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Determination of position by resection

A new solution of the resection is presented in this paper. Two methods of position determination have been developed. An indicator of possibilities of position determination has been introduced. Cases when the position cannot be determined have been considered and their geometric interpretation has been developed.

INTRODUCTION

One of the basic surveying constructions, applied for the needs of position determination, is Pothenot's construction, which is called resection. Resection allows for determination of a position basing on survey of two angles between three points of known coordinates.

Various solutions of resection are used, which are known by the names of their authors – Cassini, Collins, Delambre, Hausbrandt, Tienstra. To be precise, we should add that, as it has been proved by Hu i Kuang'a (1998), Tienstra's method, which is based on barycentric coordinates, is also justified in the case where the determined point is located outside the triangle of stable points.

Irrespective of various solutions of resection, the basic problem concerning determinability of position still exist. Establishing of non-determinable positions, as a result of graphic determination of a dangerous circle, does not seem satisfactory.

Two new methods of position determination by resection are presented in this paper. Statements have been made as a result of geometric considerations and a solution of a system of nonlinear equations, formulated in this paper. Such approach allows for general and complete solution of the discussed problem. As a result of these statements, two methods of position determination by resection, have been proposed.

The problem of nondeterminable positions has been also discussed. An indicator of position determinability has been introduced. Cases of nondeterminability of position have been specified and their geometric interpretation has been presented.

The included examples illustrate the practical application of porposed methods.

1. *Basic concepts*

Let three points of known coordinates in the *xy* coordinate system are visible from the point *P*. A point lying on the left side of the point *P* we denote by *A*, a point lying on the right side of the point *P* by *B*, and a point which is centrally located we denote by *C*. Angles measured at the point *P* between the given poitns *A*, *C* and the *C*, *B* we denote by α_1 and α_2, respectively, Fig. 1.

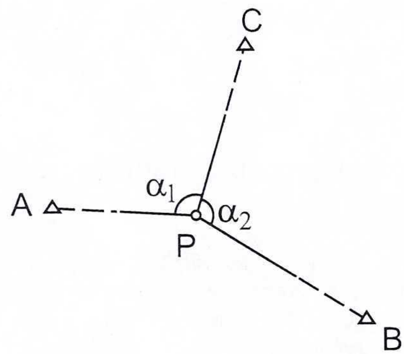

Fig. 1. Resection

In order to obtain the coordinates of the point *P* we write the known coordinates of the points *A*, *B* and *C* in the form $A(x_A, y_A)$, $B(x_B, y_B)$ and $C(x_C, y_C)$. As a result of denoting $\Delta x_{CA} = x_A - x_C$, $\Delta y_{CA} = y_A - y_C$ and $\Delta x_{CB} = x_B - x_C$, $\Delta y_{CB} = y_B - y_C$, the distances between the points *A* and *C*, and *B* and *C*, are expressed by

$$a_1 = \sqrt{\Delta x^2{}_{CA} + \Delta y^2{}_{CA}} \tag{1}$$

and

$$a_2 = \sqrt{\Delta x^2{}_{CB} + \Delta y^2{}_{CB}} \tag{2}$$

The azimuth of the lines *CA* and *CB*, which we denote by φ_1 and φ_2, respectively, can be written in the form

$$\varphi_1 = \arctan \frac{\Delta y_{CA}}{\Delta x_{CA}} \tag{3}$$

and

$$\varphi_2 = \arctan \frac{\Delta y_{CB}}{\Delta x_{CB}} \tag{4}$$

So that, the angle between the points *ACB,* which is denoted as β, Fig. 2, is as follows

$$\beta = \varphi_1 - \varphi_2 \tag{5}$$

where φ_1 and φ_2 are specified by (3) and (4).

Note, that the measured angles determine the circles from which the distances a_1 and a_2 are visible under the angles of α_1 and α_2, respectively, Fig. 2. Centres of these circles are denoted as O_1 and O_2 and their radiuses as r_1 and r_2.

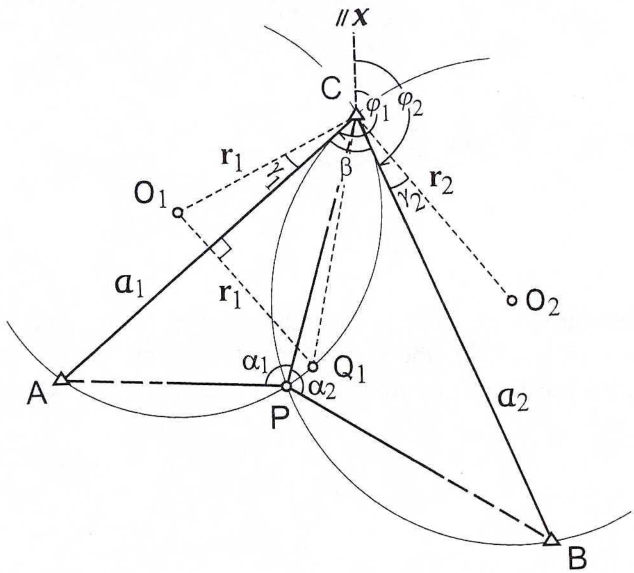

Fig. 2. Geometric elements of resection

Consider the cirle, of which centre is at the point O_1, and take into account the point Q_1 lying on this circle at the same distances from the points *A* and *C*. Since the triangle CO_1Q_1 is the isosceles triangle of which two sides equals to r_1, the angle O_1CA, which is denoted by γ_1, can be written in the form

$$\gamma_1 = 90° - (180° - \alpha_1) \tag{6}$$

thus

$$\gamma_1 = \alpha_1 - 90° \tag{7}$$

If we now note that

$$r_1 = \frac{a_1}{2\cos\gamma_1} \tag{8}$$

then, after substituting (7) into (8), we have

$$r_1 = \frac{a_1}{2\sin a_1} \tag{9}$$

Analogically, from the circle with the centre at the point O_2, we obtain

$$\gamma_2 = \alpha_2 - 90° \tag{10}$$

and

$$r_2 = \frac{a_2}{2\sin\gamma_2} \tag{11}$$

thus, after substituting (10) into (11)

$$r_2 = \frac{a_2}{2\sin\alpha_2} \tag{12}$$

Now we can determine, taking into account the radiuses r_1 and r_2, the coordinates of the centres of the circles, O_1 and O_2, in the x' y' coordinate system. The x' y' coordinate system we introduce in such a way that its origin is at the point C, and the x' axis passed through the point O_1, Fig. 3.

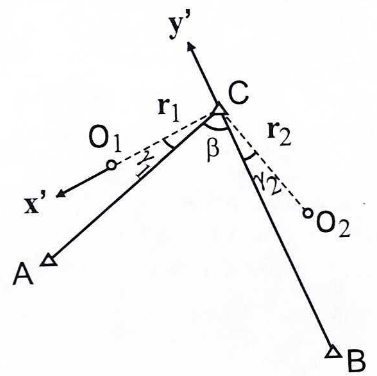

Fig. 3. Cartesian $x'y'$ coordinate system

Determining the coordinates of the point O_1, we have

$$x'_{O_1} = r_1 \tag{13}$$

and

$$y'_{O_1} = 0 \tag{14}$$

For the coordinates of the point O_2, after taking into account that the azimuth of CO_2 is equal to $360° - (\gamma_1 + \beta + \gamma_2)$, we state that

$$x'_{O_2} = r_2 \cos[360° - (\gamma_1 + \beta + \gamma_2)] \tag{15}$$

and

$$y'_{O_2} = r_2 \sin[360° - (\gamma_1 + \beta + \gamma_2)] \tag{16}$$

Substituting (7), (10) into (15), (16), after some manipulation

$$x'_{O_2} = -r_2 \cos(\alpha_1 + \beta + \alpha_2) \tag{17}$$

and

$$y'_{O_2} = r_2 \sin(\alpha_1 + \beta + \alpha_2) \tag{18}$$

If now we introduce the expression

$$\omega = \alpha_1 + \beta + \alpha_2 \tag{19}$$

and substitute (19) into (17), (18), we finally have

$$x'_{O_2} = -r_2 \cos\omega \tag{20}$$

and

$$y'_{O_2} = r_2 \sin\omega \tag{21}$$

what determines the coordinates of the point O_2 in the $x'\ y'$ coordinate system.

2. *The system of equations*

The coordinates of the point P can be obtained as a result of intersection of two circles with centres at the points O_1, O_2 and the radiuses r_1, r_2, respectively.

The equation of the circle with the centre at the point O_1, in the $x'\ y'$ coordinate system, is

$$(x' - x_{O_1})^2 + (y' - y_{O_1})^2 = r_1^2 \tag{22}$$

Substituting (13), (14) into (22) we obtain

$$(x' - r_1)^2 + y'^2 = r_1^2 \tag{23}$$

hence

$$x'^2 - 2r_1 x' + y'^2 = 0 \tag{24}$$

The equation of the circle with the centre in O_2 has the form

$$(x' - x_{O_2})^2 + (y' - y_{O_2})^2 = r_2^2 \tag{25}$$

substituting (20), (21) into (25) we have

$$(x' + r_2 \cos\omega)^2 + (y' - r_2 \sin\omega)^2 = r_2^2 \tag{26}$$

so that

$$x'^2 + 2r_2 \cos\omega \cdot x' + y'^2 - 2r_2 \sin\omega \cdot y' = 0 \tag{27}$$

Now instead of the radiuses of the circles, r_1 and r_2, we introduce their diameters

$$d_1 = 2r_1 \tag{28}$$

and

$$d_2 = 2r_2 \tag{29}$$

which after substituting (8), (9) into (26), (27), respectively, we write in the form

$$d_1 = \frac{a_1}{\sin\alpha_1} \tag{30}$$

and

$$d_2 = \frac{a_2}{\sin\alpha_2} \tag{31}$$

The equations (30) and (24) yield

$$x'^2 - d_1 x' + y'^2 = 0 \tag{32}$$

substituting (31) into (27) we have

$$x'^2 + d_2 \cos\omega \cdot x' + y'^2 - d_2 \sin\omega \cdot y' = 0 \tag{33}$$

The above equations, (32), (33), allows to determine the coordinates of the intersection points of both circles in the x' y' coordinate system.

3. *Solution of the system of equations*

Puting (32), (33) together, we obtain the following system of equations

$$x'^2 + d_2 \cos\omega \cdot x' + y'^2 - d_2 \sin\omega \cdot y' = 0 \tag{34}$$

$$x'^2 - d_1 x' + y'^2 = 0 \tag{35}$$

In order to determine the unknows x', y', after substracting the equation (34) from the equation (35), we have

$$(d_1 + d_2 \cos\omega)x' - d_2 \sin\omega \cdot y' = 0 \tag{36}$$

hence

$$y' = \frac{d_1 + d_2 \cos\omega}{d_2 \sin\omega} x' \tag{37}$$

where $\omega \neq 180°k$, $k = 0, 1$.

After introducing the expression

$$c_1 = \frac{d_1 + d_2 \cos\omega}{d_2 \sin\omega} \tag{38}$$

and substituting (38) into (37), we finally obtain

$$y' = c_1 x' \tag{39}$$

If we now substitute (39) into (35) we state that

$$x'^2 - d_1 x' + c_1^2 x'^2 = 0 \tag{40}$$

hence

$$(1 + c_1^2)x'^2 - d_1 x' = 0 \tag{41}$$

so that

$$x'[(1 + c_1^2)x' - d_1] = 0 \tag{42}$$

Considering the equation (40) we state, that the first root, of which the introduced subscript is C, yields

$$x'_C = 0 \tag{43}$$

and after substituting (43) into (39), we have

$$y'_C = 0 \tag{44}$$

what establishes the coordinates of the point C in the x' y' coordinate system.

The second root of the equation (42), of which the introduced subscript is P, yields

$$(1 + c_1^2)x'_P - d_1 = 0 \tag{45}$$

hence

$$x'_P = \frac{d_1}{1 + c_1^2} \tag{46}$$

so, after substituting (46) into (39), we write

$$y'_P = \frac{d_1}{1 + c_1^2} c_1 \tag{47}$$

what determines the coordinates of the point P in the x' y' coordinate system.

4. *Determination of coordinates*

Using the functions obtained before we establish two methods for determination of position by resection.

4.1. The first method

Note that, knowing the coordinates of the point P in x' y' coordinate system, we can determine the coordinates of the point in xy coordinate system. Since the azimuth CA equals to φ_1 in the xy system, which is denoted by δ_1, is

$$\delta_1 = \varphi_1 + \gamma_1 \tag{48}$$

Transforming the coordinates x'_P, y'_P from x' y' coordinate system to xy coordinate system, we obtain

$$x_P = x_C + x'_P \cos\delta_1 - y'_P \sin\delta_1 \tag{49}$$

and

$$y_P = y_C + x'_P \sin\delta_1 + y'_P \cos\delta_1 \tag{50}$$

what we write in the following matrix form

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \begin{bmatrix} \cos\delta_1 & -\sin\delta_1 \\ \sin\delta_1 & \cos\delta_1 \end{bmatrix} \begin{bmatrix} x'_P \\ y'_P \end{bmatrix} \tag{51}$$

If now we substitute (7) into (48)

$$\delta_1 = \varphi_1 + \alpha_1 - 90^\circ \tag{52}$$

and substituting (46), (47), (52) into (51), we obtain

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \frac{d_1}{1 + c_1^2} \begin{bmatrix} \sin(\varphi_1 + \alpha_1) & \cos(\varphi_1 + \alpha_1) \\ -\cos(\varphi_1 + \alpha_1) & \sin(\varphi_1 + \alpha_1) \end{bmatrix} \begin{bmatrix} 1 \\ c_1 \end{bmatrix} \tag{53}$$

what can be also written in the form

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \frac{d_1}{1 + c_1^2} \begin{bmatrix} 1 & c_1 \\ c_1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\varphi_1 + \alpha_1) \\ \cos(\varphi_1 + \alpha_1) \end{bmatrix} \tag{54}$$

If we now combine the obtained expressions needed for determination of the coordinates of the point *P*, we have

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \frac{d_1}{1 + c_1^2} \begin{bmatrix} 1 & c_1 \\ c_1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\varphi_1 + \alpha_1) \\ \cos(\varphi_1 + \alpha_1) \end{bmatrix} \tag{55}$$

where

$$c_1 = \frac{d_1 + d_2 \cos\omega}{d_2 \sin\omega} \tag{56}$$

$$d_1 = \frac{a_1}{\sin\alpha_1}, \qquad d_2 = \frac{a_2}{\sin\alpha_2} \tag{57}$$

$$\omega = \alpha_1 + \beta + \alpha_2 \tag{58}$$

and the distances a_1, a_2, the azimuth φ_1, the angle β, are expressed by (1), (2), (3), (5), respectively.

4.2. The second method

Introducing the $x'' y''$ coordinate system, of which the origin is at the point C and the x'' axis passed through the point O_2, Fig. 4, we obtain

$$x''_{O_2} = r_2 \tag{59}$$

and

$$y''_{O_2} = 0 \tag{60}$$

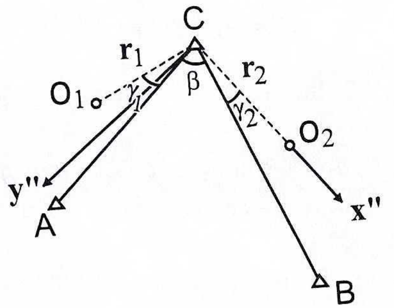

Fig. 4. Cartesian $x'' y''$ coordinate system

Since the azimuth CO_1 in the $x'' y''$ coordinate system is equal to $\gamma_1 + \beta + \gamma_2$, the coordinates of the point O_1, are

$$x''_{O_1} = r_1 \cos(\gamma_1 + \beta + \gamma_2) \tag{61}$$

and

$$y''_{O_1} = r_1 \sin(\gamma_1 + \beta + \gamma_2) \tag{62}$$

After substituting (7), (10) into (61), (62), respectively, and taking into account that $\omega = \alpha_1 + \beta + \alpha_2$ we obtain

$$x''_{O_1} = - r_1 \cos\omega \tag{63}$$

and

$$y''_{O_1} = - r_1 \sin\omega \tag{64}$$

The equations of the circles, of which the centres are at the points O_1 and O_2, have the form

$$(x'' - r_2)^2 + y''^2 = r_2^2 \tag{65}$$

and

$$(x'' + r_1 \cos\omega)^2 + (y'' + r_1 \sin\omega)^2 = r_1^2 \tag{66}$$

what establishes two square equations, given below

$$x''^2 + d_1 \cos\omega \cdot x'' + y''^2 + d_1 \sin\omega \cdot y'' = 0 \tag{67}$$

$$x''^2 - \qquad d_2 x'' + y''^2 \qquad = 0 \tag{68}$$

where $d_1 = 2r_1$, $d_2 = 2r_2$ as in (28), (29).

In order to determine the unknows x'', y'', having substracted (67) from (68), after some manipulations, we write

$$y'' = -\frac{d_2 + d_1 \cos\omega}{d_1 \sin\omega} x'' \tag{69}$$

and denoting

$$c_2 = -\frac{d_2 + d_1 \cos\omega}{d_1 \sin\omega} \tag{70}$$

we state that

$$y'' = -c_2 x'' \tag{71}$$

As a result of substitution (71) into (68), we obtain

$$x'' \lfloor (1 + c_2^2) x'' - d_2 \rfloor = 0 \tag{72}$$

hence, after introducing the subscripts C i P, we have

$$x''_C = 0 \tag{73}$$

and

$$x''_P = \frac{d_2}{1 + c_2^2} \tag{74}$$

Substituting (73), (74) into (71) we state that

$$y''_C = 0 \tag{75}$$

and

$$y''_P = -\frac{d_2}{1 + c_2^2} c_2 \tag{76}$$

so that, we obtained the coordinates of the point P in $x''\ y''$ coordinate system.

If now the azimuth of the axis x'', in xy coordinate system, is denoted by δ'' then the coordinates of the point P can be written in the matrix form

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \begin{bmatrix} \cos\delta_2 & -\sin\delta_2 \\ \sin\delta_2 & \cos\delta_2 \end{bmatrix} \begin{bmatrix} x''_P \\ y''_P \end{bmatrix} \tag{77}$$

Since the azimuth

$$\delta_2 = \varphi_2 - \gamma_2 \tag{78}$$

after substituting (10) into (78), we have

$$\delta_2 = \varphi_2 - \alpha_2 + 90° \tag{79}$$

and substituting (74), (76), (79) into (77), we obtain

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \frac{d_2}{1 + c_2^2} \begin{bmatrix} -\sin(\varphi_2 - \alpha_2) & -\cos(\varphi_2 - \alpha_2) \\ \cos(\varphi_2 - \alpha_2) & -\sin(\varphi_2 - \alpha_2) \end{bmatrix} \begin{bmatrix} 1 \\ -c_2 \end{bmatrix} \tag{80}$$

what we write in the following form

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \frac{d_2}{1 + c_2^2} \begin{bmatrix} -1 & c_2 \\ c_2 & 1 \end{bmatrix} \begin{bmatrix} \sin(\varphi_2 - \alpha_2) \\ \cos(\varphi_2 - \alpha_2) \end{bmatrix} \tag{81}$$

Combining the expressions which determine the coordinates of the point P we finally have

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \frac{d_2}{1 + c_2^2} \begin{bmatrix} -1 & c_2 \\ c_2 & 1 \end{bmatrix} \begin{bmatrix} \sin(\varphi_2 - \alpha_2) \\ \cos(\varphi_2 - \alpha_2) \end{bmatrix} \tag{82}$$

where

$$c_2 = \frac{d_2 + d_1 \cos\omega}{d_1 \sin\omega} \tag{83}$$

$$d_1 = \frac{a_1}{\sin\alpha_1}, \qquad d_2 = \frac{a_2}{\sin\alpha_2} \tag{84}$$

$$\omega = \alpha_1 + \beta + \alpha_2 \tag{85}$$

and the distances a_1, a_2, azimuth φ_2, the angle β, are expressed by (1), (2), (4), (5), respectively.

5. *Indicator of position determinability*

In the Chapter 3 the solution of the equation system (34), (35), involved the assumption that $\omega = \alpha_1 + \beta + \alpha_2 \neq 180°k$, $k = 0,1$.

Now we consider two cases of the above limitations.

1. In the first case

$$\omega = \alpha_1 + \beta + \alpha_2 = 0 \tag{86}$$

as it is easy to see, the vertex point of the angle β and the observed angles, α_1 i α_2, are lying on the same straight line. This entirely excludes the posibility of making such construction. It is imposible to do the observations becouse from the point *P* we can not distinguish which of the observed points *A, B, C* is the left, the right or the central.

2. In the second case

$$\omega = \alpha_1 + \beta + \alpha_2 = 180° \tag{87}$$

we must make further considerations. Let us assume that the points *A, B, C* and *P* are on the circle with the centre at the point *O*, Fig. 5. Denoting the angles *AOB* and *BOA* by μ_1 i μ_2, respectively, we have

$$\mu_1 = 2(\alpha_1 + \alpha_2) \tag{88}$$

and

$$\mu_2 = 2\beta \tag{89}$$

finding the sum of these angles, after taking into account that $\mu_1 + \mu_2 = 360°$, we write

$$2(\alpha_1 + \beta + \alpha_2) = 360° \tag{90}$$

and

$$\omega = \alpha_1 + \beta + \alpha_2 = 180° \tag{91}$$

so, when $\omega = 180°$ all four points are on the circle. Then, the angles α_1 and α_2, observed from the arbitrary point P lying on the circle between the points A and B, are unchangeable. Therefore the position of the point P can not be determined.

The obtained expression, introduced in this work

$$\omega = \alpha_1 + \beta + \alpha_2 \tag{92}$$

we call the indicator of position determinability.

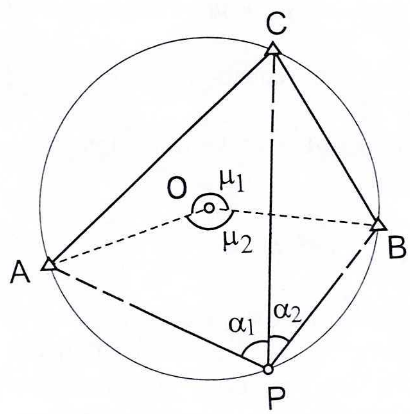

Fig. 5. Points on the circle

Basing on the above considerations we state that determination of the point is possible only if

$$\omega \neq 180° \tag{93}$$

The established indicator of position determinability, ω, gives us possibility to know whether position at the point P is determinable.

6. *Examples*

The examples of practical application of the obtained solutions are given below.

Example 1

Knowing the coordinates of the points $A(0, 0)$, $B(-2/\sqrt{3}, 1)$, $C(0, 1)$ and the observed angles $\alpha_1 = 30°$, $\alpha_2 = 30°$, determine the coordinates of the point P using the first method.

1. Position determinability

Since

$$\omega_1 = \arctan \frac{\Delta y_{CA}}{\Delta x_{CB}} = \arctan \frac{-1}{0} = 270°$$

$$\omega_2 = \arctan \frac{\Delta y_{CB}}{\Delta x_{CB}} = \arctan \frac{0}{-2\sqrt{3}} = 180°$$

then $\beta = \varphi_1 - \varphi_2 = 90°$, hence the indicator of position determinability is

$$\omega = \alpha_1 + \beta + \alpha_2 = 150° \neq 180°$$

so, the position is determinable.

2. Determination of the coordinates

Helpful expressions

$$d_1 = \sqrt{\Delta x_{CA}^2 + \Delta y_{CA}^2}/\sin\alpha_1 = \sqrt{0 + 1}/(1/2) = 2$$

$$d_2 = \sqrt{\Delta x^2{}_{CB} + \Delta y^2{}_{CB}}/\sin\alpha_2 = \sqrt{4/3 + 0}/(1/2) = 4/\sqrt{3}$$

$$c_1 = \frac{d_1 + d_2\cos\omega}{d_2\sin\omega} = \frac{2 + (4/\sqrt{3})(-\sqrt{3}/2)}{(4/\sqrt{3})(1/2)} = \frac{0}{2/\sqrt{3}} = 0$$

Coordinates of the point

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \frac{d_1}{1 + c_1^2} \begin{bmatrix} 1 & c_1 \\ c_1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\varphi_1 + \alpha_1) \\ \cos(\varphi_1 + \alpha_1) \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -\sqrt{3}/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} -\sqrt{3} \\ 0 \end{bmatrix}$$

then $P(-\sqrt{3}, 0)$.

3. Check

Since

$$[\alpha] = \alpha_1 + \alpha_2 = 30^\circ + 30^\circ = 60^\circ$$

$$[\alpha] = \text{arctg}\,\frac{\Delta y_{PB}}{\Delta x_{PB}} - \text{arctg}\,\frac{\Delta y_{PA}}{\Delta x_{PA}} =$$

$$= \text{arctg}\,\frac{1}{1/\sqrt{3}} - \text{arctg}\,\frac{0}{\sqrt{3}} = 60^\circ - 0^\circ = 60^\circ$$

what veryfies the correctness of the solution.

E x a m p l e 2

Taking into account the coordinates of the points $A(0, 0)$, $B(-2/\sqrt{3}, 1)$, $C(0, 1)$, and the angles $\alpha_1 = 30^\circ$, $\alpha_2 = 30^\circ$, given in the Example 1, determine the coordinates of the point P using the second method.

1. Determination of the coordinates

On the base of value obtained in the Example 1, ω, d_1, d_2, we state that

$$c_2 = \frac{d_2 + d_1 \cos\omega}{d_1 \sin\omega} = \frac{4/\sqrt{3} + 2(-\sqrt{3}/2)}{2(1/2)} = \frac{1}{\sqrt{3}}$$

so

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \frac{d_2}{1 + c_2^2} \begin{bmatrix} -1 & c_2 \\ c_2 & 1 \end{bmatrix} \begin{bmatrix} \sin(\varphi_2 - \alpha_2) \\ \cos(\varphi_2 - \alpha_2) \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \sqrt{3} \begin{bmatrix} -1 & 1/\sqrt{3} \\ 1/\sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} -\sqrt{3} \\ 0 \end{bmatrix}$$

then $P(-\sqrt{3}, 0)$.

2. Check

The determined coordinates, $P(-\sqrt{3}, 0)$, are identical with those obtained in the Example 1.

CONCLUSIONS

In this paper two new methods of position determination by resection have been presented. Results which have been obtained by both methods are identical.

Since the procedure establishing the final functions in the first approach is simpler, the first method is recommended.

REFERENCES

Allan A. L., *The Tienstra Method of Resection.* Int. Hydr. Rev. No. 2, 1962, pp. 71-76.

Bannister A., Baker R., *Solving Problems in Surveying.* Longman's, London 1989.

Bannister A., Raymond S., *Surveying,* 6[th] Ed., Longman's, London 1992.

Blachut T. J., Chrzanowski A., Saastamoinen J. H., *Urban Surveying and Mapping.* Springer-Verlag, New York, Heidelberg, Berlin, 1979, pp. 169-174.

Hausbrandt S., *Rachunek wyrównawczy i obliczenia geodezyjne.* Tom 1. PPWK, Warszawa 1971.

Hu W. C., Kuang J. S., *Proof of Tienstra's Formula for an External Observation Point.* J. of Surv. Engrg., ASCE, 124 (1), 1998, pp. 49-55.

Klinkenberg H., *Coordinate Systems and Three Point Problem.* Canad. Surv., Canada 1953.

Moffitt F. H., Bouchard H., *Surveying.* Intext Educational Publishers, New York, 1975, pp. 442-446.

Richardus P., *Project surveying.* North-Holland Publishing Co., Amsterdam, The Netherlands, 1966, pp. 20-29.

Skórczyński A., *Podstawy obliczeń geodezyjnych.* PPWK, Warszawa 1983.

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Wyznaczanie pozycji wcięciem wstecz

S t r e s z c z e n i e

W pracy podano nową metodę wyznaczania pozycji wcięciem wstecz. Ustaleń dokonano w wyniku rozwiązania, wprowadzonego w tej pracy, układu równań nieliniowych. Takie podejście pozwoliło na całościowe ujęcie i ogólne rozwiązanie rozpatrywanego problemu. Otrzymano dwa sposoby wyznaczania pozycji wcięciem wstecz.

Rozpatrzono zagadnienie pozycji niewyznaczalnych. Wprowadzono wskaźnik wyznaczalności pozycji. Ustalono przypadki niewyznaczalności pozycji i podano ich interpretację geometryczną.

Януш Мартусевич

Определение места методом обратной засечки

Резюме

В работе представлен новый метод определения места способом обратной засечки. Определение совершено в результате решения – введённой в этой работе – системы нелинейных уравнений. Такой подход дал возможность полного подхода и общего решения рассматриваемой проблемы. Получены два способа определени кординат методом обратной засечки.

Рассмотрена проблема неопределимых мест. Введён индекс определимости места. Определены случаи неопределимости места и представлена их геометрическая интерпретация.