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Trajectory tracking control of a mobile manipulator with an external force compensation

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Abstract. This paper considers the problem of the accurate task space finite-time control susceptible to both undesirable disturbance forces exerted on the end-effector and unknown friction forces coming from joints directly driven by the actuators as well as unstructured forces resulting from the kinematic singularities appearing on the mechanism trajectory. We obtain a class of estimated extended transposed Jacobian controllers which seem to successfully counteract the external disturbance forces on the basis of a suitably defined task-space non-singular terminal sliding manifold (TSM) and the Lyapunov stability theory. Moreover, in order to overcome (or to minimise) the undesirable chattering effects, the proposed robust control law involves the second-order sliding technique. The numerical simulations (closely related to an experiment) ran for a mobile manipulator consisting of a non-holononic platform of (2; 0) type and a holonomic manipulator of two revolute kinematic pairs show the performance of the proposed controllers and make a comparison with other well-known control schemes.

Key words: non-holonomic mobile manipulator; unstructured external forces; trajectory tracking; robust task space control; Lyapunov stability.

1. INTRODUCTION

Mobile manipulators are mechanical systems consisted of a holonomic manipulator (a mechanism with integrable differential kinematic constraints) rigidly mounted on a non-holonomic mobile platform. The working area of the platform is, in fact, not bounded. The holonomic manipulator allows the end-effector to perform the various high-tolerance assembly tasks such as precise welding or cutting, peg-in-hole assembly, material handling, etc. Nevertheless, accomplishing the tasks, which require extremely high accuracy, still represents a great challenge to mobile manipulators due to various uncertainties of payloads (e.g. parts to be assembled, end-effector tools, etc.), which are held and transferred by the end-effector along a desired trajectory mostly expressed in task (Cartesian) coordinates. In practice, for example, either known or unknown payloads (by their geometry and mass) cause structural and/or parametric uncertainties in both kinematic and dynamic equations of the mobile manipulator. Thus, transferred payloads generate undesirable external forces exerted on the end-effector, which may result in serious tracking errors. Similarly, the obstacles coming from the unstructured environment and generating artificial repulsive forces, which affect the end-effector, may lead to significant degradation of the trajectory tracking. Moreover, a movement of the end-effector on a surface of an obstacle produces undesirable friction caused by a force contact. Another important source of uncertainty is related to undesirable forces resulting from the kinematic singularities appearing on the mechanism trajectory. As is known, in neighbourhoods of singular configurations, both a platform posture and manipulator joint velocities tend to approach high values. As a result, the mobile manipulator has to generate appropriate controls to pass through singular configurations. Consequently, in order to extend the potential applications of the mobile manipulators to tasks requiring the high precision of accomplishment, it becomes necessary to control the end-effector position properly by taking into account all the aforementioned interaction forces coming from the environment. Although in most situations seen in practice, the end-effector tracking tasks are expressed in Cartesian coordinates, a vast majority of commercially available controllers are designed in the generalized (joint) coordinates. Hence, inverse or pseudo-inverse kinematics algorithms are first required to apply the aforementioned controllers. The process of kinematic inversion turns out to be time-consuming and highly complicated as the Cartesian trajectory produces kinematic and/or algorithmic singularities [1].

In the context of the motion control problem subject to external forces coming from the environment, a few approaches may be distinguished, among which two seem to be the most representative: trajectory tracking and force regulation as well as trajectory tracking and external forces compensation. The vast majority of the motion/force control algorithms is designed in the generalized (joint) coordinates of the mechanisms (see e.g. [2–8]). Unfortunately, controllers from [2–8] are not suitable to track desired trajectories specified in task (Cartesian) coordinates. Several papers have dealt with solving the problem of both trajectory tracking and force compensation in the task space. In studies [9–11], a dynamically consistent and decoupled partitioning of the mobile manipulator dynamics has been constructed within the space ranging from the external (task) one to internal (null). Nevertheless, the control laws proposed in [9–11] require both the complete knowledge of the kinematic

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and dynamic equations of the mechanisms. Moreover, controllers from [9–11] necessitate the generalized pseudo-inverse of the Jacobian matrix as well as external force measurements. In the work [12], a robust acceleration control scheme founded on the disturbance observer designed in joint space has been proposed. However, the approach from [12] needs both force measurement and the complete knowledge of the kinematic equations of the mechanism, and also a pseudo-inverse of the Jacobian matrix. Unfortunately, as is known [13], cyclic kinematic tasks may not be mostly fulfilled by means of the (generalized) pseudo-inverse based approaches. The recent works [14–17] propose adaptive control schemes in order to approximately calculate and counteract an unknown external force that makes an impact on the end-effector tracking a desired trajectory in conditions of uncertain parameters of dynamic equations. The approaches from [14–16] provide both dynamic parameter and adaptive force estimation with no demands for external force measurements. Applying the formulation of the extended task space and the inverse of the corresponding extended Jacobian matrix in [14] as well as the (generalized) pseudo-inverse Jacobian matrix of the stationary robotic manipulator in [15], the problem of mechanism redundancy has been solved in the mentioned works. However, the control laws in [14–17] involve all the adaptive elements multiplied by the regression matrix which appears to be laborious and also very complicated to put into practice. Moreover, control algorithms from [14–17] are not able to tackle singular configurations. Furthermore, the controllers proposed in [14–17] need the complete knowledge of the kinematic equations and work [17] requires additionally a force sensor.

This paper introduces a new class of controllers for mobile manipulators subject to undesirable unknown forces exerted on the end-effector. Due to an unstructured nature of the external disturbance forces, the kinematics and dynamics of the mechanism is assumed herein to be uncertain. A new non-singular terminal sliding manifold (TSM) is brought in to tackle the trajectory tracking control problem subject to the aforementioned unstructured forces. As is known [18,19], a sliding mode is precise and resistant to disturbances and (parametric, structural) uncertainties of mechanism dynamics. Nevertheless, the major disadvantage of the standard first order sliding modes is mainly connected with the undesirable chattering effect [20]. The presented TSM manifold allows the first order sliding mode technique, which is capable of the finite-time control, to join simultaneously with the second order sliding mode approach that generates the (absolutely) continuous controls. Based on the TSM proposed and partly stimulated by dynamically calculated torque techniques (see recent works [21,22]), we present a new robust controller (containing a transposed extended estimated Jacobian matrix), which seems to be efficient at counteracting unknown external forces. Satisfying an appropriate assumption which concerns the Jacobian matrix, the proposed control scheme is proved to be finite-time stable. Due to involving the second order TSM, our control algorithm appears to be more energy-saving than other well-known controllers as computer simulations have shown. It is also worth pointing out that a trajectory tracking controller, which was analysed in our recent work [21], applies a transpose Jacobian matrix. However, it differs significantly from that proposed in this paper. Namely, the controller from [21] requires a complete knowledge of the kinematic equations and fractional knowledge of the mechanism dynamics (e.g. an actuator matrix). Furthermore, the work [21] does not tackle the problem of unknown external forces. The rest of the paper is structured as follows. In Section 2 kinematic and dynamic equations are introduced, and a problem of the finite-time trajectory tracking control for a mobile manipulator susceptible to external disturbance forces is formulated. A class of robust controllers that fulfil the trajectory tracking task is analysed in Section 3. In Section 4 we present the computer examples (closely related to an experiment) of trajectory tracking by a mobile manipulator which is external disturbance forces-prone. The numerical comparison of our controller with other renowned control algorithms is also made in this section. Eventually, some conclusions are reached in Section 5.

2. PROBLEM STATEMENT

2.1. Kinematic and dynamic equations of the mobile manipulator

Let us take a mobile manipulator built from a non-holonomic platform of $(2, 0)$ type. It is expressed by the vector of generalized coordinates $x \in \mathbb{R}^l$, where $l \geq 2$. The vector of joint (generalized) coordinates $y = (y_1, \ldots, y_n)^T \in \mathbb{R}^n$ defines the configuration of holonomic manipulator; n is the number of kinematic pairs. An example of a mobile manipulator is shown in Fig. 1 defining vectors x and y. Particularly, the coordinates of the vector x define the platform posture given by variables $x_{1,c}$, $x_{2,c}$, θ and angles of driving wheels ϕ_1, ϕ_2 – see Fig. 1, where θ is the orientation angle of the platform with respect to the global coordinate system $OX_1X_2X_3$; $x_{1,c}$ and $x_{2,c}$ signify the coordinates of the mass centre of the platform; $2W$ denotes the distance between the platform wheels and $2L$ is its length; symbols y_1, y_2 stand for joint angles of the holonomic manipulator; d means the amount of space between the mass centre and the common drive wheel axis; a stands for the distance between the mass centre and the point where the holonomic manipulator is rigidly attached to the platform; R is the radius of the wheel; F stands for an external force that affects the end-effector.

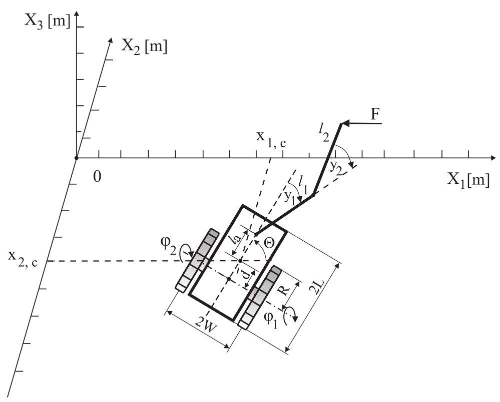

Fig. 1. A kinematic scheme of the mobile manipulator and external disturbance force F exerted on the end-effector

The movement of the mobile platform being susceptible to $1 \le k < l$ non-holonomic constraints takes frequently a Pfaffian form

$$A(x)\dot{x} = 0, \tag{1}$$

where $A(x)$ denotes the $(k \times l)$ full rank matrix (that is $\mathrm{rank}(A(x)) = k$). Non-holonomic constraints for the platform of (2, 0) type from Fig. 1 may be described as

$$A(x)\dot{x} = \begin{bmatrix} \sin(\theta) & -\cos(\theta) & d & 0 & 0 \\ \cos(\theta) & \sin(\theta) & W & -R & 0 \\ \cos(\theta) & \sin(\theta) & -W & 0 & -R \end{bmatrix} \dot{x} = 0, \tag{2}$$

where $x = (x_{1,c}, x_{2,c}, \theta, \phi_1, \phi_2)^T$; $l = 5$; $k = 3$. We assume that $\mathrm{Ker}(A(x))$ is the null space obtained by vector fields $a_1(x), \ldots, a_{l-k}(x)$, respectively. Thus, the differential constraint (1) may be equivalently expressed as a drift-less dynamic system of the form

$$\dot{x} = N(x)\alpha, \tag{3}$$

where $N(x) = [a_1(x), \ldots, a_{l-k}(x)]$; $\mathrm{rank}(N(x)) = l - k$ and $\alpha = (\alpha_1, \ldots, \alpha_{l-k})^T$ signifies the vector of quasi-velocities (presented in work [23]). Let us note that

$$A(x)N(x) = 0. \tag{4}$$

For the mobile platform shown in Fig. 1, equation (3) takes the following form:

$$\dot{x} = \begin{bmatrix} Y(W\cos(\theta) - d\sin(\theta)) & Y(W\cos(\theta) + d\sin(\theta)) \\ Y(W\sin(\theta) + d\cos(\theta)) & Y(W\sin(\theta) - d\cos(\theta)) \\ Y & -Y \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \alpha, \tag{5}$$

where $Y = \dfrac{R}{2W}$, respectively.

The location and orientation of the end-effector with respect to the global coordinate system $OX_1X_2X_3$ is expressed by a kinematic equation

$$p_e = f_e(q), \tag{6}$$

where $p_e \in \mathbb{R}^m$ denotes the coordinates of the end-effector; $q = \begin{pmatrix} cx \\ y \end{pmatrix}$ is the configuration of the mobile manipulator; $f_e\colon \mathbb{R}^l \times \mathbb{R}^n \longrightarrow \mathbb{R}^m$ represents m-dimensional mapping (in general, non-linear with respect to q) and m is the dimension of the task (Cartesian) space. Joining $\dot{q}$, $\ddot{q}$ and (3), one obtains

$$\dot{q} = Cz, \quad \ddot{q} = C\dot{z} + \dot{C}z, \tag{7}$$

where $C = \begin{bmatrix} N(x) & 0 \\ 0 & \mathbb{I}_n \end{bmatrix}$; $z = \begin{pmatrix} \alpha \\ \dot{y} \end{pmatrix} \in \mathbb{R}^{l+n-k}$ is the reduced velocity of a mobile manipulator; $\mathbb{I}_n$ signifies the $(n \times n)$ identity matrix. Note that for $N(x)$ given by eqn (5), z is equal to $z = \begin{pmatrix} \dot{\phi} \\ \dot{y} \end{pmatrix}$. As is known, mobile manipulators become, mostly in practice, redundant mechanisms with respect to tasks to be accomplished, and therefore the following inequality holds true $l + n \ge m + k$. Consequently, there is a possibility for augmenting a vector of the end-effector coordinates, describing classic (conventional) trajectory tracking, by additional auxiliary task coordinates (specified by the user) of the following general form [24]:

$$p_a = f_a(q), \tag{8}$$

where $f_a\colon \mathbb{R}^{l+n} \longrightarrow \mathbb{R}^{l+n-m-k}$ signifies no less than triple-continuously differentiable mapping with respect to q. In practical terms, redundant degrees of freedom of the mechanism may either satisfy additional task requirements (constraints) [24, 25] or optimize performance criteria reflecting the kinematic characteristics of the mobile manipulator [26]. Concatenating $f_e(q)$ with $f_a(q)$, one attains generalized kinematic-differential mappings which relate q with augmented task coordinates $p = \begin{pmatrix} p_e \\ p_a \end{pmatrix}$

$$p = f(q), \qquad \dot{p} = Jz, \tag{9}$$

where $f = \begin{pmatrix} f_e \\ f_a \end{pmatrix}$ and $J = \dfrac{\partial f}{\partial q} C$ is the $(l+n-k) \times (l+n-k)$ extended Jacobian matrix. The dynamics of a mobile manipulator characterised by generalized coordinates q has the form [6]:

$$M'(q)\ddot{q} + P'(q,\, \dot{q})\dot{q} + G'(q) + [A(x)\ 0_{k \times n}]^T \lambda = B'v + E'(t,\, q,\, \dot{q}), \tag{10}$$

where $M'(q)$ represents the $(n+l) \times (n+l)$ positive definite inertia matrix; $P'(q,\, \dot{q})\dot{q}$ denotes the $(n+l)$-dimensional vector that signifies centrifugal and Coriolis forces; $\dot{q}$ stands for the velocity of a mobile manipulator; $G'(q)$ is the $(n+l)$-dimensional vector of generalized gravity forces; $E'(t,\, q,\, \dot{q}) = D'(q)F - H'(\dot{q})$; D' represents the $(l+n) \times m$ matrix which transforms external forces F expressed in task coordinates and exerted on the end-effector into corresponding generalized forces acting in mechanism joints; $H'(\dot{q})$ is the $(l+n)$-dimensional vector of friction forces; $0_{k \times n}$ signifies the $k \times n$ zero matrix; λ is the k-dimensional vector of Lagrange multipliers (reaction forces acting on the platform) that are related to non-holonomic constraints (1); $B' = \begin{bmatrix} B'' & 0 \\ 0 & \mathbb{I}_n \end{bmatrix}$; B'' denotes the $l \times (l-k)$ matrix showing those posture variables of the platform that are directly controlled by the actuators (their elements equal 1 for variables which are directly driven by the actuators, and apart from that 0); $\mathbb{I}_n$ stands for the $n \times n$ identity matrix and $\mathbb{R}^{n+l-k} \in v$ denotes the vector of controls (torques/forces). It is worth noting that matrix B' takes a particularly simple form for a vector x representing the generalized coordinates of the platform subjected to non-holonomic constraints (2). Thus,

$$B' = \begin{bmatrix} 0_{k \times (l-k+n)} \\ \mathbb{I}_{l-k+n} \end{bmatrix}, \tag{11}$$

where $0_{k\times(l-k+n)}$ is the $k\times(l-k+n)$ null matrix. The aim of introducing auxiliary velocities α, and consequently the vector of the reduced velocity z is both to eliminate reaction forces λ from dynamic equations (10) and to reduce their dimensionality. Replacing $\dot{q}$ and $\ddot{q}$ from (10) by the reduced velocity z and acceleration $\dot{z}$ (see expressions (7)), we significantly simplify our control problem. Premultiplying left-sided the dynamic equations (10) by C^T, taking into account (11) and then using equality $C^T\left[A(x)\ 0_{k\times n}\right]^T = 0$ (see expression (4)), we obtain dynamic equations of the mobile manipulator in such a reduced statement that is convenient for our control purposes:

$$M(q)\dot{z} + P(q,\ z)z + G(q) = v + E(t,\ q,\ z), \tag{12}$$

where $M = C^T M' C \in \mathbb{R}^{(l+n-k)\times(l+n-k)}$ stands for the inertia matrix which is positive definite; $P = C^T(M'\dot{C} + P'C)$; $G = C^T G'$ and $E = C^T(D'F - H')$, respectively.

2.2. The problem of trajectory tracking control to be solved

The task completed by the mobile manipulator is to track the desired end-effector trajectory $p_d^e(t) \in \mathbb{R}^m$, $t \in [0,\ \infty)$ as well as an auxiliary (indicated by an user) trajectory $p_d^a(t) \in \mathbb{R}^{l+n-m-k}$. Vector functions $p_d^e(\cdot)$ and $p_d^a(\cdot)$ are assumed to be at least triple-continuously differentiable with respect to time. Introducing the task tracking error $e = \begin{pmatrix} e^e \\ e^a \end{pmatrix} = f(q) - p_d(t)$, where $p_d = \begin{pmatrix} p_d^e \\ p_d^a \end{pmatrix}$; $e^e = (e_1^e, \ldots, e_m^e)^T = f_e - p_d^e$; $e^a = (e_1^a, \ldots, e_{l+n-m-k}^a)^T = f_a - p_d^a$, the problem of the finite-time control in the task space may be formally formulated using the below equations:

$$\lim_{t\to T} e(t) = 0, \quad \lim_{t\to T} \dot{e}(t) = 0, \quad \lim_{t\to T} \ddot{e}(t) = 0, \tag{13}$$

where $0 \le T$ denotes finite time of convergence of $f(q)$ to p_d and $e(t) = \dot{e}(t) = \ddot{e}(t) = 0$ for $t \ge T$. Then, we introduce practically reasonable assumptions and summarise useful properties regarding both kinematic and dynamic equations that will be employed while constructing our controller. Without loss of generality of further considerations, matrix $J(\cdot)$ together with its derivatives up to the second order with respect to q are taken to be bounded

$$\|J\|_F, \ \left\|\frac{\partial J}{\partial q}\right\|_F, \ \left\|\frac{\partial^2 J}{\partial q^2}\right\|_F < \infty, \tag{14}$$

where $\|\cdot\|_F$ stands for the Frobenius (Euclidean) matrix norm. The components of dynamic equations (10) fulfil the following inequalities for revolute kinematic pairs of the holonomic manipulator [6]:

$$\begin{gathered} \|C\|_F, \ \left\|\frac{\partial C}{\partial q}\right\|_F, \ \|M'\|_F, \ \left\|\frac{\partial M'}{\partial q}\right\|_F, \ \left\|\frac{\partial P'}{\partial \dot{q}}\right\|_F, \\ \left\|\frac{\partial G'}{\partial q}\right\|_F, \ \left\|\frac{\partial D'}{\partial q}\right\|_F < \infty \end{gathered} \tag{15}$$

and

$$\left\|\frac{\partial P'}{\partial q}\right\|_F \le w\|z\|, \tag{16}$$

where w denotes a positive coefficient. The generalized external force vector E in (12) together with its time derivative $\dot{E}$ are assumed to be locally bounded Lebesgue measurable mappings. Without loss of generality, E and $\dot{E}$ are upper estimated as follows

$$\|E\| \le \beta^0(t,q,z), \qquad \|\dot{E}\| \le \beta^1(t,q,z), \tag{17}$$

where $\beta^0(\cdot)$, $\beta^1(\cdot)$ denote the time-dependent non-negative locally bounded (and not necessarily globally bounded) Lebesgue measurable functions.

3. CONTROL OF THE MOBILE MANIPULATOR IN THE AUGMENTED TASK SPACE

Before we present the control laws that solve the kinematic task (13), some useful concepts are first introduced. Let $\hat{J} = \hat{J}(q)$ denote an estimate of the uncertain extended Jacobian matrix $J(q)$. In further considerations, $\hat{J}$ is taken to satisfy the following inequalities:

$$0 < A \le \Lambda_{\min}^l \Lambda_{\min}(\hat{J}\hat{J}^T), \tag{18}$$

and

$$0 \le \left\|J - \hat{J}\right\|_F \le \frac{\rho}{\Lambda_{\max}^u}, \tag{19}$$

where $\Lambda_{\min}(M^{-1}) \ge \Lambda_{\min}^l > 0$ denotes a lower estimate of $\Lambda_{\min}(M^{-1})$; $\rho = \begin{cases} \rho' & \text{for } \|\hat{J}\|_F \le 1, \\ \rho'/\left\|\hat{J}\right\|_F & \text{otherwise;} \end{cases}$; $\Lambda_{\max}(M^{-1}) \le \Lambda_{\max}^u$ stands for un upper estimate of $\Lambda_{\max}(M^{-1})$; $\rho' \in [0,\ A)$ is the accuracy of the estimation. It is noteworthy that inequality (19) may actually be easily satisfied by choosing a sufficiently accurate device to measure kinematic parameters of a mobile manipulator, for example link lengths, radius of the wheels, joint offsets, etc. Although inequality (19) requires sufficiently accurate estimate $\hat{J}$ of the Jacobian matrix $J(q)$, the advantage of fulfilment of (19) by $\hat{J}$ is the elimination of the parameter adaptation process. If inequality (19) does not hold true, then $\hat{J}$ has to be computed based on the adaptation of kinematic and dynamic parameters of the mobile manipulator. Let us also observe that inequality (18) means non-singularity of the estimated Jacobian matrix $\hat{J}(q)$ in an operation region of the end-effector. Relations (18)–(19) are only needed in the proof of the finite-time stability of the controller to be designed. However, in what follows, we shall allow also (isolated) singular configurations appearing while trajectory tracking, which certainly do not fulfil inequality condition (18).

3.1. The dynamic controller of the mobile manipulator

In order to obtain absolutely continuous control v solving the robotic task (13), we first differentiate dynamic equations (12) with respect to time, and then determine $\ddot{z}$ (let us observe that $M(q)$ is non-singular)

$$\ddot{z} = M^{-1}\dot{v} - \mathscr{R}, \tag{20}$$

where $\mathscr{R} = M^{-1}[\dot{M}\dot{z} - \frac{d}{dt}(Pz+G) - \dot{E}]$.

In order to reduce task errors e, $\dot{e}$ and $\ddot{e}$ in finite time to zero subject to generalized external disturbance forces E, we present a new dynamic control law of the form

$$\dot{v} = \hat{J}^T u, \tag{21}$$

where $u \in \mathbb{R}^{l+n-k}$ denotes a new control to be defined. The aim is to obtain input signal $u(t)$ and, in consequence, control $v(t)$ in order that the extended location vector p from (9) follows p_d. To do so, we triply differentiate e with respect to time. Hence,

$$\dddot{e} = J\ddot{z} + 2\dot{J}\dot{z} + \ddot{J}z - \dddot{p}_d\,. \tag{22}$$

Having replaced $\ddot{z}$ from (22) by the right-hand sides of (20) and (21), we would obtain an explicit form of the task jerk error $\dddot{e}$ from u

$$\dddot{e} = JM^{-1}\hat{J}^T u + \mathscr{Q} - \dddot{p}_d\,, \tag{23}$$

where $\mathscr{Q} = -J\mathscr{R} + 2\dot{J}M^{-1}(v + E - Pz - G) + \ddot{J}z$. In the sequel, an upper estimate of $\|\mathscr{Q}\|$ will be needed. Using inequalities (14)–(17) and performing simple but time-consuming algebraic computations, we find upper estimate for $\|\mathscr{Q}\|$ of the form given below

$$\|\mathscr{Q}\| \leq \mathscr{W}(t,q,z), \tag{24}$$

where $\mathscr{W} = w_1\|z\| + w_2\|z\|^3 + w_3\|z\|\beta^0 + w_4\beta^1 + w_5\|z\|\|v\|$; $w_1,\ldots,w_5$ are positive constants (estimates of construction parameters of the mobile manipulator). Consider $s = (s_1,\ldots,s_{l+n-k})^T \in \mathbb{R}^{l+n-k}$ to be a task space sliding vector variable. Aiming to find steering signal u, we propose the following new non-singular terminal sliding manifold:

$$\mathscr{M} = \{(\ddot{e},\, \ddot{e}(0),\, \dot{e},\, e,\, s) : S(\ddot{e},\, \ddot{e}(0),\, \dot{e},\, e,\, s) = 0\}, \tag{25}$$

where $S = \ddot{e} - \ddot{e}(0) + \int\limits_0^t \left(\lambda_2\ddot{e}^{3/5} + \lambda_2\lambda_1^{3/5}(\dot{e}^{9/7} + \lambda_0^{9/7}e)^{1/3}\right) \mathrm{d}\tau - s$; $\lambda_i = \mathrm{diag}(\lambda_{i,1},\ldots,\lambda_{i,l+n-k})$; $\lambda_{i,j}$ represent positive coefficients (controller gains); $i = 0:2$; $j = 1:l+n-k$. The potency of e, $\dot{e}$, $\ddot{e}$ as well as λ_0, λ_1 is determined component-wise. It is worth noticing that unlike the terminal sliding manifolds (TSM) defined in our earlier works [21, 22], relation (25) has a great and practical property for the initial time moment $t = 0$ of control, i.e. $s(0) = 0$ (TSM $s = 0$ is attained at $t = 0$). Before we formulate our main theorems, let us recall the definition of the stable convergence in finite time [21].

Definition 1. The origin $(e,\, \dot{e},\, \ddot{e}) = (0,\, 0,\, 0)$ is said to be stably convergent in finite time if it is Lyapunov stable, and the solution of differential equations (23), (27) and (12), which starts from $(e(0),\, \dot{e}(0),\, \ddot{e}(0))$ at time $t = 0$, attains the origin in finite time $T(e(0),\, \dot{e}(0),\, \ddot{e}(0)) < \infty$, i.e. $\lim_{t\to T}(e(t),\, \dot{e}(t),\, \ddot{e}(t)) = (0,\, 0,\, 0)$ and $\lim_{(e,\, \dot{e},\, \ddot{e})\to(0,\, 0,\, 0)} T = 0$.

In the sequel, we present a useful lemma [27].

Lemma 1. If $s(t) = 0$ for $t \geq T'$, where $0 \leq T' \leq T$, then task tracking errors $(e,\, \dot{e},\, \ddot{e})$ stably converge in finite-time to the origin $(e,\, \dot{e},\, \ddot{e}) = (0,\, 0,\, 0)$.

Aiming to compute u, and consequently to satisfy equality constraints (13), the following control law is proposed:

$$u(t,\, q,\, z,\, s,\, \ddot{e}) = \begin{cases} -\dfrac{c}{A}\dfrac{s}{\|s\|}(\mathscr{Y} + c_0) & \text{for } s \neq 0, \\ 0 & \text{otherwise}, \end{cases} \tag{26}$$

where $\mathscr{Y} = \|\lambda_2\ddot{e}^{3/5} + \lambda_2\lambda_1^{3/5}(\dot{e}^{9/7} + \lambda_0^{9/7}e)^{1/3} - \dddot{p}_d\| + \mathscr{W}$; c, c_0 stand for controller gains that will be considered further on. In accordance with (21) and (26), we can determine v (in the Filippov sense [28]) by solving the following differential equation:

$$\dot{v} = \hat{J}^T u(t,\, q,\, z,\, s,\, \ddot{e}). \tag{27}$$

Our objective is to impose such conditions on controller gains λ_0, λ_1, λ_2, c and c_0 that ensure equalities (13) to be fulfilled. Making use of the Lyapunov stability theory, we provide the consequent result.

Theorem 1. If matrix $\hat{J}$ satisfied inequalities (18)–(19), λ_0, λ_1, λ_2, $c_0 > 0$, $c = \frac{c'}{1-\rho'/A}$, $c' > 1$ and $q, z, \dot{e}, \ddot{e}$ are available, then control scheme (26)–(27) results in finite-time stable convergence of task errors $(e, \dot{e}, \ddot{e})$ to the origin $(e, \dot{e}, \ddot{e}) = (0, 0, 0)$.

Proof. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\langle s, s\rangle. \tag{28}$$

Let us differentiate (28) with respect to time and take equality (25) into consideration, and as a result one gets

$$\dot{V} = \langle s, \dddot{e} + \lambda_2\ddot{e}^{3/5} + \lambda_2\lambda_1^{3/5}(\dot{e}^{9/7} + \lambda_0^{9/7}e)^{1/3}\rangle. \tag{29}$$

Based on (21) and (23), one obtains

$$\begin{aligned} \dot{V} = {} & \langle s, JM^{-1}\hat{J}^T u\rangle + \\ & \langle s, \mathscr{Q} + \lambda_2\ddot{e}^{3/5} + \lambda_2\lambda_1^{3/5}(\dot{e}^{9/7} + \lambda_0^{9/7}e)^{1/3} - \dddot{p}_d\rangle. \end{aligned} \tag{30}$$

Let us reformulate expression (30) in an equivalent and more convenient form for a further analysis

$$\begin{aligned} \dot{V} = {} & \langle s, \hat{J}M^{-1}\hat{J}^T u\rangle + \langle s, (J - \hat{J})M^{-1}\hat{J}^T u\rangle \\ & + \langle s, \mathscr{Q} + \lambda_2\ddot{e}^{3/5} + \lambda_2\lambda_1^{3/5}(\dot{e}^{9/7} + \lambda_0^{9/7}e)^{1/3} - \dddot{p}_d\rangle. \end{aligned} \tag{31}$$

Let us upper estimate the sum of the first two terms of (31). To do so, we insert the right-hand side of (26) into (31), thus obtaining

$$\begin{aligned} \dot{V} = {} & -\langle s, \hat{J}M^{-1}\hat{J}^T \frac{c}{A}\frac{s}{\|s\|}(\mathscr{Y} + c_0)\rangle - \\ & \langle s, (J - \hat{J})M^{-1}\hat{J}^T \frac{c}{A}\frac{s}{\|s\|}(\mathscr{Y} + c_0)\rangle \\ & + \langle s, \mathscr{Q} + \lambda_2\ddot{e}^{3/5} + \lambda_2\lambda_1^{3/5}(\dot{e}^{9/7} + \lambda_0^{9/7}e)^{1/3} - \dddot{p}_d\rangle. \end{aligned} \tag{32}$$

On account of inequalities (23)–(24), we get

$$\dot{V} \le -\|s\|c(\mathscr{Y}+c_0)+\|s\|\frac{\rho' c}{A}(\mathscr{Y}+c_0)+ \langle s, \mathscr{Q}+\lambda_2\ddot{e}^{3/5}+\lambda_2\lambda_1^{3/5}(\dot{e}^{9/7}+\lambda_0^{9/7}e)^{1/3}-\dddot{p}_d\rangle. \tag{33}$$

Next, we estimate the last term of $\dot{V}$ in (31). Based on definitions of $\mathscr{Q}$, $\mathscr{Y}$ and inequality (24), we attain

$$\dot{V} \le -\|s\|c(\mathscr{Y}+c_0)+\|s\|\frac{\rho' c}{A}(\mathscr{Y}+c_0)+\|s\|\mathscr{Y} \le -\|s\|c(\mathscr{Y}+c_0)+\|s\|\frac{\rho' c}{A}(\mathscr{Y}+c_0)+\|s\|(\mathscr{Y}+c_0). \tag{34}$$

Therefore in accordance with the assumptions $c=\dfrac{c'}{1-\rho'/A}$, $c'>1$ and $c_0>0$ from Theorem 1, one easily has

$$\dot{V} \le -\|s\|c_0\left(c-\frac{\rho' c}{A}-1\right)-\|s\|\mathscr{Y}\left(c-\frac{\rho' c}{A}-1\right) = -\|s\|c_0(c'-1)-\|s\|\mathscr{Y}(c'-1) \le -\|s\|c_0(c'-1). \tag{35}$$

Since $c_0(c'-1)>0$, inequality (35) proves that $s=0$ is stably achievable in finite time less or equal to $\dfrac{\sqrt{2V(0)}}{(c'-1)c_0}$, i.e. $s(t)=0$ for $t\ge T'$. From (25), it follows that $\dot{s}(t)=0$ for $t\ge T'$, i.e. $\dddot{e}+\lambda_2\ddot{e}^{3/5}+\lambda_2\lambda_1^{3/5}(\dot{e}^{9/7}+\lambda_0^{9/7}e)^{1/3}=0$. Consequently, from Lemma 1, it follows that the origin $(e,\ \dot{e},\ \ddot{e})=(0,\ 0,\ 0)$ being the solution of the above non-linear differential equations of the third order with respect to the constraint equations $e=f(q)-p_d(t)$ is stable and achievable in finite time T. □

Several remarks may be passed concerning the control law (26)–(27) and Theorem 1.

Remark 1. First, let us observe that controller (26)–(27) does not need force/torque sensors to detect external forces nor the environment model. Nevertheless, most of the works [29, 30] use these sensors to determine the values of external forces. As is known, force/torque sensors are expensive, they introduce measurement noise and are complicated to implement. Second, the adaptive estimation of both the unexpected external forces acting on the end-effector and parameters of dynamic equations is not also required by our controller. On the other hand, recent studies [14–16] estimate unexpected external forces as well as involve the adaptive terms multiplied by the regression matrix of the mobile manipulator dynamics what appears to be complex to implement and also laborious. The construction of regression matrix is not an unambiguous process nor a trivial task in practice.

Remark 2. In the particular case $\rho'=0$, the extended Jacobian matrix $\hat{J}$ fulfils equality $\hat{J}=J$ (the kinematic equations are fully known). Let $J(\cdot)$ be singular at (isolated) configuration q' and $0\ne s\notin \ker(J^T(q'))$. In such a case, equality (31) from the proof of Theorem 1 takes the follow- ing simplified form: $\dot{V}=\langle J^Ts, M^{-1}J^Tu\rangle+\langle s, \mathscr{Q}+\lambda_2\ddot{e}^{3/5}+\lambda_2\lambda_1^{3/5}(\dot{e}^{9/7}+\lambda_0^{9/7}e)^{1/3}-\dddot{p}_d\rangle$. For control law

$$u(t,\ q,\ z,\ s,\ \ddot{e}) = \begin{cases} -c_s'\dfrac{s}{\|s\|}(\mathscr{Y}+c_0) & \text{for } s\ne 0,\\ 0 & \text{otherwise,}\end{cases} \tag{36}$$

and

$$\dot{v}=J^Tu(t,\ q,\ z,\ s,\ \ddot{e}), \tag{37}$$

where $c_s'>1$ is a gain coefficient, we have $\dot{V}\le -\|J^Ts\|^2\Lambda_{\min}^l\dfrac{\mathscr{Y}+c_0}{\|s\|}c_s'+\|s\|\mathscr{Y}$. Hence, for sufficiently large c_s', $\dot{V}$ takes negative values. Consequently, control law (36)–(37) enables the mobile manipulator with known kinematics and uncertain dynamics to pass also through singular manifold $\{q':\ \det(J(q'))=0\}$ what will depict the computer simulations conducted in the next section. It is also noteworthy that control algorithms offered in the literature (see e.g. works [9–17]) cannot tackle the singular configurations.

Remark 3. Let us observe that expressions (26)–(27) or (36)–(37) present a transpose Jacobian controller. In this case, the application of the transpose of the Jacobian matrix to robotic manipulators in [31–33] is a well-known technique. However, works [31–33] present stability analysis for the set-point control problems. By contrast, Theorem 1 brings stability analysis for trajectory tracking of the non-holonomic mechanisms whose both kinematic and dynamic equations are uncertain as well as disturbances acting on the mobile manipulators are unknown. It is worth mentioning here that the authors of works [34, 35] have also presented a finite-time convergence of their controller by applying, however, the inverse of the Jacobian matrix and higher order sliding variables.

3.2. The state estimation problem

Let us observe that controllers (26)–(27) or (36)–(37) require the knowledge of task error e, task velocity error $\dot{e}$, task acceleration error $\ddot{e}$ and manipulator velocity z, respectively in order to produce appropriate steering signals. Generally, real mobile manipulators are provided with joint and wheel encoders, laser scanners and gyroscope sensors, what makes it possible to measure joint angles $y=y(t)$ and velocities $\dot{y}=\dot{y}(t)$, wheel displacements $\phi=\phi(t)$ and velocities $\dot{\phi}=\dot{\phi}(t)$, task error $e=e(t)$ and task error velocity $\dot{e}=\dot{e}(t)$, respectively. In the literature, there are a lot of approaches to reconstruct quantity $\ddot{e}$ (see for instance the work [36] where various kinds of state observers were discussed). Almost all of the observers known from the literature have to fulfil the so-called separation principle [37]. Due to discontinuity of control laws (26)–(27), (36)–(37), our controllers do not satisfy a separation principle. Consequently, a technique basing on the theoretical results presented in [38, 39] and also successfully adapted for non-holonomic mechanisms without taking into account external disturbance forces (see work [21]) will be now proposed for reconstruction of $\ddot{e}$. Let us note that the observer from [21] requires the knowledge of kinematic parameters of the mobile platform, and additionally the reconstruction of $\dot{z}$. On account of the fact that $e = e(t)$, $\dot{e} = \dot{e}(t)$ and $z = z(t) = (\dot{\phi}^T\ \dot{y}^T)^T$ are measurable, one may exactly retrace $\ddot{e}(t)$ (by disregarding a measurement noise of the device) after a transient process in finite-time, say T_e'. In our case, the first-order uniform robust exact differentiator (which is a model-free observer) can be expressed as:

$$\begin{aligned}\dot{\xi}_0 &= \xi_1 - \hat{\lambda}_1^e L_e(t)^{1/2}|\xi_0 - \dot{e}(t)|^{1/2}\mathrm{sign}(\xi_0 - \dot{e}(t)),\\ \dot{\xi}_1 &= -\hat{\lambda}_0^e L_e(t)\mathrm{sign}(\xi_0 - \dot{e}(t)),\end{aligned} \tag{38}$$

where $\hat{\lambda}_0^e$, $\hat{\lambda}_1^e$, stand for positive constants; ξ_1 signifies the output of differentiator (38) exactly reconstructing task error acceleration $\ddot{e}(t)$, i.e., $\ddot{e}(t) = \xi_1(t)$ for $t \geq T_e'$; ξ_0 represents the estimate of $\dot{e}$, and $L_e(t)$ denotes a positive continuous function which takes the form $L_e(t) = \|\bar{J}^T\|_F^2 \Lambda_{\max}^u \frac{c}{A}(\mathscr{Y} + c_0) + \mathscr{W} + \|\dddot{p}_d\|$. $L_e(t)$ is a physically upper estimate of the norm of $\dddot{e}$ (task jerk error). By substituting $\ddot{e}$ in (26)–(27) and (36)–(37) for its corresponding estimate ξ_1 from (38), one gets the following absolutely continuous trajectory tracking controller for the case of uncertain kinematics and dynamics:

$$\begin{aligned}u(t,\ q,\ z,\ s,\ \xi_1) &= \begin{cases} -\dfrac{c}{A}\dfrac{s}{\|s\|}(\mathscr{Y} + c_0) & \text{for } s \neq 0,\\ 0 & \text{otherwise,}\end{cases}\\ v &= \bar{J}^T u(t,\ q,\ z,\ s,\ \xi_1),\end{aligned} \tag{39}$$

as well as for only uncertain dynamics

$$\begin{aligned}u(t,\ q,\ z,\ s,\ \xi_1) &= \begin{cases} -c_s'\dfrac{s}{\|s\|}(\mathscr{Y} + c_0) & \text{for } s \neq 0,\\ 0 & \text{otherwise,}\end{cases}\\ v &= J^T u(t,\ q,\ z,\ s,\ \xi_1),\end{aligned} \tag{40}$$

which require measurements of $q(t)$, $z(t)$, $e(t)$ and task velocity errors $\dot{e}(t)$, respectively. Based on (39) and (40), we are now in position to give the following theorem.

Theorem 2. If q, e, $\dot{e}$ and z are obtainable from measurements and the assumptions of Theorem 1 are satisfied, then control scheme (39) ensures stable convergence in finite time of task errors $(e,\ \dot{e},\ \ddot{e})$ to the origin $(e,\ \dot{e},\ \ddot{e}) = (0,\ 0,\ 0)$.

Proof. The proof of finite-time stability of control law (39) closely resembles that presented in the proof of Theorem 1. Therefore it is omitted. □

Certainly, if controller (40) generates isolated singular configurations while trajectory tracking, then (40) also guarantees the stable finite-time convergence of $(e,\ \dot{e},\ \ddot{e})$ to the origin.

Generally, if measured task velocity error $\dot{e} = \dot{e}(t)$, attained from encoders, is also adulterated by a measurement noise, i.e., $\dot{e}(t) = \dot{e}_0(t) + \eta_e(t)$, where $\|\eta_e\| \leq \rho L_e(t)$; ρ signifies a normalized noise magnitude (practically $\rho \in [10^{-5},\ 10^{-2}]$); $\dot{e}_0(t)$ represents unknown true (noise-free) task velocity error, then the observer (38) needs to be applied to estimate quantity $\ddot{e}$ as well. Note from equations (38), that $\|\dot{e}_0(t) - \xi_0(t)\| \leq L_e(t)O(\rho)$; $\|\ddot{e}_0(t) - \xi_1(t)\| \leq L_e(t)O(\rho^{1/2})$ after finite transient time T_e' [38]. Consequently, for task position, velocity and acceleration error norm estimates $\|\hat{e}\|$, $\|\hat{\dot{e}}\| = \|\xi_0\|$, $\|\hat{\ddot{e}}\| = \|\xi_1\|$, respectively, we obtain after simple calculations the following upper estimations for controllers (39) and (40) subject to measurement noise of device: $\|\hat{e}\| \leq O(\rho)\int_0^t L_e(\tau)\,\mathrm{d}\tau$, $\|\hat{\dot{e}}\| \leq O(\rho)L_e(t)$ as well as $\|\hat{\ddot{e}}\| \leq O(\rho^{1/2})L_e(t)$.

4. NUMERICAL EXAMPLES

The performance of the controllers given by expressions (39) and (40) on the two chosen mobile manipulator tasks is presented in this section. Furthermore, numerical comparisons of our control laws to other renowned control algorithms are made as well. To achieve that objective, we utilise a mobile manipulator operating in the two-dimensional work space ($m = 2$). Figure 1 depicts its kinematic scheme with $k = 3$ and $l = 5$ as well as $n = 2$, respectively. Let us note that $l + n = 7 > m + k = 5$. Consequently, the mobile manipulator from Fig. 1 develops into redundant mechanism with $l + n - k - m = 2$ redundant degrees of freedom and $l + n - k = 4$ actuators. All numerical simulations apply the SI units. The nominal values of both kinematic and dynamic parameters of the mobile manipulator have been taken from work [14]. The nominal values of link lengths of the holonomic manipulator are equal to $l_1 = 0.514$, $l_2 = 0.362$. The wheel radius R equals $R = 0.0508$. The distance between the platform wheels $2W$ is equal to $2W = 0.364$ and the distance d between the mass centre and common drive wheels axle equals $d = 0.116$ as well as $a = 0.1$ respectively. The dynamic parameters take the following nominal values: wheel mass $m_w = 0.159$; mobile platform mass $m = 17.25$; masses of the first and second links of the holonomic manipulator $m_1 = 2.56$ and $m_2 = 1.07$, respectively; wheel inertia with respect to its axis $I_w = 0.0002$; wheel inertia I_z with respect to global axis OX_3 equals $I_z = 0.0001$; platform inertia with respect to its centre of mass $I_p = 0.297$; links inertia with respect to their centre of masses $I_1 = 0.148$ and $I_2 = 0.0228$, respectively. In all the simulations, matrix D' from (10) takes the form $D' = \left(\dfrac{\partial f_e}{\partial q}\right)^T$. Hence, generalized force $D'F$ in (10) equals

$$E_e' = \left(\frac{\partial f_e}{\partial q}\right)^T F, \tag{41}$$

where $F \in \mathbb{R}^2$ denotes external force vector (imitating the action of e.g. a payload) exerted on the end-effector. The friction forces H' in (10) assumed in computations equal

$$H' = \left(0\ 0\ 0\ \gamma_1\dot{\phi}_1\ \gamma_2\dot{\phi}_2\ \gamma_3\dot{y}_1\ \gamma_4\dot{y}_2\right)^T, \tag{42}$$

with $\gamma_i = 0.05$; $i = 1,\ldots,4$. Consequently, dynamic equations (12) are simplified as follows

$$M(q)\dot{z} + Pz = v + E, \tag{43}$$

where $E = C^T \left(\left(\frac{\partial f_e}{\partial q} \right)^T F - H' \right)$. The estimates for controller (39) are chosen as $A = 0.5$, $\Lambda^u_{\max} = 2.4$. Rough conservative estimates of w_i, $i = 1:5$ have been made to simplify numerical calculations. Therefore, positive constant coefficients w_i were chosen as follows $w_1 = 134$; $w_2 = 3$; $w_3 = 134$; $w_4 = 0$ and $w_5 = 2$, respectively. The initial configuration $q(0)$ and reduced velocity $z(0)$ are assumed to be equal to $q(0) = (0\ 0\ \pi/2\ 0\ 0\ -\pi/4\ -\pi/2)^T$, $z(0) = (0\ 0\ 0\ 0)^T$, respectively. Intending to accelerate the convergence process of differentiator (38), we have decided on an appropriate initial guess $\xi_1(0)$ in the numerical examples (which implies relation $T'_e \sim 0$) on the basis of the nominal values of the kinematic and dynamic parameters. Hence, model-free observer (38) was run with the following initial values $\xi_0(0) = (0.2\ 0.157\ 0.2\ 0)^T$; $\xi_1(0) = (60.04\ 46.7\ -0.27\ -0.13)^T$ and parameters $\hat{\lambda}^e_0 = 1.1$, $\hat{\lambda}^e_1 = 1.4$.

The aim is to numerically compare controller (39) with the following two representative algorithms: adaptive control given in [14] and robust control law proposed in [40]. Control algorithms from [14] and [40] utilize the inverse of the extended fully known Jacobian matrix. In order to compare the performance of controller (39) and the one presented in [14], we transform dynamic equations (43) into a linearly parametrizable form as follows (see [14] for the details of the parametrized dynamic equations)

$$M_p \ddot{p} + P_p \dot{p} = Y_p(q, \dot{q}, \dot{p}, \ddot{p})\Psi = B_p v + E_p F, \tag{44}$$

where $M_p \in \mathbb{R}^{4\times4}$, $P_p \in \mathbb{R}^{4\times4}$, $B_p \in \mathbb{R}^{4\times4}$, $E_p = J^{-1}E$ denote reduced components of dynamic model expressed in task coordinates p; $Y_p(q, \dot{q}, \dot{p}, \ddot{p}) \in \mathbb{R}^{4\times8}$ stands for the regression matrix and $\Psi \in \mathbb{R}^8$ is the vector of (unknown) dynamic parameters of the mobile manipulator (see also [14] for the details of the reduced components of dynamic equations (44) of the mobile manipulator from Fig. 1). Vector Ψ is equal to $\Psi = (m_p, m_w, m_1, m_2, I_p, I_w, I_1, I_2)^T$ [14]. Based on (44), the adaptive control law presented in [14] is expressed as follows:

$$\begin{aligned} v &= B_p^{-1}(Y(q, \dot{q}, \dot{X}_r, \ddot{X}_r)\hat{\Psi} + Ks - E_p \hat{F}), \\ \dot{\hat{F}} &= -K_f^{-T} E_p^T s, \\ \dot{\hat{\Psi}} &= K_u^{-T} Y^T s, \end{aligned} \tag{45}$$

where $\dot{X}_r = \dot{p}_d - \mathscr{L}e$; $s = -(\dot{e} + \mathscr{L}e)$ is the 4-dimensional sliding vector variable; K, K_f, K_u denote constant gain diagonal matrices; $\hat{F}$, $\hat{\Psi}$ mean the adaptive estimates of the unknown dynamic parameters Ψ and external forces F whose initial values equal zeros. The best numerical values for K, K_f, K_u and $\mathscr{L}$ have been chosen by trials and errors. They are equal to $K = 150\mathbb{I}_4$, $K_f = 5\cdot10^{-3}\mathbb{I}_2$, $K_u = 1.5\mathbb{I}_4$, $\mathscr{L} = 10\mathbb{I}_4$, respectively.

In order to compare the performance of controller (39) and the robust one presented in work [40], dynamic equations (43) are reformulated to the following partially linearly parametrizable form

$$M\dot{r} + Pr + Y(e, \dot{e}, q, \dot{q})\Psi - E = v, \tag{46}$$

where $r = z - J^{-1}(\dot{p}_d - \alpha e)$ denotes a filtered tracking error signal [40]; $Y = Y(e, \dot{e}, q, \dot{q}) \in \mathbb{R}^{4\times8}$ is a regression matrix; $\Psi \in \mathbb{R}^8$ stands for a vector of unknown dynamic parameters; α is a constant gain coefficient; $Y\Psi = M\frac{d}{dt}\left(J^{-1}(\dot{p}_d - \alpha e)\right) + P\left(J^{-1}(\dot{p}_d - \alpha e)\right)$. Based on dynamic equations (46), the robust control law proposed in [40] and adapted herein to mobile manipulators is expressed as follows:

$$v = Y\hat{\Psi} + Kr - J^T e + v_R, \tag{47}$$

where $v_R = \frac{r\Delta^2}{\|r\|\Delta + \varepsilon}$ stands for an auxiliary robust control component which was designed to improve the controller performance by compensating unknown disturbances E and parameter uncertainties of terms M and P, respectively; K denotes a gain coefficient; $\hat{\Psi}$ signifies the most appropriate guess estimates (of the unknown parameters Ψ). We have chosen good values for Δ, K, α, ε by trials and errors. They are equal to $\Delta = 40$, $K = 5$, $\alpha = 2.5$ and $\varepsilon = 0.01$, respectively. Aiming to demonstrate the role of the feedback amplitude adjustable term $\frac{c}{A}(\mathscr{Y} + c_0)$ of controller (39) in comparison with the corresponding constant term Δ from (47), components M and P of dynamic equations (46) are considered to be fully known, that is, the following equality is satisfied: $\hat{\Psi} = \Psi_n$, where Ψ_n stands for the vector of nominal values of dynamic parameters.

Intending to illustrate the robustness of our control schemes against singular configurations, the mobile manipulator from Fig. 1 is to fulfil two tasks.

The first task, performed by controller (39) is to track the same desired augmented singularity-free trajectory as that given in work [14]. It takes the form

$$p_d(t) = (0.2t + 0.3\ \ 0.5 + 0.25\sin(0.2\pi t)\ \ 0.2t\ \ 0)^T \in \mathbb{R}^4. \tag{48}$$

The estimate $\hat{J}(q)$ of the uncertain Jacobian matrix $J(q)$ is assumed in this simulation to be equal to

$$\hat{J} = J + \mathrm{diag}(0.015\ 0.008\ 0.02\ 0.01). \tag{49}$$

The aim of the first task is to show the ability of our control law in the effective counteracting of unknown (globally) unbounded external forces of a Brownian motion type exerted on the end-effector. Hence, external forces F take the following unstructured form:

$$F = (50 + \eta_1(t),\ 20 + \eta_2(t))^T, \tag{50}$$

where $d\eta_i = 225\sqrt{t}X(t)dt$; $X(t) \sim N(0,\ 1)$; $i = 1, 2$; $t \in [0,\ 15]$. Due to inequality $l + n - k - m = 2 > 0$, we can augment vector

$$p_e = f_e(q) = \begin{pmatrix} x_{1,c} + l_a c\theta + l_1 c\theta 1 + l_2 c\theta 12 \\ x_{2,c} + l_a s\theta + l_1 s\theta 1 + l_2 s\theta 12 \end{pmatrix} \in \mathbb{R}^2 \tag{51}$$

by additional coordinates $p_a \in \mathbb{R}^2$ of the point at which the holonomic manipulator is attached to the platform as follows

$$p_a = f_a(q) = \begin{pmatrix} x_{1,c} + l_a c\theta \\ x_{2,c} + l_a s\theta \end{pmatrix}, \tag{52}$$

where $c\theta = \cos(\theta)$, $s\theta = \sin(\theta)$, $c\theta 1 = \cos(\theta + y_1)$, $s\theta 1 = \sin(\theta + y_1)$, $c\theta 12 = \cos(\theta + y_1 + y_2)$ and $s\theta 12 = \sin(\theta + y_1 + y_2)$. The following numerical values of gain coefficients are taken for our controller: $\lambda_0 = 1$; $\lambda_1 = 11$; $\lambda_2 = 6$; $c_0 = 0.5$; $c' = 1.95$; $\rho' = 0.03$; $\beta^0 = 67$ and $\beta^1 = 0$ to obtain the convergence of task errors e not less than or equal to 10^{-3}. The measured task error velocity $\dot{e}$ has also been adulterate by a measurement noise of a Brownian motion of the form $\|d\eta_e(t)\| \leq 10^{-2}\sqrt{t}X(t)dt$ for $t \in [0,\ 15]$; $X(t) \sim N(0,1)$.

The results of a numerical comparison of control law (39) with both adaptive controller (45) and the robust one (47) are demonstrated in Figs. 2–3. In Fig. 2 can be seen that our controller generates tracking errors e, which are practically for $t \geq 3.5$ equal to zero compared with those provided by adaptive controller (45) and robust control law (47). Moreover, controller (39) provides the energy minimal solution with integral norm $\|\overline{v}\| = \sqrt{\int_0^{15} \langle v, v\rangle \, \mathrm{d}t}$ equal to $\|\overline{v}\| = 86$ whereas control schemes (45), (47) result in significantly greater torque norms equal to $\|\overline{v}\| = 108.6$ for the controller from [14] and $\|\overline{v}\| = 1818$ in the case of robust control law from [40], respectively. Furthermore, in Fig. 3 can be seen that controller (39) still generates absolutely continuous steering torques whereas those provided by control algorithms (45), (47) have taken unacceptably large values at the very beginning of the task accomplishment.

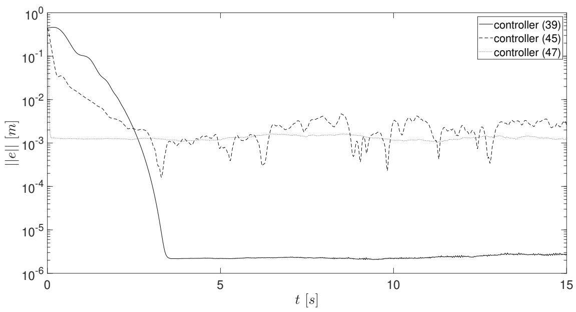

Fig. 2. Task errors $\log(\|e\|)$ for controllers (39), (45) and (47)–singularity-free desired trajectory

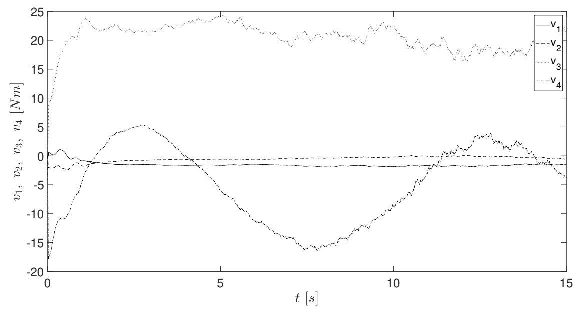

Fig. 3. Torques $v_1,\ v_2,\ v_3,\ v_4$ for controller (39)–singularity-free desired trajectory

The second task is to depict the robustness of our transposed Jacobian control law against singular movements. For this purpose, we assume a full knowledge of the mobile manipulator kinematics to show passing through singular manifold of the mechanism for control law (40). We have also assumed the numerical value of gain coefficient c_s' equal to $c_s' = 10$ in this simulation. All other controller gains are the same as those given in the first experiment. In order to force singular (isolated) configurations, we have utilised controller (40) to track the following extended desired trajectory:

$$p_d^s(t) = (0.2t+0.3\ \ 0.626+0.25\sin(0.2\pi t)\ \ 0.2t+0.2\ \ 0)^T. \quad (53)$$

It is worth mentioning that trajectory (53) produces countless singular configurations related to the set of time instances $\left\{t_k' :\ t_k' = \dfrac{0.5+2k}{0.2},\ k \text{ is integer}\right\}$ whose Lebesgue measure equals zero provided that $e = 0$. The simulation results can be seen in Figs. 4–6 that depict the stable finite-time convergence of the end-effector location p to desired trajectory p_d in the singular movement. Figure 5 presents how the mobile manipulator passes through singular manifold $\{q' :\ \det(J(q')) = 0\}$ in neighbourhoods of time instances $t' \in \{2.5,\ 12.5\}$. Such a passing can be considered as affecting the mobile manipulator by a disturbance signal in small neighbourhoods of $t' \in \{2.5,\ 12.5\}$. Visible variations of $\|e\|$ and $v_1,\ v_2,\ v_3,\ v_4$ (Figs. 4, 6) in small neighbourhoods of time instances $t' \in \{2.5,\ 12.5\}$ are a consequence of violating the assumption (18) in a set of time instances of non-zero Lebesgue measure by the equivalent numerical value of $\Lambda_{\min}(J(q')J^T(q'))$. Choosing a sufficiently large value for c_s' can make these variations utterly small. Let us observe that steering signals shown in Fig. 6 could not be feasible in the real case due to the physical limitations of the actuators. In such a case, smoothing of controls v in neighbourhoods of time instances $\{2.5,\ 12.5\}$ should be carried out.

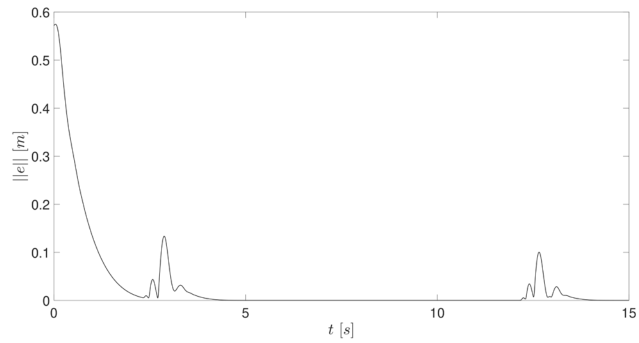

Fig. 4. Task errors $\|e\|$ for controller (40) – singular desired trajectory

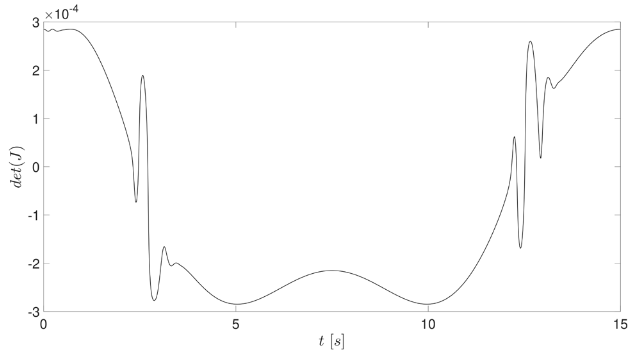

Fig. 5. Determinant of the extended Jacobian matrix $J(q)$ for controller (40) – singular desired trajectory

In order to eliminate an additional phase delay related to recursive low-pass filters which may further limit the performance of our closed-loop system (12), (27), (7), one could apply the Newton predictor enhanced Kalman filter (NPEKF) [41] which provides a wide bandwidth and significantly reduces phase lag.

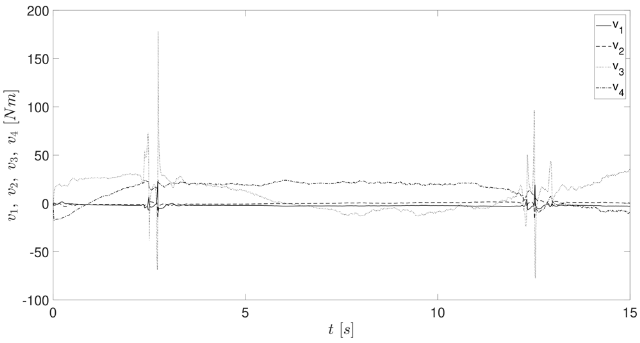

Fig. 6. Torques v_1, v_2, v_3, v_4 for controller (40) – singular desired trajectory

5. CONCLUSIONS

A new class of task space TSM controllers with finite-time stability when tracking a desired end-effector trajectory by the mobile manipulator under conditions of uncertain kinematics and dynamics as well as acting unknown external forces on the end-effector has been proposed in this paper. The main feature of the offered control laws is the elimination of both the process of external force estimation and Jacobian matrix inverse (or pseudo-inverse) from trajectory tracking. Instead, estimated Jacobian transpose matrix has been used. Applying the Lyapunov stability theory, control strategies (39) and (40) are proved to be finite-time stable by fulfilment of practically reasonable assumptions. The numerical simulations have shown that controllers (39) and (40) are superior to a well known control schemes (45), (47) in trajectory tracking accuracy. Although our transposed Jacobian controllers need the knowledge extracted from both the system kinematics and dynamics of the mechanism, the approach is able to handle uncertainties (in kinematics, dynamics and external forces acting on the end-effector) occurring in the multi-body system.

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