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Algorithm for rigorous adjustment of modular networks

Mathematical formulaes for rigorous adjustment of surveying modular networks are presented in the paper. The method is based on the idea of multigroup transformation. The solution leads to general problem of non-linear adjusting of conditional equations with unknowns by the least square method. It is a modification of the classical way of approximate adjustment of the modular networks.

INTRODUCTION

The numerical working out of modular network (according to [10]) consists of transformation adjustment and accuracy analysis for the points being determined. This algorithm gives possibility of approximate adjustment of network. At present, however, having at one's disposal computer programs and hardware, there is no obstacle to apply of the rigorous adjustment. To rigorous adjustment of a horizontal network we need to know the approximate coordinates for every point being determined. In the past, cost (time) of calculation was the essential barrier for this task. For the sake of it, the relatively high accuracy of approximate coordinates was being assumed. Network adjusting in one calculation cycle was the main goal. The repeating of whole the adjustment process when updating approximate coordinates, at present not involves expenses of computation. Such attitude to this issue gives us certainty, that the influence of systematic observational error will be considerably limited. Thus working out of modular networks could include two stages: 1) evaluation of approximate coordinates, 2) rigorous adjustment by parametric method.

Working out the methodology of determining approximate coordinates for any angular-linear network is not an easy task. The difficulty results from the fact, that varied structures of networks are unique. However, a geodetic program that is used for this type of computation, must be based on an universal predetermined algorithm. One could solve this task by analogy to block aerial triangulation (analytical photogrammetry), where the full working out consists in group transformation of the models or photographs [9]. However, the problem of undeterminate elementary modules appears in construction of algorithm for calculation of approximate coordinates. This issue is considered in works [3, 7].

The method shown in present paper is a suggestion of the rigorous adjustment of modular networks without necessity of calculating of approximate coordinates. It is based on transformational computation and it is a modification of the classical method of approximate adjustment for this type of geodetic nets. The rigorous solution of problem of modular network with evaluation of the transformation parameters of modules (2 translation parameters and 1 rotation parameter for each of modules) and corrections to direct observations of directions and distances, leads to general issue of nonlinear adjustment of the conditional equations with unknowns [1] by the least square method.

Classical modular networks on principle should be applied as the geodetic surveying nets [10]. However, practical and accuracy considerations (choosing the surveying station point in any place, elimination of centring error) cause that they can be also used as special nets (geodetic maintenance of production halls in industrial plants) or as the precision set-out nets. Thus criteria of rigorous adjustment will be diversified depending on the way of network using. In agreement to technical rules [5] we should assume the mean positional error < 0.20 m (for horizontal surveying net) and < 0.05 m (for vertical net). The errors after adjustment would not exceed these values, irrespective of method of working out the measuring data. Adjustment of observations by the transformation method following on algorithm presented in existing now guidelines G-4.1 [10], gives possibility of getting the measures of accuracy mentioned above (when complying with the surveying parameters shown in technic rules G-4 [5]). However when doing geodetic maintenance of the objects, that need higher accuracy, the rigorous adjustment by the least square method is more relevant procedure for working out the measuring data.

1. *Application of transformation computation to the modular network working out*

The classical method of modular network working out [10] is based on generally known formulas for linear transformation:

$$\begin{cases} X - X_0 = (x - x_0)\, f \cos\alpha - (y - y_0)\, f \sin\alpha \\ Y - Y_0 = (y - y_0)\, f \cos\alpha + (x - x_0)\, f \sin\alpha\,, \end{cases} \tag{1}$$

where: (x, y); (X, Y) – point coordinates in original and secondary system (respectively); (x_0, y_0); (X_0, Y_0) – coordinates of a point freely placed (transformation pole) in original and secondary system (respectively); α – angle of rotation between original system and the secondary one; f – scale factor.

Assuming that $x_0 = 0$ and $y_0 = 0$, then X_0, Y_0 are coordinates of system origin in secondary system. Thus transformation formulaes (1) will be described in simplified form:

$$\begin{cases} X = X_0 + x\, f \cos\alpha - y\, f \sin\alpha \\ Y = Y_0 + y\, f \cos\alpha + x\, f \sin\alpha\,. \end{cases} \tag{2}$$

Formulaes (2) comply with a single group of points. The issue of working out the modular networks, however, concerns multigroup transformation. The task consists in simultaneous conversion of coordinates of many groups of points to one common coordinate system [2, 6]. Every module has the coordinates given in its local system, but global system, defined by control points of network, is the target system.

2. *Functional model for modular network*

For any modular network we can specify the functional model leading to system of conditional equations with unknowns. The unknown quantities in this system are transformation parameters (varied for each of modules). When assuming that every module is fully determinable internally (every length of sight line is known), then the basic structure of observation system includes: 1) conditional equations for tie (binding) points; 2) conditions for control points.

The third type of conditional equations, also for tie points, but different than mentioned above, occurs in the case when existing a module being internally undeterminable (for example because of lack of a sight line distance). Creating such a condition consists in elimination the missing length from the equation.

For every tie point k (Fig. 1), being common for two neighbouring modules i, j we create 2 conditions including 8 unknowns altogether (4 transformation parameters per each of modules):

$$\begin{cases} X_k = X_{0i} + x_{ik}\ C_i - y_{ik}\ S_i = X_{0j} + x_{jk}\ C_j - y_{jk}\ S_j \\ Y_k = Y_{0i} + Y_{ik}\ C_i + x_{ik}\ S_i = Y_{0j} + y_{jk}\ C_j + x_{jk}\ S_j\,, \end{cases} \tag{3}$$

where: S, C – transformation quantities for both modules (respectively):

$$\begin{cases} C_i = f_i\ \cos\alpha_i \\ S_i = f_i\ \sin\alpha_i \end{cases} \qquad \begin{cases} C_j = f_j\ \cos\alpha_j \\ S_j = f_j\ \sin\alpha_j\,. \end{cases} \tag{4}$$

Relations between local system (for any module i) and target system, can be expressed in matrix form (eq. 3):

$$\mathbf{X}_k = \mathbf{X}_{0i} + \mathbf{S}_i\,\boldsymbol{x}_{ik}\,, \tag{5}$$

where:

$$\mathbf{X}_k = \begin{bmatrix} X_k \\ Y_k \end{bmatrix};\quad \mathbf{X}_{0i} = \begin{bmatrix} X_{0i} \\ Y_{0i} \end{bmatrix};\quad \mathbf{S}_i = \begin{bmatrix} C_i & -S_i \\ S_i & C_i \end{bmatrix};\quad \boldsymbol{x}_{ik} = \begin{bmatrix} x_{ik} \\ y_{ik} \end{bmatrix}$$

and: (X_{0i}, Y_{0i}), (X_{0j}, Y_{0j}) – coordinates of the origin of local system for modules i and j (respectively) in global system; X_k, Y_k – coordinates of tie point k in global system; (x_{ik}, y_{ik}), (x_{jk}, y_{jk}) – coordinates of tie point k for modules i and j (respectively) in local system; α_i, α_j

– angles of rotation between local and global system for modules i and j (respectively); f_i, f_j – scale factors for modules i and j (respectively).

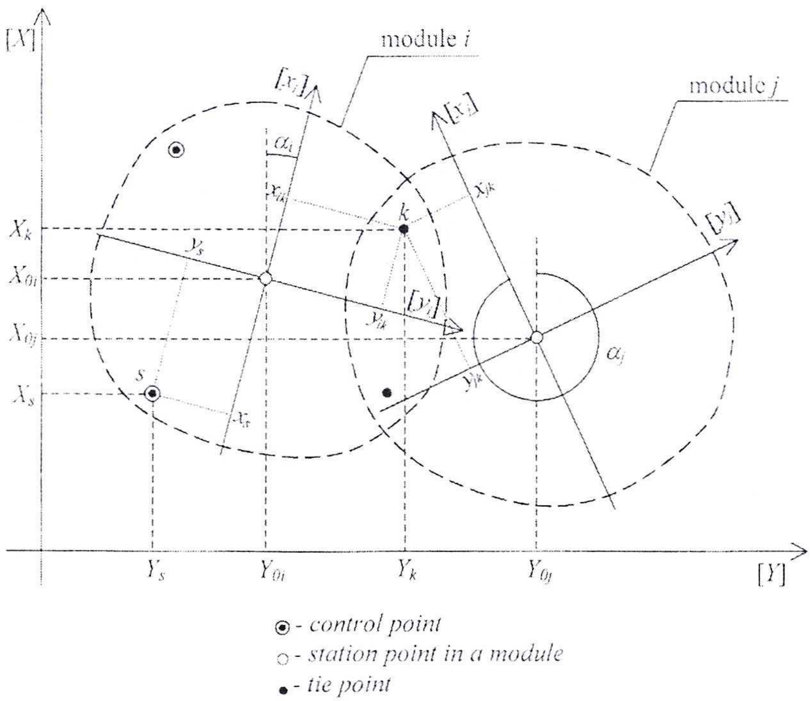

Fig. 1

Two conditions for any control point s we can describe similarly:

$$\begin{cases} X_s = X_{0i} + x_{is}\, C_i - y_{is}\, S_i \\ Y_s = Y_{0i} + y_{is}\, C_i + x_{is}\, S_i\,, \end{cases} \tag{6}$$

where the meaning of symbols is analogous to formulas (3) and (4).

The full functional model for modular network (in matrix notation) can be shown in following way:

$$\begin{cases} \mathbf{X}_{0i} + \mathbf{S}_i\, \boldsymbol{x}_{ik} = \mathbf{X}_{0j} + \mathbf{S}_j\, \boldsymbol{x}_{jk}; \qquad \mathbf{S}_i^T\, \mathbf{S}_i = I; \quad \mathbf{S}_j^T\, \mathbf{S}_j = I \\ \mathbf{X}_{0i} + \mathbf{S}_i\, \boldsymbol{x}_{is} = \mathbf{X}_s \end{cases} . \tag{7}$$

Assuming approximate unknowns: $C_i^{(0)}$, $S_i^{(0)}$, $X_{0i}^{(0)}$, $Y_{0i}^{(0)}$ (i = 1,2,...,m), m – number of modules, the observation system (7) we can solve easy, because it is linear. It is classic way of working out the modular network, being used in analytic photogrammetry as well.

3. *Rigorous solution*

The rigorous working out of modular network consists in assigning the corrections to direct observations (received on polar station of a module) or assigning relevant covariance sub-matrixes to pseudo-observations (local coordinates of a module).

Let us create a functional model including observations. Replacements like

$$X_{ik} = d_{ik} \cos\varphi_{ik}; \quad y_{ik} = d_{ik} \sin\varphi_{ik}, \tag{8}$$

yield the transformed conditions (3):

$$\begin{cases} X_{0i} + d_{ik} \cos\varphi_{ik}\, C_i - d_{ik} \sin\varphi_{ik}\, S_i = X_{0j} + d_{jk} \cos\varphi_{jk}\, C_j - d_{jk} \sin\varphi_{jk}\, S_j \\ Y_{0i} + d_{ik} \sin\varphi_{ik}\, C_i + d_{ik} \cos\varphi_{ik}\, S_i = Y_{0j} + d_{jk} \sin\varphi_{jk}\, C_j - d_{jk} \cos\varphi_{jk}\, S_j \end{cases}, \tag{9}$$

where: (d_{ik}, φ_{ik}), (d_{jk}, φ_{jk}) – polar coordinates (distance and angle) of tie point k in local system for modules i and j (respectively).

The same stage can be described in matrix notation. Inserting the following matrix into formula system (7):

$$\boldsymbol{x}_{ik} = \begin{bmatrix} x_{ik} \\ y_{ik} \end{bmatrix} = \begin{bmatrix} d_{ik} & \cos\varphi_{ik} \\ d_{ik} & \sin\varphi_{ik} \end{bmatrix} = d_{ik} \begin{bmatrix} \cos\varphi_{ik} \\ \sin\varphi_{ik} \end{bmatrix} = d_{ik}\, \boldsymbol{K}_{ik}, \tag{10}$$

results in:

$$\begin{cases} \mathbf{X}_{0i} + d_{ik}\, \mathbf{S}_i\, \mathbf{K}_{ik} = \mathbf{X}_{0j} + d_{jk}\, \mathbf{S}_j\, \mathbf{K}_{jk} \\ \mathbf{X}_{0i} + d_{is}\, \mathbf{S}_i\, \mathbf{K}_{is} = \mathbf{X}_s \end{cases}. \tag{11}$$

In formula (9) next we replace the unknowns and the observations with their relevant approximate values along with their corrections:

$$\begin{cases} d_{ik} = \tilde{d}_{ik} + \delta_{ik}; & \varphi_{ik} = \tilde{\varphi}_{ik} + \upsilon_{ik} \\ d_{jk} = \tilde{d} + \delta_{jk}; & \varphi_{jk} = \tilde{\varphi}_{jk} + \upsilon_{jk} \end{cases}, \tag{12}$$

$$\begin{cases} X_{0i} = X_{0i}^{(0)} + \delta X_{0i} \\ Y_{0i} = Y_{0i}^{(0)} + \delta Y_{0i} \end{cases} \qquad \alpha_i = \alpha_i^{(0)} + \delta\alpha_i, \tag{13}$$

where: $X_{0i}^{(0)}$, $Y_{0i}^{(0)}$; $\alpha_i^{(0)}$ – approximate values, $\tilde{d}$, $\tilde{\varphi}$ – quantities being observed (measured).

Assuming that systematic observation error had been eliminated earlier, we can take $f = 1$ (invariability of scale). Then factors C and S (from formula (9)) can be expressed as follows

$$\begin{cases} C_i = \cos(\alpha_i^{(0)} + \delta\alpha_i) \\ S_i = \sin(\alpha_i^{(0)} + \delta\alpha_i) \end{cases} . \tag{14}$$

Further development of the formula leads to the following form

$$\begin{cases} C_i = \cos\alpha_i^{(0)} - \sin\alpha_i^{(0)}\, \delta\alpha_i = \bar{C}_i^{(0)} - \bar{S}_i^{(0)}\, \delta\alpha_i \\ S_i = \sin\alpha_i^{(0)} + \cos\alpha_i^{(0)}\, \delta\alpha_i = \bar{S}_i^{(0)} + \bar{C}_i^{(0)}\, \delta\alpha_i \end{cases} . \tag{15}$$

The „new" transformations factors

$$\bar{C}_i^{(0)} = \cos\alpha_i^{(0)}; \qquad \bar{S}_i^{(0)} = \sin\alpha_i^{(0)}, \tag{16}$$

can be evaluated by approximate adjustment, in following way:

$$\bar{C}_i^{(0)} = \frac{C_i^{(0)}}{f_i^{(0)}}; \quad \bar{S}_i^{(0)} = \frac{S^{(0)}}{f^{(0)}}, \tag{17}$$

where

$$C_i^{(0)} = f_i^{(0)} \cos\alpha_i^{(0)}; \quad S_i^{(0)} = f_i^{(0)} \sin\alpha_i^{(0)}; \quad f_i^{(0)} = \sqrt{(C^{(0)})^2 + (S^{(0)})^2}.$$

As a result, the condition equation for X coordinate can be written at this stage as:

$$\begin{aligned} &X_i^{(0)} + \delta X_{0i} + d_{ik} \cos\varphi_{ik} (\bar{C}_i^{(0)} - \bar{S}_i^{(0)}\, \delta\alpha_i) - d_{ik} \sin\varphi_{ik} (\bar{S}_i^{(0)} + \bar{C}_i^{(0)}\, \delta\alpha_i) = \\ &= X_{0j} + \delta X_{0j} + d_{jk} \cos\varphi_{jk} (\bar{C}_j^{(0)} - \bar{S}_j^{(0)}\, \delta\alpha_j) - d_{jk} \sin\alpha_{jk} (\bar{S}_j^{(0)} + \bar{C}_j^{(0)}\, \delta\alpha_j). \end{aligned} \tag{18}$$

Further progress of development is been presented below. Our aim is the linear form of conditional equations with unknowns.

$$\begin{cases} d_{ik} \cos\varphi_{ik} = (\tilde{d}_{ik} + \delta_{ik}) \cos(\tilde{\varphi}_{ik} + \upsilon_{ik}) \cong \tilde{d}_{ik} \cos\tilde{\varphi}_{ik} + \delta_{ik} \cos\tilde{\varphi}_{ik} - \tilde{d}_{ik} \sin\varphi_{ik}\, \upsilon_{ik} \\ d_{ik} \sin\varphi_{ik} = (\tilde{d}_{ik} + \delta_{ik}) \sin(\tilde{\varphi}_{ik} + \upsilon_{ik}) \cong \tilde{d}_{ik} \sin\tilde{\varphi}_{ik} + \delta_{ik} \sin\tilde{\varphi}_{ik} + \tilde{d}_{ik} \cos\tilde{\varphi}_{ik}\, \upsilon_{ik}, \end{cases} \tag{19}$$

Hence, after mathematic rearrangement, we finally obtain:

$$\begin{aligned} &\delta x_{0i} - \delta_{0i} + \alpha_{1i}\, \delta\alpha_{1i} - \alpha_{1j}\, \delta\alpha_j = \\ &= - X_{0j}^{(0)} + X_{0j}^{(0)} - \tilde{x}_{ik}\, \bar{C}_i^{(0)} - \delta_{ik} \cos\tilde{\varphi}_{ik}\, \bar{C}_i + \upsilon_{ik}\, \tilde{y}_{ik}\, \bar{C}_i^{(0)} + \tilde{x}_{jk}\, \bar{C}_j^{(0)} + \\ &+ \delta_{jk} \cos\tilde{\varphi}_{jk}\, \bar{C}_j^{(0)} - \upsilon_{jk}\, \tilde{y}_{jk}\, \bar{C}_j^{(0)} + \tilde{y}_{jk}\, \bar{S}_i^{(0)} + \delta_{ik} \cos\tilde{\varphi}_{ik}\, \bar{S}_i^{(0)} + \\ &\upsilon_{ik}\, \tilde{x}_{ik}\, \bar{S}_i^{(0)} - \tilde{y}_{jk}\, \bar{S}_j^{(0)} - \delta_{jk} \cos\tilde{\varphi}_{jk}\, \bar{S}_i^{(0)} - \upsilon_{jk}\, \tilde{x}_{jk}\, \bar{S}_j^{(0)}. \end{aligned} \tag{20}$$

In formula (20) we put together the known quantities (observations, approximate unknowns):

$$\begin{cases} \cos\tilde{\varphi}_{ik} (\bar{S}_i^{(0)} - \bar{C}_i^{(0)}) = a_{2i}; \qquad \tilde{y}_{ik} \bar{C}_i^{(0)} - x_{ik} \bar{S}_i^{(0)} = a_{3i} \\ \cos\tilde{\varphi}_{jk} (\bar{C}_j^{(0)} - \bar{S}_j^{(0)} = a_{2j}; \qquad \tilde{y}_{ik} \bar{C}_j^{(0)} - \tilde{x}_{jk} \bar{S}_j^{(0)} = a_{3j} \\ - X_{0i}^{(0)} + X_{0j}^{(0)} - \tilde{x}_{ik} \bar{C}_i^{(0)} + \tilde{y}_{ik} \bar{S}_i^{(0)} + \tilde{x}_{ik} \bar{C}_j^{(0)} - \tilde{y}_{jk} \bar{S}_j^{(0)} = \omega_{1ij} \end{cases} \tag{21}$$

what leads to final form of conditional equation with unknowns for X coordinate, where left there are corrections to unknowns, and right there are corrections to observations:

$$\delta X_{0j} - \delta X_{0j} + a_{1i}\, \delta\alpha_i - a_{1j}\, \delta\alpha_j = a_{2i}\, \delta_{ik} + a_{3i}\, \upsilon_{ik} + a_{2j}\, \delta_{jk} - a_{3j}\, \upsilon_{jk} + \omega_{1ij}, \tag{22}$$

where: a_{1i}, a_{1j} – coefficients of unknowns; a_{2i}, a_{2j}, a_{3i}, a_{3j} – coefficients of corrections to observations; ω_{1ij} – free term.

Similarly we can derive the equation for Y coordinate:

$$\delta Y_{0i} + \delta Y_{0j} + b_{1i}\, \delta\alpha_i - b_{1j}\, \delta\alpha_j = - b_{2i}\, \delta_{ik} + b_{3i}\, \upsilon_{ik} + b_{2j}\, \delta_{jk} + b_{3j}\, \upsilon_{jk} + \omega_{2ji}. \tag{23}$$

By the same principle we will obtain the conditional equations for control points (Eq. 6). These equations together with equations (22) and (23) make up the functional model of modular network. In matrix form we can it describe as:

$$\mathbf{A}\,\delta\mathbf{X} - \mathbf{C}\mathbf{V} = \boldsymbol{W}, \tag{24}$$

$$\delta \boldsymbol{X} = [\delta X_{01}, \delta X_{02}, \ldots, \delta X_{0m}, \delta Y_{01}, \delta Y_{02}, \ldots, \delta Y_{0m}, \delta\alpha_1, \delta\alpha_2, \ldots, \delta\alpha_m]^T, \tag{25}$$

$$\boldsymbol{X}^{(0)} = [X_{01}^{(0)}, X_{02}^{(0)}, \ldots, X_{0m}^{(0)}, Y_{01}^{(0)}, Y_{02}^{(0)}, \ldots, Y_{0m}^{(0)}, \alpha_1^{(0)}, \alpha_2^{(0)}, \ldots, \alpha_m^{(0)}]^T, \tag{26}$$

$$\boldsymbol{V} = [V_1^T, V_2^T, \ldots, V_m^T]^T, \tag{27}$$

$$V_i = [\delta_{i1}, \upsilon_{i1}, \delta_{i2}, \upsilon_{i2}, \ldots, \delta_{ik}, \upsilon_{ik}]^T, \tag{28}$$

where: m – number of modules; k – index of tie point; $\mathbf{A}$ – matrix of coefficients of unknowns; $\mathbf{C}$ – matrix of coefficients of corrections to observations; $\boldsymbol{W}$ – vector of free terms; $\delta\boldsymbol{X}$ – vector of corrections to unknowns; $\boldsymbol{V}$ – vector of corrections to observations; $\boldsymbol{X}^{(0)}$ – vector of approximates values.

The equation system (24) can be solved by inserting pseudo-observations $\underline{\upsilon}$:

$$\mathbf{A}\,\delta\boldsymbol{X} = \boldsymbol{W} + \mathbf{C}\,\boldsymbol{V}, \tag{29}$$

$$\mathbf{A}\,\delta\boldsymbol{X} = \boldsymbol{W} + \underline{\upsilon}; \quad (\underline{\upsilon} = \mathbf{C}\,\boldsymbol{V}), \tag{30}$$

$$\underline{\upsilon}\,\mathbf{Q}_{\underline{\upsilon}}^{-1} \cdot \underline{\upsilon}^T = \min. \tag{31}$$

By the principle of variance propagation [8] we can describe

$$\mathbf{Q}_{\underline{\upsilon}} = \mathbf{C}\,\mathbf{Q}_L\,\mathbf{C}^T, \tag{32}$$

$$\mathbf{Q}_{\upsilon} = \mathbf{P}^{-1}; \quad \mathbf{P} = \left[\frac{1}{\mu_{d,\alpha}^{2}}\right] \tag{33}$$

and finally

$$\delta X = (\mathbf{A}^{T}\mathbf{Q}_{\underline{\upsilon}}^{-1}\,\mathbf{A})^{-1}\,\mathbf{A}^{T}\mathbf{Q}_{\underline{\upsilon}}^{-1}\,W, \tag{34}$$

where: $\underline{\upsilon}$ – vector of pseudo-corrections; $\mathbf{Q}_{L}$ – covariance matrix of observation vector; $\mathbf{P}$ – matrix of weight observations; $\mu_{d,\alpha}$ – mean errors of observations (of distance or angle).

The relevant submatrices $\mathbf{Q}^{(i)}$ of covariance matrix $\mathbf{Q}_{x}$ of the vector of unknowns are useful to evaluation of transformation accuracy for particular modules:

$$\mathbf{Q}_{x} = \mu_{0}^{2}(\mathbf{A}^{T}\mathbf{Q}_{\upsilon}^{-1}\cdot\mathbf{A})^{-1} = \begin{bmatrix} \mathbf{Q}^{(1)} & \mathbf{Q}^{(1,2)} & \dots & \mathbf{Q}^{(1,m)} \\ \mathbf{Q}^{(2,1)} & \mathbf{Q}^{(2)} & \dots & \mathbf{Q}^{(2,m)} \\ \dots & \dots & \dots & \dots \\ \mathbf{Q}^{(m,1)} & \mathbf{Q}^{(m,2)} & \dots & \mathbf{Q}^{(m)} \end{bmatrix}, \tag{35}$$

$$\mathbf{Q}^{(i)} = \begin{bmatrix} \mathbf{Q}_{x} & \mathbf{Q}_{xy} & \mathbf{Q}_{x\alpha} \\ \mathbf{Q}_{yx} & \mathbf{Q}_{y} & \mathbf{Q}_{y\alpha} \\ \mathbf{Q}_{\alpha x} & \mathbf{Q}_{\alpha y} & \mathbf{Q}_{\alpha} \end{bmatrix}; \quad (i = 1,2,\dots,m). \tag{36}$$

The algorithm written in agreement with formula (34) can be performed by iterative way, when updating of each of matrixes (as the functions of current vector of unknowns). In this way the iterative Gauss-Newton procedure of solving the nonlinear task by the least square method is formally realised.

4. *Heights' modular network working out by transformation adjustment*

Each of the modules has its local height h_{ik} (i = 1, 2, ..., m; m – number of modules; k – index of tie point), that are base to computation of the particular tie points in global system:

$$h_{ik} + z_{i} = H_{k}, \tag{37}$$

where: z_i – altitude of the module (origin of local coordinate system) i in global system.

Functional model of the heights' modular net working out is based (similarly to situation solution) on the conditions of two types: for tie points and for control points.

$$H_{k} = h_{ik} + z_{i} = h_{jk} + z_{j}; \quad \text{(for every tie point } k\text{)}, \tag{38}$$

$$H_s = h_{is} + z_i; \quad \text{(for every control point } s\text{)}. \tag{39}$$

In every conditional equation we replace the observations with their corrections instead of the unknown heights (height differences):

$$h_{ik} = \widetilde{h}_{ik} + \upsilon_{ik}; \quad h_{is} = \widetilde{h}_{is} + \upsilon_{is}, \tag{40}$$

what leads to the system of condition equations with unknowns:

$$\begin{cases} \widetilde{h}_{ik} + \upsilon_{ik} + z_i = \widetilde{h}_{jk} + \upsilon_{jk} + z_j, \\ \widetilde{h}_{is} + \upsilon_{is} + z_i = H_s \end{cases} \tag{41}$$

hence

$$\begin{cases} z_i - z_j = -\upsilon_{ik} + \upsilon_{jk} - \widetilde{h}_{ik} + \widetilde{h}_{jk}. \\ z_i = -\upsilon_{is} + H_s - \widetilde{h}_{is} \end{cases} \tag{42}$$

We can solve the system (42) in similar way as it was done for the horizontal net (formulae 24 and 34). Adjustment results (parameters z_i, z_j and observation corrections) are useful to final calculation of point altitudes in global (reference) coordinate system:

$$H_k = \widetilde{h}_{ik} + \upsilon_{ik} + z_i = \widetilde{h}_{jk} + \upsilon_{jk} + z_j \tag{43}$$

CONCLUSIONS

The authors of current geodetic rules in area of situation and height measurement [5] recommend the rigorous adjustment for every type of net, even in case of surveying net. Until now, modular networks as the geodetic surveying networks were adjusted by approximation (in agreement with [10]). This was caused by difficulties associated with the need to determine approximate coordinates. This task was problematic if some modules were undeterminable in their local systems. However, the method of rigorous adjustment, presented in this paper, is based on transformation calculation. It requires the creation of two type conditions: one for tie (binding) points and one for control points. The rigorous formulation of the problem leads to solution of the system of conditional equations with unknowns. As a result we obtain adjusted observation and coordinates, and also information about mean transformation errors for particular modules. The formulae shown here can be helpful when working out the computer algorithms for adjustment of this type of network.

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Algorytm wyrównania ścisłego sieci modularnych

S t r e s z c z e n i e

W pracy przedstawiono wzory obliczeniowe wyrównania ścisłego sieci modularnych. Metoda oparta jest na idei trasformacji wielogrupowej. Rozwiązanie zadania sprowadza się do ogólnego problemu nieliniowego wyrównania obserwacji zawarunkowanych z niewiadomymi metodą najmniejszych kwadratów. Proponowana metoda obliczeniowa jest modyfikacją klasycznej metody wyrównania przybliżonego sieci modularnych.

Тадэуш Гаргуля

Алгоритм строгого уравнивания модулярных сетей

Р е з ю м е

В работе представлены вычислительные формулы строгого уравнивания модулярных сетей. Метод основан на идеи многогрупповой трансформации. Решение проблемы сводится к общей проблеме нелинейного уравнивания наблюдений, обусловленных с неизвестными, методом наименьших квадратов. Предлагаемых с неизвестным, методом наименьших квадратов. Предлагаемый вычислительный метод является модификацией классического метода приблизительного уравнивания модулярных сетей.