$ being defined as:

$$\begin{aligned}
 f_{lon} &\triangleq \begin{bmatrix} -9.81 \sin(\theta) & 0 & 0 & 0 \end{bmatrix}^T + g(x), \\
 f_{lat} &\triangleq \begin{bmatrix} 0.39 \sin(\phi) & 0 & 0 & 0 \end{bmatrix}^T,
 \end{aligned} \quad (50)$$

where $g(x) \in \mathbb{R}^4$ is defined as:

$$g \triangleq \frac{U_{ds}}{2V_0} \left[1 - \cos\left(\frac{\pi d_g}{H}\right) \right] [-11.1 \ 7.2 \ 37.4 \ 0]^T, \quad (51)$$

where H denotes the distance along the airplane's flight path for the gust to reach its peak velocity, V_0 is the forward velocity of the aircraft when it enters the

gust, $d_g = \int_{t_1}^{t_2} v(t) dt$ represents the distance penetrated into the gust and U_{ds} is

the design gust velocity as specified in [24]. Parameter values were chosen as $U_{ds} = 10.12$ [m/s], $H = 15.24$ [m] and $V_0 = 25$ [m/s] [18].

Following system matrices were utilized for the reference model [18, 19]:

$$\begin{aligned}
 A_{lonm} &= \begin{bmatrix} 0.6 & -1.1 & 0 & 0 \\ 2 & -2.2 & 0 & 0 \\ 0 & 0 & -4 & -600 \\ 0 & 0 & 0.1 & -10 \end{bmatrix}, & A_{latm} &= \begin{bmatrix} -4 & -600 & 0 & 0 \\ 0.1 & -10 & 0 & 0 \\ 0 & 0 & 0.6 & -1.1 \\ 0 & 0 & 2 & -2.2 \end{bmatrix}, \\
 B_{lonm} &= \begin{bmatrix} 0 & 0.5 \\ 0 & 0 \\ 10 & 0 \\ 0 & 0 \end{bmatrix}, & B_{latm} &= \begin{bmatrix} 0 & 0 \\ 10 & 0 \\ 0 & 0.5 \\ 0 & 0 \end{bmatrix}.
 \end{aligned} \quad (52)$$

The reference control input $u_m(t) \in \mathbb{R}^4$ was designed as:

$$u_m = \begin{bmatrix} 0.2\{u_s(t-2) - u_s(t-4)\} \\ 3 \\ 0.2\{u_s(t-4) - u_s(t-6)\} \\ 0.2 \sin(t)\{u_s(t-6) - u_s(t-10)\} \end{bmatrix}. \quad (53)$$

where u_s is the unit step function and initial conditions of the system states were chosen as $x(0) = [1 \ 0 \ 0.2 \ 0 \ 0 \ 0.2 \ 0.2 \ 0]^T$.

The self tuning algorithm in [3] and [4] was used as an add-on and after the algorithm converged, numerical simulations were re-run for the final values of the control gains. Specifically, control gains β and K were obtained from the self tuning algorithm as:

$$\begin{aligned}
 \beta &= \text{diag}\{ 72.4 \ 81 \ 79.6 \ 80.8 \}, \\
 K &= \text{diag}\{ 300 \ 300.03 \ 300 \ 300.1 \}
 \end{aligned} \quad (54)$$

and $\Lambda = 2I_4$ was chosen. Weighting matrices Q and R were chosen as identity matrix.

The tracking performance of the output states are presented in Figs. 1–4 while the tracking error is given in Fig. 5. The control inputs are shown in Fig. 6. From Figs. 1–4 and 5, it is clear that the tracking objective was satisfied. Five Monte Carlo simulations are performed for different initial state values. In Tables 1 and 2, average maximum steady state error and average root mean square error

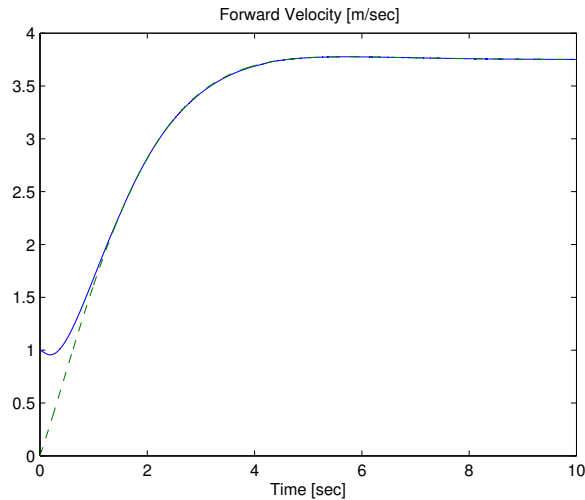


Figure 1: The reference velocity (dashed line) and the actual velocity (solid line)

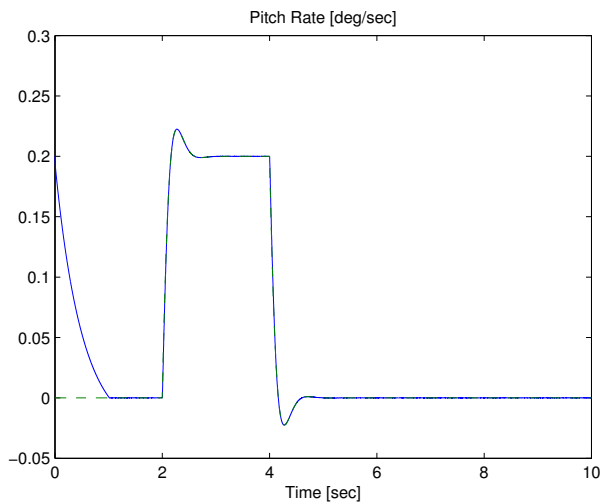


Figure 2: The reference pitch rate (dashed line) and the actual pitch rate (solid line)

are presented where asymptotic tracking was ensured for different initial values of the states. In Table 3, the values of quadratic performance index J for different values of weighting matrix R are given. In Table 4, a comparison of the proposed optimal controller and the robust controller in [32] is given for different values of weighting matrices.

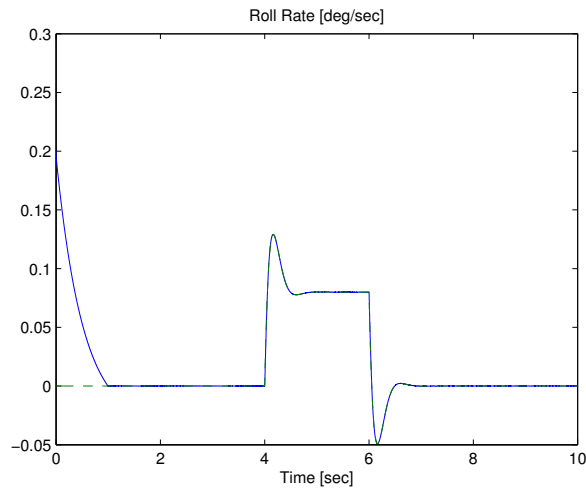


Figure 3: The reference roll rate (dashed line) and the actual roll rate (solid line)

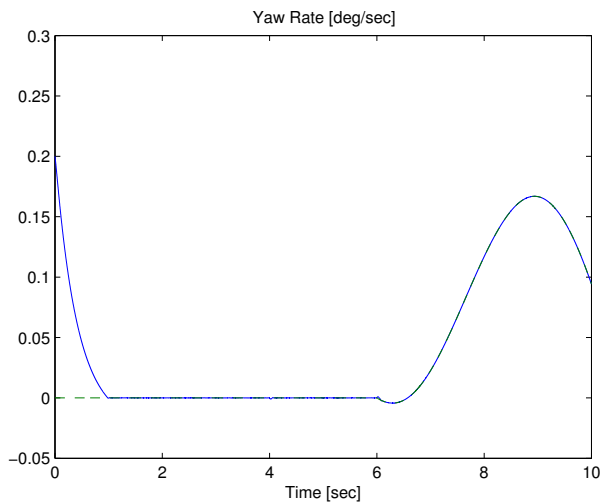


Figure 4: The reference yaw rate (dashed line) and the actual yaw rate (solid line)

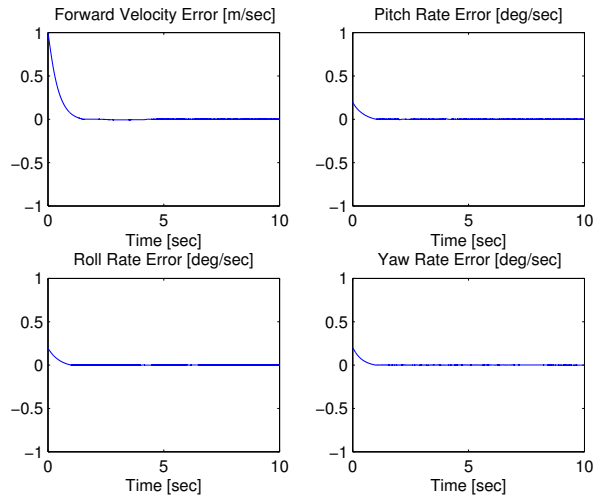
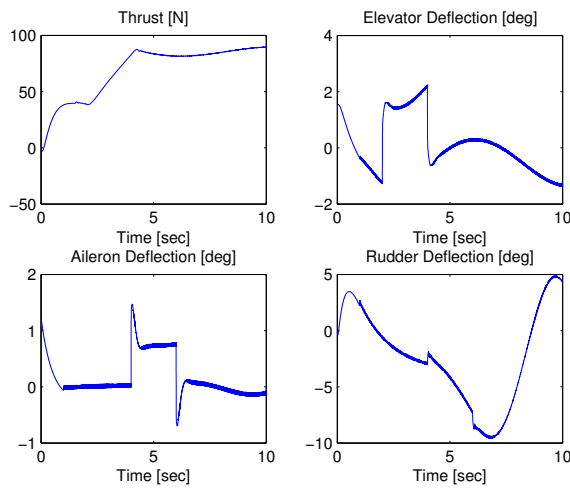
Figure 5: The output tracking error $e(t)$ Figure 6: The control input $u(t)$

Table 1: Tabulated steady state error values for 5 simulation runs

State	Average maximum steady state error
Forward velocity	3.2×10^{-4}
Pitch rate	1.0×10^{-4}
Roll rate	3.7×10^{-3}
Yaw rate	1.5×10^{-3}

Table 2: Tabulated root mean square error values for 5 simulation runs

State	Average root mean square error
Forward velocity	0.800
Pitch rate	0.089
Roll rate	0.089
Yaw rate	0.086

Table 3: Tabulated performance index J for different values of weighting matrix R

Q	R	Performance index J
$0.01I_4$	$100I_4$	2.5074×10^9
$0.1I_4$	$10I_4$	2.5077×10^8
I_4	I_4	2.5411×10^7
$10I_4$	$0.1I_4$	5.8484×10^6
$100I_4$	$0.01I_4$	3.3594×10^7

Table 4: Comparison of robust controller and robust optimal controller

Type of controller	Q	R	Mean squared error
Robust	–	–	7.1×10^{-3}
Optimal	$10I_4$	$0.1I_4$	7.0×10^{-3}
Optimal	$100I_4$	$0.01I_4$	6.5×10^{-3}
Optimal	$1000I_4$	$0.001I_4$	4.6×10^{-3}

6. Conclusions

In this work, design and accompanying analysis of an optimal controller was presented. The control design was constrained by the lack of accurate dynamic model knowledge, thus a robust approach was aimed. After the open-loop dynamics of the output tracking error was obtained, the uncertain terms were temporarily considered to be available, which is followed by the design of the optimal part of the controller. Next, an estimator term was introduced to estimate the uncertainties which were considered as available and known. Stability analysis ensured both global asymptotic stability and the asymptotic convergence of the proposed controller to the optimal one that was designed under the assumption of accurate knowledge of system dynamics. Numerical simulations were conducted that demonstrate the efficacy of the proposed robust optimal controller where ro-

bustness to variation of the initial states and comparison with the robust version of the proposed controller were also shown.

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