Formula for temperature distribution in multi-layer optical fibres for high-power fibre lasers

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Abstract

High power fibre lasers need to be cooled efficiently to avoid their thermal damage. Temperature distribution in fibre should be estimated during the fibre laser design process. The steady-state heat equation in a cylindrical geometry is solved to derive a practical formula for temperature radial distribution in multi-layered optical fibres with arbitrary number of the layers. The heat source is located in one or more cylindrical domains. The validity of the analytical formula is tested by comparison with static heat transfer simulations of typical application examples including octagonal double clad fibre, air-clad fibre, fibre with nonuniform, microstructured core. The accuracy sufficient for practical use is reported even for cases with not exactly cylindrical domains.

Keywords:
High-power fibre lasers, active optical fibres, temperature distribution, heat transfer.

1. Introduction

Thermal management of high-power fibre lasers [1] is an indispensable part of their design. As the pump power is absorbed in the active area of the fibre-doped core, the part of absorbed power is transferred to the heat that increases the fibre temperature. To avoid thermal damage of the fibre and its coating, heat should be efficiently transferred away by a medium surrounding the fibre. The cooling efficiency can be assessed based on the estimation of temperature distribution in the fibre cross-section which, in turn, depends on the intensity of the heat source, i.e., the power absorbed, and on the thermal parameters of the fibre cross-section structure.

Several works studied temperature distribution in fibre lasers from different perspectives. A time domain heat transfer analysis of doped fibres was presented [2]. A formula for the temperature distribution in a two-layer fibre was derived from a steady-state heat equation [3]. The thermo-optical behaviour of air-clad photonic crystal fibre lasers was investigated, as well [4]. Thermal influences on kilowatt Yb-doped double-clad fibre lasers were studied numerically [5]. An analytical approach for the thermal design of high-power lasers based on one dimensional heat equation was also presented [6]. Thermal effects in high-power CW fibre lasers were studied by solving the radial heat equation [7]. Transient heat conduction and thermal effects in a pulse end-pumped fibre laser were analytically solved [8]. Temperature distribution in a double-clad fibre as a function of the heat load was obtained using a finite element method [9]. Heat transfer equation was analytically solved for the case of a high-power fibre laser with a double cladding [10]. Relation for the temperature distribution in a three-layer cylindrical structure of a double-clad fibre was given [11] and an efficient heat transfer technique was discussed [12]. Considering [3] and [11], temperature dependent effects in Tm-doped fibres lasers were analysed [13]. An analytical temperature model based on a thermal conduction equation was reported for double-clad Yb-doped fibre lasers [14]. Thermal characteristics of Tm-doped fibre lasers in double-clad and triple-clad arrangements were also studied based on the analytical model of the radial temperature distribution [15]. An efficient cooling of fibre lasers by means of a metal cladding was presented, as well [16]. A thermal distribution with different heat sources and an analytical solution of the heat transfer equation in double-clad fibres were investigated [17]. An analytical thermal resistance model

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was proposed and applied to calculate heat dissipation in an Yb-doped double-clad fibre [18]. Recently, temperature distribution inside a double-cladding optical fibre laser was examined [19].

In the works cited above, the number of the considered cylindrical layers is limited or there is no explicit formula presented for cases with a higher number of layers. However, in some applications, the number of cylindrical layers forming the fibre and its surrounding is higher than two or three that has been usually assumed in theoretical approaches, so far. Numerical simulation is possible in those cases, but it is not practical for the structure design optimization, or it is not even needed for the initial estimation of the temperature distribution.

In this paper, a simple analytical formula is derived for temperature distribution in cylindrical optical fibre structures with arbitrary number of layers. The validity of the formula is tested by comparison with finite element computations of typical application examples.

2. Analytical model

The analytical formula for the radial distribution of temperature was given in Ref. 3 for a two-region static heat transfer problem in cylindrical geometry. The basic form of the model that was used in further works is as follows:

\[ T_1(r) = T_0 - \frac{Q_1 r^2}{4k_1}, \quad (1) \]

\[ T_2(r) = T_0 - \frac{Q_1 a_1^2}{4k_1} \left( 1 + 2 \ln \frac{r}{a_1} \right), \quad (2) \]

\[ T_0 = T_b + \frac{Q_1 a_1^2}{4k_1} \left( 1 + 2 \ln \frac{a_1}{a_1} \right), \quad (3) \]

where the notation of quantities involved is described below. The generalization of the problem for the case of \( N \) cylindrical layers (see Fig. 1) considered here is defined as:

\[ \nabla^2 T_n = -\frac{Q_n}{k_n} \text{ for layers } n = 1, ..., N \quad (4) \]

with interface boundary conditions:

\[ T_N(a_N) = T_b \quad (5) \]

\[ T_0(a_n) = T_{n+1}(a_n) \quad (6) \]

\[ \frac{dT_n(0)}{dr} = 0 \quad (7) \]

\[ k_n \frac{dT_n(a_n)}{dr} = k_{n+1} \frac{dT_{n+1}(a_n)}{dr} \quad (8) \]

for \( 1 \leq n < N \) where \( r \) [m] is a distance from the centre, \( T_n(r) \) [°C] is the temperature radial profile, \( Q_n \) [W/m²] is the power density of a heat source, \( a_n \) is the outer radius of the \( n \)-th layer, \( k_n \) [W/m°C] is the thermal conductivity, \( T_b \) is the boundary (background) temperature. Here, the subscript \( n \) means that the variable is evaluated in the \( n \)-th layer counted from the centre. The heat power density \( Q_n \) is assumed to be constant within the \( n \)-th layer and is zero in layers without the heat source. Generally, if the line heat power density (or heat load) \( Q_i \) [W/m] is given, the volume density is obtained as \( Q = Q_i A \) where \( A \) is the domain cross-section area of the heat source.

Assuming that all relevant quantities of the problem are isotropic and independent of the longitudinal coordinate \( z \), the left-hand side of Eq. (4) is \( \nabla^2 T = \frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} \). A homogeneous solution of Eq. (4) is obtained by a separation of variables giving \( \frac{d^2 T}{dr^2} = C_n/r \) where \( C_n \) are constants and \( 1 \leq n < N \). An inhomogeneous solution of Eq. (4) is obtained by variation of constants as \( T_n(r) = -\frac{a_n}{4k_n} r^2 + C_n \ln r + T_{0n} \) where \( C_n \) and \( T_{0n} \) are the constants. From the condition in Eq. (7), it follows \( C_1 = 0 \). From the conditions in Eq. (8), the relation for \( C_n \) in terms of \( C_{n-1} \) is determined. Finally, the conditions in Eqs. (5) and (6) are applied consecutively to determine the additive constants \( T_{0n} \). The resulting formula for the radial temperature distribution is:

\[ T_n(r) = T_{01} + K_n(r), \quad 1 \leq n \leq N \quad (9) \]

where:

\[ K_1(r) = \frac{\tau_1}{4} r^2 \quad (10) \]

\[ K_n(r) = \frac{\tau_n}{4} (r^2 - a_{n-1}^2) + C_n \ln \frac{r}{a_{n-1}} \]

\[ + \sum_{i=1}^{n-1} \frac{\tau_i}{4} (a_i^2 - a_{i-1}^2) + \sum_{i=2}^{n-1} C_i \ln \frac{a_i}{a_{i-1}} \quad (11) \]

for \( 1 < n \leq N \), with \( \tau_n = -Q_n/k_n \) and the constants:

\[ T_{01} = T_b - K_1(a_N), \quad (12) \]

\[ C_n = \frac{k_{n-1}}{k_n} C_{n-1} + \frac{a_n}{2} \left( \frac{k_{n-1}}{k_n} r_{n-1} - r_n \right) \quad (13) \]

In Eqs. (11) and (13), the quantities with zero index are assumed to be zero.

The analytical model in Eqs. (9)–(13) can be considered as a straightforward generalization of Eqs. (1)–(3) or of their particular extensions presented in literature, so far.
Nevertheless, it has a great potential for the practical usage, e.g., as a part of fibre laser/amplifier models based on the solution of laser rate equations [20,21], since quite often the higher number of cylindrical domains with different thermal conduction properties are involved in active fibre problems and numerical simulations are computationally too demanding for the purpose. In particular, the outer layers of the fibre, its coating and the surrounding environment can have significant impact on the overall temperature distribution in the cross-section of the fibre structure. This will be also demonstrated in the next section where the validity of the analytical model Eqs. (9)–(13) is checked by means of several application examples.

3. Test problems

3.1. Fibre in circular slot

The first fibre heating test problem is defined as follows. The fibre is modelled as a 5-layer cylinder with the layers 1, 2 composed of silica, the layer 3 of a polymer, the layer 4 of a Sylgard 184 kit, and the layer 5 is made of aluminium. Note that the Sylgard 184 kit is used here only as an example and there are other materials that are more suitable for the applications of high-power fibre lasers due to their higher thermal conductivity. The geometrical and material parameters of a fibre cross-section are summarized in Table 1 where the domain radius $a$ and the thermal conductivity values $k$ are stated. The boundary (or background) temperature is set to $T_b=22$ °C in all test problems.

Table 1. Cross-section geometry parameters, radius $a$ and thermal conductivity $k$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>Material/domain</th>
<th>$a_0$ [µm]</th>
<th>$k_0$ [Wm⁻¹K⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SiO$_2$ core</td>
<td>12.5</td>
<td>1.38</td>
</tr>
<tr>
<td>2</td>
<td>SiO$_2$ cladding</td>
<td>200</td>
<td>1.38</td>
</tr>
<tr>
<td>3</td>
<td>Polymer coating</td>
<td>280</td>
<td>0.18</td>
</tr>
<tr>
<td>4</td>
<td>Sylgard 184</td>
<td>400</td>
<td>0.27</td>
</tr>
<tr>
<td>5</td>
<td>Aluminium</td>
<td>450</td>
<td>238</td>
</tr>
</tbody>
</table>

Figure 2 compares the analytical solution with the numerical one computed by the finite element method (FEM) in COMSOL with the same geometry. Both solutions are identical, but one should note that the temperature distribution evaluation by an analytical model is faster by orders of magnitude than a FEM simulation. This is advantageous especially when a temperature model serves as part of some feedback mechanism inside the fibre laser model (e.g., temperature dependent absorption and emission spectra) where iterations are needed to find a solution [20,21].

It is instructive to see how the temperature at the critical point on the boundary of cladding and coating depends on the heat load. Figure 3 shows that the temperature increases with the heat load with a slope determined by the cladding radius, provided that all other parameters are fixed. Considering a polymer temperature limit of around 80 °C that should not be exceeded [16], a sufficiently large cladding radius is needed to stand a given heat load with no polymer coating damage.

Finally, let us briefly comment on the relation between the heat load and the pump power. Both the pump power evolution along the fibre and the corresponding local heat load can be estimated using numerical models of the active fibre [20,21]. The heat load can be also roughly estimated considering the pump absorption $A_p$ [db/m] that is constant along the fibre (see e.g., [22] for a more realistic model of the pump absorption) and assuming the constant power conversion efficiency $\eta$ [-]. Then the resulting heat load $Q_l$ [W/m] along the fibre as a function of the length coordinate $z$ [m] can be expressed as:

$$Q_l(z) = (1 - \eta)P_0 \langle \alpha_p \rangle a_p \exp(-a_p z), \quad (14)$$

where $a_p = (A_p/10)ln10$ and $P_0$ [W] is the input pump power. For $P_0(0) = 1200$ W, $A_p = 3$ db/m and $\eta = 0.59$, it gives $Q(0) = 340$ W/m at the beginning of the fibre.

3.2. Octagonal cladding fibre

As the second example, an octagonal cladding fibre with a flat-to-flat size $F$ [µm] = 250 µm is selected. Octagonal and other polygonal shapes are typical examples of breaking the circular symmetry of the cross-section of double-clad fibres. Such cross-sections are beneficial for mode scrambling and correspondingly improved pump
absorption [1,22]. Figure 4 shows the cross-section geometry and the spatial distribution of temperature obtained from a numerical simulation. The materials and background temperature are the same as in the previous example but a more realistic scenario with the fibre located inside a square groove is analysed.

![Spatial distribution of temperature in the octagonal cladding fiber for Q_l = 150 W/m.](image)

Figure 5 compares the analytical model (with 5 layers) with a simulation for two values of a heat power density. Here, the distance from the centre is measured along the x-axis. Analytical model slightly underestimates the temperature in a whole region of distances. When the fibre is in the corner of the square groove so that the polymer layer touches the alumina walls, which is the realistic assumption, the temperature profile becomes non-symmetrical with respect to the fibre centre.

![Radial distribution of temperature in the octagonal cladding fiber.](image)

Figure 6 compares the model and simulation in that case. The analytical formula still provides fairly accurate results with the highest error being at the outer regions where the temperature is low.

### 3.3. Fibre with nonuniform core

In the examples analyzed so far, the core is assumed to be uniformly doped resulting in a constant power density of the heat source in the entire core area. There are, however, alternative designs possible. Figure 7 shows the core cross-section of the radius r_c with 7 doped rods in a hexagonal setup.

Shaded circles are the doped areas with the diameter t, the dashed circles denote preform tubes stacked together during a fibre drawing process. Since the area of the doped domain and so of the heat source is smaller than the core area, the temperature is expected to form hot spots in doped rods. The analytical model assuming cylindrical symmetry cannot describe such structure precisely but one can approximate it by an equivalent structure with the doped layer of thickness t_l, see Fig. 7. In order to maintain the same heat source density, the area of this layer has to be the same as the total area of 6 outer rods in the original structure. In our case, it follows that t_l = 9t^2/(8r_c).

The radial distribution of temperature in the octagonal cladding fibre with 7 doped rods in the core was obtained using FEM (see Fig. 8) and in the equivalent 8-layer structure using the analytical model with the results presented in Fig. 9.

![Scheme of the core cross-section with 7 doped rods (left) and an approximately equivalent structure (right) with a doped layer with the same cross-section area as 6 outer rods.](image)

![Spatial distribution of temperature in the octagonal cladding fiber with a core with 7 doped rods. Left: entire cross-section, right: core area.](image)
different values of heat load are related by the ratio of doped areas in both scenarios.

### 3.4. Air-clad fibre

As a final example, an air-clad fibre is analysed with the geometry depicted in Fig. 11, where also the spatial temperature distribution obtained from the FEM simulation is shown. In the simulation of the air-clad fibre, the thermal conductivity of air [27] was defined using the polynomial formula:

\[
k_{\text{air}}(T) = \sum_{i=0}^{3} c_i T^i, \tag{15}
\]

with coefficients \(c_0 = -3.9333 \times 10^{-4}\), \(c_1 = 1.0184 \times 10^{-4}\), \(c_2 = -4.8574 \times 10^{-6}\), \(c_3 = 1.5207 \times 10^{-11}\) where the temperature \(T\) is expressed in Kelvins. A typical value of the thermal conductivity of air is \(k_{\text{air}} = 0.034\) Wm\(^{-1}\)K\(^{-1}\) for \(T = 140^\circ\)C.

However, the layer of air cladding contains also narrow silica bridges that greatly increase the heat transfer away from the fibre centre area compared with the pure air layer case. Therefore, when an analytical model is to be applied in the case of the air-clad fibre, the effective value of the thermal conductivity \(k_{\text{eff}}\) of the air-clad layer should be used:

\[
k_{\text{eff}} = k_{\text{air}} \frac{w_{\text{air}}}{w_{\text{air}} + w_{\text{SiO2}}} + k_{\text{SiO2}} \frac{w_{\text{SiO2}}}{w_{\text{air}} + w_{\text{SiO2}}}, \tag{16}
\]

where in our example, \(w_{\text{SiO2}} = \alpha = 0.72\) \(\mu\)m (\(\alpha = 1.3\)) is the effective thickness of the silica bridges and \(w_{\text{air}} = 20.35\) \(\mu\)m is the respective width of the air domains in the azimuth direction. Note that the effective thickness \(w_{\text{SiO2}}\) is larger than the smallest bridge thickness since the rounded shape of the air holes should be considered, see Fig. 11 (right). Also, the application of Eq. (16) is of great importance because about 90°C higher core temperature value is obtained from the model using the simple relationship \(k_{\text{eff}} = k_{\text{air}}\). Furthermore, in order to respect Eq. (15) in the analytical model, several iterations are applied where the temperature determined in the centre of the air-clad layer is chosen as the input of Eq. (15) in the next iteration.

Figure 12 compares an analytical solution (with 7 layers) with a numerical one computed by FEM. The solution is fully consistent with the FEM simulation of the air-clad fibre presented in Ref. 28. The analytical solution agrees very well with the numerical one, but it underestimates temperature in layers outside the air-clad

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**Fig. 9.** Radial distribution of temperature in the octagonal cladding fiber. 7 doped rods with the diameter \(t = 1\) \(\mu\)m vs. the uniformly doped core with the same heat load \(Q = 150\) W/m.

**Fig. 11.** Spatial distribution of temperature in the air-clad fiber for \(Q = 200\) W/m. Left: entire cross-section, right: detail of air hole shapes.

**Fig. 10.** Radial distribution of temperature in the octagonal cladding fiber. 7 doped rods with \(t = 5\) \(\mu\)m vs. the uniform core with the same heat density \(Q = 1.1 \times 10^{12}\) W/m\(^2\).*
layer. This could again be attributed to the square shape of the groove.

Finally, the similar methodology presented here for the case of the air-clad fibre can be in principle applied even for other similar fibre structures, e.g., for large-pitch photonic crystal fibres [29,30], provided that the effective thermal conductivity is determined also in the micro-structured part of the fibre.

4. Conclusions

The analytical formula for the radial temperature distribution in multi-layer optical fibres was presented and tested in comparison with the FEM simulation results. A detailed review of the previous works on analytical formulas for the temperature distribution in optical fibres for high-power fibre lasers is provided. Contrary to previous works, devoted to the cases with specific number of layers (typically three), a generalized formula that comprises an arbitrary number of cylindrical layers and a heat source located in arbitrary layers is presented in the paper. The derived formula is practical for a range of optical fibre arrangements; three specific examples known from applications are reported to demonstrate the applicability of the formula. Although the formula was derived for the case of completely cylindrical geometry including a cooling medium, it was demonstrated by the analysis of octagon shape and air-clad fibres that it was fairly accurate even in more realistic scenarios with a fibre inside a square groove. The presented model can be in principle also applied in the case of nonuniform heat sources if they can be approximated by several cylindrical layers with the uniform heat distribution. It is envisaged that the presented formula can serve well not only for a fast estimation of the radial temperature distribution in multi-layer fibres, but it can also be part of more complex temperature-dependent fibre laser models.

Authors’ statement

Research concept and designs, P. P. and P. H.; collection and/or assembly of simulation data, M. G.; data analysis and interpretation, M. G.; analytical model development M. G.; writing the article, M. G.; critical revision of the article, P. P. and P. H.; final approval of article, M. G., P. P. and P. H.

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